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CARBON EMISSIONS REGULATIONS FOR INVENTORY SYSTEM SENSITIVE TO GREEN TECHNOLOGY INVESTMENT AND PROMOTIONS

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ABSTRACT
Background: Over the past decade, there has been a significant surge in the imperative for manufacturing organizations to integrate environmentally friendly practices and innovate toward the development of sustainable and eco-conscious products. By escalating the environmental concerns, this study investigates the efficacy of green technology with distinct carbon reduction regulations: i) Carbon tax regulations, ii) Carbon cap-and-trade regulations, and iii) Limited carbon emissions regulations.
Methodology: This research utilizes classical optimization techniques to maximize total profit across various scenarios encompassing different carbon reduction policies and green technology investment plans. The study analyzes the concave nature of the profit function graphically using Maple 18.
Results: Through the application of optimization methodologies, this study delves into the optimal investment strategies and replenishment cycles for retailers under different regulations. Research proves limited carbon regulations cut emissions, and boost efficiency, and profitability for a sustainable transition.
Conclusion: In conclusion, this study underscores the critical role of green technology investment in aligning manufacturing practices with environmental objectives amidst evolving regulatory landscapes. The findings highlight the viability of limited carbon emissions regulations as a pragmatic approach to balancing environmental stewardship with business imperatives.

KEYWORDS: Carbon emissions (CE); Carbon tax (CT); Carbon cap-and-trade (CCAT); Limited Carbon emissions; Green technology investment (GTI)

MSC 90B05

RESUMEN
Antecedentes: Durante la última década, ha habido un aumento significativo en el imperativo de que las organizaciones manufactureras integren prácticas respetuosas con el medio ambiente e innoven hacia el desarrollo de productos sostenibles y con conciencia ecológica. Al intensificar las preocupaciones ambientales, este estudio investiga la eficacia de la tecnología verde con distintas regulaciones de reducción de carbono: i) regulaciones de impuestos al carbono, ii) regulaciones de límites máximos y comercio de carbono, y iii) regulaciones de emisiones de carbono limitadas.
Metodología: Esta investigación utiliza técnicas de optimización clásicas para maximizar el beneficio total en varios escenarios que abarcan diferentes políticas de reducción de carbono y planes de inversión en tecnología verde. El estudio analiza gráficamente la naturaleza cóncava de la función de beneficio utilizando Maple 18.
Resultados: Mediante la aplicación de metodologías de optimización, este estudio profundiza en las estrategias de inversión y ciclos de reposición óptimos para los minoristas bajo diferentes regulaciones. La investigación demuestra que las regulaciones limitadas sobre el carbono reducen las emisiones, aumentan la eficiencia y la rentabilidad para una transición sostenible.
Conclusión: En conclusión, este estudio subraya el papel fundamental de la inversión en tecnología verde a la hora de alinear las prácticas de fabricación con los objetivos ambientales en medio de panoramas regulatorios en evolución. Los hallazgos resaltan la viabilidad de regulaciones limitadas sobre emisiones de carbono como un enfoque pragmático para equilibrar la gestión ambiental con los imperativos comerciales.

1. INTRODUCTION

Carbon emissions regulation is crucial for mitigating climate change, as excessive carbon dioxide contributes to global warming. By setting limits on emissions, these regulations promote cleaner energy sources and sustainable practices, safeguarding the environment and the well-being of future generations. Additionally, they incentivize innovation and the development of technologies that reduce our carbon footprint. The numerous projects and investment programs

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needed to maintain low-carbon development are being intensified and accelerated by worldwide organizations and governments. In the early 1990s, Northern Europe started imposing carbon taxes. In 1990, Finland was the pioneer in this regard. After 1990, the Netherlands (1990), Sweden (1991), Norway (1991) and Denmark (1992) implemented this carbon policy. A CT policy is a regulatory measure implemented by governments to address climate change and reduce carbon dioxide emissions. It comprises employing a tax on fossil fuels containing carbon, such as oil, coal, and natural gas, or the amount of carbon dioxide emitted from industrial processes. The primary goal is to internalize the social cost of carbon by making businesses and individuals financially accountable for their contribution to climate change. This policy plays a crucial role in incentivizing the reduction of carbon emissions, fostering sustainable practices, and contributing to the broader goal of mitigating climate change.

Carbon cap-and-trade regulation is a governmental strategy aimed at curbing greenhouse gas emissions, particularly carbon dioxide. It involves establishing an overall emissions cap for a defined period, with corresponding allowances issued to entities operating within the cap. These allowances can be exchanged on a market, signifying the permission to release a certain amount of greenhouse gases. Companies that can reduce their emissions below their allocated allowances can sell surplus allowances to those exceeding their limits. This market-driven system encourages emissions reductions where it is most cost-effective, fostering flexibility, economic incentives for cleaner practices, and a gradual transition to a low-carbon economy. Examples include the European Union Emissions Trading System and provincial programs like the California Cap-and-Trade Program.

