REVISTA INVESTIGACION OPERACIONAL VOL. xx , NO.x, xxx-xxx, 201x

FORTHCOMING PAPER ·#90B05-19-03-2

EOQ MODEL UNDER THE POLICY OF REGULATED DELIVERY & PARTIAL BACKLOGGING

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ABSTRACT
This paper develops an economic ordering policy model for non-instantaneous deteriorating items with price dependent demand under the effect of permissible delay in payments. The deterioration is assumed to follow a three parameter Weibull distribution. Shortages are allowed and partially backlogged. The main objective is to determine the optimal selling price and optimal replenishment cycle time so as to maximize the total profit. An efficient algorithm is presented to find the optimal solution of the developed model. The validation of the aforesaid model is done with numerical examples. Comprehensive sensitivity analyses have been given to invent the effect of changing parameter value on the optimal solution. The management can accurately determine the order quantity to regulate the delivery keeping in view of the delivery period and quantum of determination.

KEYWORDS: Price dependent demand, non – instantaneous deterioration, shortage, Permissible delay in payments.

MSC: 90B05

RESUMEN
Este documento desarrolla un modelo de política de orden económico para artículos deteriorados no instantáneos con demanda dependiente del precio bajo el efecto de una demora permisible en los pagos. Se supone que el deterioro sigue una distribución de Weibull de tres parámetros. Las carencias son permitidas y parcialmente atrasadas. El objetivo principal es determinar el precio de venta óptimo y el tiempo óptimo del ciclo de reposición para maximizar el beneficio total. Se presenta un algoritmo eficiente para encontrar la solución óptima del modelo desarrollado. La validación del modelo mencionado se realiza con ejemplos numéricos. Se han realizado análisis de sensibilidad completos para inventar el efecto de cambiar el valor de los parámetros en la solución óptima. La administración puede determinar con precisión la cantidad del pedido para regular el mantenimiento de la entrega en vista del período de entrega y la cantidad de determinación.

PALABRAS CLAVE: Demanda dependiente del precio, deterioro no instantáneo, escasez, demora permisible en los pagos.

1. INTRODUCTION

Many authors have developed their models by assuming that the products delivered are non – deteriorating or the deterioration of the product is instantaneous. Product starts deteriorating as soon as the retailer gets the product delivered. But, in real life there are very limited products which do not deteriorate at all or starts deteriorating immediately. Generally, products start deteriorating after certain time. This kind of deterioration is known as non– instantaneous deterioration. Jaggi C. K. [7] has developed the inventory models for non – instantaneous deteriorating products. Wu et al. [16] proposed an inventory model for non –instantaneous deteriorating items considering stock dependent demand and partial backlogging. Yang et al [17] developed on inventory model for non –instantaneous deteriorating items with price dependent demand and shortages. Gupta et al. [3] proposed an optimal ordering policy for stock dependent demand inventory model with non-instantaneous deteriorating items. Then Kapoor [9] studied an inventory model for non-instantaneous deteriorating products with price and time dependent demand. Again Kaur et al. [8] proposed an optimal ordering policy for inventory model with non-instantaneous deteriorating items and stock dependent demand.
Furthermore, when the shortages occur, it is assumed that it is either completely backlogged or completely lost. But practically some customers are willing to wait for backorder and others would turn to buy from other sellers.

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Researchers such as Park [11], Hollier [5] and Wee [15] developed inventory models with partial backorders. Goyal and Giri [4] developed production-inventory problem of a product with time varying demand, production and deterioration rates. Recently, Mohanty and Tripathy [10] developed a fuzzy inventory model for deteriorating items with exponentially decreasing demand under fuzzified cost and partially backlogging.

In addition to deterioration, price has a great impact on demand. In general, a decrease in selling price leads to increased customer demand and results in a high sales volume. Therefore, pricing strategy is a primary tool that sellers or retailers use to maximize profit and consequently models with price-dependent demand occupy a prominent place in the inventory literature. Cohen [2] determined both the optimal replenishment cycle and price for inventory that was subject to continuous decay over time at a constant rate. Wee [14] studied joint pricing and replenishment policy for a deteriorating inventory with a price elastic demand rate that declined over time. Amutha and Chadrasekharan [1] introduced an inventory model for deteriorating items with three parameter weibull deterioration and price dependent demand. Then, Sahoo and Tripathy [12] developed an EOQ model with three parameter weibull deterioration, trended demand and time varying holding cost with salvage.

In deriving the economic order quantity (EOQ) formula, it is tacitly assumed that the retailer (buyer) must pay for the items as soon as he receives them from a supplier. However, in practice, a supplier will allow a certain fixed period (Credit Period) for settling the amount the retailer owes to him for the items supplied. Delivery period would play an important role in the conduct of business. Firstly, Hwang and Shinn [6] examined the retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Tung et. al. [13] introduced a note on inventory models with a permissible delay in payments. Table-1 focuses on a glance of research works undertaking various aspects of the discussed criteria.

In this paper, an inventory model is developed with regulated delivery in payments. Firstly, the demand rate of the items is assumed to be dependent on the commodity price. The deterioration is assumed to follow a three parameter Weibull distribution. Shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. The theoretical results reflected in this paper are also studied through numerical examples and sensitivity analysis. Quantitative agreement with experimental data is observed.

Table-1: Major characteristics on inventory models on selected areas

Authors & Publication year	**Deterioration**	**Varying Demand**	**Backlogging**	**Regulated Delivery is allowed**
M. A. Cohen (1997)	Constant	Exponential	No	No
K. S. Park (1982)	Constant	Constant	Partial Backlogged	No
Hollier & Mak (1983)	Constant	Negative Exponential	No	No
H. M. Wee (1995)	Two Parameter Weibull Deterioration	Price Dependent Demand	Fully Backlogged	No
Hwang & Shinn (1997)	Exponential	Constant	No	Yes
Goyal & Giri (2003)	Time Dependent	Time-varying Demand	Partially Backlogged	No
Wu et. al. (2006)	Constant	Stock Dependent Demand	Fully Backlogged	No
Yang et. al. (2009)	Constant	Price Dependent Demand	Partially Backlogged	No
Kaur et. al. (2013)	Constant	Stock Dependent Demand	No	No
Amutha & Chandrasekaran (2013)	Three Parameter Weibull	Price Dependent Demand	No	No
Proposed Paper (2017)	Three Parameter Weibull Deterioration	Price Dependent Demand	Partially Backlogged	Yes

2. ASSUMPTIONS AND NOTATIONS

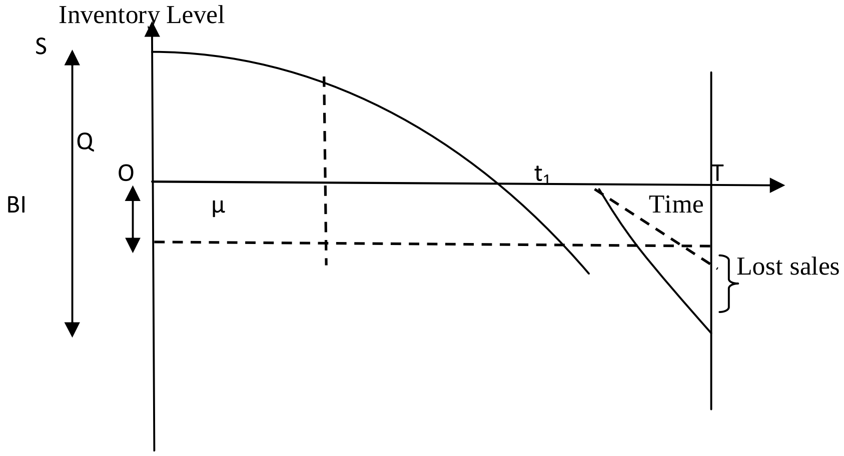
The following assumptions and notations are used throughout the paper.

- The demand rate is a function of selling price.

