FORTHCOMING 62D05-5-23-01

OPTIMUM ESTIMATION OF POPULATION MEANS IN STRATIFIED RANDOM SAMPLING USING REGRESSION TYPE ESTIMATOR –NON-RESPONSE SITUATION.

*Zakir Hussain Wani, S.E.H. Rizvi
Division of Statistics and Computer Science, Main Campus SKUAST-J, Chatha Jammu-180009, India

ABSTRACT:
The population mean of separate and combined regression estimators under non-response in two different situations is estimated in this study. Situation I arise when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known, and situation II rises when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known. The proposed estimator's bias and MSE are calculated up to the first degree of approximation. In terms of mean square errors, the proposed estimators perform better than existing estimators, as proven by numerical data that backs up the theoretical predictions.

KEYWORDS: Mean Square error, Bias, Efficiency, Non-response, Auxiliary variable, Separate regression estimator, Combined regression estimator,

MSC: 62D05

RESUMEN
En este estudio se estima la media poblacional de estimadores de regresión separados y combinados bajo falta de respuesta en dos situaciones diferentes. La situación I surge cuando no hay respuesta tanto en la variable de estudio Y como en la variable auxiliar X, y se conoce la media poblacional de la variable auxiliar ($\bar{X}$), y la situación II surge cuando no hay respuesta en la variable de estudio Y solamente y la información sobre la variable auxiliar X se obtiene de todas las unidades muestrales, y se conoce la media poblacional de la variable auxiliar ($\bar{X}$) . El sesgo del estimador propuesto y el MSE se calculan hasta el primer grado de aproximación. En términos de errores cuadráticos medios, los estimadores propuestos funcionan mejor que los estimadores existentes, como lo demuestran los datos numéricos que respaldan las predicciones teóricas.

PALABRAS CLAVE: Error cuadrático medio, Sesgo, Eficiencia, Falta de respuesta, Variable auxiliar, Estimador de regresión separado, Estimador de regresión combinado,

1. INTRODUCTION:

Non-response is a relatively common problem in practice. The estimators of the parameters produce biased results due to non-response. The concept of non-response was initially addressed by Hansen and Hurwitz (1946): Later, various authors, such as Cochran (1977), Rao (1986), Khare and Srivastava (1997), Singh and Kumar (2008, 2009), Singh et al (2010), discussed the problem of estimating the population mean of the study variable using information on an auxiliary variable in the presence of non-response using the Hansen and Hurwitz (1946) technique under the SRSWOR scheme. When the population units are homogeneous, the SRSWOR sampling strategy is usually utilized. However, in practice, heterogeneous populations are also commonly encountered. In such cases, stratified random sampling is advised. With this in mind, Chaudhary et al. (2009) investigated non-response in stratified random sampling, assuming that non-response happens only on the study variable. Sanaullah et al. (2015), Saleem et al. (2018), Onyeka et al. (2019) and Shabbir et al. (2019) have studied the problem of non-response in stratified single and two-phase sampling where non-response occurs on both the study and auxiliary variable, as well as on the study variable only. For further study, Singh et al. (2010), Singh et al. (2018), Singh and Sharma (2015),and Singh and Vishwakarma (2019) are all recommended for further study. In this paper we have made an effort to propose a separate and combined regression estimators for estimating the population mean of the study variable y using information on auxiliary variable x in presence of non-response under stratified random sampling in two situations i.e., Situation I arise when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known, and situation II arises when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known. The properties of proposed estimators were investigated under large sample approximation. The proposed estimators have been demonstrated to outperform all other estimators studied in the literature. To support the current study, a numerical illustration is provided.

Notations:

Let us consider a finite heterogeneous population of N units prearranged into L homogeneous sub-classes called strata, each of which contains N_h units, with h=1,2,3, ... The population of size N is made up of two mutually exclusive groups: response group and non-response. N_{1h} and N_{2h} are the responding and non-responding units in the h^{th} stratum, respectively. SRSWOR is used to pick a sample of size n_h from N_h units in the h^{th} stratum, with the assumption that n_{1h} units respond and n_{2h} units do not. From the n_{2h} non-responding units in the h^{th} stratum, we select a sub-sample of size $r_h = (n_{2h} / k_h \,; k_h > 1)$.

Table1 To find bias and mean square error (MSE) of the proposed separate and combined regression estimators, we use the following approximations.

For Separate Case	**For Combined Case**
$\xi_{0h}^* = \frac{\bar{y}_h^* - \bar{Y}_h}{\bar{Y}_h}$: $\quad \xi_{1h}^* = \frac{\bar{x}_h^* - \bar{X}_h}{\bar{X}_h}$: $\xi_{1h} = \frac{\bar{x}_h - \bar{X}_h}{\bar{X}_h}$ $E(\xi_{0h}^*) = E(\xi_{1h}^*) = E(\xi_{1h})$ $E(\xi_{0h}^{*2}) = \frac{1}{\bar{Y}_h^2}\left[\theta_h S_{hy}^2 + \frac{W_{2h}(k_h - 1)}{n_h} S_{hy(2)}^2\right] = A_h$ $E(\xi_{1h}^{*2}) = \frac{1}{\bar{X}_h^2}\left[\theta_h S_{hx}^2 + \frac{W_{2h}(k_h - 1)}{n_h} S_{hx(2)}^2\right] = B_h$ $E(\xi_{0h}^* \xi_{1h}^*) = \frac{1}{\bar{Y}_h \bar{X}_h}\left[\theta_h S_{hxy} + \frac{w_{2h}(k_h - 1)}{n_h} S_{hxy(2)}\right] = C_h$ $E(\xi_{1h}^2) = \frac{1}{\bar{X}_h^2}\theta_h S_{hx}^2 = D_h$ $E(\xi_{0h}^* \xi_{1h}) = \frac{1}{\bar{Y}_h \bar{X}_h}\theta_h S_{hxy} = E_h$	$\xi_{0st}^* = \frac{\bar{y}_{st}^* - \bar{Y}}{\bar{Y}}$: $\quad \xi_{1st}^* = \frac{\bar{x}_{st}^* - \bar{X}}{\bar{X}}$: $\xi_{1st} = \frac{\bar{x}_{st} - \bar{X}}{\bar{X}}$ $E(\xi_{0st}^*) = E(\xi_{1st}^*) = E(\xi_{1st}) = 0$ $E(\xi_{0st}^{*2}) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L} W_h^2\left[\theta_h S_{hy}^2 + \frac{W_{2h}(k_h - 1)}{n_h} S_{hy(2)}^2\right] = A$ $E(\xi_{1st}^{*2}) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2\left[\theta_h S_{hx}^2 + \frac{W_{2h}(k_h - 1)}{n_h} S_{hx(2)}^2\right] = B$ $E(\xi_{0st}^* \xi_{1st}^*) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h^2\left[\theta_h S_{hxy} + \frac{w_{2h}(k_h - 1)}{n_h} S_{hxy(2)}\right] = C$ $E(\xi_{1st}^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2\, \theta_h S_{hx}^2 = D$ $E(\xi_{0st}^* \xi_{1st}) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h^2\, \theta_h S_{hxy} = E$

Where $\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$ and $\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$ are the population means of the study and auxiliary variables, respectively for the h^{th} stratum. $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$ and $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$ are combined population means of the study and auxiliary variable respectively. $S_{hy}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2$ and $S_{hx}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)^2$ are the population variances of the study and auxiliary variables respectively.

$S_{hy(1)}^2 = \frac{1}{N_{h(1)} - 1}\sum_{i=1}^{N_{h(1)}}\left(y_{hi} - \bar{Y}_{h(1)}\right)^2$ and $S_{hx(1)}^2 = \frac{1}{N_{h(1)} - 1}\sum_{i=1}^{N_{h(1)}}\left(X_{hi} - \bar{X}_{h(1)}\right)^2$ are the variances of response groups of the study and auxiliary variables respectively.

$S_{hy(2)}^2 = \frac{1}{N_{h2} - 1}\sum_{i=1}^{N_{h(2)}}\left(Y_{hi} - \bar{Y}_{h(2)}\right)^2$ and $S_{hx(2)}^2 = \frac{1}{N_{h(2)} - 1}\sum_{i=1}^{N_{h(2)}}\left(Y_{hi} - \bar{X}_{h(2)}\right)^2$ are the variances of non-response groups of the study and auxiliary variables respectively.

$\rho_{hxy} = \frac{S_{hxy}}{S_{hy} S_{hx}}$ is the correlation coefficient between the auxiliary and study variables in h^{th} stratum. $\rho_{hxy(2)} = \frac{S_{hxy(2)}}{S_{hy(2)} S_{hx(2)}}$ is the correlation coefficient between the auxiliary and study variables of non-response group in h^{th} stratum. $\sum_{h=1}^{L} N_h = N, L_h = \frac{\bar{X}_h}{\bar{Y}_h}, \quad R_h = \frac{\bar{Y}_h}{\bar{X}_h}, L = \frac{\bar{X}}{\bar{Y}}$ and $R = \frac{\bar{Y}}{\bar{X}}, f_h = \frac{n_h}{N_h}$ is the sampling fraction of h^{th} stratum etc.

$\theta_h = \frac{1}{n_h} - \frac{1}{N_h}, \hat{\beta}_h^* = \frac{s_{hxy}^*}{s_{hx}^{*2}}$ and $\hat{\beta}_c^* = \frac{\sum_{h=1}^{L} W_h^2 \theta_h s_{hxy}^*}{\sum_{h=1}^{L} W_h^2 \theta_h s_{hx}^{*2}}, \hat{\beta}_h = \frac{s_{hxy}^*}{s_{hx}^2}$ and $\hat{\beta}_c = \frac{\sum_{h=1}^{L} W_h^2 \theta_h s_{hxy}^*}{\sum_{h=1}^{L} W_h^2 \theta_h s_{hx}^2}; s_{hxy}^* = \frac{1}{n_h - 1}\left(\sum_{i=1}^{n_h} x_{hi} y_{hi} + k_h \sum_{i=1}^{r_h} x_{hi} y_{hi} - n_h \bar{x}_h \bar{y}_h^*\right), s_{hx}^{*2} = \frac{1}{n_h - 1}\left(\sum_{i=1}^{n_h} x_{hi}^2 + k_h \sum_{i=1}^{r_h} x_{hi}^2 - n_h \bar{x}_h \bar{x}_h^*\right), s_{hx}^2 = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}(x_{hi} - \bar{X}_h)^2, \beta_{2h(x)}$ is the coefficient of Kurtosis for the h^{th} stratum

2. EXISTING ESTIMATORS

This section gives a brief introduction of some well-known estimators/classes of estimators from the literature. The Hansen and Hurwitz unbiassed estimator $\bar{y}_h^*$ for h[th] stratum is given as $\bar{y}_h^* = \frac{n_{h1}\bar{y}_{nh1}+n_{h2}\bar{y}_{rh2}}{n_h}$, $MSE(\bar{y}_h^*) = \theta_h S_{hy}^2 + \frac{W_{2h}(k_h-1)}{n_h} S_{hy(2)}^2$

Therefore, the Hansen and Hurwitz unbiassed estimator in stratified random sampling is given as

1. $\bar{y}^* = \sum_{h=1}^{L} W_h \bar{y}_h^*$

The mean square error of $\bar{y}^*$ is given as

$$MSE(\bar{y}^*) = \bar{Y}^2 A \quad .... \quad (1)$$

2. The usual separate ratio estimator when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known is given by $\bar{y}_{SR}^* = \sum_{h=1}^{L} W_h \frac{\bar{y}_h^*}{\bar{x}_h^*} \bar{X}_h$

