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OPTIMAL PRICING AND LOT-SIZE POLICIES FOR INVENTORY MODELS FOR DETERIORATIVE ITEMS WITH BERTRAND AND COURNOTS' MODELS –IN THIRD ORDER EQUATION

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ABSTRACT
This research examines the optimal ordering and pricing of deteriorating inventory models with Bertrand-Dependent demand along with Cournot-dependent price in third order equations. Many other types of demand models, such as stock dependent, price dependent, exponential, quadratic, linear, and constant, may be found in the academic literature. To wit, there is a lack of research that employs a pricing strategy reliant on Bertrand demand and Cournot prices. Two models are created: The Bertrand-dependent demand is used in the first model for both sellers, The second model implements a pricing strategy that is reliant on Cournot for both of the sellers. Both models take into account the point at which sales are profitable. In order to clarify the proposed method, mathematical models for every model are outlined, and applicable examples are presented. The price break even is presented and the law of demand is verified. In this case, the objective of this paper is to acquire the ideal order quantities at the optimal price in order to achieve maximum profit. For both models, we also provide a sensitivity analysis. visual basic 6.0 was used to create the required data.

KEY WORDS: Inventory, deteriorating, Bertrand-demand, Cournot price, cycle time, and sensitivity analysis.

MSC : 90B05

RESUMEN
Esta investigación examina el orden y el precio óptimos de los modelos de inventario deteriorados con la demanda dependiente de Bertrand , junto con el precio dependiente de Cournot en ecuaciones de tercer orden. Muchos otros tipos de modelos de demanda, como el dependiente de acciones, dependiente del precio, exponencial, cuadrático, lineal y constante, se pueden encontrar en la literatura académica. Hay una carencia de investigaciónes que empleen una estrategia de precios que dependa de la demanda de Bertrand y los precios de Cournot. Se crean dos modelos: la demanda dependiente de Bertrand se utiliza en el primer modelo para ambos vendedores, El segundo modelo implementa una estrategia de precios que depende de Cournot para ambos vendedores. Ambos modelos tienen en cuenta el punto en el que las ventas son rentables. Con el fin de aclarar el método propuesto, se esbozan modelos matemáticos para cada modelo y se presentan ejemplos aplicables. Se presenta el punto de equilibrio del precio y se verifica la ley de la demanda. En este caso, el objetivo de este documento, es obener las cantidades de pedido ideales al precio óptimo, para lograr el máximo beneficio. Para ambos modelos, también proporcionamos un análisis de sensibilidad. Se utilizó Visual Basic 6.0 para crear los datos necesarios.

PALABRAS CLAVE: Inventario, deterioro, demanda de Bertrand, precio de Cournot, tiempo de ciclo y análisis de sensibilidad.

1. INTRODUCTION

Typical EOQ (Economic Order Quantity)-based inventory models assume a constant holding cost and demand rate and a constant unit purchase cost across all order sizes. However, in practical contexts, several factors—including linearity, quadratic, exponentially, availability of stock selling price, and seasonality, —can influence the rate of demand for a particular item. In addition, the average cost per unit of holding space is likely to be greater over longer durations of storage. In addition, the cost per unit of acquisition is often reduced for bigger order sizes owing to discounts offered for greater quantities. During a certain time frame and for a specific price, demand is the amount of a product or service which customers are looking to purchase. A product's selling price, that should be reflective of reality, is the sole factor in determining the product's demand. If the price is raised,

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fewer people will buy it, whereas if it's lowered, more people would want to buy it. As a result, inventory models for perishable goods take into account both Bertrand-dependent demand and Cournot-dependent pricing. Under the Cournot-type competitive model, it is assumed that each seller seeks to maximize his profit per unit time by optimizing both his order quantity and his demand (sale) per unit time, given a given level of demand (sale) per unit time of his competition. Bertrand competition, on the other hand, assumes that each seller seeks to maximize his profit per unit time by optimizing both his order quantity and his pricing per unit at a certain level of price per unit for his rival. In both the Cournot and Bertrand models, the setting of prices implies the determination of demands and the other way around. Sales (as measured by demand per unit of time) policies are therefore determined by pricing policies and inversely so. Both models assume that requests and prices are linear functions of one another. There are several studies in the inventory literature that analyse the financial effects of different pricing and inventory strategies. Inventory model with price-dependent demand in third-order equation was developed by Whitin (1955). T.M. cannot resolve the third order equation. There is no consideration given to things that are worn or broken. However, the resultant third order equation is solved in this study using 6.0, and deteriorating objects are taken into account. The uniform and quantity discount pricing and inventory strategies under competition are studied by Min (1992). Otake and Min (1992) by taking into account a scenario in which the price of one product causes a rise in demand for a second product that is a replacement. According to the research of Otake and Min (1995), as the price of one good goes up, consumers buy less of its complementary good (for example, tennis balls and tennis rackets). Since the trigonometric notion used to solve the third-order equation is foreign to most readers, the problem was programmed in visual basic 6.0 and deterioration impacts were also taken into consideration. Rabbani et al. (2017) investigated the best joint inventory dynamic pricing and marketing strategies for non-instantaneously decaying commodities. Given that demand for perishable goods is affected by factors such as price, freshness, as well as on-hand stock, Feng et al. (2017) calculate the optimal selling price, non-zero ending inventory level and cycle length. Li et al. (2021) modelled SRC (supplier-retailer-customer) supply chain where retailer gets an upstream ACC payment from supplier and in return provides a down-stream cash-credit payment to customers, where demand is affected by cumulative effects of stock and selling price and where deteriorative rate varies over time. Bertrand-dependent demand and Cournot-dependent price in third order equations are implemented in this paper as a result of the late-stage realization that price-dependent demand, stock-dependent, ramp type, exponential, quadratic, linear, and other demand patterns did not accurately depict the demand for certain products. That is, Bertrand-dependent demand is $(a-bP_1+cP_2)$ and Cournot-dependent price is ($a-bD_1-cD_2)$. No author to my knowledge has investigated Bertrand-dependent demand and Cournot-dependent pricing in third order equations. Because of this property, two models are constructed: Both sellers in the first scenario have demand that is contingent on Bertrand, whereas both sellers in the second model have pricing strategies that are conditional on Cournot. Both models take into account the point at which sales are profitable. Visual Basic 6.0 is used to build the mathematical derivation, display numerical examples, and create the data for two different models. Finding the optimal quantity and unit pricing to maximize profit is the primary focus of this research. the objective of this paper is to acquire the ideal order quantities at the optimal price in order to achieve maximum profit. In addition, the paper is laid out as follows: The literature review is presented in section 2, while section 3 details the underlying assumptions as well as notations utilized to create the model. In section 4, the inventory model is constructed, as well as the optimal solution procedure is created. section 5 concludes with a brief overview and suggestions for further study.

2. LITERATURE REVIEW:

There has been much study devoted to improving inventory management in order to create models, methodologies, and practical solutions. Current inventory management and lot-sizing models are discussed, along with related recent studies. Economic order quantity (EOQ) models, as introduced by Cheng (1990), combine product price with order size choices in order to maximise profits. When resources like money and space are at a premium, the Kuhn-Tucker criteria help choose the best course of action. This research aims to investigate how linking price and order size choices might improve optimality. Despite its existence in practise, the conventional EOQ model has always disregarded this connection. Two duopoly models with rival vendors were developed and analyzed by Otake (1998). It is believed that all retailers make rational, profit-maximizing EOQ-based decisions in response to demand curves that are linear in nature. Hou and Lin (2006) developed an inventory model for perishable products where the demand rate is a function of both stock-level as well as selling price to maximise the net profit value. Under the assumptions of demand dependency and the possibility of price modifications before the end of the sales season to alter demand and enhance revenue, you (2005) examine the problem of determining the order size and optimal pricing for a perishable inventory system. You and Hsieh's (2007) inventory system study found that the demand rate is affected by both supply and pricing. When a product's general price and time dependant season of demand rate is taken into account, Banerjee and Sharma (2010) propose a deterministic inventory model that integrates these factors. Goh et al. (2012) examined a combined pricing and inventory management strategy for slowly degrading goods when payment delays are acceptable. We use a demand curve that changes with both time and cost. Delay in addressing the shortage is permitted. A combined pricing and inventory management model was studied by Maihami et al. (2012) for non-instantaneously degrading products with allowable delay in payments as well as an item-specific demand function that is time - and price -dependent, with shortages permitted. A combined ordering and price issue was investigated by Jianmaishi et al. (2012) for a shop whose supplier offers a quantity discount on all units ordered. The issue may be formulated using either a generalised disjunctive programming model or a mixed integer non-linear programming model. Inventory modelling for perishable goods with stock dependent demand rate was established by Ghoreishi et al. (2015). Because of the model's allowance for shortages, when a store's stock is depleted, it will give a price break to consumers who are prepared to back-order their needs. Here, both the wholesaler and the retailer use a customer trade credit scheme to boost competitiveness. For goods that degrade over time yet may be paid for late, Jaggi et al. (2015) developed a fuzzy inventory model. This study explores the many scenarios that may occur but were overlooked by earlier inventory models with forgivable payment delays. In addition, this study takes into account the potential of an interest earn rate that is greater than an interest payable rate. Demand rate's volatility, unit holding cost's fluctuation, along with unit purchase cost's variability were all taken into account at the same time by Hesham et al. (2016). An inventory model is described, including a demand rate that differs with selling price, a holding cost that varies with time an item is in storage, and a purchasing cost that varies with order size due to an all-units-quantity discount. Since a retailer's storage and display space for perishable goods is likely to be restricted, Jaggi et al.(2018) examined two-tier supply chain model. As a result, the store keeps unsold items in the warehouse, where space is not an issue. In this model, the retail price and on-shelf availability of an item are supposed to have an effect on the rate of demand. The suggested method clearly simulates the connection between product pricing, product demand, and retailer and manufacturer integration under 4 distinct policies: retailer-led Stackelberg policy, supplier-led Stackelberg policy, integrated, non-integrated policy. Management of a short-lived, deteriorating product was studied by Chena et al. (2019). We look at a multi-period, finite-horizon scenario in which demand is determined by stock levels, fluctuates over time in response to price changes, and is price-sensitive. The optimal selling price, quantity, and advertising budget for Veblen goods were calculated by Agarwal et al. (2020). With the help of the LINGO optimization programme, we were able to find the optimal balance between advertising costs for the specialised and mainstream sectors. Nasui and Esmaelli (2021) examined the interplay between pricing and stock management choices in a hybrid manufacturing setting. There are Poisson distributions for demand that vary with the selling prices of individual goods. The predicted profit function in the long term may be calculated by modelling the system as a Markov chain. A discounted cash flow technique is used in the EOQ inventory model for degrading items that was explored by Tsao et al. (2021). Maximum present value of sales is achieved by simultaneously determining the ideal selling price, credit period rules, and replenishment cycle duration. In the presence of stock display and reference price impacts, Duan and Cao (2021) examined the combined dynamic pricing as well as deteriorating inventory management issue. Over an unlimited horizon of time, maximise profits for the product in random prospective market with convex replenishment/ordering and holding/shortages cost functions. Li et al. (2021) developed a supplier-retailer-customer chain in which the retailer receives an upstream ACC payment from the supplier while in return offers a down-stream cash-credit payment to customers, the demand is influenced by the combined effect of selling price and stock age, and the deteriorative rate is time-varying.