- Shortages are allowed and are partially backlogged. During the stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. So that the backlogging rate for negative inventory is $\beta(t) = \ell^{-\lambda(T-t)}$, λ is the backlogging parameter and $(T-t)$ is the waiting time $(t_1 \le t \le T)$.
- The inventory system deals with single item.
- The planning horizon is infinite.
- The lead time is zero.
- The replenishment cycle is infinite and replenishment amount is constant.
- A is the ordering cost per order.
- v_1 is the holding cost per unit time.
- v_2 is the shortage cost per unit time.
- v_3 is the unit cost of lost sales per unit time.
- p is the selling price per unit time , where $p > c$.
- c is the purchase cost per unit.
- $\theta = \alpha\beta(t-\gamma)^{\beta-1}$, Where $0 \le \alpha < 1$ is the scale parameter, $\beta \ge 1$ is the shape parameter, $\gamma > 0$ is the location parameter.
- μ is the length of time in which the product exhibits no deterioration.
- t_1 is the length of time in which there is no inventory shortage.
- Q is the order quantity.
- S is the initial inventory level.
- T is the length of the replenishment cycle time.
- P^* is the optimal selling price per unit time.
- T^* is the optimal length of the replenishment cycle time.
- $q_1(t)$ is the inventory level at time $t \in [0, \mu]$.
- $q_2(t)$ is the inventory level at time $t \in [\mu, t_1]$.
- $q_3(t)$ is the inventory level at time $t \in [t_1, T]$.
- $\pi(P, T)$ is the total profit per unit time.
- $\pi(P^*, T^*)$ is the optimal total profit per unit time of the inventory system.
- M is the length of trade credit period per year.
- I_e is the interest earned per rupee per year.
- I_p is the interest paid per rupee per year.
- Selling price P follows an increasing trend and demand rate posses the negative derivative throughout its domain and demand rate is $f(t) = (a-p) > 0$.
- During the trade credit period, M, the account is not settled; generated sales revenue is deposited in an interest-bearing account. At the end of the period, the retailer pays off all units bought, and starts to pay the capital opportunity cost for the items in stock.

3. DEVELOPMENT OF THE MODEL

Here, the inventory model as follows: S units of item arrive at the inventory system at the beginning of each cycle. During the time interval $(0, \mu)$, the inventory level decreases due to demand only. Afterwards, during the time interval (μ, t_1), the inventory level drops to zero due to both demand and deterioration. Finally, a shortage occurs due to demand and partial backlogging during the time interval (t_1, T). (See Figure: 1).



(Figure-1)

The equation representing the inventory status in the system for the first interval is derived as follows:
During the time interval $(0, \mu)$, the differential equation representing the inventory status is given by

$$\frac{dq_1(t)}{dt} = -(a-p), \qquad (0 \le t \le \mu) \tag{1}$$

With the boundary condition $q_1(0) = S$, solving Eq. (1) yields

$$q_1(t) = (a-p)\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left\{(t_1-\gamma)^{\beta+1} - (\mu-\gamma)^{\beta+1}\right\} + \frac{\alpha^2}{2(2\beta+1)}\left\{(t_1-\gamma)^{2\beta+1} - (\mu-\gamma)^{2\beta+1}\right\}\right] \tag{2}$$

In the second interval (μ, t_1), the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status:

$$\frac{dq_2(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1} q_2(t) = -(a-p), \qquad (\mu \le t \le t_1) \tag{3}$$

With the condition, $q_2(t_1) = 0$, the solution of Eq. (3) is

$$q_2(t) = (a-p)\left[(t_1 - t) + \frac{\alpha}{\beta+1}\left\{(t_1-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}\right\} + \frac{\alpha^2}{2(2\beta+1)}\left\{(t_1-\gamma)^{2\beta+1} - (t-\gamma)^{2\beta+1}\right\}\right] \tag{4}$$

It is clear from Figure-1 that $q_1(\mu) = q_2(\mu)$, therefore, the maximum inventory level (S) can be obtained

$$S = (a-p)\left[t_1 + \frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1} - (\mu-\gamma)^{\beta+1}\} + \frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1} - (\mu-\gamma)^{2\beta+1}\}\right] \tag{5}$$

In third interval (t_1, T), shortage is partially backlogged. Therefore, the inventory level t is obtained by the following equation:

$$\frac{dq_3(t)}{dt} = -(a-p)\ell^{-\lambda(T-t)}, \qquad (t_1 \le t \le T) \tag{6}$$

The solution of the above differential equation, after applying the initial value condition $q_3(t_1)=0$, is

$$q_3(t) = (a-p)\left[(1-\lambda T)(t_1-t) + \frac{\lambda}{2}(t_1^2-t^2) - \frac{\lambda^2}{2}\left(T^2(t_1-t) + \frac{1}{3}(t_1^3-t^3) - T(t_1^2-t^2)\right)\right] \tag{7}$$

If we put $t=T$ into $q_3(t)$, the maximum backordered inventory BI will be obtained:

$$BI = -q_3(T) = -(a-p)\left[(t_1-T)(1-\lambda T) + \frac{\lambda}{2}(t_1^2-T^2) - \frac{\lambda^2}{2}\{T^2(t_1-T) + \frac{1}{3}(t_1^3-T^3) - T(t_1^2-T^2)\}\right] \tag{8}$$

The order quantity per cycle (Q) is sum of S and BI i.e

$$Q = (a-p)\left[\left\{t_1 + \frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1} - (\mu-\gamma)^{\beta+1}\} + \frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1} - (\mu-\gamma)^{2\beta+1}\}\right\}\right.$$
$$\left. -(t_1-T)(1-\lambda T) + \frac{\lambda}{2}(t_1^{\,2}-T^2) - \frac{\lambda^2}{2}\{T^2(t_1-T) + \frac{1}{3}(t_1^{\,3}-T^3) - T(t_1^{\,2}-T^2)\}\right] \tag{9}$$

It is obvious that the element comprising the profit function per cycle of the retailer is listed below:

1. Ordering cost is A
2. Inventory holding cost (denoted by HC) is

$$HC = v_1\left[\int_0^{\mu} q_1(t)dt + \int_{\mu}^{t_1} q_2(t)dt\right] = v_1(a-p)\left[\frac{t_1^{\,2}}{2} + \frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}t_1 - (\mu-\gamma)^{\beta+1}\mu\} + \right.$$
$$\left.\frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1}t_1 - (\mu-\gamma)^{2\beta+1}\} + \frac{\alpha}{(\beta+1)(\beta+2)}\{-(t_1-\gamma)^{\beta+2} - (\mu-\gamma)^{\beta+2}\}\right] \tag{10}$$

3. Purchase cost (denoted by PC) is

$$PC = C*Q = C*(a-p)\left\{t_1 + \frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1} - (\mu-\gamma)^{\beta+1}\} + \frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1}\right.$$
$$\left. -(\mu-\gamma)^{2\beta+1}\right\} - (t_1-T)(1-\lambda T) + \frac{\lambda}{2}(t_1^{\,2}-T^2) - \frac{\lambda^2}{2}\{T^2(t_1-T) + \frac{1}{3}(t_1^{\,3}-T^3) - T(t_1^{\,2}-T^2)\}\Big] \tag{11}$$

4. Opportunity cost due to lost sales (denoted by OC) is

$$OP = v_3(a-p)\int_{t_1}^{T}\left[1-\ell^{-\lambda(T-t)}\right]dt = v_3(a-p)\left[-\lambda T(T-t_1) + \lambda\left(\frac{T^2}{2} - \frac{t_1^{\,2}}{2}\right)\right.$$
$$\left. + \frac{\lambda^2}{2}\{T^2(T-t_1) + \left(\frac{T^3}{2} - \frac{t_1^{\,3}}{2}\right) - 2T\left(\frac{T^2}{2} - \frac{t_1^{\,2}}{2}\right)\}\right] \tag{12}$$

5. Sales revenue (denoted by SR) is

$$SR = p\left[\int_0^{t_1}(a-p)dt - q_3(T)\right] = p\left[(a-p)t_1 - (a-p)\{(t_1-T)(1-\lambda T) + \frac{\lambda}{2}(t_1^{\,2}-T^2)\right.$$

$$-\frac{\lambda^2}{2}\{T^2(t_1-T)+\frac{1}{3}(t_1^{\ 3}-T^3)-T(t_1^{\ 2}-T^2)\}\}\Big] \tag{13}$$

6. Shortage cost due to backlog (denoted by SC) is

$$SC=-v_2\int_{t_1}^{T}q_3(t)dt=v_2(a-p)\Big[(t_1^{\ 2}-t_1T)(1-\lambda T)-\frac{\lambda}{2}(Tt_1^{\ 2}-t_1^{\ 3})-\frac{1}{3}(T^3-t_1^{\ 3})$$
$$+\frac{\lambda^2}{2}(T^3t_1-T^2t_1-T^2+t_1)+\frac{2t_1^{\ 3}T}{3}-\frac{t_1^{\ 4}}{4}+\frac{T^4}{4}-T^2t_1^{\ 2}+Tt_1^{\ 3}\Big] \tag{14}$$