The mean square error of $\bar{y}_{SR}^*$ is given by

$$MSE(\bar{y}_{SR}^*) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 (A_h + B_h - 2C_h) \quad (2)$$

The separate ratio estimator when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known, is given by $\bar{y}'_{SR} = \sum_{h=1}^{L} W_h \frac{\bar{y}_h^*}{\bar{x}_h} \bar{X}_h$. The mean square error of $\bar{y}'_{SR}$ is given by

$$MSE(\bar{y}'_{SR}) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 (A_h + D_h - 2E_h) \quad (3)$$

3. The usual combined ratio estimator when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known is given by $\bar{y}_{CR}^* = \frac{\bar{y}_{st}^*}{\bar{x}_{st}^*} \bar{X}$

The mean square error of $\bar{y}_{CR}^*$ is given by

$$MSE(\bar{y}_{CR}^*) = \bar{Y}^2 (A + B - 2C) \quad (4)$$

The combined ratio estimator when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known is given by $\bar{y}'_{CR} = \frac{\bar{y}_{st}^*}{\bar{x}_{st}} \bar{X}$. The mean square error of $\bar{y}'_{CR}$ is given by

$$MSE(\bar{y}'_{CR}) = \bar{Y}^2 (A + D - 2E) \quad (5)$$

.4 The usual separate regression estimator when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known is given by $\bar{y}_{Sreg}^* = \sum_{h=1}^{L} W_h^2 \left(\bar{y}_h^* + \hat{\beta}_h^* (\bar{X}_h - \bar{x}_h^*)\right)$. The mean square error of $\bar{y}_{Sreg}^*$ is given by

$$MSE\left(\bar{y}_{Sreg}^*\right) = \sum_{h=1}^{L} W_h^2 \, [\bar{Y}_h^2 A_h + \beta_h^2 \bar{X}_h^2 B_h - 2\beta_h \bar{X}_h \bar{Y}_h C_h] \quad (6).$$

The separate regression estimator when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known, is given by $\bar{y}'_{Sreg} = \sum_{h=1}^{L} W_h^2 \left(\bar{y}_h^* + \hat{\beta}_h (\bar{X}_h - \bar{x}_h)\right)$. The mean square error of $\bar{y}'_{Sreg}$ is given by

$$MSE\left(\bar{y}'_{Sreg}\right) = \sum_{h=1}^{L} W_h^2 \, [\bar{Y}_h^2 A_h + \beta_h^2 \bar{X}_h^2 D_h - 2\beta_h \bar{X}_h \bar{Y}_h E_h] \quad (7)$$

5. The usual combined regression estimator when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known is given by $\bar{y}_{Creg}^* = \bar{y}_{st}^* + \hat{\beta}_c^* (\bar{X} - \bar{x}_{st}^*)$. The mean square error of $\bar{y}_{Creg}^*$ is given by

$$MSE\left(\bar{y}_{Creg}^*\right) = \bar{Y}^2 A + \beta_c^2 \bar{X}^2 B - 2\beta_c \bar{X} \bar{Y} C \quad (8)$$

The Combined regression estimator when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) known is given by $\bar{y}'_{Creg} = \bar{y}_{st}^* + \hat{\beta}_c (\bar{X} - \bar{x}_{st})$. The mean square error of $\bar{y}'_{Creg}$ is given by

$$MSE\left(\bar{y}'_{Creg}\right) = \bar{Y}^2 A + \beta_c^2 \bar{X}^2 D - 2\beta_c \bar{X}\bar{Y}E \tag{9}$$

6. The following are the stratified modified estimators in presence of non-response developed by Onyeka et al. (2019) using known values of coefficient of correlation, kurtosis, and coefficient of variation.

When there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean $(\bar{X})$ is known. $\bar{y}_{ok}^{*(i)} = \sum_{h=1}^{L} W_h \bar{y}_h^* exp\left[\frac{\Psi_1(\bar{X}_h - \bar{x}_h^*)}{\Psi_1(\bar{X}_h - \bar{x}_h^*) + 2\Psi_2}\right]$. The mean square error of $\bar{y}_{ok}^{*(i)}$ is given by

$$MSE\left(\bar{y}_{ok}^{*(i)}\right) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[A_h + \frac{1}{4}\lambda_i^2 B_h - \lambda_i C_h\right] \tag{10}$$

When there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean $(\bar{X})$ is known. $\bar{y}_{ok}^{\prime(i)} = \sum_{h=1}^{L} W_h \bar{y}_h^* exp\left[\frac{\Psi_1(\bar{X}_h - \bar{x}_h)}{\Psi_1(\bar{X}_h - \bar{x}_h) + 2\Psi_2}\right]$. The mean square error of $\bar{y}_{ok}^{\prime(i)}$ is given by

$$MSE\left(\bar{y}_{ok}^{\prime(i)}\right) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[A_h + \frac{1}{4}\lambda_i^2 D_h - \lambda_i E_h\right] \tag{11}$$

Where,

$\lambda_i = \frac{\Psi_1 \bar{X}_h}{\Psi_1 \bar{X}_h + \Psi_2}$, $\lambda_1 = 1$, $= \frac{\bar{X}_h}{\bar{X}_h + C_{(x)h}}$, $\lambda_3 = \frac{\bar{X}_h}{\bar{X}_h + \beta_{2(x)h}}$, $\lambda_4 = \frac{C_{(x)h}\bar{X}_h}{C_{(x)h}\bar{X}_h + \rho_{yxh}}$, $\lambda_5 = \frac{\beta_{2(x)h}\bar{X}_h}{\beta_{2(x)h}\bar{X}_h + C_{(x)h}}$, $\lambda_6 = \frac{\bar{X}_h}{\bar{X}_h + \rho_{yxh}}$, $\lambda_7 = \frac{\rho_{yxh}\bar{X}_h}{\rho_{yxh}\bar{X}_h + \beta_{2(x)h}}$,

3. PROPOSED ESTIMATORS

Motivated from classical regression estimator and Singh and Espejo (2003), we have to develop a separate and combined regression estimator in presence of non-response in two situations shown in theorem 1. 2, 3 and 4.

3.1 Proposed Separate Regression Estimator

In the theorem 1 and 2 we propose a separate regression estimator in two situations of non-response

Situation I: This situation arises when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean $(\bar{X})$ is known,

Situation II: This situation arises when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean $(\bar{X})$ is known.

Theorem 1: In this theorem we propose a separate regression estimator when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean $(\bar{X})$ is known.

$$t_1^{*(i)} = \sum_{h=1}^{L} W_h \left\{\left[\bar{y}_h^* + \hat{\beta}_h^*(\bar{X}_h - \bar{x}_h^*)\right]\left[\alpha^*\left(\frac{\pi_1\bar{X}_h + \pi_2}{\pi_1\bar{x}_h^* + \pi_2}\right) + (1-\alpha^*)\left(\frac{\pi_1\bar{x}_h^* + \pi_2}{\pi_1\bar{X}_h + \pi_2}\right)\right]\right\} \tag{12}$$

Where, α^* is the real constant that must be calculated in order MSE $t_1^{*(i)}$ to be the least. $\pi_1 \neq 0$ and π_2 are either real numbers or be the known parameters such coefficient of variation $C_{h(x)}$, Kurtosis $\beta_{2h(x)}$ for the h[th] stratum.

The bias and MSE of the estimator $t_1^{*(i)}$ are given by

$$Bias\left(t_1^{*(i)}\right) = \sum_{h=1}^{L} W_h \bar{Y}_h \{(\alpha^*\varphi_{hi}^2 + 2\alpha^*\beta_h L_h \varphi_{hi} - \beta_h L_h \varphi_{hi})B_h + (\varphi_{hi} - 2\alpha^*\varphi_{hi})C_h\} \tag{13}$$

$$MSE\left(t_1^{*(i)}\right) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \begin{bmatrix} A_h + (\varphi_{hi} - 2\alpha^*\varphi_{hi} - \beta_h L_h)^2 B_h \\ +2(\varphi_{hi} - 2\alpha^*\varphi_{hi} - \beta_h L_h) C_h \end{bmatrix} \tag{14}$$

$$MSE\left(t_1^{*(i)}\right) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left\{A_h - \frac{C_h^2}{B_h}\right\} \tag{15}$$

Where,

$\varphi_{hi} = \frac{\pi_{h1}\bar{X}_h}{\pi_{h1}\bar{X}_h + \pi_{h2}}$, $\varphi_{h1} = 1$, $\pi_{h1} = 1$ and $\pi_{h2} = 0$, $\varphi_{h2} = \frac{\bar{X}_h}{\bar{X}_h + C_{hx}}$, $\pi_{h1} = 1$ and $\pi_{h2} = C_{hx,}\varphi_{h3} = \frac{\bar{X}_h}{\bar{X}_h + \beta_{2h(x)}}$, $\pi_{h1} = 1$ and $\pi_{h2} = \beta_{2h(x)}$, $\varphi_{h4} = \frac{\beta_{2h(x)}\bar{X}_h}{\beta_{2h(x)}\bar{X}_h + C_{hx}}$, $\pi_{h1} = \beta_{2h(x)}$ and $\pi_{h2} = C_{hx}$

$$\varphi_{h5}=\frac{C_{hx}\bar{X}_h}{C_{hx}\bar{X}_h+\beta_{2h(x)}}\ ,\quad \pi_{h1}=C_{hx}\ \ and\ \pi_{h2}=\beta_{2h(x)}$$

Proof: We rewrite the proposed estimator $t_1^{*(i)}$ as.

$$t_1^{*(i)}=\sum_{h=1}^{L}W_h J_h \tag{16}$$

Where,

$$J_h=\left[\bar{y}_h^*+\hat{\beta}_h^*(\bar{X}_h-\bar{x}_h^*)\right]\left[\alpha^*\left(\frac{\pi_1\bar{X}_h+\pi_2}{\pi_1\bar{x}_h^*+\pi_2}\right)+(1-\alpha^*)\left(\frac{\pi_1\bar{x}_h^*+\pi_2}{\pi_1\bar{X}_h+\pi_2}\right)\right] \tag{17}$$

To obtained the bias and MSE of J_h up to first order of approximations by using the error terms ξ_{0h}^* and ξ_{1h}^*

$$J_h=\left[\bar{Y}_h(1+\xi_{0h}^*)-\hat{\beta}_h^*\bar{X}_h\xi_{1h}^*\right][\alpha^*(1+\varphi_{hi}\xi_{1h}^*)^{-1}+(1-\alpha^*)(1+\varphi_{hi}\xi_{1h}^*)]$$

$$J_h-\bar{Y}_h=\bar{Y}_h\begin{Bmatrix}\xi_{0h}^*-2\alpha^*\varphi_{hi}\xi_{1h}^*+\varphi_{hi}\xi_{1h}^*+\alpha^*\varphi_{hi}^2\xi_{1h}^{*2}-2\alpha^*\varphi_{hi}\xi_{0h}^*\xi_{1h}^*+\\ \varphi_{hi}\xi_{0h}^*\xi_{1h}^*-\hat{\beta}_h^*L_h\xi_{1h}^*+2\alpha^*\hat{\beta}_h^*L_h\varphi_{hi}\xi_{1h}^{*2}-\hat{\beta}_h^*L_h\varphi_{hi}\xi_{1h}^{*2}\end{Bmatrix} \tag{18}$$