3. **ASSUMPTIONS AND NOTATIONS**

3.1 Assumptions

The basic assumptions for traditional EOQ model applied in this paper are as follows: i) the initial inventory level is zero, ii) buyer's demand rate is linear over time, iii) the replenishment rate is infinite, iv) no shortages is allowed, v) there is no delivery lag, vi) Cournot-type inventory model, demands are linear functions of prices that is, $P_1 = a - bD_1 - cD_2$ (for seller -1) and $P_2 = a - cD_1 - bD_2$ (for seller-2), vii) The demand rate is Bertrand-type inventory model, prices are linear functions of demands that is $D_1 = a - bP_1 + cP_2$ (for seller-

1) and $D_2 = a + cP_1 - bP_2$ (for seller-2), viii) the deteriorative rate is constant and the planning horizon is finite.

3.2 Notations

i) P_i - the per unit price of product i, i = 1,2, (Decision variables), ii) D_i - the per unit time demand of product i, i = 1,2, (Decision variables), iii) a – the intercept of the demand (inverse demand) function, iv) b – the own price effect, v) c – cross price effect, vi) Q_i - the order quantity of product i for seller i, (Decision variables) vi) C_0 - the set up cost of product, vii) C_P - the purchase cost per unit time product, viii) C_h - the inventory holding cost per unit per unit time of product, ix) T - the cycle length for product , x) θ - rate of deteriorative items, x) C_d - the inventory deteriorative cost per unit per unit time of product.

4. MATHEMATICAL MODELS

4.1 Optimal Pricing and lot-size Policies with Bertrand- Type model with Linear Demand

The Bertrand-type model, the linear demand function is employed as follows $D_1 = a - bP_1 + cP_2$ (for seller 1) and $D_2 = a + cP_1 - bP_2$ (for seller 2) . In this model, each seller maximizes his profit per unit time over his order quantity and his price per unit assuming a given level of price per unit of his competitor.

4.1. (a) Bertrand-type with linear demand for seller 1

In Bertrand-type model involves the price levels instead of the demand levels as the decision variable. The rate of decrease in the inventory level I(t) is equal to the rate of deteriorative items and the demand rate. The relationship is expressed by the following differential equation.

$$\frac{d}{dt} I(t) + \theta I(t) = -(a - bP_1 + cP_2), \quad 0 \le t \le T_1 \qquad (1)$$

with the boundary conditions $I(0) = Q_1$, I(0) = 0 (2)

Integrating the above equation and plugging in the end-of-cycle boundary condition I(T) =0, then the solution for the differential equation (1) is given below

$$I(t) = \frac{(a - bP_1 + cP_2)}{\theta}\left(e^{\theta(T-t)} - 1\right) \qquad (3)$$

To find Q: $I(0) = Q_1$, then $Q_1 = \frac{(a - bP_1 + cP_2)}{\theta}(e^{\theta T} - 1)$

Re-arranging equation,

$$Q_1 = (a - bP_1 + cP_2)T \text{ , Therefore, } T = \frac{Q_1}{a - bP_1 + cP_2} \qquad (4)$$

Suggested model's goal is to maximize the cumulative profit per elapsed period $TP(Q_1, P_1)$. Sales income and three types of expenses are included in the profit function: holding cost, deteriorative cost, purchasing and ordering cost. In a time period of unit T cycles, an order at cost C_0 is placed once, hence the ordering cost is $\frac{C_0}{T}$. Calculating the total cost of a purchase is as simple as multiplying the unit price by the demand rate $(a - bP_1 + cP_2)$, which is the number of units bought in a certain time frame. The overall holding cost per cycle is calculated by first summing holding costs throughout each period of the inventory cycle C_h , and then by multiplying each period's holding costs by stock level at time t, I(t). Total holding costs are divided by the cycle length T to get the holding cost per unit time. The total cost per cycle $TC(Q,P)$ is calculated by adding the following four factors: deteriorative cost, sum of ordering, holding as well as purchasing cost components.

1. Purchase cost = $DC_P = (a - bP_1 + cP_2)C_P$ where demand is a constant price. (5)

2. Setup cost = $\frac{C_0}{T} = \frac{(a - bP_1 + cP_2)C_0}{Q_1}$ (from the equation (4)) (6)

3. Holding cost (HC) = $\frac{C_h}{T}\int_0^T \frac{(a - bP_1 + cP_2)}{\theta}\left(e^{\theta(T-t)} - 1\right)dt$ = $\frac{(a - bP_1 + cP_2)C_h}{\theta^2 T}\left(e^{\theta T} - \theta T - 1\right)$

On simplification with using the equation (4), HC= $\frac{C_h Q_1}{2}$ (7)

4. Deteriorative cost = $\frac{\theta C_d Q_1}{2}$ (8)

Therefore, TC = $(a - bP_1 + cP_2)C_P + \frac{(a - bP_1 + cP_2)C_0}{Q_1} + \frac{(C_h + \theta C_d)Q_1}{2}$

Total Profit (TP)= Total sales – total cost

In the Bertrand-type inventory model involves the price levels instead of the demand levels a the decision variables. Thus, for seller 1, per unit time profit maximization problem for the Bertrand-type inventory model is formed as follows". To get the total profit per cycle, $TP(Q_1, P_1)$ we subtract $TC(Q_1, P_1)$ from the sales revenue for the cycle, which is equal to the unit selling price, P_1, times the number of units sold per time period, which is equal to the demand rate $(a - bP_1 + cP_2)$

Total Sales = Demand × Price per unit (P) = $(a - bP_1 + cP_2)P_1$

Therefore, $$TP(Q_1, P_1) = \begin{bmatrix} (a - bP_1 + cP_2)P_1 - (a - bP_1 + cP_2)C_P \\ -\frac{(a - bP_1 + cP_2)C_0}{Q_1} - \frac{(C_h + \theta C_d)Q_1}{2} \end{bmatrix} \quad (9)$$

Where P_2 denotes a given level of per unit price for seller – 2

In order to maximize the total profit, the first partial derivatives of $TP(Q_1, P_1)$ with respect to Q_1 and P_1 are set equal to zero, leading to following systems of nonlinear equations. Partially differential the total profit (9) with respect to Q_1, then $\frac{\partial}{\partial Q_1}TP(Q_1, P_1) = \frac{(a - bP_1 + cP_2)C_0}{Q_1^2} - \frac{C_h + \theta C_d}{2} = 0$

On simplification,

$$Q_1^2 = \frac{2(a - bP_1 + cP_2)C_0}{C_h + \theta C_d} \quad (10)$$

Partially differential the total profit $TP(Q_1, P_1)$ with respect to P_1, then

$$\frac{\partial}{\partial P_1}TP(Q_1, P_1) = a - 2bP_1 + cP_2 + \frac{bC_0}{Q_1} + bC_P = 0$$

$$(a - 2bP_1 + cP_2 + bC_P)Q_1 + bC_0 = 0$$

on simplification, $Q_1 = \frac{bC_0}{a - 2bP_1 + cP_2 + bC_p}$ (11)

substitute the value of (11) in the equation (10) then,

$$\frac{b^2 C_0}{(a-2bP_1+cP_2+bC_P)^2}=\frac{2(a-bP_1+cP_2)}{C_h+\theta C_d}$$

$$2(a-bP_1+cP_2)(a-2bP_1+cP_2+bC_P)^2=b^2C_0(C_h+\theta C_d)$$

$$2(a-bP_1+cP_2)\begin{bmatrix}a^2+4b^2P_1^2+c^2P_2^2+b^2C_P^2-4abP_1+2acP_2\\+2abC_P-4bcP_1P_2-4b^2P_1C_P+2bcP_2C_P\end{bmatrix}=b^2C_0(C_h+\theta C_d)$$

On simplification,

$$\begin{bmatrix}8b^3P_1^3-8(2ab^2+2b^2cP_2+b^3C_P)P_1^2+2\begin{pmatrix}5a^2b+10abcP_2+6ab^2C_P+5bc^2P_2^2\\+b^3C_P^2+6b^2cP_2C_P\end{pmatrix}P_1\\-2\begin{pmatrix}a^3+3ac^2P_2^2+ab^2C_P^2+3a^2cP_2+2a^2bC_P\\+4abcP_2C_P+c^3P_2^3+b^2cP_2C_P^2+2bc^2P_2^2C_P\end{pmatrix}-b^2C_0(C_h+\theta C_d)\end{bmatrix}=0 \quad (12)$$

which is optimum solution in P in third order equation. On solving equation (12), the price (P_1) of the product can determine then substitute the value of P_1 in the equation (10), then the optimum quantity (Q_1) can determine.