In response to evolving carbon regulations, industries are increasingly compelled to adopt green industry practices, driven by legal mandates and government encouragement. This paradigm shift necessitates the exploration and implementation of efficient methods for emission reduction, prompting substantial investments in environmentally friendly technologies and enhanced operational planning. Moreover, to comply with stringent emission limits imposed by regulatory authorities, industries are proactively investing in cutting-edge, greener technologies. These technologies are designed to optimize resource utilization, minimize waste, and significantly reduce carbon footprints. Examples include advancements in energy-efficient machinery, sustainable manufacturing processes, and the integration of renewable energy sources into industrial operations. The shift towards greener technologies and improved operational planning is not solely driven by regulatory compliance; it is increasingly becoming a strategic business imperative. Companies recognize the long-term benefits of supporting ecologically friendly activities to save money, improve brand recognition, and get access to new markets with a preference for eco-friendly products and services. Due to such investment, the company's overall expenses rise. But nowadays the increased investment in GT can be compensated due to increasing consumer awareness and willingness to buy green products. The Kyoto Protocol was established to combat climate change by imposing legally mandated reduction goals on industrialized nations' emissions of greenhouse gases, aiming to mitigate the impact of human activities on the Earth's climate.

Over the past decade, there has been a notable surge in research endeavors aimed at addressing carbon emissions regulations, particularly in the realm of logistics and supply chain inventory systems. Numerous studies have delved into identifying diverse sources of emissions within these systems, reflecting a widespread effort to mitigate environmental impact in response to regulatory measures.

2. LITERATURE REVIEW

The appropriate research on the model of sustainable inventory, which served as the study's foundation, is presented in this part.

In Min *et al.* (2010) article, demand is stock-dependent and trade credit of two levels is considered. To promote market competition, suppliers provide credit periods to retailers and retailers provide credit periods to customers. Pal *et al.* (2015) studied a supply chain under different model structures with selling price, promotional efforts, and product quality-dependent demand. The described demand pattern is frequently observed in the market for electronic products. Chen *et al.* (2019) considered stock and price-sensitive deterministic demand to maximize profit function. Mashud *et al.* (2020) presented an article with considers selling price and advertisement-dependent demand to maximize total profit function by finding the best trade credit period and reduce deterioration by preservation technology investment. In Pando *et al.* (2020) article, the profit/cost ratio is maximized where demand is stock-dependent. Under the CCAT regulation framework, Ghosh *et al.* (2020) examined the best marketing approaches thinking about a stochastic demand pattern that is sensitive to emissions. Chaudhary and Bali (2021) conducted optimization of both price and promotional efforts within the constraints of a limited budget, particularly in scenarios where a company employs multiple promotional programs.

Xu *et al.* (2017) investigated, under CCAT regulation, the production and emission abatement choices made by a manufacturer-retailer in a make-to-order supply chain. They concluded that a producer and store may work together to cut carbon emissions without having to give up revenue. Tao and Xu (2019) formulated economic order quantity models incorporating both CT and CCAT policies. They supposed that the clients are environmentally conscious, thereby impacting the demand rate. Yu *et al.* (2020) employed to construct an economic order quantity model using CT and CCAT policies. But they were concentrating on a failing product, for which demand depends on both selling price and stock level. Huang *et al.* (2020) looked at how three different carbon emission regulatory policies CCAT, CT, and restricted carbon emissions affected supply chain model parameters such as ideal lot size, ideal delivery amount, and investment in green technology. They considered demand as a constant function.

Vachon (2007) highlighted the positive correlation between supplier cooperation on environmental issues and increased investment in pollution prevention technologies. Wang *et al.* (2021) suggested that the positive impact of high levels of green technology investments on environmental sustainability varies depending on the specific sensitivity of the environmental impact to the adopted green technologies. Li *et al.* (2021) investigate the green decisions effected by fixed GTI cost and emission reduction amount-based subsidies under the CCAT scheme of a two-echelon supply chain. The analytical findings presented by them reveal that manufacturers and retailers are inclined to engage in collaborative efforts for green marketing when there is investment and subsidization in green technology. Luo *et al.* (2022) studied the effect of two scenarios such as with GTI and without GTI under the CT regulations to optimize the manufacturer's decision. They demonstrated how the CT regulations gave firms a reason to invest in environmentally friendly technologies. Paul *et al*. (2022) studied an inventory model, incorporating a flexible demand function influenced by both price and environmental consciousness. Jauhari *et al.* (2023) presented a model with selling price and green technology-dependent stochastic demand by considering CT policy. They examined that the increased GTI and CT policy has a positive impact on reducing CE. Consequently, GTI emerges as crucial for boosting customer purchases.

Promotion should be taken into account in this study since the sustainability performance motion draws in more clients and thus increases demand. Demand is also affected positively by green technology as an investment in green technology uses environmentally friendly products and technologies which can increase demand by awareness of consumers and sustainability. Similarly, the stock level should be taken into account in this study because businesses need to strike a balance and implement effective inventory management strategies to optimize their stock levels and have a positive effect on demand. These scenarios with three different carbon emissions policies considering CT policy, CCAT policy, and limited carbon emissions policy combined make the problem more realistic. These all scenarios are considered in this presented study and analyze the effect of carbon policies. This study maximizes total profit by optimizing cycle time and green technology investment.

In this study, section 3 provides notations and assumptions that are considered and taken throughout this article. The mathematical model with necessary and sufficient conditions is presented in section 4 and its numerical analyses are given in section 5. Section 6 describes managerial insights including the effect of different inventory parameters and different scenarios. The conclusion is given in section 7.

3. NOTATIONS AND ASSUMPTIONS

3.1 Notations

The following notations are used throughout this article.