Where $L_h=\frac{\bar{X}_h}{\bar{Y}_h}$

Taking Expectation on equation (18) both sides we get the bias of J_h

$$Bias(J_h)=\bar{Y}_h\{(\alpha^*\varphi_{hi}^2+2\alpha^*\beta_hL_h\varphi_{hi}-\beta_hL_h\varphi_{hi})B_h+(\varphi_{hi}-2\alpha^*\varphi_{hi})C_h\} \tag{19}$$

The Bias of the proposed estimator $t_1^{*(i)}$ is given by using equation (16)

$$Bias\left(t_1^{*(i)}\right)=\sum_{h=1}^{L}W_h\,Bias(J_h)$$

$$Bias\left(t_1^{*(i)}\right)=\sum_{h=1}^{L}W_h\,\bar{Y}_h\{(\alpha^*\varphi_{hi}^2+2\alpha^*\beta_hL_h\varphi_{hi}-\beta_hL_h\varphi_{hi})B_h+(\varphi_{hi}-2\alpha^*\varphi_{hi})C_h\} \tag{20}$$

Squaring equation (18) and then taking expectation we get the MSE of J_h

$$MSE(J_h)=\bar{Y}_h^2[A_h+(\varphi_{hi}-2\alpha^*\varphi_{hi}-\beta_hL_h)^2B_h+2(\varphi_{hi}-2\alpha^*\varphi_{hi}-\beta_hL_h)C_h] \tag{21}$$

The MSE of the proposed estimator $t_1^{*(i)}$ is given by using equation (16)

$$MSE\left(t_1^{*(i)}\right)=\sum_{h=1}^{L}W_h^2MSE(J_h)$$

$$MSE\left(t_1^{*(i)}\right)=\sum\nolimits_{h=1}^{L}W_h^2\,\bar{Y}_h^2\begin{bmatrix}A_h+(\varphi_{hi}-2\alpha^*\varphi_{hi}-\beta_hL_h)^2B_h\\ +2(\varphi_{hi}-2\alpha^*\varphi_{hi}-\beta_hL_h)C_h\end{bmatrix} \tag{22}$$

Where, $\varphi_{hi}=\frac{\pi_{h1}\bar{X}_h}{\pi_{h1}\bar{X}_h+\pi_{h2}}$

For obtaining the optimum values of α^* , differentiating equation (22)) w.r.t α^* and equating to zero we have

$$\frac{\partial MSE\left(t_1^{*(i)}\right)}{\partial\alpha^*}=0$$

$$\alpha_{opt}^*=\frac{1}{2}\left[1-\frac{1}{\varphi_{hi}}\left(\beta_hL_h-\frac{C_h}{B_h}\right)\right] \tag{23}$$

Using the value of α_{opt}^* in equation (22), the minimal MSE of the proposed estimators is the given as

$$MSE\left(t_1^{*(i)}\right)=\sum_{h=1}^{L}W_h^2\,\bar{Y}_h^2\left\{A_h-\frac{C_h^2}{B_h}\right\} \tag{24}$$

From the above equation(23) it is clear that all the proposed estimators$t_1^{*(i)}$ have same MSE because the MSE is independent of φ_{hi}

Theorem 2. In this theorem we propose a separate regression estimator when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known

$$t_1'^{(i)} = \sum_{h=1}^{L} W_h \left\{ \left[\bar{y}_h^* + \hat{\beta}_h(\bar{X}_h - \bar{x}_h)\right] \left[\alpha' \left(\frac{\pi_1 \bar{X}_h + \pi_2}{\pi_1 \bar{x}_h + \pi_2}\right) + (1-\alpha')\left(\frac{\pi_1 \bar{x}_h + \pi_2}{\pi_1 \bar{X}_h + \pi_2}\right)\right]\right\} \tag{25}$$

Where, α' is the real constant that must be calculated in order MSE $t_1'^{(i)}$to be the least.$\pi_1 \neq 0$ and π_2 are either real numbers or be the known parameters such coefficient of variation $C_{h(x)}$, Kurtosis$\beta_{2h(x)}$ for the h[th] stratum

The bias and MSE of the estimator $t_1'^{(i)}$ are given by

$$Bias\left(t_1'^{(i)}\right) = \sum_{h=1}^{L} W_h \bar{Y}_h \{(\alpha' \varphi_{hi}^2 + 2\alpha' \beta_h L_h \varphi_{hi} - \beta_h L_h \varphi_{hi}) D_h + (\varphi_{hi} - 2\alpha' \varphi_{hi}) E_h\} \tag{26}$$

$$MSE\left(t_1'^{(i)}\right) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \begin{bmatrix} A_h + (\varphi_{hi} - 2\alpha' \varphi_{hi} - \beta_h L_h)^2 D_h \\ +2(\varphi_{hi} - 2\alpha' \varphi_{hi} - \beta_h L_h) E_h \end{bmatrix} \tag{27}$$

$$MSE\left(t_1'^{(i)}\right) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[A_h - \frac{E_h^2}{D_h}\right] \tag{28}$$

Where,φ_{hi} has the same values used in theorem1

Proof: We rewrite the proposed estimator $t_1'^{(i)}$ as

$$t_1'^{(i)} = \sum_{h=1}^{L} W_h J_h' \tag{29}$$

Where $J_h' = \left[\bar{y}_h^* + \hat{\beta}_h(\bar{X}_h - \bar{x}_h)\right] \left[\alpha' \left(\frac{\pi_1 \bar{X}_h + \pi_2}{\pi_1 \bar{x}_h + \pi_2}\right) + (1-\alpha')\left(\frac{\pi_1 \bar{x}_h + \pi_2}{\pi_1 \bar{X}_h + \pi_2}\right)\right]$

To obtained the bias and MSE of J_h' up to first order of approximations by using the error terms ξ_{0h}^* and ξ_{1h}

$J_h' = \left[\bar{Y}_h(1+\xi_{0h}^*) - \hat{\beta}_h \bar{X}_h \xi_{1h}\right]\left[\alpha'(1+\varphi_{hi}\xi_{1h})^{-1} + (1-\alpha')(1+\varphi_{hi}\xi_{1h})\right]$

$$J_h' - \bar{Y}_h = \bar{Y}_h \begin{Bmatrix} \xi_{0h}^* - 2\alpha' \varphi_{hi} \xi_{1st} + \varphi_{hi}\xi_{1h} + \alpha' \varphi_{hi}^2 \xi_{1h}^2 - 2\alpha' \varphi_{hi} \xi_{0h}^* \xi_{1h} + \\ \varphi_{hi} \xi_{0h}^* \xi_{1h} - \hat{\beta}_h L_h \xi_{1h} + 2\alpha' \hat{\beta}_h L_h \varphi_{hi} \xi_{1h}^2 - \hat{\beta}_h L_h \varphi_{hi} \xi_{1h}^2 \end{Bmatrix} \tag{30}$$

Where $L_h = \frac{\bar{X}_h}{\bar{Y}_h}$

Taking Expectation on equation (30) both sides we get the bias of J_h'

$Bias(J_h') = \bar{Y}_h\{(\alpha' \varphi_{hi}^2 + 2\alpha' \beta_h L_h \varphi_{hi} - \beta_h L_h \varphi_{hi}) D_h + (\varphi_{hi} - 2\alpha' \varphi_{hi}) E_h\}$

Therefore, the Bias of the proposed estimator $t_1'^{(i)}$ is given by using equation (29) $Bias\left(t_1'^{(i)}\right) = \sum_{h=1}^{L} W_h \, Bias(J_h')$

$$Bias\left(t_1'^{(i)}\right) = \sum_{h=1}^{L} W_h \bar{Y}_h \{(\alpha' \varphi_{hi}^2 + 2\alpha' \beta_h L_h \varphi_{hi} - \beta_h L_h \varphi_{hi}) D_h + (\varphi_{hi} - 2\alpha' \varphi_{hi}) E_h\} \tag{31}$$

Squaring equation (30) and then taking expectation we get the MSE of J_h'

$$MSE(J_h') = \bar{Y}_h^2 [A_h + (\varphi_{hi} - 2\alpha' \varphi_{hi} - \beta_h L_h)^2 D_h + 2(\varphi_{hi} - 2\alpha' \varphi_{hi} - \beta_h L_h) E_h]$$

The MSE of the proposed estimator $t_1'^{(i)}$ is given by using equation (29) $MSE\left(t_1'^{(i)}\right) = \sum_{h=1}^{L} W_h^2 \, MSE(J_h')$

$$MSE\left(t_1'^{(i)}\right) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \begin{bmatrix} A_h + (\varphi_{hi} - 2\alpha' \varphi_{hi} - \beta_h L_h)^2 D_h \\ +2(\varphi_{hi} - 2\alpha' \varphi_{hi} - \beta_h L_h) E_h \end{bmatrix} \tag{32}$$

For obtaining the optimum values of α' , differentiating equation (32) w.r.t α' and equating to zero we have

$$\frac{\partial MSE\left(t_1'^{(i)}\right)}{\partial \alpha'} = 0$$

$$\alpha'_{opt} = \frac{1}{2}\left[1 - \frac{1}{\varphi_{hi}}\left(\beta_h L_h - \frac{E_h}{D_h}\right)\right] \tag{33}$$

Using the value of α'_{opt} in equation (32), the minimal MSE of all proposed estimators is given as

$$MSE\left(t_1'^{(i)}\right) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[A_h - \frac{E_h^2}{D_h}\right] \tag{34}$$

From the above equation (32)it is clear that all the proposed estimators$t_1'^{(i)}$ have same MSE because the MSE is independent of φ_{hi}

3.2. Proposed Combined Regression Estimator

In the theorem 3 and 4 we propose a combined regression estimator in two situations of non-response.

Situation I: This situation arises when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known,

Situation II: This situation arises when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known.