Numerical example 1

To illustrate the solution procedure and the results, let us apply the proposed algorithms to solve the following numerical example. The results can be found by apply visual basic 6.0. This example is based on the following parameters and assumptions. The cost parameters are

P_2 = 80.4, Demand D = 57.14, a = 100, b = 1.0, c = 0.5, Setup cost = 1000, Holding cost per unit/time = 4, Purchase cost per unit = 20, Deteriorative cost per unit (C_d) = 100, Rate of Deteriorative items (θ) = 0.01.

Optimum Solution

The third order equation is $8P_1^3-2403.2P_1^2+231008.4P_1-7191796.81=0$ which has three real roots in which one positive and two negative roots. The positive root 83.20 is considered in this model. Optimum price (P_1) = 83.20, Optimum Quantity= 160.95, Optimums Demand = 56.99, Optimum cycle time (T) = 2.8240,, Purchase cost = 1139.87, Setup cost = 354.09, Holding cost = 321.90, Deteriorative cost = 32.19, Total cost = 1848.06, Total sales = 4742.23, Total profit = 2894.16

Result of sensitivity analysis with respect of Price (equal) of the Sellers

At the current price of 84.10, the company is not making a profit or loss, as shown in Table 1. To put it another way, total sales are the same as total expenses. The business will make a profit if and only if the price of the product is more than 84.10.

Table – 1 Result of sensitivity analysis with respect to Price of the Seller

P_2	Price (P_1)	D	Q_1	T	Purchase cost	Setup cost	Holding cost	DC	Total cost	Total Sales	Total Profit
80	83.11	56.89	160.80	2.8266	1137.81	353.77	321.61	32.16	1845.37	4728.14	2882.77
81	83.35	57.14	161.17	2.8202	1142.95	354.57	322.34	32.23	1852.10	4763.39	2911.28
82	83.59	57.40	161.53	2.8139	1148.09	355.57	323.06	32.30	1858.84	4798.76	2939.92
83	83.83	57.65	161.89	2.8076	1153.23	356.16	323.78	32.37	1865.56	4834.25	2968.68
84	84.08	57.91	162.25	2.8014	1158.36	356.96	324.50	32.45	1872.28	4869.87	2997.58
84.10	**84.10**	**57.94**	**162.29**	**2.8008**	**1158.88**	**357.03**	**324.58**	**32.45**	**1872.96**	**4873.44**	**3000.48**
85	84.32	58.17	162.61	2.7952	1163.50	357.75	325.22	32.52	1879.00	4905.61	3026.60
86	84.56	58.43	162.97	2.7890	1168.63	358.53	325.94	32.59	1885.71	4941.47	3055.75
87	84.81	58.68	163.32	2.7829	1173.77	359.32	326.65	32.66	1892.42	4977.46	3085.03
88	85.05	58.94	163.68	2.7769	1178.90	360.11	327.37	32.73	1899.12	5013.57	3114.44
89	85.29	59.20	164.04	2.7708	1184.04	360.89	328.08	32.80	1905.82	5049.81	3143.98

Sensitivity analysis and discussion

In order to assess the relative impact of the different input parameters on the solution quantity, systematic sensitivity analysis was performed on the above example.

Sensitivity Analysis with respect to Rate of Deteriorative items (θ)

The findings of the rate of deterioration ϕ sensitivity analysis are shown in Table 2. Table 1 shows the results of an investigation into the relationship between the rate of deterioration and the unit price, optimal cycle time, optimal quantity, cost to acquire the item, the cost to set it up, the cost to store it, the cost to lose money as it deteriorates, total profit, total sales, total cost. The rate of deterioration has a positive correlation with unit price, total cost, setup cost, deteriorative cost, but negative correlation with demand, total sales, total profit, holding cost, buy cost, cycle duration, and optimal quantity.

Table – 2 Result of sensitivity analysis with respect to Deteriorative items

θ	Price (P_1)	D	Q_1	T	Purchase cost	Setup cost	Holding cost	DC	Total cost	Total Sales	Total Profit
0.01	83.20	56.99	160.95	2.8240	1139.87	354.09	321.90	32.19	1848.06	4742.23	2894.16
0.02	83.34	56.85	153.90	2.7072	1137.02	369.38	307.81	61.56	1875.79	4738.48	2862.69
0.03	83.48	56.71	147.69	2.6041	1134.29	384.06	295.38	88.61	1902.29	4734.84	2832.54
0.04	83.61	56.58	142.15	2.5123	1131.65	398.03	284.31	113.72	1927.72	4731.29	2803.56
0.05	83.74	56.45	137.18	2.4298	1129.10	411.54	274.36	137.18	1952.18	4727.82	2775.64
0.06	83.86	56.33	132.67	2.3553	1126.62	424.57	265.35	159.21	1975.77	4724.43	2748.66
0.07	83.98	56.21	128.57	2.2874	1124.22	437.17	257.15	180.01	1998.56	4721.15	2722.54
0.08	84.10	56.09	124.82	2.2253	1121.88	449.37	249.65	199.72	2020.64	4717.85	2697.21
0.09	84.21	55.98	121.37	2.1681	1119.61	461.22	242.74	218.47	2942.05	4714.65	2672.59
0.10	84.33	55.86	118.18	2.1153	1117.38	472.73	236.36	236.36	2062.85	4711.49	2648.64

The graphical representation between rate of deteriorative items and total profit is given below. Is it observed that the total profit in downward curve.

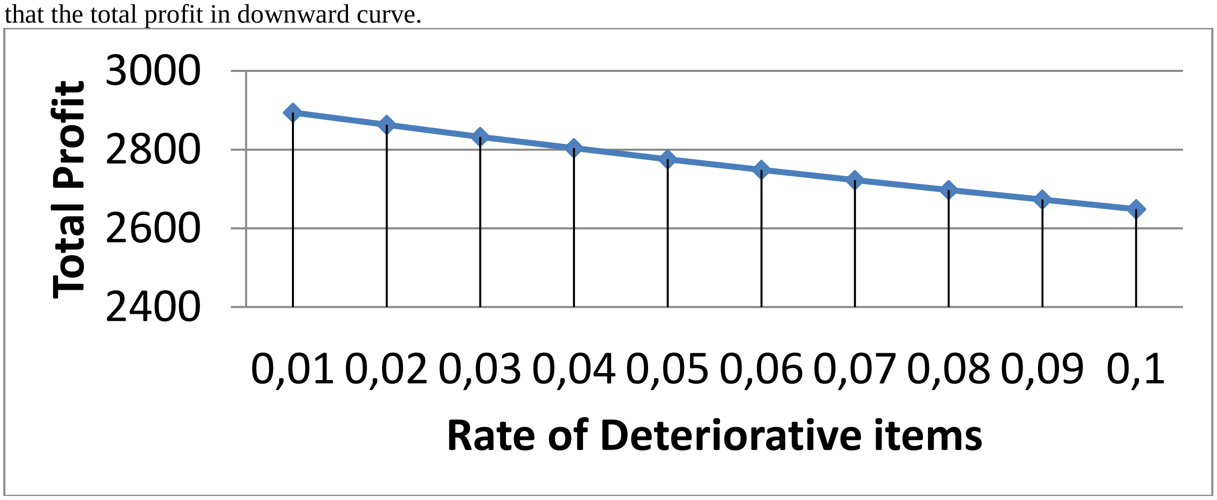

Figure 1 Relationship between total profit with rate of Deteriorative items

Sensitivity Analysis with respect to cost parameters:

Table 3 Result of sensitivity analysis with respect to the inventory parameters (Bertrand-type model for seller 1)