Inventory parameters	
$i = 1,2,3$	Index of scenarios
$f(G)$	Carbon decline function
p	Selling price per unit ($/unit)
α	Mark-up of $f(G)$
a	Green technology's efficiency factor
b	Green technology's emissions factor
R	Demand rate (units)
m	Sensitivity of promotion $0 < m < 1$
υ	Level of promotion
d_1	Delivery distance (km)
$E(T,G)$	Total carbon emissions ($/unit/year)
C_E	Carbon emissions from distribution (unit of carbon emissions/km)
C_h	Carbon emissions for storing a unit product (unit of carbon emissions /unit)
A	Ordering cost per unit ($/order)

c	Purchase cost ($/unit)
C_T	Carbon tax for unit carbon emissions ($/unit ton carbon emissions)
C_{td}	Carbon trading price ($/unit ton carbon emissions)
C_{Tr}	Transportation cost per unit ($/unit)
h	Holding cost per unit ($/unit)
U	Carbon emissions limit in the cap-and-trade policy
W	Carbon emissions limit in the limited carbon emissions policy
Decision variables	
T	Cycle time (year)
G	Green technology investment ($/year)

3.2 Assumptions

The following assumptions are considered throughout this article.

1) Infinite replacement without shortages is considered. Similar to the buyer's inventory system, all supplies are received in one delivery at the start of the inventory cycle in this system.
2) Investment in green technology yields good results in reducing carbon emissions. The function $f(G)$ presents the reduction in the following way:
   $f(G) = aG - bG^2$, where $G < \frac{a}{b}$ (Toptal *et al.* (2014))
   Investment in green technology reduces carbon emissions by aG. Also adding consumed energy and carbon emissions during operation as bG^2.
3) Demand is GTI G dependent with positive impact as in Zanoni *et al.* (2014). Additionally, the promotion level raises consumer demand for sustainable efficiency (Xia *et al.* (2018)). Also, demand is positively affected by inventory stock level as in Min *et al.* (2010). In this analysis, demand is taken as dependent on all these three factors such as stock level, investment in green technology, and its promotion, which is taken as
   $R_1(G,\upsilon,I(t)) = R + \alpha f(G) + m\upsilon + \beta I(t)$, where $\alpha, m, \beta > 0$.
4) To reduce carbon emissions, we take into account investing in green technology in the following scenarios:
   i) GTI with a carbon tax policy
   ii) GTI with a cap-and-trade policy
   iii) GTI with limited carbon emissions policy
5) The causes of carbon emissions from the warehouse and the transportation method used to move inventory from one place to another place are addressed here. The vehicle is transporting the entire order, so emissions are unit distance emissions rate and delivery distance-dependent, which were motivated by Hasan *et al.* (2021).

4. MATHEMATICAL MODEL

In this section, the proposed inventory model has been methodically formulated taking specified assumptions delineated in section 3 to ensure a robust and technically sound framework.
The supply vehicle travels the distance d_1 and according to distance, transportation cost is applied to the retailer.
The inventory system in the retailer's warehouse is characterized by a maximum stock level at the initiation of each cycle $(t=0)$ with a lot size denoted by Q. Throughout the time interval $[0,T]$, the inventory experiences depletion attributable to demand effects. By the end of the cycle time T, the stock level reaches zero and the replenishment process begins for the subsequent cycle. Sales are impacted when a business commits to sustainability and marks its goods more aggressively than other companies. This impact is encapsulated in the inventory system's behavior, represented by the differential equation

$$\frac{dI(t)}{dt} = -R_1(G,\upsilon,I(t)), \quad 0 \le t \le T \qquad (1)$$

with $I(0)=Q$ and $I(T)=0$.

Solving the equation (1), the inventory level at time t is,

$$I(t)=\frac{R+\alpha\left(aG-bG^2\right)+m\upsilon}{\beta}\left(e^{\beta(T-t)}-1\right)$$

and the maximum quantity of stock Q is

$$Q=\frac{R+\alpha\left(aG-bG^2\right)+m\upsilon}{\beta}\left(e^{\beta T}-1\right)$$

The total CE resulting from shipping is expressed as the product of the unit distance of carbon emissions due to transportation and delivery distance.

Thus, the relationship is represented as,

$$T_{CEd}=C_E d_1$$

Here, C_E represents the emissions associated with delivering products for each unit of distance traveled and d_1 denotes the delivery distance.

The total carbon emissions attributed to holding management are considered by incorporating the CE per unit holding inventory. The total CE for holding the total quantity of inventories can be expressed as

$$T_{CEh}=C_h\int_0^T I(t)\,dt$$

$$=C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\frac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}$$

Here, C_h represents the CE units due to holding unit inventory.

The implementation of carbon pricing regulations and the encouragement of carbon emissions reduction empower buyers to make investments in green technologies. The function describing the extent of emissions reduction can be denoted as,

$$f(G)=aG-bG^2\text{, where } G<\frac{a}{b}.$$

Hence, the entire amount of carbon emissions from these three mentioned processes are,

$$E(T,G)=C_E d_1+C_h\int_0^T I(t)\,dt-f(G)$$

$$=C_E d_1+\frac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-aG+bG^2$$

This inventory model considers costs associated with ordering, purchasing, transportation, and holding inventory. Also, GTI is considered.

1) The ordering cost is $OC=A$
2) Transportation cost is included in the purchasing cost. The supplier adds a transportation charge from the retailer per unit of inventory. The purchase cost per inventory is

$$PC=\left(c+C_{Tr}\right)Q$$

where c is the initial price for purchasing a single unit and C_{Tr} is per unit transportation cost to the retailer from the supplier.