Theorem 3 In this theorem we propose a combined regression estimator when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known.

$$t_2^{*(i)} = \left[\bar{y}_{st}^* + \hat{\beta}_c^*(\bar{X} - \bar{x}_{st}^*)\right]\left[\omega^*\left(\frac{\tau_1\bar{X} + \tau_2}{\tau_1\bar{x}_{st}^* + \tau_2}\right) + (1-\omega^*)\left(\frac{\tau_1\bar{x}_{st}^* + \tau_2}{\tau_1\bar{X} + \tau_2}\right)\right] \quad (35)$$

Where, ω^* is the real constant that must be calculated in order MSE $t_2^{*(i)}$to be the least. $\tau_1 \neq 0$ and τ_2 are either real numbers or be the known parameters such as: Coefficient of variation $C_{st(x)} = \sum_{h=1}^{L} W_h C_{h(x)}$, Kurtosis $\beta_{2st(x)} = \sum_{h=1}^{L} W_h \beta_{2h(x)}$

The bias and MSE of the estimator $t_2^{*(i)}$ are given by

$$Bias\left(t_2^{*(i)}\right) = \bar{Y}\{(\omega^*\gamma_i^2 + 2\omega^*\beta_c L\gamma_i - \beta_c L\gamma_i)B + (\gamma_i - 2\omega^*\gamma_i)C\}$$

$$MSE\left(t_2^{*(i)}\right) = \bar{Y}^2\begin{bmatrix} A + (\gamma_i - 2\omega^*\gamma_i - \beta_c L)^2 B \\ +2(\gamma_i - 2\omega^*\gamma_i - \beta_c L)C \end{bmatrix} \quad (36)$$

$$MSE\left(t_2^{*(i)}\right) = \bar{Y}^2\left[A - \frac{C^2}{B}\right] \quad (37)$$

Where,

$\gamma_i = \frac{\tau_1\bar{X}}{\tau_1\bar{X}+\tau_2}$,$\gamma_i = 1, \quad \tau_1 = 1\ and\ \tau_2 = 0, \gamma_2 = \frac{\bar{X}}{\bar{X}+C_{st(x)}}\tau_1 = 1\ and\ \tau_2 = C_{st(x)}, \gamma_3 = \frac{\bar{X}}{\bar{X}+\beta_{2st(x)}}\tau_1 = 1\ and\ \tau_2 = \beta_{2st(x)}, \gamma_4 = \frac{\beta_{2st(x)}\bar{X}}{\beta_{2st(x)}\bar{X}+C_{st(x)}}\tau_1 = \beta_{2st(x)}\ and\ \tau_2 = C_{st(x)}, \gamma_5 = \frac{C_{st(x)}\bar{X}}{C_{st(x)}\bar{X}+\beta_{2st(x)}}\tau_1 = C_{st(x)}\ and\ \tau_2 = \beta_{2st(x)}$

Proof: To obtain the bias and MSE of $t_2^{*(i)}$ up to first order of approximations by using the error terms ξ_{0st}^* and ξ_{1st}^*

$$t_2^{*(i)} = \left[\bar{Y}(1+\xi_{0st}^*) - \hat{\beta}_c^*\bar{X}\xi_{1st}^*\right]\left[\omega^*(1+\gamma_i\xi_{1st}^*)^{-1} + (1-\omega^*)(1+\gamma_i\xi_{1st}^*)\right]$$

$$t_2^{*(i)} - \bar{Y} = \bar{Y}\begin{Bmatrix} \xi_{0st}^* - 2\omega^*\gamma_i\xi_{1st}^* + \gamma_i\xi_{1st}^* + \omega^*\gamma_i^2\xi_{1st}^{*2} - 2\omega^*\gamma_i\xi_{0st}^*\xi_{1st}^* + \\ \gamma_i\xi_{0st}^*\xi_{1st}^* - \hat{\beta}_c^* L\xi_{1st}^* + 2\omega^*\hat{\beta}_c^* L\gamma_i\xi_{1st}^{*2} - \hat{\beta}_c^* L\gamma_i\xi_{1st}^{*2} \end{Bmatrix} \quad (38)$$

Taking Expectation on equation (38) both sides we get the bias of the proposed estimator $t_2^{*(i)}$ as

$$Bias\left(t_2^{*(i)}\right) = \bar{Y}\{(\omega^*\gamma_i^2 + 2\omega^*\beta_c L\gamma_i - \beta_c L\gamma_i)B + (\gamma_i - 2\omega^*\gamma_i)C\} \quad (39)$$

Squaring equation (38) and then taking expectation we get the MSE of $t_2^{*(i)}$

$$MSE\left(t_2^{*(i)}\right) = \bar{Y}^2\begin{bmatrix} A + (\gamma_i - 2\omega^*\gamma_i - \beta_c L)^2 B \\ +2(\gamma_i - 2\omega^*\gamma_i - \beta_c L)C \end{bmatrix} \quad (40)$$

For obtaining the optimum values of ω^* , differentiating equation (40) w.r.t ω^* and equating to zero we have

$$\frac{\partial MSE\left(t_2^{*(i)}\right)}{\partial\omega^*} = 0$$

$$\omega^* = \frac{1}{2}\left[1 - \frac{1}{\gamma_i}\left(\beta_c L - \frac{C}{B}\right)\right] \quad (41)$$

Using the value of ω^*in equation (40), the minimal MSE of the proposed estimators$t_2^{*(i)}$ is given as

$$MSE\left(t_2^{*(i)}\right) = \bar{Y}^2\left[A - \frac{C^2}{B}\right] \quad (42)$$

From the above equation (41)it is clear that all the proposed estimators $t_2^{*(i)}$ have same MSE because the MSE is independent of γ_i

Theorem 4. In this theorem we propose a combined regression estimator when there is non-response on the study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean ($\bar{X}$) is known.

$$t_2^{\prime(i)} = \left[\bar{y}_{st}^* + \hat{\beta}_c(\bar{X} - \bar{x}_{st})\right]\left[\omega'\left(\frac{\tau_1\bar{X} + \tau_2}{\tau_1\bar{x}_{st} + \tau_2}\right) + (1-\omega')\left(\frac{\tau_1\bar{x}_{st} + \tau_2}{\tau_1\bar{X} + \tau_2}\right)\right] \quad (43)$$

Where, ω' is the real constant that must be calculated in order MSE $t_2^{\prime(i)}$to be the least. $\tau_1 \neq 0$ and τ_2 are either real numbers or be the known parameters such as: Coefficient of variation $C_{st(x)} = \sum_{h=1}^{L} W_h C_{h(x)}$, Kurtosis $\beta_{2st(x)} = \sum_{h=1}^{L} W_h \beta_{2h(x)}$

The bias and MSE of the estimator $t_2^{\prime(i)}$ are given by

$$Bias\left(t_2^{\prime(i)}\right) = \bar{Y}\{(\omega'\gamma_i^2 + 2\omega'\beta_c L\gamma_i - \beta_c L\gamma_i)D + (\gamma_i - 2\omega'\gamma_i)E\}$$

$$MSE\left(t_2^{\prime(i)}\right) = \bar{Y}^2[A + (\gamma_i - 2\omega'\gamma_i - \beta_c L)^2 D + 2(\gamma_i - 2\omega'\gamma_i - \beta_c L)E] \tag{44}$$

$$MSE\left(t_2^{\prime(i)}\right) = \bar{Y}^2\left[A - \frac{E^2}{D}\right] \tag{45}$$

Where, γ_i has the same values used in theorem 3

Proof: To obtained the bias and MSE of $t_2^{\prime(i)}$ up to first order of approximations we use the error terms ξ_{0st}^* and ξ_{1st}

$$t_2^{\prime(i)} = \left[\bar{Y}(1 + \xi_{0st}^*) - \hat{\beta}_c \bar{X}\xi_{1st}\right]\left[\omega'(1 + \gamma_i\xi_{1st})^{-1} + (1 - \omega')(1 + \gamma_i\xi_{1st})\right]$$

$$t_2^{\prime(i)} - \bar{Y} = \bar{Y}\left\{\begin{matrix} \xi_{0st}^* - 2\omega'\gamma_i\xi_{1st} + \gamma_i\xi_{1st} + \omega'\gamma_i^2\xi_{1st}^2 - 2\omega'\gamma_i\xi_{0st}^*\xi_{1st} + \\ \gamma_i\xi_{0st}^*\xi_{1st} - \hat{\beta}_c L\xi_{1st} + 2\omega'\hat{\beta}_c L\gamma_i\xi_{1st}^2 - \hat{\beta}_c L\gamma_i\xi_{1st}^2 \end{matrix}\right\} \tag{46}$$

Taking Expectation on equation (46) both sides we get the bias of the proposed estimator $t_2^{\prime(i)}$ as

$$Bias\left(t_2^{\prime(i)}\right) = \bar{Y}\{(\omega'\gamma_i^2 + 2\omega'\beta_c L\gamma_i - \beta_c L\gamma_i)D + (\gamma_i - 2\omega'\gamma_i)E\} \tag{47}$$

Squaring equation (46) and then taking expectation we get the MSE of $t_2^{\prime(i)}$

$$MSE\left(t_2^{\prime(i)}\right) = \bar{Y}^2[A + (\gamma_i - 2\omega'\gamma_i - \beta_c L)^2 D + 2(\gamma_i - 2\omega'\gamma_i - \beta_c L)E] (48)$$

For obtaining the optimum values of ω' , differentiating equation (48) w.r.t ω' and equating to zero we have,

$$\frac{\partial MSE\left(t_2^{\prime(i)}\right)}{\partial\omega'} = 0$$

$$\omega' = \frac{1}{2}\left[1 - \frac{1}{\gamma_i}\left(\beta_c L - \frac{E}{D}\right)\right]$$

$$\omega' = \frac{1}{2}\left[1 - \frac{1}{\gamma_i}\left(\beta_c L - \frac{E}{D}\right)\right] \tag{49}$$

Using the value of ω'in equation (48), the minimal MSE of the proposed estimators is given as

$$MSE\left(t_2^{\prime(i)}\right) = \bar{Y}^2\left[A - \frac{E^2}{D}\right] \tag{50}$$

From the above equation (50)it is clear that all the proposed estimators $t_2^{\prime(i)}$ have same MSE because the MSE is independent of γ_i.

3. EFFICIENCY COMPARISON

This section compares the optimal MSE of the proposed estimators with some existing estimators:

4.1 Situation I.

When there is non-response on both the study variable Yand the auxiliary variable X, and the auxiliary variable's population mean ($\bar{X}$) is known.

4.1.1. For Separate Regression Estimator

1. Comparison with Hansen and Hurwitz estimator

$t_1^{*(i)}$, $i = 1, 2, \ldots, 5$. Perform better than $\bar{y}^*$ if:

$$MSE\left(t_1^{*(i)}\right)_{min} < MSE(\bar{y}^*) \Rightarrow \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left\{A_h - \frac{C_h^2}{B_h}\right\} < \bar{Y}^2 A \Rightarrow \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left\{A_h - \frac{C_h^2}{B_h}\right\} - \bar{Y}^2 A < 0$$

2. Comparison with usual classical separate ratio estimator

$t_1^{*(i)}$, $i = 1, 2, \ldots, 5$ Perform better than $\bar{y}_{SR}^*$ if:

$$MSE\left(t_1^{*(i)}\right)_{min} < MSE(\bar{y}_{SR}^*) \Rightarrow \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left\{A_h - \frac{C_h^2}{B_h}\right\} < \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 (A_h + B_h - 2C_h) \Rightarrow \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left\{2C_h - B_h - \frac{C_h^2}{B_h}\right\} < 0$$

3. Comparison with usual classical combined ratio estimator

$t_1^{*(i)}$, $\;i = 1, 2, \dots, 5$ Perform better than $\bar{y}_{CR}^*$ if:

$$MSE\left(t_1^{*(i)}\right)_{min} < MSE(\bar{y}_{CR}^*) \Rightarrow \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left\{A_h - \frac{C_h^2}{B_h}\right\} < \bar{Y}^2(A + B - 2C) \Rightarrow \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left\{A_h - \frac{C_h^2}{B_h}\right\} - \bar{Y}^2(A + B - 2C) < 0$$

4. Comparison with usual classical separate regression estimator

$t_1^{*(i)}$, $\;i = 1, 2, \dots, 5$, performs better than $\bar{y}_{Sreg}^*$ if:

$$MSE\left(t_1^{*(i)}\right)_{min} < MSE\left(\bar{y}_{Sreg}^*\right) \Rightarrow \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left\{A_h - \frac{C_h^2}{B_h}\right\} < \sum_{h=1}^{L} W_h^2\left[\bar{Y}_h^2 A_h + \beta_{hyx}^2\bar{X}_h^2 B_h - 2\beta_{hyx}\bar{X}_h\bar{Y}_h C_h\right] \Rightarrow \sum_{h=1}^{L} W_h^2\left\{2\beta_{hyx}\bar{X}_h\bar{Y}_h C_h - \beta_{hyx}^2\bar{X}_h^2 B_h - \frac{\bar{Y}_h^2 C_h^2}{B_h}\right\} < 0$$