7. According to above assumptions, there are three cases to occur in interest payable in each order cycle:

Case 1: $0<M\le\mu$**.** In this case, the trade credit period M occurs before deteriorating time of the item μ. The payment for items is settled and the retailer starts paying the interest charged for all unsold items in inventory with rate I_p. (See Figure-2)

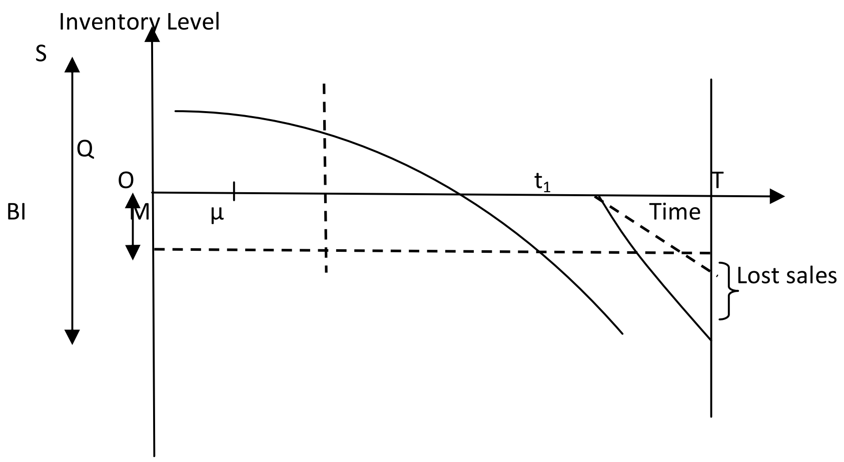


Figure-2

The present value of the interest payable during the one replenishment cycle is given by

$$IP_1=cI_p\int_{M}^{t_1}q(t)dt=cI_p\left[\int_{M}^{\mu}q_1(t)dt+\int_{\mu}^{t_1}q_2(t)dt\right]=cI_p\Big[\frac{t_1^2}{2}+\frac{M^2}{4}-t_1M+\frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}M-(\mu-\gamma)^{\beta+1}\mu$$
$$+(\mu-\gamma)^{\beta+1}M+(t_1-\gamma)^{\beta+1}t_1\}+\frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1}M-(\mu-\gamma)^{2\beta+1}\mu+(\mu-\gamma)^{2\beta+1}M+(t_1-\gamma)^{2\beta+1}t_1\}$$
$$-\frac{\alpha}{(\beta+1)(\beta+2)}\{(t_1-\gamma)^{\beta+2}-(\mu-\gamma)^{\beta+2}\}-\frac{\alpha^2}{2(2\beta+1)(2\beta+2)}\{(t_1-\gamma)^{2\beta+2}-(\mu-\gamma)^{2\beta+2}\}\Big] \tag{15}$$

Case 2 : $\mu<M\le t_1$**.** In this case, the trade credit period M occurs after deteriorating time and before the length of time in which there is no inventory shortage. The condition of this case is similar to those for Case 1. (See Figure-3) Thus, the present value of the interest payable during the one replenishment cycle is given below:

$$IP_2 = cI_p \int_M^{t_1} q_2(t)dt = cI_p (a-p)\left[t_1(t_1 - M) - (\frac{t_1^{\,2}}{2} - \frac{M^2}{2}) + \frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}(t_1 - M) - \frac{(t_1-\gamma)^{\beta+2}}{\beta+2} + \frac{(M-\gamma)^{\beta+2}}{\beta+2} + \frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1}(t_1-M) - \frac{(t_1-\gamma)^{2\beta+2}}{2\beta+2} + \frac{(M-\gamma)^{2\beta+2}}{2\beta+2}\}\right] \quad (16)$$

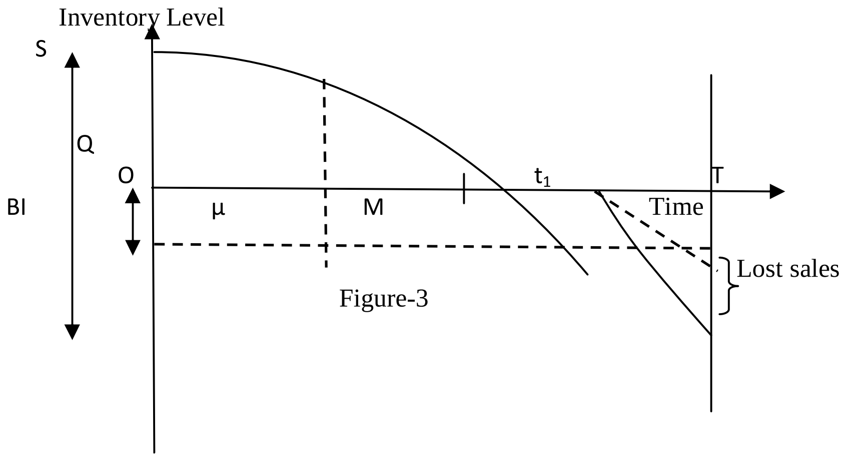


Figure-3

Case 3 : $t_1 < M \le T$. In this case, the trade credit period (M) is more than period with positive inventory (t_1) (See Figure-4). So, that interest paid during the time period $(0,T)$ is equal to zero because the supplier can be paid in full at the trade credit period (M). The interest earned during the time period $(0,t_1)$ plus interest earned from the cash invested during the time period (t_1, M) after the inventory is exhausted. So, there is no opportunity cost. Therefore, $IP_3 = 0$. (See Figure-4) (17)

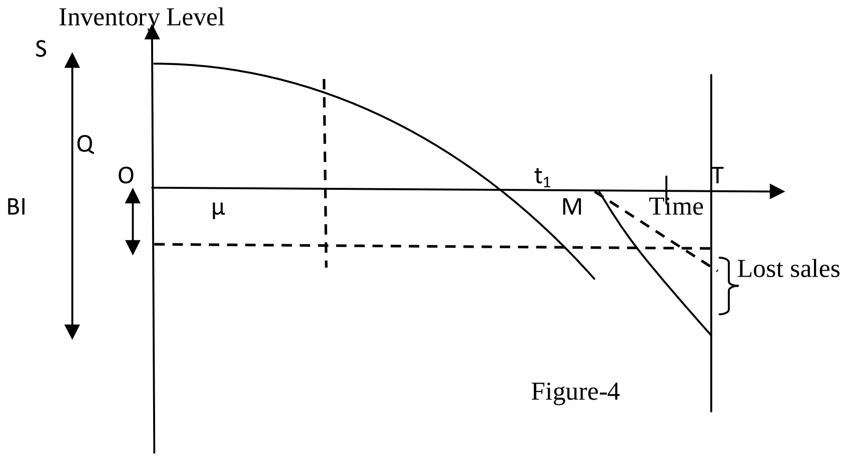


Figure-4

8. Similarly, there exists three cases to occur in interest earned in each order cycle.

Case 1: $0 < M \le \mu$. During the delay period, when the account is not settled, the retailer sales the goods and continues to accumulate sales revenue and earns the interest with rate I_e . Therefore, the present value of the interest earned during the replenishment cycle is

$$IE_1 = pI_e\int_0^M D(t)t dt = pI_e\alpha\beta\left[\frac{(M-\gamma)^{\beta}(\beta M+\gamma)}{\beta(\beta+1)}\right] \tag{18}$$

Case 2: $\mu < M \le t_1$. The condition of this case is similar to those for Case 1. Thus, the present value of the interest payable during the replenishment cycle is given below:

$$IE_2 = pI_e\int_0^M D(t)t dt = pI_e\alpha\beta\left[\frac{(M-\gamma)^{\beta}(\beta M+\gamma)}{\beta(\beta+1)}\right] \tag{19}$$

Case 3: $t_1 < M \le T$. In this case, the retailer earns interest on the revenue of sales up to the delay period and no interest is payable during the period for the item in stock. Thus, the present value of the interest payable during the replenishment cycle is given by

$$IE_3 = pI_e\left[\int_0^{t_1} D(t)t dt + (M-t_1)\int_0^{t_1} D(t)dt\right]$$

$$= pI_e\alpha\beta\left[\frac{(t_1-\gamma)^{\beta}(\beta t_1+\gamma)}{\beta(\beta+1)} + (M-t_1)\frac{(t_1-\gamma)^{\beta}}{\beta}\right] \tag{20}$$