Cost Parameters		Optimum Values										
		Price	D	Q_1	T	Purchase cost	Setup cost	Holding cost	DC	Total cost	Total Sales	Total Profit
C_0	800	82.87	57.32	144.38	2.5185	1146.59	317.64	288.77	28.87	1781.88	4750.92	2969.03
	900	83.04	57.15	152.91	2.6753	1143.14	336.40	305.82	30.58	1815.96	4746.49	2930.53
	1000	83.20	56.99	160.95	2.8240	1139.87	354.09	321.90	32.19	1848.06	4742.23	2894.16
	1100	83.36	56.83	168.57	2.9659	1136.74	370.87	337.15	33.71	1878.49	4738.11	2859.62
	1200	83.51	56.68	175.84	3.1019	1133.75	386.85	351.68	35.16	1907.46	4734.12	2826.65
C_h	2	82.37	57.82	219.51	3.7963	1156.44	263.41	219.51	43.90	1683.27	4763.26	3079.99
	3	82.82	57.37	183.71	3.1018	1147.56	312.31	275.57	36.74	1772.20	4752.17	2979.96
	4	83.20	56.99	160.95	2.8240	1139.87	354.09	321.90	32.19	1848.06	4742.23	2894.16
	5	83.55	56.64	144.84	2.5569	1132.96	391.08	362.11	28.96	1915.13	4733.05	2817.91
	6	83.86	56.33	132.67	2.3553	1126.62	424.57	398.03	26.53	1975.77	4724.43	2748.66
C_d	36	83.19	57.00	161.71	2.8366	1140.16	352.53	323.42	29.10	1845.22	4742.61	2897.38
	38	83.19	57.00	161.33	2.8303	1140.01	353.31	322.66	30.65	1846.64	4742.42	2895.77
	40	83.20	56.99	160.95	2.8240	1139.87	354.09	321.90	32.19	1848.06	4742.23	2894.16
	42	83.21	56.98	160.57	2.8178	1139.72	354.87	321.15	33.72	1849.48	4742.04	2892.55
	44	83.22	56.97	160.20	2.8116	1139.58	355.65	320.41	35.24	1850.89	4741.85	2890.95
	18	82.17	58.02	162.39	2.7989	1044.38	357.27	324.79	32.47	1758.93	4768.11	3009.17

C_P	19	82.69	57.50	161.67	2.8114	1092.64	355.69	323.35	32.33	1804.02	4755.43	2951.41
	20	83.20	56.99	160.95	2.8240	1139.87	354.09	321.90	32.19	1848.06	4742.23	2894.16
	21	83.72	56.47	160.22	2.8368	1186.06	352.49	320.45	32.04	1891.06	4728.48	2837.42
	22	84.23	55.96	159.49	2.8499	1231.23	350.89	318.89	31.89	1933.01	4714.21	2781.20
a	96	81.26	54.93	158.02	2.8764	1098.71	347.64	316.04	31.60	1794.01	4464.31	2670.30
	98	82.23	55.96	159.49	2.8499	1119.30	350.88	318.99	31.89	1821.08	4602.28	2781.20
	100	83.20	56.99	160.95	2.8240	1139.87	354.09	321.90	32.19	1848.06	4742.23	2894.16
	102	84.17	58.02	162.39	2.7989	1160.42	357.27	324.79	32.47	1874.97	4884.15	3009.17
	104	85.15	59.04	163.82	2.7745	1180.96	360.42	327.65	32.76	1901.80	5028.05	3126.24
b	0.8	100.66	59.67	164.69	2.7599	1193.42	362.32	329.38	32.93	1918.06	6006.56	4088.49
	0.9	90.95	58.33	162.83	2.7913	1166.73	358.24	325.67	32.56	1803.22	5306.25	3423.03
	1.0	83.20	56.99	160.95	2.8240	1139.87	354.09	321.90	32.19	1848.06	4742.23	2894.16
	1.1	76.87	55.64	159.03	2.8581	1112.83	349.87	318.06	31.80	1812.57	4277.24	2464.66
	1.2	71.59	54.28	157.07	2.8937	1085.60	345.56	314.15	31.41	1776.73	3886.45	2109.71
c	0.3	75.42	48.69	148.78	3.0551	973.98	327.32	297.56	29.75	1628.62	3672.93	2044.30
	0.4	79.30	52.85	154.99	2.9325	1057.08	340.99	209.99	30.99	1739.07	4191.64	2452.56
	0.5	83.20	56.99	160.95	2.8240	1139.87	354.09	321.90	32.19	1848.06	4742.23	2894.16
	0.6	87.11	61.12	166.67	2.7270	1222.46	366.69	333.35	33.33	1955.79	5324.78	3368.98
	0.7	91.04	65.23	172.20	2.6396	1304.72	378.84	344.40	34.44	2062.40	5939.35	3876.94

Managerial insights: A sensitivity analysis is conducted for studying the impacts of changes in system parameters, purchase cost per unit (C_P), deteriorating cost per unit (C_d), ordering cost per order (C_0), holding cost per unit per unit time (C_h),deteriorating rate, intercept of demand function (a), own price effect (b), cross price effect (c) on optimal price per unit, total profit, total sales, total cost, deteriorative cost, holding cost, setup cost, purchase cost, cycle time, optimum quantity, and demand. To do a sensitivity analysis, one parameter is varied (increased or decreased) at a time, while the others are held constant. The following effects are derived from table 3 using sensitivity analysis.

1. An increase in the setup cost per set is positively correlated with unit price (C_o), the optimal production run size, cycle duration, the holding cost, the deteriorative cost, along with total cost, whereas it is inversely correlated with demand, the purchase price, sales volume, and profits.
2. There is positive relationship between increase in the holding cost per unit per unit time (C_h) with total cost, holding cost per unit per unit time, setup cost per set, and optimum price per unit, there is negative relationship between increase in the holding cost per unit per unit time (C_h) with demand, total profit, total sales, optimum quantity, deteriorative cost, purchase cost, cycle time.
3. Increases in the rate of purchase cost per unit are positively correlated with unit price (C_P), total cost, and cycle time, and negatively correlated with total profit (C_P), total sales, deteriorative cost, holding cost, setup cost, demand, and optimal quantity.
4. Parameters such as the rate of deterioration (C_d), the intercept of the demand function (a), the own price impact (b), and the cross price effect (c) may also be seen in table 3.

4.1(b). Bertrand-type model with linear demand for seller 2

For seller 2, profit maximization per unit time is derived below:

$$\frac{d}{dt}I(t) + \theta I(t) = -(a + cP_1 - bP_2),\ \ 0 \le t \le T_1 \tag{13}$$

with the boundary conditions I(0) = Q, I(0) = 0
From the differential equation (13)

$$I(t) = \frac{(a + cP_1 - bP_2)}{\theta}\left(e^{\theta(T-t)} - 1\right) \tag{14}$$

To find Q $_2$: I(0) = Q, then $Q_2 = \dfrac{(a + cP_1 - bP_2)}{\theta}(e^{\theta T} - 1)$

On simplification, $Q_2 = (a + cP_1 - bP_2)T$, therefore, $T = \dfrac{Q_2}{a + cP_1 - bP_2}$ (15)

Total cost (TC) : purchase cost + setup cost + holding cost + deteriorative cost

1. Purchase cost = $DC_P = (a + cP_1 - bP_2)C_P$ where demand is a constant price. (16)

2. Setup cost = $\frac{C_0}{T}$ = $\frac{(a + cP_1 - bP_2)C_0}{Q_2}$ (from the equation (15)) (17)

3. Holding cost (HC) = $\frac{C_h}{T}\int_0^T \frac{(a + cP_1 - bP_2)}{\theta}(e^{\theta(T-t)} - 1)dt$ = $\frac{(a + cP_1 - bP_2)}{\theta^2 T}(e^{\theta T} - \theta T - 1)$

On simplification with using the equation (15), HC = $\frac{C_h Q_2}{2}$ (18)

4. Deteriorative cost = $\frac{\theta C_d Q_2}{2}$ (19)

Therefore, TC = $(a + cP_1 - bP_2)C_P + \frac{(a + cP_1 - bP_2)C_0}{Q_2} + \frac{(C_h + \theta C_d)Q_2}{2}$

Total Profit (TP)= Total sales – total cost

Total Sales = demand × price per unit (P) = $(a + cP_1 - bP_2)P_2$

$$TP(Q_2, P_2) = \left[(a + cP_1 - bP_2)P_1 - (a + cP_1 - bP_2)P_P - \frac{(a + cP_1 - bP_2)C_0}{Q_2} - \frac{(C_h + \theta C_d)Q_2}{2}\right] \quad (20)$$

Partially differential the total profit (20) with respect to Q_2, then

$$\frac{\partial}{\partial Q_2}TP(Q_2, P_2) = \frac{(a + cP_1 - bP_2)C_0}{Q_2^2} - \frac{C_h + \theta C_d}{2} = 0$$

On simplification, $Q_2^2 = \frac{2(a + cP_1 - cP_2)C_0}{C_h + \theta C_d}$ (21)

Partially differential the total profit $TP(Q_2, P_2)$ with respect to P_2, then

$$\frac{\partial}{\partial P_2}TP(Q_2, P_2) = a + cP_1 - 2bP_2 + \frac{bC_0}{Q_2} + bC_P = 0$$

$$(a + cP_1 - 2bP_2 + bC_P)Q_2 + bC_0 = 0$$

on simplification, $Q_2 = \frac{bC_2}{a + cP_1 - 2bP_2 + bC_P}$ (22)

substitute the value of (11) in the equation (10) then,

$$\frac{b^2 C_0}{(a + cP_1 - 2bP_2 + bC_P)^2} = \frac{2(a + cP_1 - bP_2)}{C_h + \theta C_d}$$

$$2(a + cP_1 - bP_2)(a + cP_1 - 2bP_2 + bC_P)^2 = b^2 C_0 (C_h + \theta C_d)$$

On simplification,

$$\left[\begin{array}{l} 8b^3P_2^3 - 8\left(2ab^2 + 2b^2cP_1 + b^3C_P\right)P_2^2 + 2\begin{pmatrix} 5a^2b + 10abcP_1 + 6ab^2C_P + 5bc^2P_1^2 \\ + b^3C_P^2 + 6b^2cP_1C_P \end{pmatrix}P_2 \\ -2\begin{pmatrix} a^3 + 3ac^2P_1^2 + ab^2C_P^2 + 3a^2cC_1 + 2a^2bC_P \\ + 4abcP_1C_P + c^3P_1^3 + b^2cP_1C_P^2 + 2bc^2P_1^2C_P \end{pmatrix} - b^2C_0(C_h + \theta C_d) \end{array}\right] = 0 \quad (23)$$

which is optimum solution in P in third order equation. On solving equation (23), the price (P_1) of the product can determine then substitute the value of P_2 in the equation (21), then the optimum quantity (Q_1) can determine.