3) The inventory is kept in a warehouse for a specific period leads to a charge for handling the inventory during that time. Thus, the holding cost is

$$HC = h\int_0^T I(t)\,dt$$

$$= h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\frac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}$$

4) The GTI costs G.
5) Sales revenue is obtained from the selling price and demand. Thus, sales revenue is

$$SR = p\int_0^T R_1(t)\,dt$$

$$= p\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\frac{e^{\beta T}-1}{\beta}\right)$$

In this article, we are considering three different scenarios in which the total profit function is taken under three different carbon pricing policies.

Scenario 1: Total profit under CT policy.

In this scenario, CT is levied on total CE. Thus, the total cost due to CE under this policy is, $T_{CE1} = C_T E(T,G)$

The total profit under the CT policy with GTI is,

$$TP_1 = \frac{1}{T}\left(SR-HC-PC-OC-T_{CE1}-G\right)$$

$$= \frac{1}{T}\left(\begin{array}{l}\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\frac{e^{\beta T}-1}{\beta}\right)-h\frac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right)-A\\ -C_T\left(C_E d_1+\frac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-aG+bG^2\right)-G\end{array}\right) \quad (2)$$

Scenario 2: Total profit under CCAT policy

In this scenario, the total amount of permitted carbon emissions may be limited by the government is U. Extra carbon emissions are sold by C_{td} rate. When it is over the limit U more allowances are bought from other organizations or make investments in green technology.

Thus, the total CE cost under the CCAT policy is,

$$T_{CE2} = C_T E(T,G) - C_{td}U$$

The total profit under the CCAT policy with GT1 is,

$$TP_2 = \frac{1}{T}\left(SR-HC-PC-OC-T_{CE2}-G\right)$$

$$= \frac{1}{T}\left(\begin{array}{l}\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\frac{e^{\beta T}-1}{\beta}\right)-h\frac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right)-A\\ -C_T\left(C_E d_1+\frac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-aG+bG^2\right)+C_{td}U-G\end{array}\right) \quad (3)$$

Scenario 3: Total profit under limited carbon emissions policy

In this policy, W is a limit of strict CE which must be complied by the retailer. Any additional CE must be mitigated through the allocation of investments in GT. The total CE emitted from all associated sources, subtracted by the reduction achieved through investments in GT, must equal the specified CE limit W.

Hence $E(T,G)=W$

$$C_E d_1 + C_h \int_0^T I(t)\,dt - aG + bG^2 = W$$

To find optimal GTI and maximize profit, the Lagrange multiplier method is applied.
The total profit under this limited CE policy is,

$$TP_3 = \frac{1}{T}\left(SR - HC - PC - OC - G - \lambda\left(E(T,G) - W\right)\right)$$

$$= \frac{1}{T}\left(\begin{array}{l} \left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\dfrac{e^{\beta T}-1}{\beta}\right)-h\dfrac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right)-A \\ -\lambda\left(C_E d_1+\dfrac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-aG+bG^2-W\right)-G \end{array}\right) \tag{4}$$

Necessary Condition

To maximize total profit, the necessary condition is the partial derivatives of total profit with respect to decision variables, namely cycle time and green technology investment are zero.

i.e. $\dfrac{\partial TP_i}{\partial G} = 0$ & $\dfrac{\partial TP_i}{\partial T} = 0,\ i = 1,2,3$

For total profit under carbon tax regulations:

$$\frac{\partial TP_1}{\partial G} = \frac{1}{T}\left(\begin{array}{l} \alpha\left(a-2bG\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\dfrac{e^{\beta T}-1}{\beta}\right)-h\dfrac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right) \\ -C_T\left(\dfrac{C_h\alpha\left(a-2bG\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-a+2bG\right)-1 \end{array}\right)$$

$$\frac{\partial TP_1}{\partial T} = \frac{1}{T}\left(\begin{array}{l} \left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)e^{\beta T}-h\dfrac{\left(e^{\beta T}-1\right)}{\beta}\right) \\ -C_T\left(\dfrac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-1\right)}{\beta}\right) \end{array}\right)$$

$$-\frac{1}{T^2}\left(\begin{array}{l} \left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\dfrac{e^{\beta T}-1}{\beta}\right)-h\dfrac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right)-A \\ -C_T\left(C_E d_1+\dfrac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-aG+bG^2\right)-G \end{array}\right)$$

For total profit under carbon cap-and-trade regulations:

$$\frac{\partial TP_2}{\partial G} = \frac{1}{T}\left(\begin{array}{l} \alpha\left(a-2bG\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\dfrac{e^{\beta T}-1}{\beta}\right)-h\dfrac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right) \\ -C_T\left(\dfrac{C_h\alpha\left(a-2bG\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-a+2bG\right)-1 \end{array}\right)$$

$$\frac{\partial TP_2}{\partial T}==\frac{1}{T}\begin{pmatrix}\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)e^{\beta T}-\frac{h\left(e^{\beta T}-1\right)}{\beta}\right)\\-C_T\left(\frac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-1\right)}{\beta}\right)\end{pmatrix}$$

$$-\frac{1}{T^2}\begin{pmatrix}\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\frac{e^{\beta T}-1}{\beta}\right)-h\frac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right)-A\\-C_T\left(C_E d_1+\frac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-aG+bG^2\right)+C_{td}U-G\end{pmatrix}$$