From the above argument, the total average profit per unit time for the retailer can be expressed as

$$\pi(T) = \frac{1}{T}\left\{\begin{array}{l}\text{sales revenue - ordering cost - holding cost - shortage cost - purchase cost}\\ \text{- opportunity cost - interest payable + interest earned}\end{array}\right\}$$, i.e $\pi(T)$ =

$$\begin{cases}\pi_1(P, t_1, T), & 0 < M \le \mu,\\ \pi_2(P, t_1, T), & \mu < M \le t_1,\\ \pi_3(P, t_1, T), & t_1 < M \le T,\end{cases} \tag{21}$$

On simplification, we get

$$\pi_1(p,t_1,T) = p(a-p) - \frac{1}{T}\Bigg[p\Bigg\{(a-p)t_1 - (a-p)\{(t_1-T)(1-\lambda T) + \frac{\lambda}{2}(t_1^2-T^2) - \frac{\lambda^2}{2}\{T^2(t_1-T) +$$

$$\frac{1}{3}(t_1^{\,3}-T^3) - T(t_1^{\,2}-T^2)\}\Bigg\} - A - v_1(a-p)\Bigg\{\frac{t_1^{\,2}}{2} + \frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}t_1 - (\mu-\gamma)^{\beta+1}\mu\} + \frac{\alpha^2}{2(2\beta+1)}$$

$$\{(t_1-\gamma)^{2\beta+1}t_1 - (\mu-\gamma)^{2\beta+1}\} + \frac{\alpha}{(\beta+1)(\beta+2)}\{-(t_1-\gamma)^{\beta+2} - (\mu-\gamma)^{\beta+2}\}\Bigg\} - v_2(a-p)\Big\{(t_1^{\,2} - t_1 T)$$

$$(1-\lambda T) - \frac{\lambda}{2}(Tt_1^{\,2} - t_1^{\,3}) - \frac{1}{3}(T^3 - t_1^{\,3}) + \frac{\lambda^2}{2}(T^3 t_1 - T^2 t_1 - T^2 + t_1) + \frac{2t_1^{\,3}T}{3} - \frac{t_1^{\,4}}{4} + \frac{T^4}{4} - T^2 t_1^{\,2} + Tt_1^{\,3}\Big\}$$

$$-C*(a-p)\Bigg\{\Bigg\{t_1 + \frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1} - (\mu-\gamma)^{\beta+1}\} + \frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1} - (\mu-\gamma)^{2\beta+1}\}\Bigg\} - (t_1 - T)$$

$$(1-\lambda T)+\frac{\lambda}{2}(t_1{}^2-T^2)-\frac{\lambda^2}{2}\{T^2(t_1-T)+\frac{1}{3}(t_1{}^3-T^3)-T(t_1{}^2-T^2)\}\Big\}-v_3(a-p)\Big\{\lambda T(T-t_1)$$

$$\lambda(\frac{T^2}{2}-\frac{t_1{}^2}{2})+\frac{\lambda^2}{2}\{T^2(T-t_1)+(\frac{T^3}{2}-\frac{t_1{}^3}{2})-2T(\frac{T^2}{2}-\frac{t_1{}^2}{2})\}\Big\}-cI_p\Big\{\frac{t_1{}^2}{2}+\frac{M^2}{4}-t_1M+\frac{\alpha}{\beta+1}$$

$$\{(t_1-\gamma)^{\beta+1}M-(\mu-\gamma)^{\beta+1}\mu+(\mu-\gamma)^{\beta+1}M+(t_1-\gamma)^{\beta+1}t_1\}+\frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1}M$$

$$-(\mu-\gamma)^{2\beta+1}\mu+(\mu-\gamma)^{2\beta+1}M+(t_1-\gamma)^{2\beta+1}t_1\}-\frac{\alpha}{(\beta+1)(\beta+2)}\{(t_1-\gamma)^{\beta+2}-(\mu-\gamma)^{\beta+2}\}$$

$$-\frac{\alpha^2}{2(2\beta+1)(2\beta+2)}\{(t_1-\gamma)^{2\beta+2}-(\mu-\gamma)^{2\beta+2}\}\Big\}+pI_e\alpha\beta\left\{\frac{(M-\gamma)^{\beta}(\beta M+\gamma)}{\beta(\beta+1)}\right\}\Bigg]\ ,$$

$$\pi_2(p,t_1,T)=p(a-p)-\frac{1}{T}\Bigg[p\Big\{(a-p)t_1-(a-p)\{(t_1-T)(1-\lambda T)+\frac{\lambda}{2}(t_1{}^2-T^2)-\frac{\lambda^2}{2}\{T^2(t_1-T)+$$

$$\frac{1}{3}(t_1{}^3-T^3)-T(t_1{}^2-T^2)\}\Big\}-A-v_1(a-p)\Big\{\frac{t_1{}^2}{2}+\frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}t_1-(\mu-\gamma)^{\beta+1}\mu\}+\frac{\alpha^2}{2(2\beta+1)}$$

$$\{(t_1-\gamma)^{2\beta+1}t_1-(\mu-\gamma)^{2\beta+1}\}+\frac{\alpha}{(\beta+1)(\beta+2)}\{-(t_1-\gamma)^{\beta+2}-(\mu-\gamma)^{\beta+2}\}\Big\}-v_2(a-p)\Big\{(t_1{}^2-t_1T)$$

$$(1-\lambda T)-\frac{\lambda}{2}(Tt_1{}^2-t_1{}^3)-\frac{1}{3}(T^3-t_1{}^3)+\frac{\lambda^2}{2}(T^3t_1-T^2t_1-T^2+t_1)+\frac{2t_1{}^3T}{3}-\frac{t_1{}^4}{4}+\frac{T^4}{4}-T^2t_1{}^2+Tt_1{}^3\Big\}$$

$$-C*(a-p)\Big\{\Big\{t_1+\frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}-(\mu-\gamma)^{\beta+1}\}+\frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1}-(\mu-\gamma)^{2\beta+1}\}\Big\}-(t_1-T)$$

$$(1-\lambda T)+\frac{\lambda}{2}(t_1{}^2-T^2)-\frac{\lambda^2}{2}\{T^2(t_1-T)+\frac{1}{3}(t_1{}^3-T^3)-T(t_1{}^2-T^2)\}\Big\}-v_3(a-p)\Big\{\lambda T(T-t_1)$$

$$\lambda(\frac{T^2}{2}-\frac{t_1{}^2}{2})+\frac{\lambda^2}{2}\{T^2(T-t_1)+(\frac{T^3}{2}-\frac{t_1{}^3}{2})-2T(\frac{T^2}{2}-\frac{t_1{}^2}{2})\}\Big\}-cI_p(a-p)\Big\{t_1(t_1-M)-(\frac{t_1{}^2}{2}-\frac{M^2}{2})+$$

$$\frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}(t_1-M)-\frac{(t_1-\gamma)^{\beta+2}}{\beta+2}+\frac{(M-\gamma)^{\beta+2}}{\beta+2}+\frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1}(t_1-M)-\frac{(t_1-\gamma)^{2\beta+2}}{2\beta+2}$$

$$+\frac{(M-\gamma)^{2\beta+2}}{2\beta+2}\}\Big\}+pI_e\alpha\beta\left\{\frac{(M-\gamma)^{\beta}(\beta M+\gamma)}{\beta(\beta+1)}\right\}\Bigg]$$

and

$$\pi_3(p,t_1,T)=p(a-p)-\frac{1}{T}\Bigg[p\Big\{(a-p)t_1-(a-p)\{(t_1-T)(1-\lambda T)+\frac{\lambda}{2}(t_1{}^2-T^2)-\frac{\lambda^2}{2}\{T^2(t_1-T)+$$

$$\frac{1}{3}(t_1{}^3-T^3)-T(t_1{}^2-T^2)\}\Bigg\}-A-v_1(a-p)\Bigg\{\frac{t_1{}^2}{2}+\frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}t_1-(\mu-\gamma)^{\beta+1}\mu\}+\frac{\alpha^2}{2(2\beta+1)}$$

$$\{(t_1-\gamma)^{2\beta+1}t_1-(\mu-\gamma)^{2\beta+1}\}+\frac{\alpha}{(\beta+1)(\beta+2)}\{-(t_1-\gamma)^{\beta+2}-(\mu-\gamma)^{\beta+2}\}\Bigg\}-v_2(a-p)\Big\{(t_1{}^2-t_1T)$$