Numerical example 2

To illustrate the solution procedure and the results, let us apply the proposed algorithms to solve the following numerical example. The results can be found by apply visual basic 6.0. This example is based on the following parameters and assumptions. The cost parameters are

P_2 = 83.05, Demand D = 57.14, a = 100, b = 1.0, c = 0.5, Setup cost = 750, Holding cost per unit/time = 3, Purchase cost per unit = 15, Deteriorative cost per unit (C_d) = 30, Rate of Deteriorative items (θ) = 0.01.

Optimum Solution: The third order equation is

$$8P_2^3 - 2384.8P_2^2 + 226293.02P_2 - 6935712.19 = 0$$

which has three real roots in which one positive and two negative roots. The positive root 83.20 is considered in this model. Optimum price (P_1) = 80.52, Optimum Quantity= 166.54, Optimums Demand = 56.99, Optimum cycle time (T) = 2.7293,, Purchase cost = 915.35, Setup cost = 274.80, Holding cost = 249.82, Deteriorative cost = 24.98, Total cost = 1464.95, Total sales = 4914.00, Total profit = 3449.05

Result of sensitivity analysis with respect of price (equal) of the sellers : From the table 4, it is observed that at the price per unit 79.68,

Table –4 Result of sensitivity analysis with respect to Price of the Seller

P_2	Price (P_1)	D	Q_1	T	Purchase cost	Setup cost	Holding cost	DC	Total cost	Total Sales	Total Profit
75	78.54	58.95	163.70	2.7765	884.38	2.7011	245.55	24.55	1424.61	4630.70	3206.08
76	78.78	59.21	164.05	2.7706	888.21	270.69	246.08	24.60	1429.61	4665.23	3235.62
77	79.03	59.46	164.41	2.7646	892.03	271.28	246.61	24.66	1434.59	4699.89	3265.30
78	79.27	59.72	164.76	2.7587	895.86	271.86	247.14	24.71	1439.58	4734.68	3295.09
79	79.52	59.97	165.11	2.7528	899.68	272.44	247.67	24.76	1444.56	4769.58	3325.02
79.68	**79.68**	**60.15**	**165.35**	**2.7489**	**902.28**	**272.83**	**248.03**	**24.80**	**1447.94**	**4793.39**	**3345.44**
80	79.76	60.23	165.46	2.7470	903.50	273.01	248.19	24.81	1449.54	4804.61	3355.07
81	80.01	60.48	165.81	2.7412	907.32	273.59	248.72	24.87	1454.51	4839.77	3385.25
82	80.25	60.74	166.16	2.7355	911.14	274.17	249.24	24.92	1459.48	4875.05	3415.56
83	80.50	60.99	166.51	2.7298	914.96	2.7474	249.76	24.97	1464.45	4910.45	3446.00
84	80.74	61.25	166.85	2.7241	918.78	275.31	250.28	25.02	1469.42	4945.98	3476.56

Sensitivity analysis with respect to rate of deteriorative items (θ):

Table 5 shows the results of the sensitivity analysis with respect to the rate of deteriorative items θ . From the above table 5, it is observed that a study of rate of the deteriorative items with price per unit, demand, optimal quantity, optimal cycle time, purchasing cost, setup cost, holding cost, deteriorative cost, total cost, total sales and total profit. There is positive relationship between price per unit, setup cost, deteriorative cost and total cost when the rate of the deteriorating items increases and there is negative relationship between demand, optimum quantity, cycle time, purchase cost, holding cost and total sales when the rate of deteriorative items increases.

Table – 5 Result of sensitivity analysis with respect to deteriorative items

θ	Price P_2	D	Q_2	T	Purchase cost	Setup cost	Holding cost	DC	Total cost	Total Sales	Total Profit
0.01	80.52	61.02	166.54	2.7292	915.35	274.80	249.82	24.98	1464.95	4914.00	3449.05
0.02	80.62	60.92	159.32	2.6152	913.81	286.78	238.98	47.79	1487.38	4912.00	3424.62
0.03	80.72	60.82	152.94	2.5146	912.34	298.25	229.42	68.82	1508.85	4910.06	3401.21
0.04	80.82	60.72	147.27	2.4250	910.93	309.26	220.90	88.36	1529.46	4908.17	3378.70
0.05	80.91	60.63	142.17	2.3446	909.55	319.88	213.25	106.62	1549.32	4903.32	3357.00
0.06	81.00	60.54	137.55	2.2718	908.23	330.13	206.33	123.79	1568.49	4904.52	3336.02
0.07	81.08	60.46	133.35	2.2055	906.94	340.05	200.03	140.02	1589.04	4902.76	3315.71
0.08	81.17	60.37	129.50	2.1448	905.64	349.66	194.26	155.40	1605.02	4901.03	3296.00
0.09	81.25	60.29	125.96	2.0890	904.47	359.00	188.95	170.05	1622.48	4899.33	3276.84
0.10	81.33	60.21	122.69	2.0375	903.28	368.09	184.04	184.04	1639.46	4897.65	3258.19

The graphical representation between rate of deteriorative items and total profit is given below. It is observed that the total profit in downward curve.

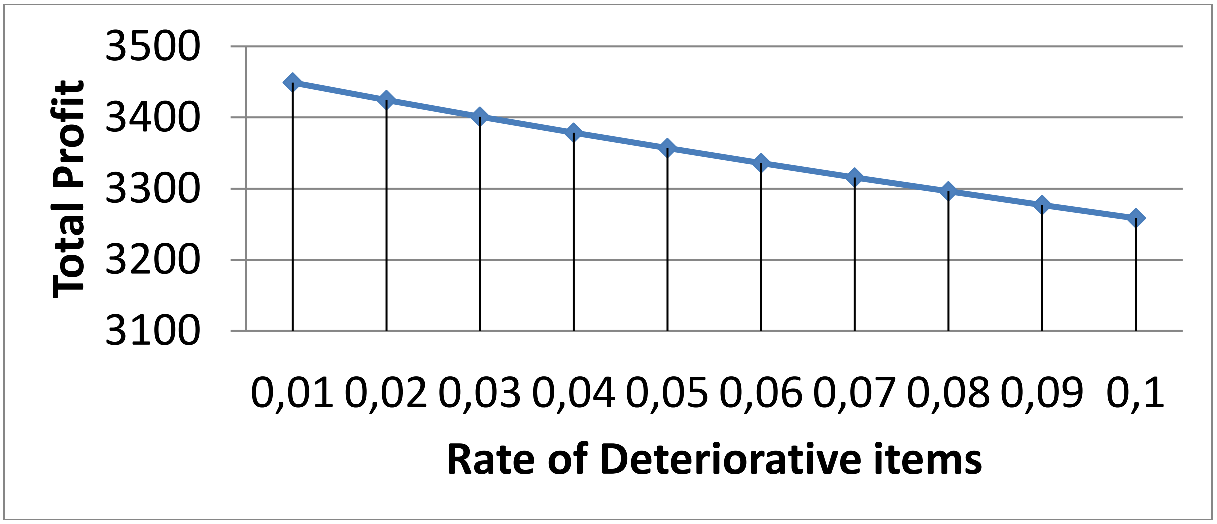

Figure 2 Relationship between total profit with rate of deteriorative items

4.2. Optimal pricing and procurement policies with Cournot- Type model with linear price (inverse demand)

The Cournot-type competitive model, each seller maximizes his profit per unit time over his order quantity and his demand per unit time assuming a given level of demand per unit time of his competitor. The linear inverse demand function is as follows:

$P_1 = a - bD_1 - cD_2$ and $P_2 = a - cD_1 - bD_2$

4.2 (a). Cournot-type model with linear inverse demand

The rate of decrease in the inventory level I(t) is equal to the rate of deteriorative items and the demand rate. The relationship is expressed by the following differential equation.

The profit maximization per unit time for seller 1 is given below:

$$\frac{d}{dt} I(t) + \theta I(t) = -D_1 \tag{24}$$

with the boundary conditions, $I(0) = Q$ and $I(T) = 0$

Integrating the above equation and plugging in the end-of-cycle boundary condition I(T) =0, then the solution for the differential equation (24) is given below. From the equation (24), the solution of the differential equation

$$I(t) = \frac{D_1}{\theta}(e^{\theta(T-t)} - 1) \tag{25}$$

To find Q_1: We know that $I(0) = Q_1$

$$Q_1 = D_1 T \text{ , therefore, } T = \frac{Q_1}{D_1} \tag{26}$$

Total cost consist of purchase cost, setup cost, holding cost and deteriorative cost

(i) Purchase cost = $D_1 C_P$ (27)

(ii) Setup cost = $\dfrac{D_1 C_0}{Q_1}$ (28)

(iii) Holding cost = $\dfrac{C_h}{T}\int_0^T \dfrac{D_1}{\theta}(e^{\theta(T-t)} - 1)dt = \dfrac{D_1 C_h}{\theta^2 T}(e^{\theta T} - \theta T - 1) = \dfrac{D_1 C_h T}{T}$

From the equation (26),

(29)

(iv) Deteriorative cost = $\frac{\theta C_d Q_1}{2}$ (30)