For total profit under limited carbon emissions regulations:

$$\frac{\partial TP_3}{\partial G}=\frac{1}{T}\begin{pmatrix}\alpha\left(a-2bG\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\frac{e^{\beta T}-1}{\beta}\right)-h\frac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right)\\-\lambda\left(\frac{C_h\alpha\left(a-2bG\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-a+2bG\right)-1\end{pmatrix}$$

$$\frac{\partial TP_3}{\partial T}=\frac{1}{T}\begin{pmatrix}\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)e^{\beta T}-h\frac{\left(e^{\beta T}-1\right)}{\beta}\right)\\-\lambda\left(\frac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-1\right)}{\beta}\right)\end{pmatrix}$$

$$-\frac{1}{T^2}\begin{pmatrix}\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(\left(p-\left(c+C_{Tr}\right)\right)\left(\frac{e^{\beta T}-1}{\beta}\right)-h\frac{\left(e^{\beta T}-\beta T-1\right)}{\beta^2}\right)-A\\-\lambda\left(C_E d_1+\frac{C_h\left(R+\alpha\left(aG-bG^2\right)+m\upsilon\right)\left(e^{\beta T}-\beta T-1\right)}{\beta^2}-aG+bG^2-W\right)-G\end{pmatrix}$$

Now, check the necessary condition for the total profit under all carbon emissions regulations using assumed inventory parameters using Maple 18 software and find out the decision variables values. To validate the sufficient condition, proceed further.

Sufficient condition

To verify the sufficiency condition, evaluate the decision variable values and employ the graphical method to examine the nature of the graph, specifically determining whether it exhibits concavity or another characteristic. The sufficient condition is illustrated after obtaining the solution in the numerical analysis section.

5. NUMERICAL ANALYSIS

In the indicated portion, a mathematical model is validated by considering the numerical example. The goal of this inventory model is to maximize the total profit in all the scenarios. Here, a function of total profit contains investment in green technology G and cycle time T. To compute decision variables, we use the below method.

Step 1: Take partial derivatives of the profit function from equations (2)-(4) with respect to decision variables.

Step 2: Allot hypothetical values to all considered inventory parameters.

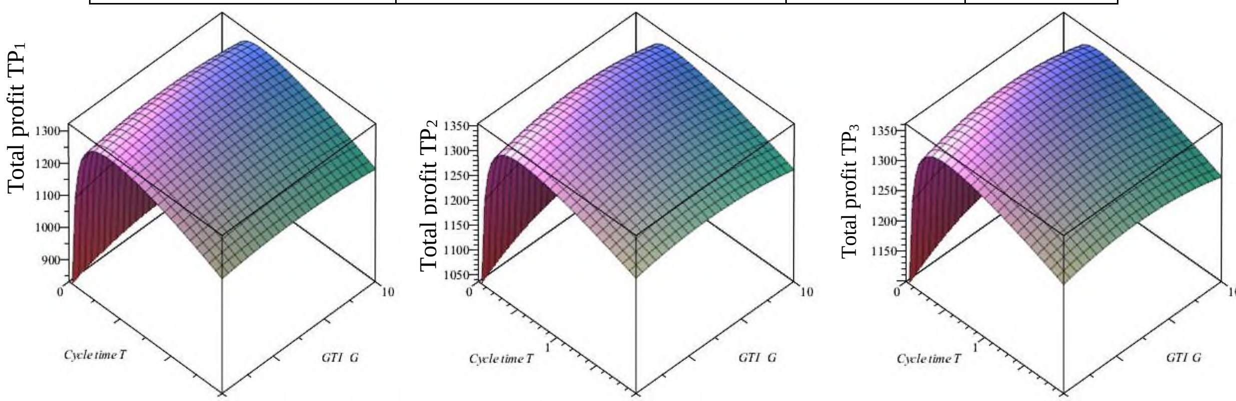
$$\left[\begin{array}{l} a=7, b=0.3, A=10, \upsilon=30, m=1.3, C_p=\left(c+C_{Tr}\right)=13, h=0.7, d_1=160, C_E=0.3, C_h=0.02, R=50, P=15, \alpha=15, \\ \beta=0.3, C_T=1.5, W=20, \lambda=0.9, C_{td}=10, U=2 \end{array}\right]$$

Step 3: Take all partial derivatives with respect to decision variables equal to zero.
Step 4: Using the classical optimization method solve these equations and obtain the solutions for each model.
Step 5: Check the optimality of the profit function by graphical method.

Table 1: Solution of decision variables and total profit

Using the solution from Table 1, the concavity of the profit function of three scenarios is mentioned below:

Carbon regulations policy	Green Technology Investment G (in \$)	Cycle time T (in year)	Total Profit (in \$)
Carbon tax	11.6	0.78	1322.97
Carbon cap-and-trade	11.56	0.49	1354.29
Limited carbon emissions	11.55	0.46	1361.19



Carbon tax policy Carbon cap-and-trade policy Limited carbon emissions policy

Fig. 1 Concavity of total profit functions with respect to decision variables

This above figure 1 shows that the obtained solutions in all scenarios are optimum. Hence obtained total profit is maximum in all scenarios.