5. Comparison with usual classical combined regression estimator

$t_1^{*(i)}, 1, 2, \dots, 5$, performs better than $\bar{y}_{Creg}^*$ if:

$$MSE\left(t_1^{*(i)}\right)_{min} < MSE\left(\bar{y}_{Creg}^*\right) \Rightarrow \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left\{A_h - \frac{C_h^2}{B_h}\right\} < \bar{Y}^2 A + \beta_c^2\bar{X}^2 B - 2\beta_c\bar{X}\bar{Y}C \Rightarrow \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left\{A_h - \frac{C_h^2}{B_h}\right\} - \{\bar{Y}^2 A + \beta_c^2\bar{X}^2 B - 2\beta_c\bar{X}\bar{Y}C\} < 0$$

6. Comparison with Onyeka *et al.* (2019) estimator

$t_1^{*(i)}$, $\;i = 1, 2, \dots, 5$, performs better than $\bar{y}_{ok}^{*(i)}, i = 1, 2, \dots, 7$ if:

$$MSE\left(t_1^{*(i)}\right)_{min} < MSE\left(\bar{y}_{ok}^{*(i)}\right) \Rightarrow \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left\{A_h - \frac{C_h^2}{B_h}\right\} < \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\lambda_i^2 B_h - \lambda_i C_h\right] \Rightarrow \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left\{\lambda_i C_h - \frac{1}{4}\lambda_i^2 B_h - \frac{C_h^2}{B_h}\right\} < 0$$

4.1.2. For Combined Regression Estimator

1. Comparison with Hansen and Hurwitz estimator

$t_2^{*(i)}$, $\;i = 1, 2, \dots, 5$, performs better than $\bar{y}^*$ if:

$$MSE\left(t_2^{*(i)}\right)_{min} < MSE(\bar{y}^*) \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] < \bar{Y}^2 A \Rightarrow -\frac{C^2}{B} < 0$$

2. Comparison with usual classical separate ratio estimator

$t_2^{*(i)}$, $\;i = 1, 2, \dots, 5$, performs better than $\bar{y}_{SR}^*$ if:

$$MSE\left(t_2^{*(i)}\right)_{min} < MSE(\bar{y}_{SR}^*) \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] < \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2(A_h + B_h - 2C_h) \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] - \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2(A_h + B_h - 2C_h) < 0$$

3. Comparison with usual classical combined ratio estimator

$t_2^{*(i)}$, $\;i = 1, 2, \dots, 5$, performs better than $\bar{y}_{CR}^*$ if:

$$MSE\left(t_2^{*(i)}\right)_{min} < MSE(\bar{y}_{CR}^*) \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] < \bar{Y}^2(A + B - 2C) \Rightarrow \bar{Y}^2\left[2C - B - \frac{C^2}{B}\right] < 0$$

4. Comparison with usual classical separate regression estimator

$t_2^{*(i)}$, $\;i = 1, 2, \dots, 5$, performs better than $\bar{y}_{Sreg}^*$ if:

$$MSE\left(t_2^{*(i)}\right)_{min} < MSE\left(\bar{y}_{Sreg}^*\right) \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] < \sum_{h=1}^{L} W_h^2\left[\bar{Y}_h^2 A_h + \beta_{hyx}^2\bar{X}_h^2 B_h - 2\beta_{hyx}\bar{X}_h\bar{Y}_h C_h\right] \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] - \sum_{h=1}^{L} W_h^2\left[\bar{Y}_h^2 A_h + \beta_{hyx}^2\bar{X}_h^2 B_h - 2\beta_{hyx}\bar{X}_h\bar{Y}_h C_h\right] < 0$$

5. Comparison with usual classical combined regression estimator

$t_2^{*(i)}, 1, 2, \dots, 5$, performs better than $\bar{y}_{Creg}^*$ if:

$$MSE\left(t_2^{*(i)}\right)_{min} < MSE\left(\bar{y}_{Creg}^*\right) \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] < \bar{Y}^2 A + \beta_c^2\bar{X}^2 B - 2\beta_c\bar{X}\bar{Y}C \Rightarrow \left[2\beta_c\bar{X}\bar{Y}C - \beta_c^2\bar{X}^2 B - \frac{\bar{Y}^2 C^2}{B}\right] < 0$$

6. Comparison with Onyeka *et al.* (2019) estimator

$t_2^{*(i)}$, $\;i = 1, 2, \dots, 5$, performs better than $\bar{y}_{ok}^{*(i)}, i = 1, 2, \dots, 7$ if:

$$MSE\left(t_2^{*(i)}\right)_{min} < MSE\left(\bar{y}_{ok}^{*(i)}\right) \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] < \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\lambda_i^2 B_h - \lambda_i C_h\right] \Rightarrow \bar{Y}^2\left[A - \frac{C^2}{B}\right] - \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[A_h + \frac{1}{4}\lambda_i^2 B_h - \lambda_i C_h\right] < 0$$

4.2 Situation II.

When there is non-response on study variable Y only and information on the auxiliary variable X is obtained from all sample units, and the auxiliary variable's population mean $(\bar{X})$ is known.

4.2.1. For Separate Regression Estimator.

1. Comparison with Hansen and Hurwitz estimator

$t_1'^{(i)}$, $i = 1, 2, \ldots, 5$, performs better than $\bar{y}^*$ if:

$$MSE\left(t_1'^{(i)}\right)_{min} < MSE(\bar{y}^*) \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{E_h^2}{D_h}\right\} < \bar{Y}^2 A \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{E_h^2}{D_h}\right\} - \bar{Y}^2 A < 0$$

2. Comparison with usual classical separate ratio estimator

$t_1'^{(i)}$, $i = 1, 2, \ldots, 5$, performs better than $\bar{y}'_{SR}$ if:

$$MSE\left(t_1'^{(i)}\right)_{min} < MSE(\bar{y}'_{SR}) \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{E_h^2}{D_h}\right\} < \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 (A_h + D_h - 2E_h) \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{2E_h - D_h - \frac{E_h^2}{D}\right\} < 0$$

3. Comparison with usual classical combined ratio estimator

$t_1'^{(i)}$, $i = 1, 2, \ldots, 5$, performs better than $\bar{y}'_{CR}$ if:

$$MSE\left(t_1'^{(i)}\right)_{min} < MSE(\bar{y}'_{CR}) \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{E_h^2}{D_h}\right\} < \bar{Y}^2 (A + D - 2E) \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{E_h^2}{D_h}\right\} - \bar{Y}^2 (A + D - 2E) < 0$$

4. Comparison with usual classical separate regression estimator

$t_1'^{(i)}$, $i = 1, 2, \ldots, 5$, performs better than $\bar{y}'_{Sreg}$ if:

$$MSE\left(t_1'^{(i)}\right)_{min} < MSE\left(\bar{y}'_{Sreg}\right) \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{E_h^2}{D_h}\right\} < \sum_{h=1}^{L} W_h^2 \left[\bar{Y}_h^2 A_h + \beta_{hyx}^2 \bar{X}_h^2 D_h - 2\beta_{hyx} \bar{X}_h \bar{Y}_h E_h\right] \Rightarrow \sum_{h=1}^{L} W_h^2 \left\{2\beta_{hyx} \bar{X}_h \bar{Y}_h E_h - \beta_{hyx}^2 \bar{X}_h^2 D_h - \frac{\bar{Y}_h^2 E_h^2}{D_h}\right\} < 0$$

5. Comparison with usual classical combined regression estimator

$t_1'^{(i)}, 1, 2, \ldots, 5$, performs better than $\bar{y}'_{Creg}$ if:

$$MSE\left(t_1'^{(i)}\right)_{min} < MSE\left(\bar{y}'_{Creg}\right) \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{E_h^2}{D_h}\right\} < \bar{Y}^2 A + \beta_c^2 \bar{X}^2 D - 2\beta_c \bar{X}\bar{Y} E \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{C_h^2}{B_h}\right\} - \{\bar{Y}^2 A + \beta_c^2 \bar{X}^2 D - 2\beta_c \bar{X}\bar{Y} E\} < 0$$

6. Comparison with Onyeka et al. (2019) estimator

$t_1'^{(i)}$, $i = 1, 2, \ldots, 5$, performs better than $\bar{y}_{ok}'^{(i)}, i = 1, 2, \ldots, 7$ if:

$$MSE\left(t_3'^{(i)}\right)_{min} < MSE\left(\bar{y}_{ok}'^{(i)}\right) \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{A_h - \frac{E_h^2}{D_h}\right\} < \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left[A_h + \frac{1}{4}\lambda_i^2 D_h - \lambda_i E_h\right] \Rightarrow \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left\{\lambda_i E_h - \frac{1}{4}\lambda_i^2 D_h - \frac{E_h^2}{D_h}\right\} < 0$$

4.2.2. For Combined Regression Estimator.

1. Comparison with Hansen and Hurwitz estimator

$t_2'^{(i)}$, $i = 1, 2, \ldots, 5$ Performs better than $\bar{y}^*$ if:

$$MSE\left(t_2'^{(i)}\right)_{min} < MSE(\bar{y}^*) \Rightarrow \bar{Y}^2 \left[A - \frac{E^2}{D}\right] < \bar{Y}^2 A \Rightarrow -\frac{E^2}{D} < 0$$

2. Comparison with usual classical separate ratio estimator

$t_2'^{(i)}$, $i = 1, 2, \ldots, 5$, performs better than $\bar{y}'_{SR}$ if:

$$MSE\left(t_2'^{(i)}\right)_{min} < MSE(\bar{y}'_{SR}) \Rightarrow \bar{Y}^2 \left[A - \frac{E^2}{D}\right] < \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 (A_h + D_h - 2E_h) \Rightarrow \bar{Y}^2 \left[A - \frac{E^2}{D}\right] - \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 (A_h + D_h - 2E_h) < 0$$

3. Comparison with usual classical combined ratio estimator

$t_2'^{(i)}$, $i = 1, 2, \ldots, 5$, performs better than $\bar{y}'_{CR}$ if:

$$MSE\left(t_2'^{(i)}\right)_{min} < MSE(\bar{y}'_{CR}) \Rightarrow \bar{Y}^2 \left[A - \frac{E^2}{D}\right] < \bar{Y}^2 (A + D - 2E) \Rightarrow \bar{Y}^2 \left[2E - D - \frac{E^2}{D}\right] < 0$$

4. Comparison with usual classical separate regression estimator

$t_2'^{(i)}$, $i = 1, 2, \ldots, 5$, performs better than $\bar{y}'_{Sreg}$ if:

$$MSE\left(t_2'^{(i)}\right)_{min} < MSE\left(\bar{y}'_{Sreg}\right) \Rightarrow \bar{Y}^2 \left[A - \frac{E^2}{D}\right] < \sum_{h=1}^{L} W_h^2 \left[\bar{Y}_h^2 A_h + \beta_{hyx}^2 \bar{X}_h^2 D_h - 2\beta_{hyx} \bar{X}_h \bar{Y}_h E_h\right] \Rightarrow \bar{Y}^2 \left[A - \frac{E^2}{D}\right] - \sum_{h=1}^{L} W_h^2 \left[\bar{Y}_h^2 A_h + \beta_{hyx}^2 \bar{X}_h^2 D_h - 2\beta_{hyx} \bar{X}_h \bar{Y}_h E_h\right] < 0$$

5. Comparison with usual classical combined regression estimator

$t_2'^{(i)}, 1, 2, \ldots, 5$, performs better than $\bar{y}'_{Creg}$ if:

$$MSE\left(t_2'^{(i)}\right)_{min} < MSE\left(\bar{y}'_{Creg}\right) \Rightarrow \bar{Y}^2 \left[A - \frac{E^2}{D}\right] < \bar{Y}^2 A + \beta_c^2 \bar{X}^2 D - 2\beta_c \bar{X}\bar{Y} E \Rightarrow \left[2\beta_c \bar{X}\bar{Y} E - \beta_c^2 \bar{X}^2 D - \frac{\bar{Y}^2 E^2}{D}\right] < 0$$

6. Comparison with Onyeka *et al.* (2019) estimator

$t_2'^{(i)}$, $i = 1, 2, \dots, 5$, performs better than $\bar{y}_{ok}'^{(i)}, i = 1, 2, \dots, 7$ if:

$$MSE(t_2'^{(i)})_{min} < MSE(\bar{y}_{ok}'^{(i)}) \Rightarrow \bar{Y}^2\left[A - \frac{E^2}{D}\right] < \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2\left[A_h + \frac{1}{4}\lambda_i^2 D_h - \lambda_i E_h\right] \Rightarrow \bar{Y}^2\left[A - \frac{E^2}{D}\right] - \sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2\left[A_h + \frac{1}{4}\lambda_i^2 D_h - \lambda_i E_h\right] < 0$$

5. EMPIRICAL STUDY

In this section we investigate our theoretical results with the help of two real data sets.