Total cost = $D_1 C_P + \frac{D_1 C_0}{Q_1} + \frac{(C_h + \theta C_d)Q_1}{2}$

In order to maximize the total profit, the first partial derivatives of $TP(Q_1, D_1)$ with respect to Q_1 and D_1 are set equal to zero, leading to following systems of nonlinear equations. Partially differential the total profit (31) with respect to Q_1, then

Total Profit $TP(Q_1, D_1) = (a - bP_1 - cD_2)D_1 - D_1 C_P - \frac{D_1 C_0}{Q_1} - \frac{(C_h + \theta C_d)Q_1}{2}$ (31)

Partially differentiate with respect to Q_1

$$\frac{\partial}{\partial Q_1} TP(Q_1, D_1) = \frac{D_1 C_0}{Q_1^2} - \frac{(C_h + \theta C_d)}{2} = 0$$

Therefore, $Q_1^{\,2} = \frac{2D_1 C_0}{C_h + \theta C_d}$

(32)

$$\frac{\partial}{\partial D_1} TP(Q_1, D_1) = (a - 2bD_1 - cD_2) - C_P - \frac{C_0}{Q_1} = 0$$

$$(a - 2bD_1 - cD_2 - C_P)Q_1 = C_0$$

Therefore, $Q_1 = \frac{C_0}{a - 2bD_1 - cD_2 - C_P}$

(33)

Substitute the value of Q_1 in the equation (32)and simplify, then

$$\frac{C_0}{(a - 2bD_1 - cD_2 - C_P)^2} = \frac{2D_1}{C_h + \theta C_d}$$

$$2D_1(a - 2bD_1 - cD_2 - C_P)^2 = C_0(C_h + \theta C_d)$$

$$2D_1\begin{bmatrix} a^2 + 4b^2 D_1^2 + c^2 D_2^2 + C_P^2 - 4abD_1 + 4bcD_1 D_2 \\ + 2cD_2 C_P - 2acD_2 - 2aC_P + 4bC_P D_1 \end{bmatrix} = C_0(C_h + \theta C_d)$$

On simplification,

$$8b^2 D_1^3 - 8b(a - cD_2 - C_P)D_1^2 + 2\begin{pmatrix} a^2 + c^2 D_2^2 + C_P^2 + 2cC_P D_2 \\ - 2acD_2 - 2aC_P \end{pmatrix} D_1 - C_0(C_h + \theta C_d) = 0 \quad (34)$$

Which is the third order equation in the Demand (D_1). Solving this equation for (D_1) and substitute in the equation (32), then we can get the value for Q_1 which both (Q_1, D_1) are decision variables in this model.

Numerical example 3

To illustrate the solution procedure and the results, let us apply the proposed algorithms to solve the following numerical example. The results can be found by apply visual basic 6.0. This example is based on the following parameters and assumptions. The cost parameters are

D_1 = 32.7, a = 100, b = 1.0, c = 0.5, Setup cost (C_0) = 1000, Holding cost per unit/time (C_h) = 4, Purchase cost per unit = 20, Deteriorative cost per unit (C_d) = 40, Rate of Deteriorative items (θ) = 0.01.

Optimum Solution: The third order equation is $8P_1^3 - 509.2P_1^2 + 8102.64P_1 - 337.61 = 0$

which has three real roots in which one positive and two negative roots. The positive root 83.20 is considered in this model. D_2 = 27.34, optimum price (P_1) = 56.31, optimum quantity= 111.47, optimum cycle time (T) = 4.0774,, purchase cost = 546.79, setup cost = 245.24, holding cost = 222.95, deteriorative cost = 22.29, total cost = 1037.29, total sales = 1539.50, total profit =502.21

Result of sensitivity analysis with respect of price (equal) of the sellers From the table 6, it is observed that at demand 28.48

Table – 6 Result of sensitivity analysis with respect to price of the seller

D_2	D_1	P_1	Q_1	**T**	**purchase cost**	**setup cost**	**holding cost**	**DC**	**Total cost**	**Total Sales**	**Total Profit**
25	29.42	58.04	115.65	3.9302	588.53	254.43	231.30	23.13	1097.41	1708.90	611.49
26	29.15	57.84	115.12	3.9483	583.13	253.26	230.24	23.02	1089.67	1686.52	596.84
27	28.88	57.61	114.58	3.9668	577.73	252.09	229.17	22.91	1081.91	1664.25	582.33
28	28.61	57.38	114.04	3.9855	572.31	250.90	228.09	22.80	1074.13	1642.09	567.96
29	28.34	57.15	113.50	4.0045	566.90	249.71	227.01	22.70	1066.33	1620.05	553.72
28.48	28.48	57.27	113.78	3.9946	569.71	250.33	227.57	22.75	1070.39	1631.50	561.11
30	28.07	56.92	112.96	402.38	561.47	248.52	225.92	22.59	1058.51	1588.13	539.61
31	27.80	56.69	112.41	4.0434	556.04	247.31	224.83	22.48	1050.67	1576.32	525.64
32	27.53	56.46	111.86	4.0633	550.60	246.10	223.73	22.37	1042.81	1554.62	511.81
33	27.25	56.24	111.31	4.0835	545.16	244.88	222.62	22.26	1034.92	1533.04	498.11
34	26.98	56.01	110.75	4.1041	539.70	243.65	221.50	22.15	1027.01	484.55	

Sensitivity analysis with respect to rate of Deteriorate items (θ):

Table 6 shows the results of the sensitivity analysis with respect to the rate of deteriorative items θ . From the above table 6, it is observed that a study of rate of the deteriorative items with price per unit, demand, optimal quantity, optimal cycle time, purchasing cost, setup cost, holding cost, deteriorative cost, total cost, total sales and total profit. There is positive relationship between price per unit, setup cost, deteriorative cost and total cost when the rate of the deteriorating items increases and there is negative relationship between demand, optimum quantity, cycle time, purchase cost, holding cost and total sales when the rate of deteriorative items increases.

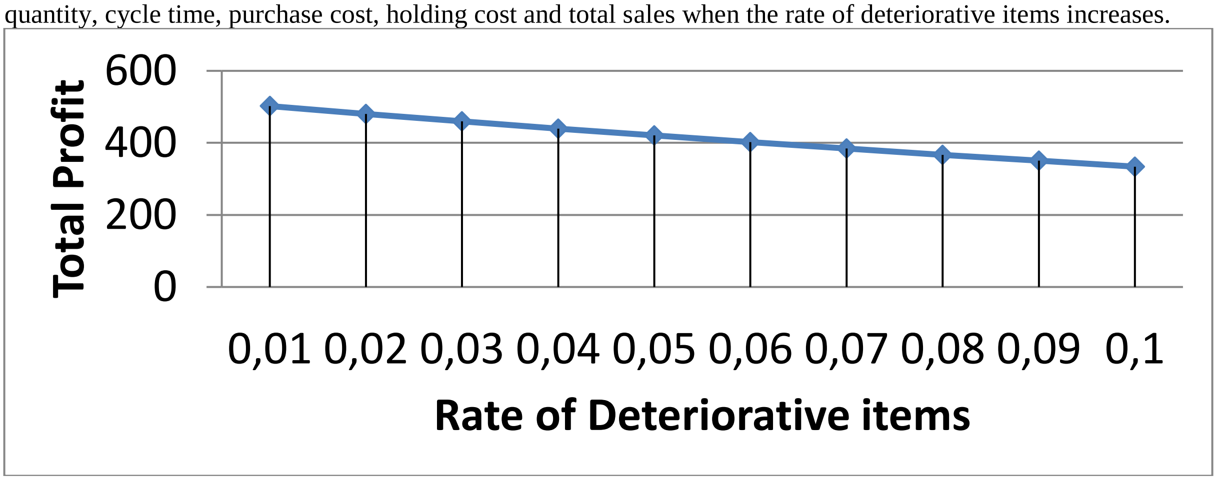

Figure 3 Relationship between total profit with rate of deteriorate items

Sensitivity analysis and discussion

Table – 6 Result of sensitivity analysis with respect to deteriorative items

θ	D_1	P_1	Q_1	**T**	**Purchase cost**	**Setup cost**	**Holding cost**	**DC**	**Total cost**	**Total Sales**	**Total Profit**
0.01	27.34	56.31	111.47	4.0774	546.79	245.24	222.95	22.29	1037.29	1539.50	502.21
0.02	27.12	56.52	106.30	3.9195	542.49	255.13	212.60	42.52	1052.69	1533.13	480.44
0.03	26.91	56.74	101.73	3.7805	538.20	264.51	203.47	61.04	1067.23	1526.88	459.65
0.04	26.70	56.94	97.66	3.6569	534.10	273.44	195.32	78.12	1081.00	1520.72	439.72
0.05	26.50	57.14	93.99	3.5462	530.11	281.98	187.99	93.99	1094.08	1514.64	420.56
0.06	26.31	57.33	90.67	3.4463	526.21	290.16	181.35	108.81	1106.54	1508.64	402.09
0.07	26.12	57.52	87.64	3.3555	522.40	298.00	175.30	122.70	1118.42	1502.69	384.27
0.08	25.93	57.71	84.87	3.2727	518.68	305.55	169.75	135.08	1129.78	1496.80	367.02
0.09	25.75	57.89	82.32	3.1967	515.02	312.81	164.64	148.17	1140.65	1490.96	350.30
0.10	25.57	58.07	79.95	3.1267	511.43	319.82	159.91	159.91	1151.07	1485.15	334.08

The graphical representation between rate of deteriorative items and total profit is given below. It is observed that the total profit in downward curve.

In order to assess the relative impact of the different input parameters on the solution quantity, systematic sensitivity analysis was performed on the above example.