5.1. Managerial Insights

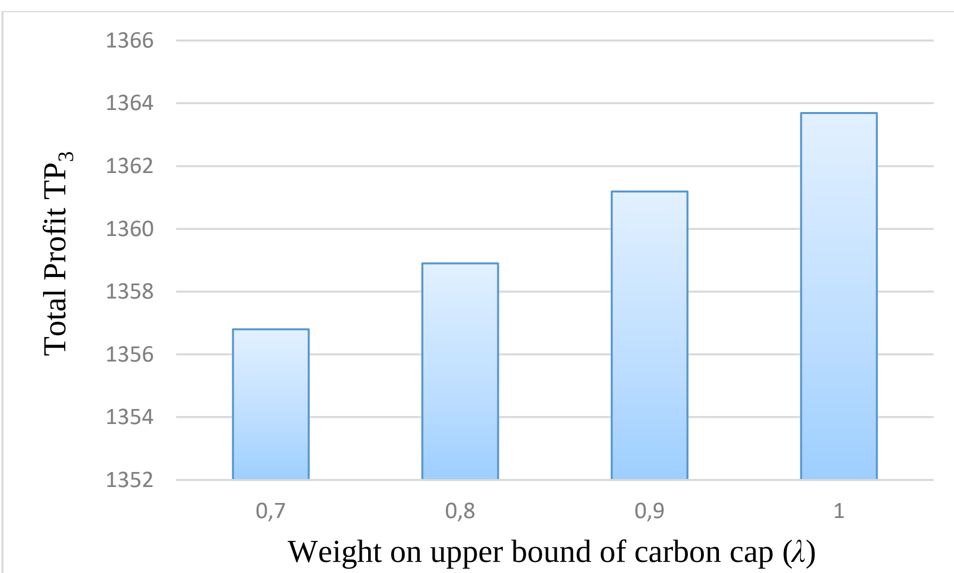


Fig. 2 Variation in total profit with respect to different weights on carbon cap.

From Figure 2, it is shown that the increase in weight on the upper bound of the carbon cap increases the total profit of the system.

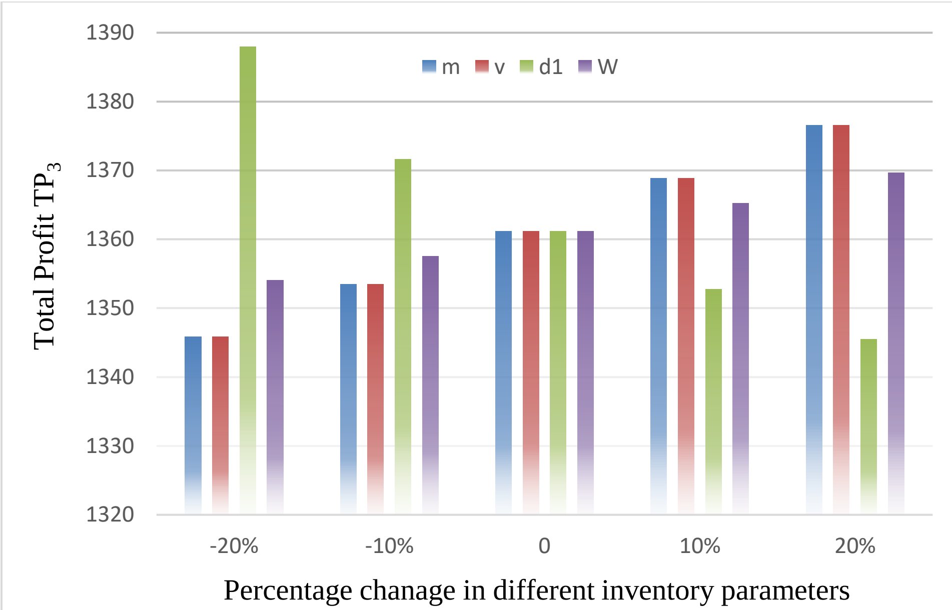


Fig. 3 Variation in total profit with respect to changes in inventory parameters

From this figure 3, several results can be derived. More and more regulations are needed on maximum available carbon emissions to increase the total profit. An increase in promotion level will increase total profit. Similarly, promotion elasticity also increases total profit. Hence, promotion level and promotion elasticity attract more customers resulting in higher profit. Increasing distance means it gives more emissions and that results in reducing profit.

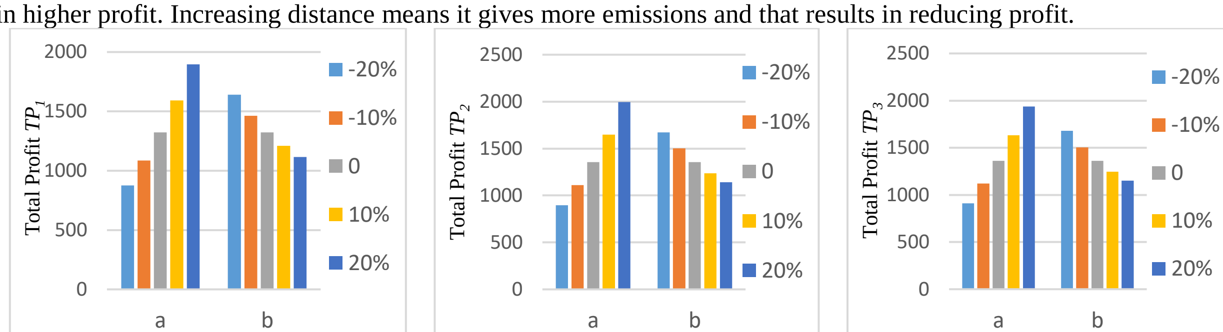


Fig. 4 Variation in total profit by changing the values of efficiency and emissions factor of green technology

Figure 4, shows that the emissions and efficiency factors of green technology have different proportional effects on the total profit function. In all scenarios, the efficiency factor of green technology has a positive effect on total profit, and the emissions factor of green technology hurts total profit. The increasing rate of efficiency factor is higher in CT policy. Similarly decreasing rate of emissions factor is higher in CT policy.