$$(1-\lambda T)-\frac{\lambda}{2}(Tt_1{}^2-t_1{}^3)-\frac{1}{3}(T^3-t_1{}^3)+\frac{\lambda^2}{2}(T^3t_1-T^2t_1-T^2+t_1)+\frac{2t_1{}^3T}{3}-\frac{t_1{}^4}{4}+\frac{T^4}{4}-T^2t_1{}^2+Tt_1{}^3\Bigg\}$$

$$-C*(a-p)\Bigg\{\Bigg\{t_1+\frac{\alpha}{\beta+1}\{(t_1-\gamma)^{\beta+1}-(\mu-\gamma)^{\beta+1}\}+\frac{\alpha^2}{2(2\beta+1)}\{(t_1-\gamma)^{2\beta+1}-(\mu-\gamma)^{2\beta+1}\}\Bigg\}-(t_1-T)$$

$$(1-\lambda T)+\frac{\lambda}{2}(t_1{}^2-T^2)-\frac{\lambda^2}{2}\{T^2(t_1-T)+\frac{1}{3}(t_1{}^3-T^3)-T(t_1{}^2-T^2)\}\Bigg\}-v_3(a-p)\Big\{\lambda T(T-t_1)$$

$$\lambda(\frac{T^2}{2}-\frac{t_1{}^2}{2})+\frac{\lambda^2}{2}\{T^2(T-t_1)+(\frac{T^3}{2}-\frac{t_1{}^3}{2})-2T(\frac{T^2}{2}-\frac{t_1{}^2}{2})\}\Bigg\}+pI_e\alpha\beta\Bigg\{\frac{(t_1-\gamma)^{\beta}(\beta t_1+\gamma)}{\beta(\beta+1)}$$

$$(M-t_1)\frac{(t_1-\gamma)^{\beta}}{\beta}\Bigg\}\Bigg]$$

Let $t_1=vT$, $0<v<1$

Hence, we have the profit functions

$$\pi_1(p,T)=p(a-p)-\frac{1}{T}\Bigg[p\Bigg\{(a-p)vT-(a-p)\{(vT-T)(1-\lambda T)+\frac{\lambda}{2}(v^2T^2-T^2)-\frac{\lambda^2}{2}\{T^2(vT-T)+$$

$$\frac{1}{3}(v^3T^3-T^3)-T(v^2T^2-T^2)\}\Bigg\}-A-v_1(a-p)\Bigg\{\frac{v^4T^4}{2}+\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}vT-(\mu-\gamma)^{\beta+1}\mu\}+\frac{\alpha^2}{2(2\beta+1)}$$

$$\frac{1}{3}(v^3T^3-T^3)-T(v^2T^2-T^2)\}\Bigg\}-A-v_1(a-p)\Bigg\{\frac{v^4T^4}{2}+\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}vT-(\mu-\gamma)^{\beta+1}\mu\}+\frac{\alpha^2}{2(2\beta+1)}$$

$$\{(vT-\gamma)^{2\beta+1}vT-(\mu-\gamma)^{2\beta+1}\}+\frac{\alpha}{(\beta+1)(\beta+2)}\{-(vT-\gamma)^{\beta+2}-(\mu-\gamma)^{\beta+2}\}\Bigg\}-v_2(a-p)\Big\{(v^4T^4-vT^2)$$

$$(1-\lambda T)-\frac{\lambda}{2}(v^4T^5-v^3T^3)-\frac{1}{3}(T^3-v^3T^3)+\frac{\lambda^2}{2}(vT^4-vT^3-T^2+vT)+\frac{2v^3T^4}{3}-\frac{v^4T^4}{4}+\frac{T^4}{4}-T^2v^2T^2+Tv^3T^3\Bigg\}$$

$$-C*(a-p)\Bigg\{\Bigg\{vT+\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}-(\mu-\gamma)^{\beta+1}\}+\frac{\alpha^2}{2(2\beta+1)}\{(vT-\gamma)^{2\beta+1}-(\mu-\gamma)^{2\beta+1}\}\Bigg\}-(vT-T)$$

$$(1-\lambda T)+\frac{\lambda}{2}(v^2T^2-T^2)-\frac{\lambda^2}{2}\{T^2(vT-T)+\frac{1}{3}(v^3T^3-T^3)-T(v^2T^2-T^2)\}\Bigg\}-v_3(a-p)\Big\{\lambda T(T-vT)$$

$$\lambda(\frac{T^2}{2}-\frac{v^2T^2}{2})+\frac{\lambda^2}{2}\{T^3(1-v)+(\frac{T^3}{2}-\frac{v^3T^3}{2})-2T(\frac{T^2}{2}-\frac{v^2T^2}{2})\}\Bigg\}-cI_p\Bigg\{\frac{v^2T^2}{2}+\frac{M^2}{4}-vTM+\frac{\alpha}{\beta+1}$$

$$\{(vT-\gamma)^{\beta+1}M-(\mu-\gamma)^{\beta+1}\mu+(\mu-\gamma)^{\beta+1}M+(vT-\gamma)^{\beta+1}vT\}+\frac{\alpha^2}{2(2\beta+1)}\{(vT-\gamma)^{2\beta+1}M-(\mu-\gamma)^{2\beta+1}\mu+$$

$$(\mu-\gamma)^{2\beta+1}M+(vT-\gamma)^{2\beta+1}vT\}-\frac{\alpha}{(\beta+1)(\beta+2)}\{(vT-\gamma)^{\beta+2}-(\mu-\gamma)^{\beta+2}\}-\frac{\alpha^2}{2(2\beta+1)(2\beta+2)}\{(vT-\gamma)^{2\beta+2}$$

$$-(\mu-\gamma)^{2\beta+2}\}\Big\}+pI_e\alpha\beta\left\{\frac{(M-\gamma)^\beta(\beta M+\gamma)}{\beta(\beta+1)}\right\}\Bigg],$$

$$\pi_2(p,T)=p(a-p)-\frac{1}{T}\Bigg[p\Big\{(a-p)vT-(a-p)\{(vT-T)(1-\lambda T)+\frac{\lambda}{2}(v^2T^2-T^2)-\frac{\lambda^2}{2}\{T^2(vT-T)+$$

$$\frac{1}{3}(v^3T^3-T^3)-T(v^2T^2-T^2)\}\Big\}-A-v_1(a-p)\Big\{\frac{v^4T^4}{2}+\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}vT-(\mu-\gamma)^{\beta+1}\mu\}+\frac{\alpha^2}{2(2\beta+1)}$$

$$\frac{1}{3}(v^3T^3-T^3)-T(v^2T^2-T^2)\}\Big\}-A-v_1(a-p)\Big\{\frac{v^4T^4}{2}+\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}vT-(\mu-\gamma)^{\beta+1}\mu\}+\frac{\alpha^2}{2(2\beta+1)}$$

$$\{(vT-\gamma)^{2\beta+1}vT-(\mu-\gamma)^{2\beta+1}\}+\frac{\alpha}{(\beta+1)(\beta+2)}\{-(vT-\gamma)^{\beta+2}-(\mu-\gamma)^{\beta+2}\}\Big\}-v_2(a-p)\Big\{(v^4T^4-vT^2)$$

$$(1-\lambda T)-\frac{\lambda}{2}(v^4T^5-v^3T^3)-\frac{1}{3}(T^3-v^3T^3)+\frac{\lambda^2}{2}(vT^4-vT^3-T^2+vT)+\frac{2v^3T^4}{3}-\frac{v^4T^4}{4}+\frac{T^4}{4}-T^2v^2T^2+Tv^3T^3\Big\}$$

$$-C*(a-p)\Big\{\Big\{vT+\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}-(\mu-\gamma)^{\beta+1}\}+\frac{\alpha^2}{2(2\beta+1)}\{(vT-\gamma)^{2\beta+1}-(\mu-\gamma)^{2\beta+1}\}\Big\}-(vT-T)$$

$$(1-\lambda T)+\frac{\lambda}{2}(v^2T^2-T^2)-\frac{\lambda^2}{2}\{T^2(vT-T)+\frac{1}{3}(v^3T^3-T^3)-T(v^2T^2-T^2)\}\Big\}-v_3(a-p)\Big\{\lambda T(T-vT)$$

$$\lambda(\frac{T^2}{2}-\frac{v^2T^2}{2})+\frac{\lambda^2}{2}\{T^3(1-v)+(\frac{T^3}{2}-\frac{v^3T^3}{2})-2T(\frac{T^2}{2}-\frac{v^2T^2}{2})\}\Big\}-cI_p(a-p)\Big\{vT(vT-M)-(\frac{v^2T^2}{2}-\frac{M^2}{2})+$$