Table 8 Result of sensitivity analysis with respect to the inventory parameters (Cournot-type model for seller 1)

Cost Parameters		Optimum Values										
		Price	D	Q_1	T	Purchase cost	Setup cost	Holding cost	DC	Total cost	Total Sales	Total Profit
C_0	800	27.85	55.79	100.63	3.6134	557.00	221.39	201.26	20.12	999.79	1554.03	554.23
	900	27.58	56.06	106.23	3.8507	551.78	233.72	212.47	21.24	1019.23	1556.67	527.44
	1000	27.34	56.31	111.47	4.0774	546.79	245.24	222.95	22.29	1037.29	1539.50	502.21
	1100	27.10	56.54	116.40	4.2953	542.00	256.09	232.80	23.28	1054.18	1532.50	478.32
	1200	26.86	56.78	121.06	4.5056	537.37	266.33	242.12	24.21	1070.04	1525.64	455.60
C_h	2	28.58	55.06	154.34	5.3992	571.70	185.20	154.34	30.86	942.12	1574.04	631.91
	3	27.92	55.72	128.16	4.5897	558.47	217.87	192.24	25.63	994.22	1556.08	561.85
	4	27.34	56.31	111.47	4.0774	546.79	245.24	222.95	22.29	1037.29	1539.50	502.21
	5	26.80	56.84	99.64	3.7170	536.14	269.03	249.10	19.92	1074.20	1523.97	449.58
	6	26.31	57.33	90.67	3.4463	526.21	290.16	272.02	18.13	1106.54	1508.64	402.09
C_d	36	27.36	56.28	112.03	4.0944	547.24	244.23	224.06	20.16	1035.70	1540.15	504.44
	38	27.35	56.30	111.75	4.0859	547.01	244.74	223.50	21.23	1036.50	1539.83	503.32
	40	27.34	56.31	111.47	4.0774	546.79	245.24	222.95	22.29	1037.29	1539.50	502.21
	42	27.32	56.32	111.20	4.0690	546.57	245.75	222.40	23.52	1038.08	1539.18	501.10
	44	27.31	56.33	110.92	4.0607	546.35	246.26	221.85	24.40	1038.87	1538.86	499.98
C_P	18	28.42	55.22	113.67	3.9987	511.67	250.07	227.34	22.73	1011.82	1569.80	557.98
	19	27.88	55.76	112.58	4.0375	529.79	247.67	225.16	22.51	1025.14	1554.97	529.82
	20	27.34	56.31	111.47	4.0774	546.79	245.24	222.95	22.29	1037.29	1539.50	502.21
	21	26.79	56.85	110.35	4.1187	562.68	242.79	220.71	22.07	1048.26	1523.41	475.14
	22	26.24	57.40	109.22	4.1614	577.44	240.30	218.45	21.84	1058.04	1506.66	448.62
a	96	25.14	54.50	106.91	4.2514	502.92	235.21	213.83	21.38	973.40	1370.62	397.22
	98	26.24	55.40	109.22	4.1614	524.94	240.30	218.45	21.84	1005.54	1454.17	448.62
	100	27.34	56.31	111.47	4.0774	546.79	245.24	222.95	22.29	1037.29	1539.50	502.21
	102	28.42	57.22	113.67	3.9987	568.52	250.07	227.34	22.73	1068.67	1626.65	557.98
	104	29.50	58.14	115.81	3.9248	590.15	254.78	231.62	23.16	1099.72	1715.64	615.91
b	0.8	34.81	55.79	125.79	3.6134	696.25	276.74	251.58	25.15	1249.74	1942.54	692.79
	0.9	30.65	56.06	118.04	3.8507	613.09	259.69	236.08	23.60	1132.47	1718.52	586.04
	1.0	27.34	56.31	111.47	4.0774	546.79	245.24	222.95	22.29	1037.29	1539.50	502.21
	1.1	24.63	56.54	105.82	4.2953	492.72	232.81	211.64	21.16	958.34	1393.18	434.84
	1.2	22.39	56.78	100.88	4.5056	447.81	221.94	201.74	20.17	891.70	1271.36	379.66
c	0.3	30.87	59.31	118.46	3.8369	617.48	260.62	236.92	23.69	1138.72	1831.33	692.60
	0.4	29.11	57.80	115.03	3.9513	582.27	253.08	230.07	23.00	1088.43	1682.95	594.51
	0.5	27.34	56.31	111.47	4.0774	546.79	245.24	222.95	22.29	1037.29	1539.50	502.21
	0.6	25.55	54.82	107.76	4.2178	511.00	237.08	215.53	21.55	985.18	1400.91	415.73
	0.7	23.74	53.36	103.88	4.3755	474.83	228.54	207.76	20.77	931.92	1267.06	335.13

Managerial insights: A sensitivity analysis is performed to study the effects of changes in the system parameters, rate of deteriorating items (θ), ordering cost per order (C_0), holding cost per unit per unit time (C_h), deteriorating cost per unit per unit time (C_d), purchase cost per unit (C_P), the intercept of the demand function (a), the own price effect (b), cross price effect (c) on optimal price per unit, demand, optimum quantity, cycle time, purchase cost, setup cost, holding cost, deteriorative cost, total cost, total sales and total profit. The sensitivity analysis is performed by changing (increasing or decreasing) the parameter taken at a time, keeping the remaining parameters at their original values. The following influences can be obtained from sensitivity analysis based on table 8.

1. There is positive relationship between increase in the setup cost per set (C_o) with price per unit, optimum quantity, cycle time, setup cost, holding cost, deteriorative cost , total cost and there is negative relationship between increase in the setup cost per set with demand, purchase cost, total sales and total profit.
2. There is positive relationship between increase in the holding cost per unit per unit time (C_h) optimum price per unit, setup cost per set, holding cost per unit per unit time, and total cost and there is negative relationship between increase in the holding cost per unit per unit time (C_h) demand, optimum quantity, cycle time, purchase cost, deteriorative cost, total sales, and total profit.
3. There is positive relationship between increase in the rate of purchase cost per unit (C_P) with price per unit, cycle time, purchasing cost, total cost and there is negative relationship between the increase in the rate of purchase cost per unit (C_P) with demand, optimum quantity, setup cost, holding cost, deteriorative cost, total sales and total profit.
4. Similarly, other parameters, rate of deteriorating items (C_d), intercept of the demand function (a), the own price effect (b), cross price effect (c) can also be observed from the table 8.
5.

4.2(b). Cournot- type model with linear price (inverse demand) for seller 2

$$\frac{d}{dt}I(t)+\theta I(t)=-D_2 \qquad (35)$$

With the boundary conditions, $I(0)=Q$ and $I(T)=0$

From the equation (1), the solution of the differential equation

$$I(t)=\frac{D_2}{\theta}(e^{\theta(T-t)}-1) \tag{36}$$

To find Q_2 : We know that $I(0)=Q_2$

$$Q_2 = D_2T \text{ , therefore, } T=\frac{Q_2}{D_2} \tag{37}$$

Total cost:

(i) Purchase cost = D_2C_P (38)

(ii) Setup cost = $\frac{D_2C_0}{Q_2}$ (39)

(iii) Holding cost = $\frac{C_h}{T}\int_0^T \frac{D_2}{\theta}(e^{\theta(T-t)}-1)dt$

$=\frac{D_2C_h}{\theta^2T}(e^{\theta T}-\theta T-1) = \frac{D_2C_hT}{T}$

From the equation ( ), HC = $\frac{C_hQ_2}{2}$ (40)

(iv) Deteriorative cost = $\frac{\theta C_dQ_2}{2}$ (41)

Total cost = $D_2C_P+\frac{D_2C_0}{Q_2}+\frac{(C_h+\theta C_d)Q_2}{2}$

Total Profit $TP(Q_2,D_2)=(a-cD_1-bD_2)D_2-D_2C_P-\frac{D_2C_0}{Q_2}-\frac{(C_h+\theta C_d)Q_2}{2}$ (42)

Partially differentiate with respect to Q_2

$$\frac{\partial}{\partial Q_2}TP(Q_2,D_2)=\frac{D_2C_0}{Q_2^2}-\frac{(C_h+\theta C_d)}{2}=0$$

Therefore, $Q_2{}^2=\frac{2D_2C_0}{C_h+\theta C_d}$ (43)

$$\frac{\partial}{\partial D_2}TP(Q_2,D_2)=(a-cD_1-2bD_2)-C_P-\frac{C_0}{Q_2}=0$$

$$(a-cD_1-2bD_2-C_P)Q_2=C_0$$

Therefore, $Q_2=\frac{C_0}{a-cD_1-2bD_2-C_P}$ (44)

Substitute the value of Q_2 in the equation (43)and simplify, then

$$\frac{C_0}{(a-cD_1-2bD_2-C_P)^2}=\frac{2D_2}{C_h+\theta C_d}$$

$$2D_2(a-cD_1-2bD_2-C_P)^2=C_0(C_h+\theta C_d)$$

$$2D_2\begin{bmatrix}a^2+4b^2D_2^2+c^2D_1^2+C_P^2-4abD_2+4bcD_1D_2\\+2cD_1C_P-2acD_1-2aC_P+4bC_PD_2\end{bmatrix}=C_0(C_h+\theta C_d)$$

On simplification,

$$8b^2D_2^3 - 8b(a - cD_1 - C_P)D_2^2 + 2\begin{pmatrix} a^2 + c^2D_1^2 + C_P^2 + 2cC_PD_1 \\ -2acD_1 - 2aC_P \end{pmatrix}D_2 - C_0(C_h + \theta C_d) = 0 \quad (45)$$

which is the third order equation in the demand D_2. Solving this equation for D_2 and substitute in the equation (43), then we can get the value for Q_2 which both (Q_2, D_2) are decision variables in this model.