Population I: (Source: Koyuncu and Kadilar (2009)). We consider No. of teachers as study variable *(Y)* and No. of classes in primary and secondary schools as auxiliary variable *(X)*, for 923 districts at six 6 regions (1: Marmara, 2: Agean, 3: Mediterranean,4: Central Anatolia, 5: Black Sea, and 6: East and Southeast Anatolia) in Turkey in2007.

Table 2 The descriptive statistics for Population I

h	1	2	3	4	5	6
N_h	127	117	103	170	205	201
n_h	31	21	29	38	22	39
S_{hy}	883.84	644.92	1033.40	810.58	403.65	771.72
S_{hx}	555.58	365.46	612.95	458.03	260.85	397.05
$\bar{Y}_h$	703.74	413	573.17	424.66	267.03	393.84
$\bar{X}_h$	498.28	318.33	431.36	311.32	227.2	313.71
$\beta_{2h(x)}$	2.3149	11.1909	10.7864	8.6241	9.7209	14.407
ρ_{hxy}	0.979	0.976	0.984	0.983	0.964	0.983
$\mathbf{W_{2h} = 10\%\ Non-response}$						
$S_{hy(2)}$	510.57	386.77	1872.88	1603.3	264.19	497.84
$S_{hx(2)}$	303.92	278.51	960.71	821.29	190.85	287.99
$\rho_{hxy(2)}$	0.9931	0.9871	0.9972	0.9942	0.985	0.9647
$\mathbf{W_{2h} = 20\%\ Non-response}$						
$S_{hy(2)}$	396.77	406.15	1654.40	1333.35	335.83	903.91
$S_{hx(2)}$	244.56	274.42	965.42	680.28	214.49	469.86
$\rho_{hxy(2)}$	0.9898	0.9798	0.9846	0.9940	0.9818	0.9874
$\mathbf{W_{2h} = 30\%\ Non-response}$						
$S_{hy(2)}$	500.26	356.95	1383.70	1193.47	289.41	825.24
$S_{hx(2)}$	284.44	247.63	811.21	631.28	188.30	437.90
$\rho_{hxy(2)}$	0.9739	0.9793	0.9839	0.9904	0.9799	0.9829

Population II:

(Source: Singh and Mangat (p no.369)). We consider No. of tube wells as study variable *(Y)* and No. of tractors as auxiliary variable*(X)*. We divided the population of size 69 into three strata and a sample is selected from each stratum by proportional allocation.

Table 3 The descriptive statistics for Population II

h	1	2	3
N_h	22	27	20
n_h	8	10	7
S_{yh}	122.73	117.55	102.68
S_{xh}	16.67	18.02	16.41
$\bar{Y}_h$	149.14	126.70	131.55
$\bar{X}_h$	22.59	20.26	21.05
$\beta_{2h(x)}$	3.95	6.49	1.49
ρ_{hxy}	0.939	0.961	0.800
$\mathbf{W_{2h} = 10\%\ Non-response}$			
$S_{hy(2)}$	31.82	56.32	153.44
$S_{hx(2)}$	6.36	9.54	24.04
$\rho_{hxy(2)}$	1.00	0.999	1.00
$\mathbf{W_{2h} = 20\%\ Non-response}$			
$S_{hy(2)}$	92.65	24.06	186.55
$S_{hx(2)}$	13.96	3.91	27.27
$\rho_{hxy(2)}$	0.920	0.726	0.900
$\mathbf{W_{2h} = 30\%\ Non-response}$			
$S_{hy(2)}$	70.76	98.84	127.72
$S_{hx(2)}$	12.05	16.16	11.76
$\rho_{hxy(2)}$	0.774	0.976	0.950

The %RE of the existing and proposed estimators with Hansen and Hurwitz estimator in the situation 1 and situation II for Population I are respectively in Table 4 and Table 5.

Table 4%RE of the existing and proposed estimators with Hansen and Hurwitz estimator. Situation I

$\mathbf{W_{2h} = 10\%\ Non-response}$					
Estimators	$K_h = 2$	$K_h = 2.5$	$K_h = 3$	$K_h = 3.5$	$K_h = 4$
$\bar{y}^*$	100	100	100	100	100
$\bar{y}^*_{SR}$	1021.72	1022.33	1019.68	1023.33	1023.73
$\bar{y}^*_{CR}$	1031.44	1023.39	1013.42	1010.73	1005.66
$\bar{y}^*_{Sreg}$	2525.79	2571.18	2603.44	2647.60	2680.06
$\bar{y}^*_{Creg}$	2266.25	2314.00	2367.05	2411.91	2450.27
$\bar{y}_{ok}^{*(1)}$	245.18	244.57	243.28	243.58	243.19
$\bar{y}_{ok}^{*(2)}$	244.13	243.52	242.24	242.54	242.14
$\bar{y}_{ok}^{*(3)}$	237.43	236.89	235.70	236.04	235.69
$\bar{y}_{ok}^{*(4)}$	244.56	243.95	242.68	242.98	242.59
$\bar{y}_{ok}^{*(5)}$	245.05	244.44	243.15	243.46	243.06
$\bar{y}_{ok}^{*(6)}$	244.39	243.78	242.50	242.80	242.41
$\bar{y}_{ok}^{*(7)}$	237.27	236.74	235.55	235.89	235.55

$\bar{y}_{ok}^{*(8)}$	1021.72	1022.33	1019.68	1023.33	1023.73
$t_1^{*(i)}$ $i = 1 - 5$	2564.04	2644.96	2717.59	2803.98	2884.16
$t_2^{*(i)}$ $i = 1 - 5$	2198.89	2256.98	2304.71	2363.76	2412.76
$\mathbf{W_{2h} = 20\%\ Non-response}$					
$\bar{y}^{*}$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_{SR}^{*}$	1019.46	1019.28	1019.10	1018.97	1018.85
$\bar{y}_{CR}^{*}$	1045.18	1032.84	1029.08	1026.05	1023.57
$\bar{y}_{Sreg}^{*}$	2662.10	2725.73	2794.82	2853.33	2903.71
$\bar{y}_{Creg}^{*}$	2280.98	2339.91	2405.31	2462.32	2512.65
$\bar{y}_{ok}^{*(1)}$	246.43	243.97	243.43	243.00	242.64
$\bar{y}_{ok}^{*(2)}$	245.37	242.92	242.38	241.95	241.59
$\bar{y}_{ok}^{*(3)}$	238.48	236.10	235.59	235.17	234.83
$\bar{y}_{ok}^{*(4)}$	245.80	243.36	242.83	242.40	242.05
$\bar{y}_{ok}^{*(5)}$	246.30	243.84	243.31	242.88	242.52
$\bar{y}_{ok}^{*(6)}$	245.63	243.18	242.65	242.22	241.86
$\bar{y}_{ok}^{*(7)}$	238.32	235.95	235.43	235.02	234.68
$\bar{y}_{ok}^{*(8)}$	1026.86	1019.28	1019.11	1018.97	1018.85
$t_1^{*(i)}$ $i = 1 - 5$	2694.45	2784.22	2880.78	2966.92	3043.94
$t_2^{*(i)}$ $i = 1 - 5$	2280.98	2339.91	2405.31	2462.32	2512.65
$\mathbf{W_{2h} = 30\%\ Non-response}$					
$\bar{y}^{*}$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_{SR}^{*}$	1027.85	1030.21	1032.04	1033.48	1034.66
$\bar{y}_{CR}^{*}$	1043.22	1040.35	1038.15	1036.42	1035.02
$\bar{y}_{Sreg}^{*}$	2573.49	2625.46	2666.63	2700.29	2728.23
$\bar{y}_{Creg}^{*}$	2231.68	2284.93	2327.42	2362.31	2391.28
$\bar{y}_{ok}^{*(1)}$	245.42	245.01	244.70	244.45	244.25
$\bar{y}_{ok}^{*(2)}$	244.36	243.95	243.63	243.35	243.18
$\bar{y}_{ok}^{*(3)}$	237.49	237.09	236.78	236.54	236.34
$\bar{y}_{ok}^{*(4)}$	244.80	244.39	244.08	243.84	243.64
$\bar{y}_{ok}^{*(5)}$	245.29	244.88	244.57	244.32	244.12
$\bar{y}_{ok}^{*(6)}$	244.62	244.21	243.90	243.66	243.46
$\bar{y}_{ok}^{*(7)}$	237.33	236.93	236.63	236.39	236.19
$\bar{y}_{ok}^{*(8)}$	1027.85	1030.21	1032.04	1033.48	1034.66
$t_1^{*(i)}$ $i = 1 - 5$	2595.87	2664.30	2722.05	2771.39	2814.02
$t_2^{*(i)}$ $i = 1 - 5$	2241.89	2302.20	2351.88	2393.46	2428.83