$$\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}(vT-M)-\frac{(vT-\gamma)^{\beta+2}}{\beta+2}+\frac{(M-\gamma)^{\beta+2}}{\beta+2}+\frac{\alpha^2}{2(2\beta+1)}\{(vT-\gamma)^{2\beta+1}(vT-M)-\frac{(vT-\gamma)^{2\beta+2}}{2\beta+2}$$

$$+\frac{(M-\gamma)^{2\beta+2}}{2\beta+2}\}\Big\}+pI_e\alpha\beta\left\{\frac{(M-\gamma)^\beta(\beta M+\gamma)}{\beta(\beta+1)}\right\}\Bigg]$$

and $\pi_3(p,T)=p(a-p)-\frac{1}{T}\Bigg[p\Big\{(a-p)vT-(a-p)\{(vT-T)(1-\lambda T)+\frac{\lambda}{2}(v^2T^2-T^2)-\frac{\lambda^2}{2}\{T^2(vT-T)+$

$$\frac{1}{3}(v^3T^3-T^3)-T(v^2T^2-T^2)\}\Big\}-A-v_1(a-p)\Big\{\frac{v^4T^4}{2}+\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}vT-(\mu-\gamma)^{\beta+1}\mu\}+\frac{\alpha^2}{2(2\beta+1)}$$

$$\{(vT-\gamma)^{2\beta+1}vT-(\mu-\gamma)^{2\beta+1}\}+\frac{\alpha}{(\beta+1)(\beta+2)}\{-(vT-\gamma)^{\beta+2}-(\mu-\gamma)^{\beta+2}\}\Big\}-v_2(a-p)\Big\{(v^4T^4-vT^2)$$

$$(1-\lambda T)-\frac{\lambda}{2}(v^4T^5-v^3T^3)-\frac{1}{3}(T^3-v^3T^3)+\frac{\lambda^2}{2}(vT^4-vT^3-T^2+vT)+\frac{2v^3T^4}{3}-\frac{v^4T^4}{4}+\frac{T^4}{4}-T^2v^2T^2+Tv^3T^3\Big\}$$

$$-C*(a-p)\Big\{\Big\{vT+\frac{\alpha}{\beta+1}\{(vT-\gamma)^{\beta+1}-(\mu-\gamma)^{\beta+1}\}+\frac{\alpha^2}{2(2\beta+1)}\{(vT-\gamma)^{2\beta+1}-(\mu-\gamma)^{2\beta+1}\}\Big\}-(vT-T)$$

$$(1-\lambda T)+\frac{\lambda}{2}(v^2T^2-T^2)-\frac{\lambda^2}{2}\{T^2(vT-T)+\frac{1}{3}(v^3T^3-T^3)-T(v^2T^2-T^2)\}\Big\}-v_3(a-p)\Big\{\lambda T(T-vT)$$

$$\lambda(\frac{T^2}{2}-\frac{v^2T^2}{2})+\frac{\lambda^2}{2}\{T^3(1-v)+(\frac{T^3}{2}-\frac{v^3T^3}{2})-2T(\frac{T^2}{2}-\frac{v^2T^2}{2})\}\Bigg\}+pI_e\alpha\beta\Bigg\{\frac{(vT-\gamma)^\beta(\beta vT+\gamma)}{\beta(\beta+1)}$$

$$(M-vT)\frac{(vT-\gamma)^\beta}{\beta}\Bigg\}\Bigg]$$

4. SOLUTION PROCEDURE

In order to find the optimal solutions of P^* and T^* so as to maximize the total profit function, the first and second order derivatives of $\pi_i(P\,,\,T)$ with respect to P and T are taken, where $i=\{1\,,2\,,3\}$. In other words, the necessary and sufficient conditions for maximization of $\pi_i(P\,,\mathrm{T})$ are respectively $\frac{d\pi_i(P\,,\,T)}{dP}=0$ *and* $\frac{d\pi_i(P\,,\,T)}{dT}=0$.

and

$$\frac{d^2\pi_i(P\,,\,T)}{dP^2}<0\,,\ \frac{d^2\pi_i(P\,,\,T)}{dT^2}<0 \text{ and } \left(\frac{d^2\pi_i(P\,,\,T)}{dP\,dT}=0\right)^2-\left(\frac{d^2\pi_i(P\,,\,T)}{dP^2}\right)\left(\frac{d^2\pi_i(P\,,\,T)}{dT^2}\right)<0\cdot$$

The solution can be very complicated and hence solutions for the optimal problem with maximum constraints can be exhusbitantly expensive in computations. So, it is solved using Mathematica 5.1 software.

Case 1: $0<M\leq\mu$ **.**

The necessary conditions for the total profit $\pi_1(P\,,\mathrm{T})$ to be maximum is $\frac{d\pi_1(P\,,\mathrm{T})}{dP}=0$,

$\frac{d\pi_1(P\,,\mathrm{T})}{dT}=0$. The sufficient conditions for the total profit to be maximum is

$$\frac{d^2\pi_1(P\,,\,T)}{dP^2}<0\,,\ \frac{d^2\pi_1(P\,,\,T)}{dT^2}<0 \text{ and } \left(\frac{d^2\pi_1(P\,,\,T)}{dP\,dT}\right)^2-\left(\frac{d^2\pi_1(P\,,\,T)}{dP^2}\right)\left(\frac{d^2\pi_1(P\,,\,T)}{dT^2}\right)<0\,.$$

Case 2 : $\mu<M\leq t_1$.

Similarly for the total profit $\pi_2(P\,,\mathrm{T})$ to be maximum is $\frac{d\pi_2(P\,,\,T)}{dP}=0$, $\frac{d\pi_2(P\,,\,T)}{dT}=0$.

The sufficient conditions for the total profit to be maximum is

$$\frac{d^2\pi_2(P\,,\,T)}{dP^2}<0\,,\ \frac{d^2\pi_2(P\,,\,T)}{dT^2}<0 \text{ and } \left(\frac{d^2\pi_2(P\,,\,T)}{dP\,dT}\right)^2-\left(\frac{d^2\pi_2(P\,,\,T)}{dP^2}\right)\left(\frac{d^2\pi_2(P\,,\,T)}{dT^2}\right)<0\cdot$$

Case 3: $t_1<M\leq T$.

The necessary conditions for the total profit $\pi_3(P\,,\,T)$ to be maximum is

$$\frac{d\pi_3(P\,,\,T)}{dP}=0\,,\ \frac{d\pi_3(P\,,\,T)}{dT}=0$$

The sufficient conditions for the total profit to be maximum is

$$\frac{d^2\pi_3(P,\mathrm{T})}{d\mathrm{P}^2}<0,\ \frac{d^2\pi_3(P,\mathrm{T})}{d\mathrm{T}^2}<0 \text{ and } \left(\frac{d^2\pi_3(P,\mathrm{T})}{dP\,d\mathrm{T}}\right)^2-\left(\frac{d^2\pi_3(P,\mathrm{T})}{d\mathrm{P}^2}\right)\left(\frac{d^2\pi_3(P,\mathrm{T})}{d\mathrm{T}^2}\right)<0.$$

5. NUMERICAL ILLUSTRATIONS

In this section, numerical examples to illustrate the models obtained in the presented paper have been provided. The sensitivity analysis of major parameters on the optimal solution are also carried out. The denoising performances of numerical experiments are compared.

Example 1:

$A=100$ units, $\mathrm{a}=132$, $\mathrm{v}_1=5$, $\mathrm{v}_2=4$, $v_3=6$, $\alpha=0.4$, $\beta=2$, $\gamma=0.2$, $\lambda=0.3$, $\mathrm{v}=0.2$, $\mathrm{M}=0.5$ per year, $\mu=0.4$, $\mathrm{I_e}=0.15$ per year, $I_p=0.10$ per year, $p=8, c=5$.

$p^*=1.3447$, $T^*=5.5482$, Total Profit (TP)=20,688.0, Order Quantity(Q) =182.45

Example 2:

$A=100$ units, $\mathrm{a}=132$, $\mathrm{v}_1=5$, $\mathrm{v}_2=4$, $v_3=6$, $\alpha=0.4$, $\beta=2$, $\gamma=0.2$, $\lambda=0.3$, $\mathrm{v}=0.2$, $\mathrm{M}=3$ per year, $\mu=0.4$, $\mathrm{I_e}=0.15$ per year, $I_p=0.10$ per year, $p=8, c=5$.