Numerical example 4

To illustrate the solution procedure and the results, let us apply the proposed algorithms to solve the following numerical example. The results can be found by apply visual basic 6.0. This example is based on the following parameters and assumptions. The cost parameters are D_2 = 32.51, $P_2 = 53.68$ 3.05, demand D = 57.14, a = 100, b = 1.0, c = 0.5, setup cost = 1000, holding cost per unit/time = 4, purchase cost per unit = 30, deteriorative cost per unit (C_d) = 100, rate of deteriorative items (θ) = 0.01.

Optimum Solution:

The third order equation is $8P_2^3 - 569.6P_2^2 + 10138.88P_2 - 2475 = 0$ which has three real roots in which one positive and two negative roots. The positive root 83.20 is considered in this model. D_2 = 32.51, P_2 = 53.68, Q_2 = 121.57, optimum cycle time (T) = 3.7389, purchase cost = 487.73, setup cost = 200.59, holding cost = 182.35, deteriorative cost = 18.23, total cost = 888.91, Total sales = 1745.57, total profit = 856.65

Result of sensitivity analysis with respect of price (equal) of the sellers Table 10 shows the results of the sensitivity analysis with respect to the price per unit. From the above table 10, it is observed that a study of price per unit with price per unit, demand, optimal quantity, optimal cycle time, purchasing cost, setup cost, holding cost, deteriorative cost, total cost, total sales and total profit. There is positive relationship between price per unit, demand, optimum quantity, purchase cost, setup cost, holding cost, deteriorative cost, total cost and total profit when the rate price increases and there is negative relationship between cycle time when the rate of deteriorative items increases.

Table – 10 Result of sensitivity analysis with respect to price of the seller

D_1	D_2	P_2	Q_2	**T**	**Purchase cost**	**Setup cost**	**Holding cost**	**DC**	**Total cost**	**Total Sales**	**Total Profit**
25	33.19	54.30	122.83	3.7003	497.95	202.68	184.25	18.42	903.33	1802.70	899.37
26	32.96	54.06	122.35	3.7150	494.02	201.88	183.53	18.35	897.79	1780.63	882.83
27	32.67	53.82	121.86	3.7298	490.09	201.07	182.79	18.27	892.25	1758.68	866.43
28	32.41	53.58	121.37	3.7449	486.15	200.26	182.06	18.20	886.69	1736.86	850.16
29	32.14	53.35	120.88	3.7602	482.21	199.45	181.32	18.13	881.13	1715.15	834.02
30	31.88	53.11	120.38	3.7756	478.27	198.63	180.58	18.05	875.55	1693.57	818.01
31	31.62	52.87	119.89	3.7913	474.32	197.81	179.83	17.98	869.97	1672.11	802.14
31.49	31.49	52.76	119.64	3.7990	472.39	197.41	176.46	17.94	867.22	1661.63	794.40
32	31.35	52.64	119.39	3.8072	470.38	196.99	179.08	17.90	864.37	1650.77	786.39
33	31.09	52.40	118.88	3.8232	466.43	196.16	178.33	17.83	858.76	1629.55	770.78
34	30.83	52.16	118.38	3.8395	462.48	195.33	177.53	17.75	853.15	1608.45	755.29
35	30.56	51.93	117.87	3.8561	458.53	194.49	176.81	17.68	847.52	1587.47	739.94

Sensitivity Analysis with respect to rate of deteriorative items (θ):

Table 9 shows the results of the sensitivity analysis with respect to the rate of deteriorative items θ . From the above table 9, it is observed that a study of rate of the deteriorative items with price per unit, demand, optimal quantity, optimal cycle time, purchasing cost, setup cost, holding cost, deteriorative cost, total cost, total sales and total profit. There is positive relationship between price per unit, setup cost, deteriorative cost and total cost when the rate of the deteriorating items increases and there is negative relationship between demand, optimum quantity, cycle time, purchase cost, holding cost and total sales when the rate of deteriorative items increases.

Table – 9 Result of sensitivity analysis with respect to deteriorative items

θ	D_2	P_2	Q_2	**T**	**Purchase cost**	**Setup cost**	**Holding cost**	**DC**	**Total cost**	**Total Sales**	**Total Profit**

0.01	32.52	53.69	121.59	3.7382	487.88	200.62	182.38	18.23	889.14	1746.44	857.30
0.02	32.38	53.83	116.15	3.5871	485.72	209.08	174.23	34.84	903.88	1743.37	839.48
0.03	32.24	53.97	111.35	3.4538	483.63	217.15	167.03	50.11	917.94	1740.37	822.42
0.04	32.10	54.11	107.08	3.3351	481.62	224.87	160.62	64.25	931.37	1737.43	806.05
0.05	31.97	54.24	103.24	3.2286	479.66	232.29	154.86	77.43	944.26	1734.54	790.28
0.06	31.85	54.36	99.76	3.1322	477.76	239.44	149.65	89.76	956.65	1731.71	775.06
0.07	31.72	54.49	96.60	3.0446	475.92	246.33	144.90	101.43	968.58	1728.92	760.33
0.08	31.60	54.61	93.70	2.9644	474.11	252.99	140.55	112.44	980.10	1726.17	746.06
0.09	31.49	54.72	91.03	2.8907	472.35	259.44	136.55	122.89	991.24	1723.46	732.21
0.10	31.37	54.84	88.56	2.8227	470.63	265.69	132.84	132.84	1002.03	1720.78	718.74

The graphical representation between rate of deteriorative items and total profit is given below. It is observed that the total profit in downward curve.

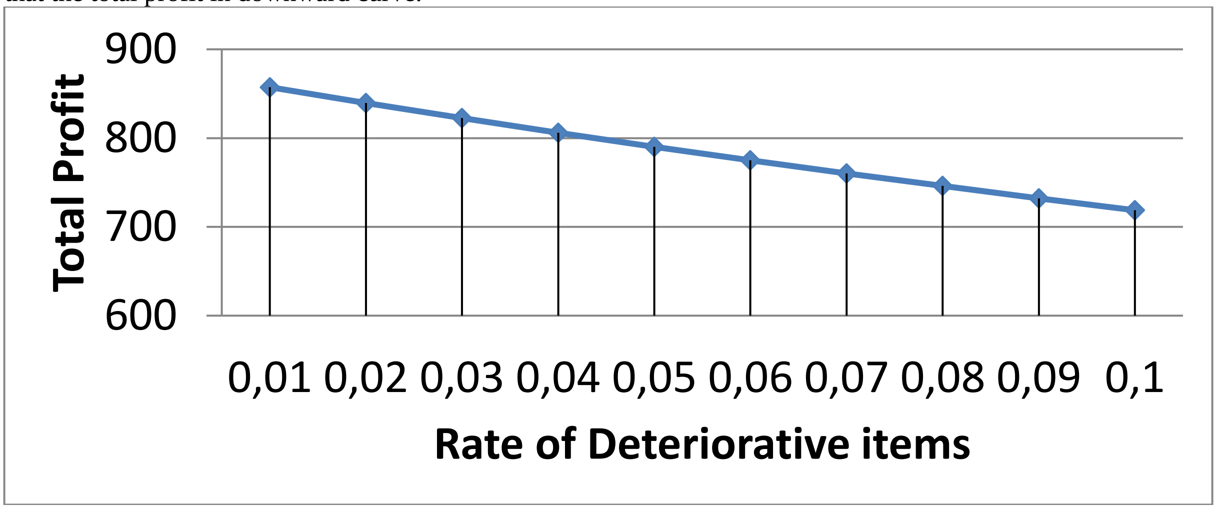

Figure 5 Relationship between total profit with rate of deteriorative items

6. CONCLUSION AND FUTURE STUDIES

This research examine the optimal ordering and optimal pricing models of inventory for perishable products with Bertrand-dependent Demand and Cournot-Dependent price in higher order equations. In order to clarify the proposed method, mathematical models for each model are outlined, and applicable examples are presented. The goal here is to determine the optimum order size and price in terms of total cost. For both models, we also provide a sensitivity analysis. Visual Basic 6.0 was used to create the required data. Table-based sensitivity analysis reveals the following effects: **(i)** When the rate of deterioration of the goods accelerates, there is a positive link between the price per unit, the cost to set up, the cost to replace decaying components, and the overall cost. **(ii)** The unit price, holding cost, optimal quantity, deteriorative cost, cycle duration, setup cost, and overall cost are all positively correlated with the setup cost per set. **(iii)** The optimal unit price, setup cost per set, holding cost per unit per unit time, and total cost all rise in a positive correlation with the holding cost per unit per unit time. **(iv)** Overall cost, cycle time, as well as unit price, purchasing cost, are all positively correlated with rise rate of the purchase cost per unit.

The suggested model is useful for both manufacturers and retailers since it helps them pinpoint the ideal production run length, inventory turnover rate, and overall cost. The suggested inventory model is also applicable to the management of stock in other areas, including grocery stores, boutiques, office supply shops, and other retail establishments. It is possible to further develop this study by:

1. Demand is modelled as a continuous compound in this analysis of inventory models. Another thing that may be included to this study is an examination of probabilistic demand.
2. This study's inventory models are only applicable to a specific time period. For in-depth analysis, we may think about using multi-item inventory models.
3. In this research, a single new idea is introduced at a time using the generated inventory models. Models that include many ideas may be utilized to establish best practices in the near future.

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