Table 5 -Situation II

$\mathbf{W_{2h} = 10\%\ Non-response}$					
Estimators	$K_h = 2$	$K_h = 2.5$	$K_h = 3$	$K_h = 3.5$	$K_h = 4$
$\bar{y}^{*}$	2776.10	3003.14	3220.19	3457.23	3684.28
$\bar{y}_{SR}'$	681.71	908.76	1135.80	1362.85	1589.89
$\bar{y}_{CR}'$	674.64	901.68	1128.73	1355.77	1582.82
$\bar{y}_{Sreg}'$	550.21	777.25	1004.30	1231.34	1458.37
$\bar{y}_{Creg}'$	566.14	793.18	1020.24	1247.29	1474.30
$\bar{y}_{ok}'^{(1)}$	1394.96	1622.01	1849.05	2076.10	2303.14
$\bar{y}_{ok}'^{(2)}$	1399.00	1626.04	1853.09	2080.13	2307.18
$\bar{y}_{ok}'^{(3)}$	1426.35	1653.40	1880.44	2107.49	2334.53
$\bar{y}_{ok}'^{(4)}$	1397.43	1624.47	1851.52	2078.56	2305.61
$\bar{y}_{ok}'^{(5)}$	1395.48	1622.52	1849.57	2076.61	2303.66
$\bar{y}_{ok}'^{(6)}$	1398.06	1625.11	1852.15	2079.20	2306.24
$\bar{y}_{ok}'^{(7)}$	1427.02	1654.10	1881.11	2108.16	2335.20
$\bar{y}_{ok}'^{(8)}$	681.71	908.76	1135.80	1362.85	1589.89
$t_1'^{(i)}$ $i = 1 - 5$	550.21	777.25	1004.30	1231.35	1458.39
$t_2'^{(i)}$ $i = 1 - 5$	490.36	378.62	315.63	277.18	249.90
$\mathbf{W_{2h} = 20\%\ Non-response}$					
$\bar{y}^{*}$	3169.92	3593.87	4017.83	4441.78	4865.74
$\bar{y}_{SR}'$	1075.53	1499.49	1923.44	2347.39	2771.35
$\bar{y}_{CR}'$	1068.46	1492.41	1916.37	2340.32	2764.27
$\bar{y}_{Sreg}'$	944.03	1367.98	1791.94	2215.89	2639.85
$\bar{y}_{Creg}'$	959.96	1383.91	1807.87	2231.82	2655.77
$\bar{y}_{ok}'^{(1)}$	1788.78	2212.74	2636.69	3060.65	3484.60
$\bar{y}_{ok}'^{(2)}$	1792.81	2216.77	2640.72	3064.68	3488.63
$\bar{y}_{ok}'^{(3)}$	1820.17	2244.12	2668.08	3092.03	3515.99
$\bar{y}_{ok}'^{(4)}$	1791.24	2215.20	2639.15	3063.11	3487.06
$\bar{y}_{ok}'^{(5)}$	1789.29	2213.25	2637.20	3061.16	3485.11
$\bar{y}_{ok}'^{(6)}$	1791.88	2215.84	2639.79	3063.75	3487.70
$\bar{y}_{ok}'^{(7)}$	1820.84	2244.80	2668.75	3092.71	3516.66
$\bar{y}_{ok}'^{(8)}$	1075.53	1499.49	1923.44	2347.39	2771.35
$t_1'^{(i)}$ $i = 1 - 5$	944.03	1367.98	1791.94	2215.89	2639.85
$t_2'^{(i)}$ $i = 1 - 5$	330.21	259.69	222.24	199.02	183.21
$\mathbf{W_{2h} = 30\%\ Non-response}$					
$\bar{y}^{*}$	3331.90	3836.85	4341.80	4846.75	5351.69
$\bar{y}_{SR}'$	1237.52	1742.46	2247.41	2752.36	3257.31
$\bar{y}_{CR}'$	1230.44	1735.39	2240.34	2745.28	3250.23
$\bar{y}_{Sreg}'$	1106.01	1610.96	2115.91	2620.86	3125.80
$\bar{y}_{Creg}'$	1121.93	1626.89	2131.89	2636.83	3141.77

$\bar{y}'^{(1)}_{ok}$	1950.77	2455.72	2960.66	3465.61	3970.56
$\bar{y}'^{(2)}_{ok}$	1954.80	2459.75	2964.69	3469.64	3974.59
$\bar{y}'^{(3)}_{ok}$	1982.15	2487.10	2992.05	3497.00	4001.94
$\bar{y}'^{(4)}_{ok}$	1953.23	2458.18	2963.12	3468.07	3973.02
$\bar{y}'^{(5)}_{ok}$	1951.28	2456.23	2961.17	3466.12	3971.07
$\bar{y}'^{(6)}_{ok}$	1953.87	2458.81	2963.76	3468.71	3973.66
$\bar{y}'^{(7)}_{ok}$	1982.83	2487.78	2992.72	3497.67	4002.62
$\bar{y}'^{(8)}_{ok}$	1237.52	1742.46	2247.41	2752.36	3257.31
$t'^{(i)}_1$ $i = 1-5$	1106.01	1610.96	2115.91	2620.86	3125.80
$t'^{(i)}_2$ $i = 1-5$	296.98	235.84	203.66	183.81	170.34

The %RE of the existing and proposed estimators with Hansen and Hurwitz estimator in the situation 1 and situation II for Population II are respectively in Table 6 and table 7

Table 6. Situation I

$\mathbf{W_{2h} = 10\%}$ **Non − response**					
Estimators	$K_h = 2$	$K_h = 2.5$	$K_h = 3$	$K_h = 3.5$	$K_h = 4$
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}^*_{SR}$	525.26	524.85	524.39	523.99	523.53
$\bar{y}^*_{CR}$	531.64	531.28	530.91	530.55	530.24
$\bar{y}^*_{Sreg}$	768.79	751.22	734.53	718.57	703.20
$\bar{y}^*_{Creg}$	572.98	568.65	564.39	560.14	556.00
$\bar{y}^{*(1)}_{ok}$	227.40	223.38	219.49	215.74	212.12
$\bar{y}^{*(2)}_{ok}$	219.54	215.51	211.64	207.90	204.28
$\bar{y}^{*(3)}_{ok}$	192.05	188.61	185.28	182.07	178.98
$\bar{y}^{*(4)}_{ok}$	217.10	213.08	209.21	205.48	201.88
$\bar{y}^{*(5)}_{ok}$	225.27	221.15	217.19	213.36	209.66
$\bar{y}^{*(6)}_{ok}$	218.73	214.72	210.86	207.13	203.59
$\bar{y}^{*(7)}_{ok}$	190.57	187.09	183.74	180.50	177.38
$\bar{y}^{*(8)}_{ok}$	525.26	524.85	524.39	523.99	523.53
$t^{*(i)}_1$ $i = 1-5$	773.28	759.96	748.21	737.63	728.03
$t^{*(i)}_2$ $i = 1-5$	573.89	588.54	603.39	562.13	633.41
$\mathbf{W_{2h} = 20\%}$ **Non − response**					
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}^*_{SR}$	498.34	500.46	502.32	503.89	505.31
$\bar{y}^*_{CR}$	503.23	505.04	506.54	507.88	509.03
$\bar{y}^*_{Sreg}$	674.60	648.31	627.48	610.57	596.58
$\bar{y}^*_{Creg}$	523.13	515.35	508.86	503.40	498.74
$\bar{y}^{*(1)}_{ok}$	227.49	230.10	232.36	234.34	236.10
$\bar{y}^{*(2)}_{ok}$	219.82	222.28	224.42	226.30	227.96
$\bar{y}^{*(3)}_{ok}$	194.36	197.45	200.16	202.55	204.67
$\bar{y}^{*(4)}_{ok}$	218.04	219.75	221.82	223.65	225.27
$\bar{y}^{*(5)}_{ok}$	225.17	227.55	229.61	231.43	233.03
$\bar{y}^{*(6)}_{ok}$	219.03	221.50	223.64	225.53	227.20
$\bar{y}^{*(7)}_{ok}$	192.79	195.77	198.38	200.68	202.73
$\bar{y}^{*(8)}_{ok}$	498.34	500.46	502.32	503.89	505.31
$t^{*(i)}_1$ $i = 1-5$	683.29	668.00	645.66	632.52	621.76
$t^{*(i)}_2$ $i = 1-5$	527.67	523.42	520.64	518.82	517.61
$\mathbf{W_{2h} = 30\%}$ **Non − response**					
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}^*_{SR}$	487.87	486.07	484.61	483.37	482.28
$\bar{y}^*_{CR}$	495.85	494.87	494.05	493.35	492.76
$\bar{y}^*_{Sreg}$	550.27	500.41	465.16	438.87	418.55
$\bar{y}^*_{Creg}$	525.03	518.10	512.43	507.71	503.69
$\bar{y}^{*(1)}_{ok}$	223.85	224.93	225.85	226.64	227.32
$\bar{y}^{*(2)}_{ok}$	216.34	217.36	218.23	218.98	219.62
$\bar{y}^{*(3)}_{ok}$	189.91	191.12	192.15	193.03	193.79
$\bar{y}^{*(4)}_{ok}$	214.04	215.07	215.93	216.68	217.32
$\bar{y}^{*(5)}_{ok}$	221.79	222.78	223.61	224.33	224.94
$\bar{y}^{*(6)}_{ok}$	215.58	216.62	217.50	218.25	218.91
$\bar{y}^{*(7)}_{ok}$	188.46	189.63	190.62	191.46	192.19
$\bar{y}^{*(8)}_{ok}$	487.87	486.07	484.61	483.37	482.28
$t^{*(i)}_1$ $i = 1-5$	588.09	560.52	545.41	537.27	533.31
$t^{*(i)}_2$ $i = 1-5$	526.79	521.22	516.96	513.60	510.86

Table 7 . Situation II

$\mathbf{W_{2h} = 10\%}$ **Non − response**					
Estimators	$K_h = 2$	$K_h = 2.5$	$K_h = 3$	$K_h = 3.5$	$K_h = 4$
$\bar{y}^*$	100.00	100.00	100.00	100.00	109.00
$\bar{y}'_{SR}$	394.49	361.74	335.55	314.13	296.27
$\bar{y}'_{CR}$	397.99	364.57	337.91	316.13	298.02
$\bar{y}'_{Sreg}$	533.52	470.79	423.92	387.56	358.53
$\bar{y}'_{Creg}$	425.00	386.40	355.99	331.43	311.16
$\bar{y}'^{(1)}_{ok}$	205.20	198.76	193.08	188.01	183.47
$\bar{y}'^{(2)}_{ok}$	199.24	193.34	188.10	183.42	179.22
$\bar{y}'^{(3)}_{ok}$	176.96	172.88	169.20	165.87	162.86
$\bar{y}'^{(4)}_{ok}$	197.37	191.63	186.53	181.97	177.87
$\bar{y}'^{(5)}_{ok}$	203.74	197.45	191.87	186.90	182.43
$\bar{y}'^{(6)}_{ok}$	198.59	192.75	187.56	182.92	178.75
$\bar{y}'^{(7)}_{ok}$	175.85	171.84	168.24	164.98	162.01