$p^*=0.8192$, $T^*=5.5584$, Total Profit (TP) = 20,894.3, Order Quantity (Q) = 181.47

Example 3:

	% Change	P	T	TP	Q
	-10	1.3250	5.5482	18598.6	164.09
A	-5	1.3550	5.5485	19621.6	172.38
	0	1.3447	5.5482	20688.0	182.78
	+5	4.4237	5.5487	21593.5	186.73
	+10	7.9446	5.5432	22639.0	191.66
	-10	1.3150	5.5492	20686.0	180.95
	-5	1.3296	5.5485	20687.5	180.96
α	0	1.3447	5.5482	20688.0	182.78
	+5	1.3592	5.5477	20688.7	182.48
	+10	1.3741	5.5476	20689.2	183.33
	-10	1.3337	5.5483	20792.4	183.01
	-5	1.3395	5.5487	20790.2	182.33
β	0	1.3447	5.5482	20688.0	182.78
	+5	1.3492	5.5465	20788.3	181.10
	+10	1.3532	5.5431	20787.5	180.54
	-10	1.3532	5.5478	20786.9	182.71
	-5	1.3489	5.5408	20699.9	182.85
γ	0	1.3447	5.5482	20688.0	182.78
	+5	1.3404	5.5485	20594.1	181.12
	+10	1.3362	5.5487	20496.1	180.72
	-10	7.2539	5.4231	18180.0	248.71
	-5	4.4861	5.4820	18075.8	217.22
λ	0	1.3447	5.5482	20688.0	182.78
	+5	2.2359	5.2662	17984.9	171.69
	+10	1.3378	5.2063	17815.6	146.02
	-10	1.34712	5.5482	20788.8	181.69
	-5	1.3459	5.5482	20989.8	182.69
M	0	1.3447	5.5482	20688.0	182.78
	+5	1.3434	5.5483	21790.0	184.69
	+10	1.3422	5.5483	22770.8	185.65

$A = 100$ units, $a = 132$, $v_1 = 5$, $v_2 = 4$, $v_3 = 6$, $\alpha = 0.4$, $\beta = 2$, $\gamma = 0.2$, $\lambda = 0.3$, $v = 0.2$, $M = 1.5$ per year, $\mu = 0.4$, $I_e = 0.15$ per year, $I_p = 0.10$ per year, $p = 8$, $c = 5$.

$p^* = 1.4070\ T^* = 5.5461$, Total Profit (TP) = 20,749.0, Order Quantity (Q) = 181.80.

6. SENSITIVITY ANALYSIS

Next, the effects of changes of parameters $a, \alpha, \beta, \gamma, \lambda$ and M on the optimal solutions are studied. Sensitivity analysis is performed by changing (increasing /decreasing) the parameters by 10 % and 5 % and taking one parameter at a time, keeping the remaining parameters at their original value.

The following inferences can be made based on Table-1, Table-2 Table-3.

Case-1: $0 < M \leq \mu$

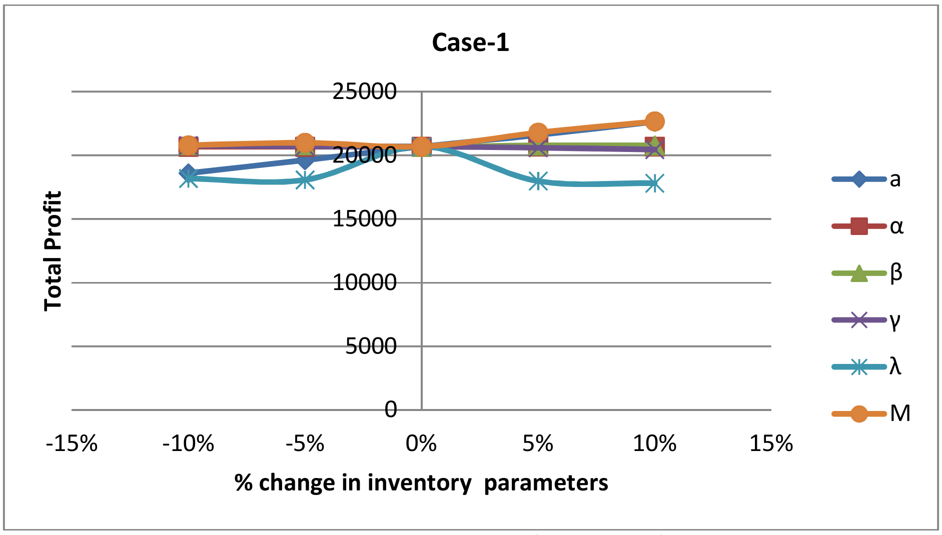


Figure – 5. Sensitivity analysis for total profit (TP))

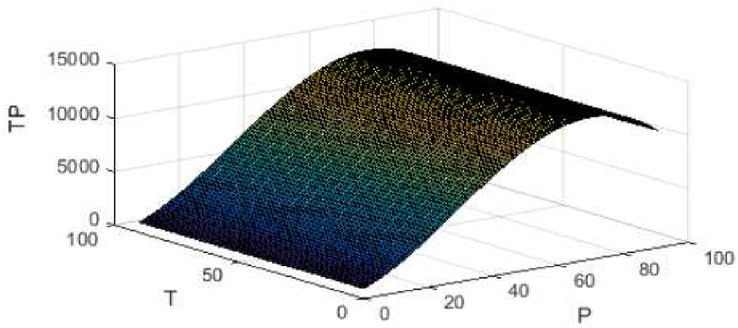


Figure-6, Concavity of total profit (TP) w. r. t. cycle time (T) and credit period (M)

Case-2: : $\mu < M \leq t_1$

	% Change	P	T	TP	Q
	-10	5.7805	5.5586	18546.0	153.90
A	-5	2.4806	5.5586	19766.4	170.04
	0	0.8192	5.5586	20894.3	181.47
	+5	3.9191	5.5586	21788.1	185.76
	+10	7.4190	5.5586	22838.7	190.60
	-10	0.7942	5.5491	20885.2	179.90
	-5	0.8067	5.5489	20782.3	180.68
α	0	0.8192	5.5586	20894.3	181.47
	+5	0.8319	5.5583	20673.9	182.27
	+10	0.8446	5.5581	20471.0	183.07
	-10	0.8103	5.5479	20873.6	182.86
	-5	0.8152	5.5482	20875.2	179.95
β	0	0.8192	5.5586	20894.3	181.47
	+5	0.8225	5.5590	20880.9	180.84
	+10	0.8250	5.5594	20883.8	180.24
	-10	0.8260	5.5483	20875.0	182.49
	-5	0.8226	5.5484	20775.9	181.98
γ	0	0.8192	5.5586	20894.3	181.47
	+5	0.8158	5.5587	20523.6	180.98
	+10	0.8124	5.5489	20480.5	180.50
	-10	6.7445	5.4336	18165.9	248.68
	-5	3.9690	5.4925	17363.3	217.36
λ	0	0.8192	5.5586	20894.3	181.47
	+5	2.7701	5.6329	17269.7	134.03
	+10	1.8512	5.7143	17100.0	125.19
	-10	0.9282	5.5554	20850.1	180.61
	-5	0.8751	5.5569	21863.3	191.55
M	0	0.8172	5.5486	20894.3	181.47
	+5	0.7604	5.5606	22895.6	275.37
	+10	0.6985	5.5628	24914.5	290.25