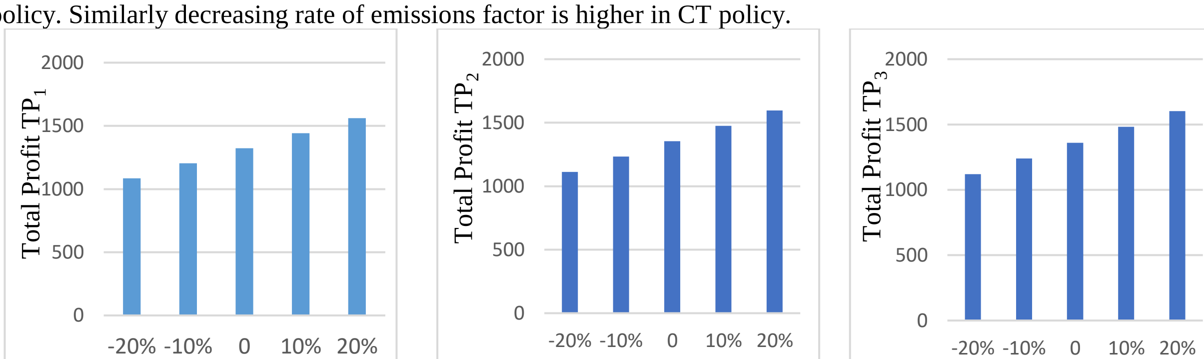


Fig. 5 Values of total profit by changing the mark-up value of $f(G)$

Figure 5 shows the effect of the variations in the mark-up value of $f(G)$ on total profit for all three scenarios. The carbon reduction function has a positive effect on total profit. Hence, increasing the value of markup gives an increasing value of total profit in all carbon policies.

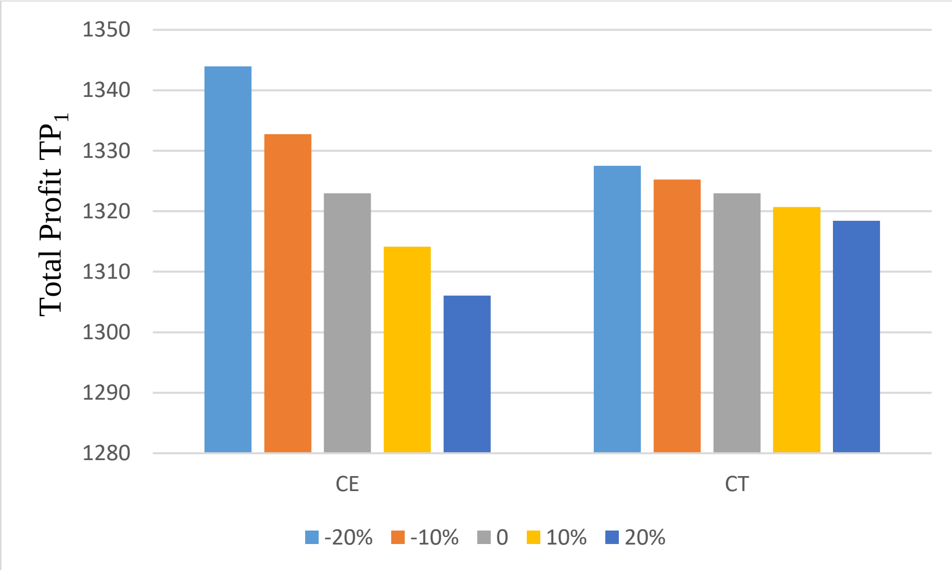


Fig. 6 Values of total profit TP_1 with respect to changes in carbon emissions from distributions (C_E) and carbon tax (C_T)

Figure 6 describes the values of total profit under CT regulations with respect to changes in carbon emissions from distributions and carbon tax. An increasing value of carbon emissions results in to decreasing value of total profit. Similarly increasing the value of carbon tax results in increasing total profit. Hence, increasing amounts of CE and CT are adverse to the total profit function.