$\bar{y}_{ok}^{\prime(8)}$	394.49	361.74	335.55	314.13	296.27
$t_1^{\prime(i)}$ $i = 1 - 5$	533.52	470.79	423.92	387.56	358.53
$t_2^{\prime(i)}$ $i = 1 - 5$	425.00	386.40	355.90	331.43	311.16
$\mathbf{W_{2h} = 20\%}$ **Non − response**					
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}'_{SR}$	292.61	253.50	227.59	209.16	195.39
$\bar{y}'_{CR}$	294.29	254.66	228.45	209.84	195.95
$\bar{y}'_{Sreg}$	352.72	293.34	256.55	231.52	213.40
$\bar{y}'_{Creg}$	307.03	263.35	234.89	214.88	200.04
$\bar{y}_{ok}^{\prime(1)}$	182.50	171.19	162.60	155.86	150.44
$\bar{y}_{ok}^{\prime(2)}$	178.32	167.79	159.76	153.43	148.31
$\bar{y}_{ok}^{\prime(3)}$	162.20	154.51	148.50	143.69	139.75
$\bar{y}_{ok}^{\prime(4)}$	176.99	166.71	158.85	152.65	147.63
$\bar{y}_{ok}^{\prime(5)}$	181.48	170.36	161.91	155.27	149.92
$\bar{y}_{ok}^{\prime(6)}$	177.86	167.41	159.44	153.15	148.07
$\bar{y}_{ok}^{\prime(7)}$	161.38	153.81	147.91	143.17	139.29
$\bar{y}_{ok}^{\prime(8)}$	292.61	253.50	227.59	209.16	195.39
$t_1^{\prime(i)}$ $i = 1 - 5$	352.72	293.34	256.55	231.52	213.40
$t_2^{\prime(i)}$ $i = 1 - 5$	307.03	263.35	234.89	214.88	200.04
$\mathbf{W_{2h} = 30\%}$ **Non − response**					
$\bar{y}^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}'_{SR}$	279.32	241.00	216.18	198.79	185.93
$\bar{y}'_{CR}$	280.82	242.02	216.93	199.38	186.41
$\bar{y}'_{Sreg}$	332.00	275.37	240.96	217.83	201.23
$\bar{y}'_{Creg}$	292.09	249.58	222.48	203.70	189.91
$\bar{y}_{ok}^{\prime(1)}$	178.85	167.17	158.50	151.81	146.50
$\bar{y}_{ok}^{\prime(2)}$	174.93	164.03	155.90	149.61	144.58
$\bar{y}_{ok}^{\prime(3)}$	159.75	151.71	145.58	140.75	136.85
$\bar{y}_{ok}^{\prime(4)}$	173.69	163.04	155.08	148.90	143.97
$\bar{y}_{ok}^{\prime(5)}$	177.90	166.41	157.87	151.28	146.03
$\bar{y}_{ok}^{\prime(6)}$	174.50	163.69	155.62	149.36	144.37
$\bar{y}_{ok}^{\prime(7)}$	158.97	151.07	145.04	140.28	136.43
$\bar{y}_{ok}^{\prime(8)}$	300.02	262.35	237.40	219.66	206.39
$t_1^{\prime(i)}$ $i = 1 - 5$	332.00	275.37	240.96	217.83	201.23
$t_2^{\prime(i)}$ $i = 1 - 5$	292.09	249.58	222.48	203.70	189.91

6. CONCLUSION

In this article we have considered the issue of determining the population mean in stratified random sampling in the situation of non-response under two conditions. The properties of improved versions of separate and combined regression type estimators for finite population means have been examined up to the first degree of approximation under large sample conditions, and their properties have been discussed. The proposed estimators $t_1^{*(i)}; i = 1 - 5$have the same optimal MSE and also $t_2^{*(i)}; i = 1 - 5$ have the same MSE. Also the proposed estimators $t_1^{\prime(i)}; i = 1 - 5$have the same optimal MSE and also $t_2^{\prime(i)}; i = 1 - 5$ have the same MSE. The %RE of the estimators $t_1^{*(i)}; i = 1 - 5$and $t_2^{*(i)}; i = 1 - 5$have been compared with existing estimators for different values of K and response rates. Also, for Situation II the estimator $t_1^{\prime(i)}; i = 1 - 5$and $t_2^{\prime(i)}; i = 1 - 5$have also been compared with other existing estimators for different values of K and response rates. The proposed estimators $t_1^{*(i)}; i = 1 - 5$ is more efficient than$\bar{y}^*$, $\bar{y}^*_{SR}, \bar{y}^*_{CR}, \bar{y}^*_{Sreg}, \bar{y}^*_{Creg}, \bar{y}_{ok}^{*(1)}, \bar{y}_{ok}^{*(2)}, \bar{y}_{ok}^{*(3)}, \bar{y}_{ok}^{*(4)}, \bar{y}_{ok}^{*(5)}, \bar{y}_{ok}^{*(6)}, \bar{y}_{ok}^{*(7)}, \bar{y}_{ok}^{*(8)}$and the proposed estimators$t_2^{*(i)}; i = 1 - 5$ is more efficient than$\bar{y}^*$, $\bar{y}^*_{SR}, \bar{y}^*_{CR}, , \bar{y}^*_{Creg}, \bar{y}_{ok}^{*(1)}, \bar{y}_{ok}^{*(2)}, \bar{y}_{ok}^{*(3)}, \bar{y}_{ok}^{*(4)}, \bar{y}_{ok}^{*(5)}, \bar{y}_{ok}^{*(6)}, \bar{y}_{ok}^{*(7)}, \bar{y}_{ok}^{*(8)}$ for situation I of non-response. Also The proposed estimators $t_1^{\prime(i)}; i = 1 - 5$ is more efficient than $\bar{y}^*$, $\bar{y}'_{SR}, \bar{y}'_{CR}$, $\bar{y}'_{Creg}, \bar{y}_{ok}^{\prime(1)}, \bar{y}_{ok}^{\prime(2)}, \bar{y}_{ok}^{\prime(3)}, \bar{y}_{ok}^{\prime(4)}, \bar{y}_{ok}^{\prime(5)}, \bar{y}_{ok}^{\prime(6)}, \bar{y}_{ok}^{\prime(7)}, \bar{y}_{ok}^{\prime(8)}$ but are equally efficient to $\bar{y}'_{Sreg}$and the proposed estimators $t_2^{\prime(i)}; i = 1 - 5$ is more efficient than$\bar{y}^*$, $\bar{y}'_{SR}, \bar{y}'_{CR}$, $\bar{y}_{ok}^{\prime(1)}, \bar{y}_{ok}^{\prime(2)}, \bar{y}_{ok}^{\prime(3)}, \bar{y}_{ok}^{\prime(4)}, \bar{y}_{ok}^{\prime(5)}, \bar{y}_{ok}^{\prime(6)}, \bar{y}_{ok}^{\prime(7)}, \bar{y}_{ok}^{\prime(8)}$ but are equally efficient to , $\bar{y}'_{Creg}$ for situation II of non-response. The areas where the proposed estimators to be more efficient than competitors are obtained. To evaluate the benefits of the proposed estimators over others, a real data sets was investigated in support of the current study. According to empirical study, the proposed estimators are more efficient than existing estimators.

RECEIVED: MARCH, 2023.
REVISED: MAY, 2023.

REFERENCES

[1] BOUZA, C.N. (2008): Estimation of population mean with missing observations using product type estimator. **Revista Investigacion Operacional**,29, 207-223
[2] CHAUDHARY, M. K. and KUMAR, A. (2015): Estimating the population mean in stratified sampling using two phase sampling in the presence of non-response. **World Applied Sciences Journal**, 33 ,874–82.
[3] CHAUDHURY, M. K., SINGH, R., SHUKLA, R. K., KUMAR, M. and SMARANDACHE, F. (2009): A family of estimators for estimating population mean in stratified sampling under non-response. **Pakistan Journal of Statistics and Operation Research**, 5, 47-54.
[4] COCHRAN, W. G. (1977): **Sampling techniques**, 3rd Edition, John Wiley: New York.
[5] HANSEN, M. H. and HURWITZ, W. N. (1946): The problem of non-response in sample surveys. **Journal of the American Statistical Association**, 41, 517-529.
[6] ISMAIL, M. and SHAHBAZ, M.Q. (2017): Combination of Ratio and Regression Estimator of Population Mean in Presence of Non–response. **GU J. Sci**., 30, 634-642
[7] KADILAR, C. and H.CINGI(2005): A new ratio estimator in stratified random sampling.**Communications in Statistics-Theory and Methods**, 34, 597-602.
[8] KHARE, B. B. and SRIVASTAVA, S. (1997): Transformed ratio-type estimators for the population mean in the presence of non-response, **Communications in Statistics-Theory and Methods**, 26, 1779- 1791.
[9] KOYUNCU, N. and KADILAR, C. (2009): Ratio and product estimators in stratified random sampling, **Journal of Statistical Planning and Inference**, 139, 2552-2558.
[10]SING, R. and MANGAT, N.S. (1996): **Elements of survey sampling**. Kluwer Academic Publisher-Dordrecht-Boston.
[11] ONYEKA, A.C., OGBUMUO, D.T. and IZUNOBI, C. (2019): Estimation of population mean in stratified random sampling when using auxiliary information in the presence of non-response. **Far East Journal of Theoretical Statistics**, 55, 151-167
[12] RAO, P. S. R. S. (1986): Ratio estimation with sub-sampling the non-respondents, **Survey Methodology**, 12, 217-230.
[13] SINGH, H.P. and ESPEJO, M.R. (2003): On linear regression and ratio-product estimation of a finite population mean, **The statistician**, 52, 59-67, 2003
[14] SALEEM, I., SANAULLAH, A. and HANIF, M. (2018): A generalized class of estimators for estimating population mean in the presence of non-response, **Journal of Statistical Theory and Applications**, 17, 616-626.
[15] SANAULLAH, A., NOOR-UL-AMIN, M. and HANIF, M. (2015): Generalized exponential-type ratio-cum-ratio and product-cum-product estimators for population mean in the presence of non-response under stratified two-phase random sampling. **Pakistan Journal of Statistics and Operation Research**, 31, 71-94.
[16] SEARLS, D. T. (1964): The utilization of known coefficient of variation in the estimation procedure, **Journal of the American Statistical Association**, 59, 1225-1226.
[17]SHABBIR, J., GUPTA, S. and AHMED, S. (2019): A generalized class of estimators under two-phase stratified sampling for non-response. **Communications in Statistics-Theory and Methods**, 48, 3761- 3777.
[18]SINGH, H. P. and KUMAR, S. (2008): A general family of estimators of finite population ratio, product and mean using two-phase sampling scheme in presence of non-response, **Journal of Statistical Theory and Practice**, 2, 677-692.
[19]SINGH, H. P. and KUMAR, S. (2009): A general procedure of estimating the population mean in the presence of non-response under double sampling using auxiliary information, **SORT**, 33, 71-84.
[20]SINGH, H. P., KUMAR, S. and KOZAK, M. (2010): Improved estimation of finite population mean using sub-sampling to deal with non-response in two-phase sampling scheme. **Communications in Statistics-Theory and Methods**, 39, 791-802.
[21]SINGH, N. and VISHWAKARMA, G. K. (2019): A generalized class of estimator of population mean with the combined effect of measurement errors and non-response in sample survey, **Investigación Operacional**, 40, 275-285.
[22]SINGH, R. and SHARMA, P. (2015): Method of estimation in the presence of non-response and measurement errors simultaneously, **Journal of Modern Applied Statistical Methods**, 14, 107-114.
[23]SINGH, R., SINGH, R. and BOUZA, C. N. (2018): Effect to measurement error and non-response on estimation of population mean, **Revista Investigación Operacional**, 39, 108-120.
[24] WANI, Z.H., RIZVI, S.E.H. SHARMA, M. and BHAT, M.I.J. (2021): Efficient Class of Combined Ratio Type Estimators for Estimating the Population Mean under Non-response. **International Journal of Scientific Research in Mathematical and Statistical Sciences**.8(6) 01-06.

[25] WANI, Z.H., YOUSUF, R., RIZVI, S.E.H. and JEELANI, M.I. (2022): Efficient Class of exponential type estimators of finite population mean in presence of non-response. **International Journal of Statistics and Reliability Engineering**. 9(1), 1-10.
[26]WANI, Z.H., RIZVI, S.E.H., (2022):Enhanced Generalized class of exponential estimators for estimating the population mean in the case of non-response**. International Journal of Statistics and Reliability Engineering**. 9, 61-77.
[27] WANI, Z.H., YOUSUF, R., RIZVI, S.E.H., MUSHTAQ, S. (2022): Modified Ratio-cum- Exponential Estimator for Appraising the Population Mean using Conventional and non-Conventional Parameters. **An International Journal of Statistics Applications & Probability Letters**. 9, 49-61.

.

.