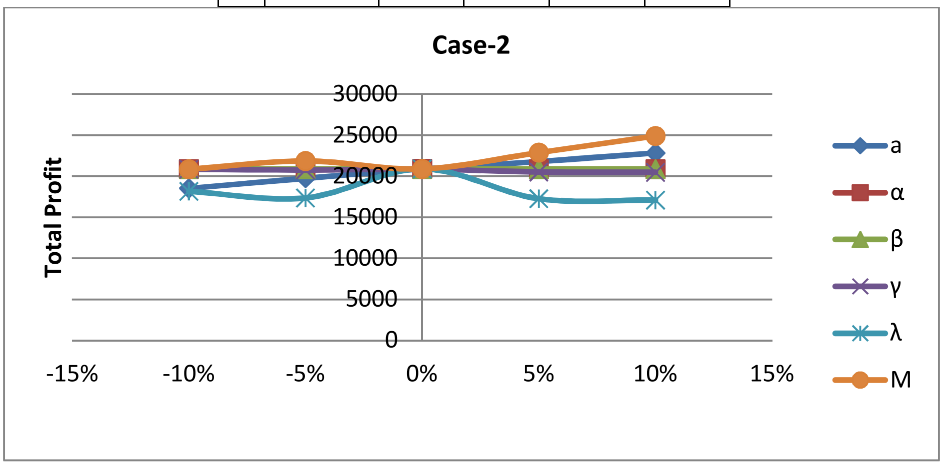


Figure-7, Sensitivity analysis for total profit

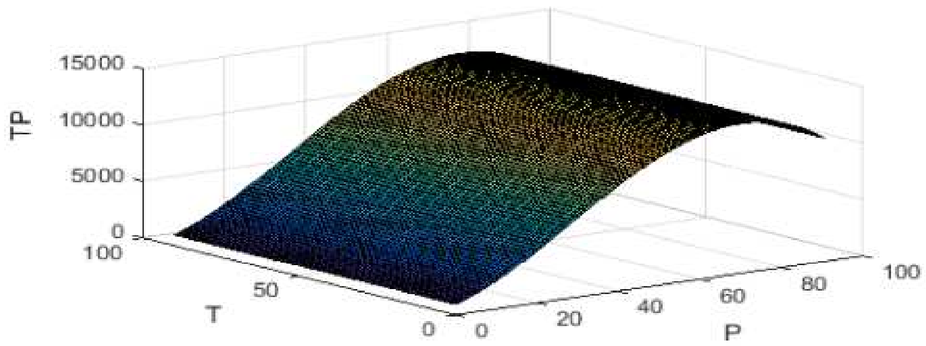


Figure-8. Concavity of total profit (TP) w. r. t. cycle time (T) and credit period (M)

Case-3: $t_1 < M \le T$

	% Change	P	T	TC	Q
	-10	5.5950	5.5596	18603.7	156.52
A	-5	1.9550	5.5596	19826.0	170.21
	0	1.4076	5.5461	20749.5	181.80
	+5	4.4237	5.5596	21840.2	185.41
	+10	7.9446	5.5596	22892.3	190.25
	-10	1.0090	5.5596	20927.6	179.56
	-5	1.0068	5.5596	20927.6	180.34
α	0	1.4076	5.4076	20749.5	181.80
	+5	1.0023	5.5597	20931.3	181.91
	+10	1.0000	5.5597	20932.5	182.13
	-10	1.3968	5.5461	20751.6	181.88
	-5	1.4025	1.5461	20750.5	182.44
β	0	1.4076	5.4076	20749.5	181.80
	+5	1.4121	5.5461	20748.5	181.20
	+10	1.4160	5.5461	20747.6	180.64
	-10	1.4170	5.5456	20744.4	182.82
	-5	1.4123	5.5459	20747.6	182.30
γ	0	1.4076	5.4076	20749.5	181.80
	+5	1.4029	5.5463	20751.4	181.31
	+10	1.3983	5.5466	20754.6	180.82
	-10	7.3158	5.4209	18142.7	248.20
	-5	4.5486	5.4798	20993.1	217.27
λ	0	1.4076	5.4076	20749.5	181.80
	+5	2.1725	5.6206	20266.1	136.04
	+10	1.2739	5.7043	20097.0	126.94
	-10	1.4076	5.4209	20749.6	151.80
	-5	1.4076	5.5461	21540.5	135.56
M	0	1.4076	5.4076	20749.5	182.80
	+5	1.4076	5.5461	23623.3	171.45
	+10	1.4076	5.5461	24700.5	172.21

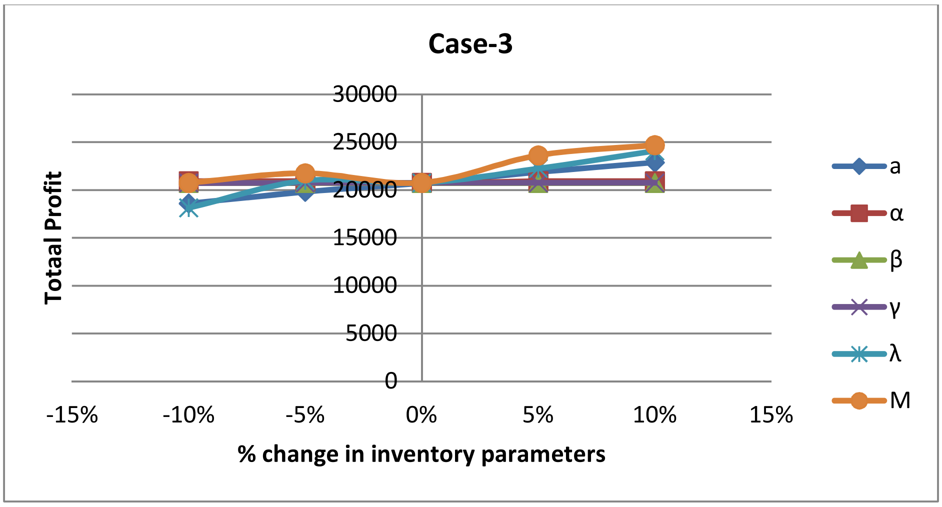


Figure-9. Sensitivity analysis for total profit (TP)

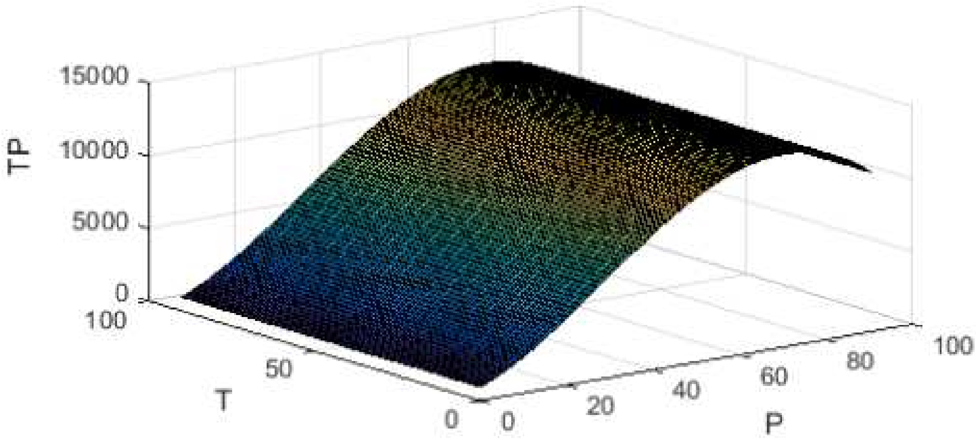


Figure-10. Concavity of total profit (TP) w. r. t. cycle time (T) and credit period (M)

7. OBSERVATIONS

Based on the sensitivity analysis the following managerial insights can be obtained.

1) Increase in the value of the parameter a, associates itself with increase in total profit and order quantity.
2) Increase in the value of the parameter α, induces both total profit and order quantity is slightly increase.
3) Increase in the value of the parameter β, induces total profit and order quantity is slightly decrease.
4) Increase in the value of the parameter γ, associates itself with total profit as well as order quantity to slightly decrease.
5) Increase in the value of the parameter λ, acts the total profit and order quantity to decrease.
6) Increase in the value of the parameter M, comes total profit as well as order quantity to increase.

8. CONCLUSION

In this paper, an economic order quantity model for non-instantaneous three parameter Weibull deteriorating items with price dependent demand under permissible delay in payments is developed. A model is developed in which shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. There are three possible scenarios in the study: $(1)\ \ 0 < \mathrm{M} \le \mu$ $(2)\ \ \mu < \mathrm{M} \le t_1$ $(3)\ \ \mathrm{t}_1 < \mathrm{M} \le T$. Several numerical examples are provided to illustrate the results under various situations and sensitivity analysis of the optimal solution with respect to major parameters is also carried out. There are some managerial insights derived from Table-2, Table-3 and Table-4, we get 1) a higher value of deterioration rate causes lower value of order quantity and total profit. 2) a higher value of retailer credit period causes higher value of total profit and order quantity. The analogy with the EOQ model is enlightened but the main difference with EOQ in presence of deterioration and credit period is also put forward. Problems with this kind of constraint arise in a variety of applications.

The presented model can be further extended to some practical situations, such as price discount, inflation, time value of money and fuzzy demand may be added.

Acknowledgement: The research work is supported by Ministry of Science and Technology, Government of India, under the Grant INSPIRE Fellowship (IF150161).

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