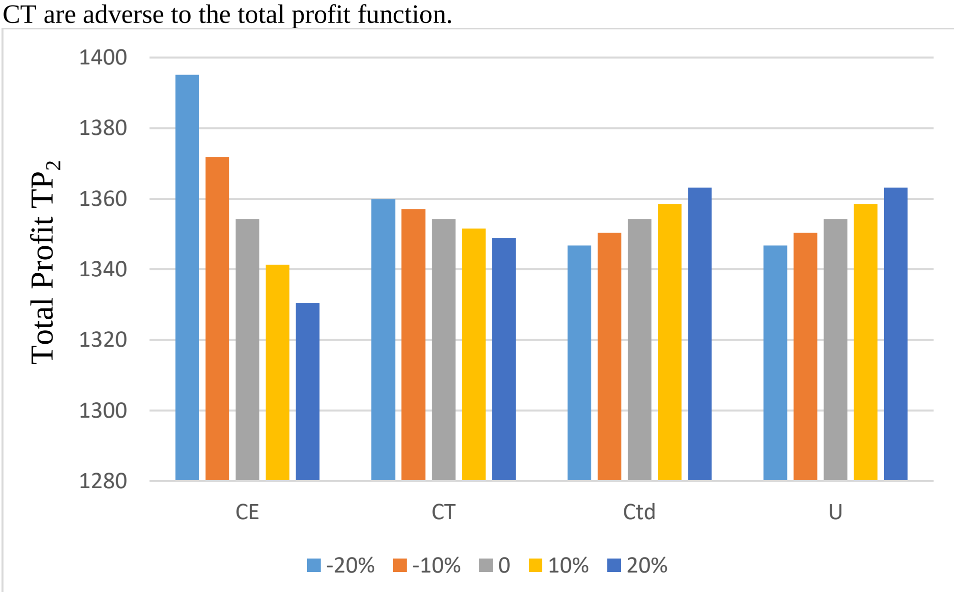


Fig. 7 Values of total profit TP_2 with respect to changes in terms associated with carbon emissions

From Figure 7 shown above, the changes in carbon emissions-related inventory parameters to total profit under CCAT regulations can be seen. The increasing amount of the carbon cap results in increasing the value of total profit because increasing the carbon cap leads to sales of more carbon units which results in increased profit. Also, increasing the price for trading carbon emissions results in more profit. However, increasing the value of carbon tax is adverse to total profit. As more tax reduces total profit and from this figure, it's shown that small change in carbon emissions due to distributions will lead to a high change in total profit compared to all other carbon emissions-related parameters.

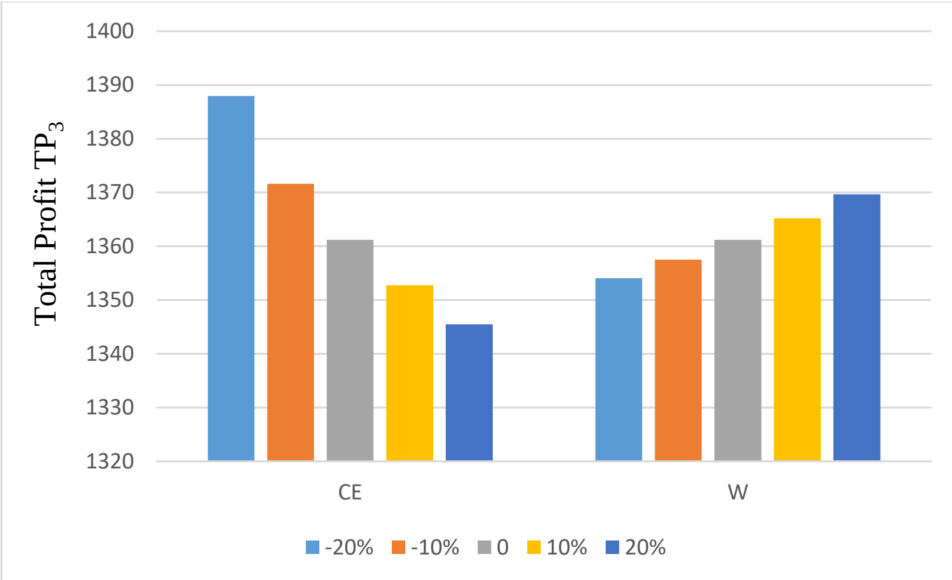


Fig. 8 Values of total profit TP_3 with respect to changes in carbon emissions from distributions (C_E) and carbon emissions limit (W)

Figure 8 represents the value of total profit under limited CE with respect to different values of CE associated with distributions and CE limits in the limited CE policy. Carbon emissions are inversely proportional to the total profit function as it has the opposite effect on the total profit function. The increasing value of CE may result in a decreasing value of total profit. As we increase the constraints value say the limit of CE in limited CE policy, the value of total profit is increasing.

6. CONCLUSION

In this research paper, three different carbon policies are considered, namely carbon tax, carbon cap-and-trade, and limited CE. This research delves into the strategies adopted by a company to mitigate carbon emissions by focusing on inventory planning and technological investments within the framework of carbon emissions regulations. The analysis takes into account the potential rise in customer demand resulting from the company's environmentally friendly technology investments, stock level, and the promotion of its eco-friendly performance. The study aims to optimize decisions related to cycle time and green investment amounts per cycle, all while ensuring the company maximizes its profit. A mathematical model was created from the problem, and a solution process was given along with an example to help explain the model. The optimal overall profit, considering all carbon emissions policies, exhibits a concave relationship with green technology investment and cycle time as the key decision variables.

Carbon emissions due to distributions are more effective compared to all other parameters related to carbon emissions in all the scenarios. Also, increasing the amount of CT results in decreasing total profit, so the government has to carefully set the tax level. Increasing the value of carbon limit parameters is more beneficial. Comparing all the carbon policies, the limited carbon emissions policy is most appropriate in the mentioned scenarios. Such investments in green technology with carbon policy are necessary to adhere to government regulations, safeguard the environment, and generate profits for corporations or the government. Increasing the value of the green technology's efficiency factor has a positive effect on total profit, while the emission factor hurts the total profit function. The demand of the presented scenarios is considered in linear equations. Hence for future work, this demand function can also be considered in non-linear form. In assumptions, it is given that the shortages are not permitted. Hence, this model is also studied while considering shortages.

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