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A NEW APPROACH FOR MODELLING OF THE GLOBAL WARMING DATA USING TRIGONOMETRIC REGRESSION MODELS: INFERENCE AND PROPERTIES

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ABSTRACT
Climate change is one of the biggest challenges that we face in this life, and especially Global Warming, which is one aspect of climate change taking place in this world with huge impacts globally and harmful to all of humanity. Among the causes of this problem are human actions that lead to an increase in harmful gases and a rise in the temperature of the earth and the atmosphere, which is the result of Global Warming. The effects of climate change and Global Warming require the concerted efforts of countries and researchers in addressing and studying the causes that lead to this problem and mitigating its harmful effects.
In this paper, the modelling of a big data set of Global Warming is developed using of the trigonometric regression models with heavy statistical analyses. The data of daily temperatures is collected from several cities over a 50 year period and estimating the parameters of the Sine model for the biggest three cities of Iraq (Baghdad, Mosul and Basra), testing each parameter estimator, testing the overall estimated model of the whole period and for each period/each city, and for Max, and Min temperatures are developed. In addition, fitting the Sine model for each city/period and for Max, and Min temperatures, estimating the parameters by the least squares principle, and testing each model by ANOVA are also developed for getting accurate fitting.

KEYWORDS: Daily Temperatures; Global Warming; Trigonometric Regression Models; Modeling; Fitting

MSC: 60E99, 62F10, 62P12

RESUMEN
El cambio climático es uno de los mayores desafíos que enfrentamos en esta vida, y especialmente el calentamiento global, que es un aspecto del cambio climático que está teniendo lugar en este mundo con enormes impactos a nivel global y perjudiciales para toda la humanidad. Entre las causas de este problema se encuentran las acciones humanas que conllevan un aumento de gases nocivos y un aumento de la temperatura de la tierra y la atmósfera, que es resultado del Calentamiento Global. Los efectos del cambio climático y el calentamiento global requieren esfuerzos concertados de países e investigadores para abordar y estudiar las causas que conducen a este problema y mitigar sus efectos nocivos.
En este artículo, se desarrolla el modelado de un gran conjunto de datos sobre el Calentamiento Global utilizando modelos de regresión trigonométrica con intensos análisis estadísticos. Los datos de temperaturas diarias se recopilan de varias ciudades durante un período de 50 años y se estiman los parámetros del modelo Sine para las tres ciudades más grandes de Irak (Bagdad, Mosul y Basora), probando cada estimador de parámetros, probando el modelo estimado general de la Se desarrollan todo el período y para cada período/cada ciudad, y para las temperaturas máximas y mínimas. Además, también se desarrollan el ajuste del modelo sinusoidal para cada ciudad/período y para temperaturas máximas y mínimas, la estimación de los parámetros mediante el principio de mínimos cuadrados y la prueba de cada modelo mediante ANOVA para obtener un ajuste preciso.

PALABRAS CLAVE: Temperaturas Diarias; Calentamiento global; Modelos de Regresión Trigonométrica; Modelado; Adecuado

1. INTRODUCTION

The world is going through a number of difficult circumstances such as wars with economic and political motives; and a number of climatic changes such as global warming, which have a significant impact on the overall results of social and economic processes of societies, which require high attention from the governments to the dangers of these conditions on the future of life on this planet, and at the same time require focus on the scientific research on these phenomena, and assessment of their potential risks.

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There is much evidence of the occurrence of Global Warming, as a result of unregulated human industrial actions, which have led to an increase of harmful gases in the atmosphere. Also, many studies have expected an increase in these gases, which leads to global warming, and these studies show that that this phenomenon has a global impact, and the world will be exposed to waves of high and frequent heat (see, Schliep, et. al., 2021; Soldatenko, et. Al., 2021; Lynas, et al., 2021; Anderegg, et. al., 2020; Daneshvar, et al., 2019; Mudelsee, 2019; Ollila, 2019; Tsokos, 2014).

It is necessary to distinguish simply, between climate change and Global Warming. A plethora of literature has suggested that Global Warming refers only to an increase in the surface temperature of the Earth, while climate change is more general than Global Warming, it consists of Global Warming and its effects - such as melting glaciers, heavy rainstorms, or frequent droughts. Thus, Global Warming is just one symptom of a much bigger problem of climate change (https://www.nationalgeographic.com/environment/article/global-warming-overview, ibid). In addition, there are several definitions of Global Warming, and it can be defined in different forms; but a simple definition can be a "set of changes to the earth's climate" (ibid).

Now, the world is unanimously witnessing that the climate change in general and Global Warming in particular have very important effects on society. Unfortunately, many studies have shown that there is no doubt that the climate change is directly or indirectly man-made. Moreover, the issue of mitigating Global Warming is one of the necessities agreed upon globally. In addition, the issues of addressing Global Warming, changing rainfall and mitigating it have become political issues that witness more discussions and less work. Although many researchers have made various efforts to study the problem of Global Warming and climate change by publishing various papers and holding scientific conferences, the efforts of the governments are meagre by comparison and do not fit the problem (ibid).

It is possible to represent humans and the environment with two dependent variables as humans work on the earth and coexist with different environmental and climatic conditions. On the one hand, some human activities, especially the industrial and construction, have multiple and fundamental effects on the environment, and these effects may lead to changing the characteristics of the environment. On the other hand, the environmental and climatic conditions affect humans in many aspects, in particular, it affects their health and viability and changes their ability to survive (William, et al., 2010; Butler, 2018; Cianconi, et al., 2021; Lynas, et al., 2021; Soldatenko, et al., 2021).

Human activities, especially the industrial and construction have led to a dangerous and unprecedented change in the characteristics of the current climate, and many regions of the Earth have experienced unusual and yet expected increases in average surface temperature. Among the most obvious consequences of unreasonable human activities is the high concentration of carbon dioxide (CO_2) in the atmosphere. The combustion of fossil fuels generates gases that lead to the same result, which caused the emergence of the Global Warming phenomenon and its interrelated and unexpected problems (ibid).

There is no doubt that there is significant change in climate in general and in Global Warming particularly. These changes have altered the climatic conditions in almost all regions of the world and thus have negatively affected all regions of the world, but this effect varies greatly from one region to another. It is necessary to study that impact and its risks in each region that has been affected or will be affected by it later. For this purpose, it is necessary to develop mathematical modelling that should be free of the complexity of Global Warming, in order to study the phenomenon accurately (ibid).

Mathematical sciences are very powerful and accurate tools, methods, and methodologies for understanding the complex problems that exist in various aspects of life, and helping all other sciences achieve the same purpose, including many practical problems in the planets, including environmental problems, and climate change (Soldatenko, et al., 2021). In addition, the mathematical modelling/ model fitting has remained effective and useful methods for studying climate phenomena in general, and also accurate tools used to predict environmental phenomena, climate change, and Global Warming over several decades (William, et al., 2010; Butler, 2018; Cianconi, et al., 2021; Lynas, et al., 2021; Soldatenko, et al., 2021).

The importance of applying trigonometric models in modelling the data of climate change and global warming is very clear in studying the data of various phenomena, especially the data of weather change and Global Warming. However, where many of the properties of the other statistical methods and measures are not assured in the data of weather change and global warming, such as the homogeneity of the data, and the phenomenon, then the data is affected by many reasons that change its nature. Therefore, trigonometric models are used because they are more compatible with the data and thus give efficient fitting (see, Arbela´ez et al. 2021; Zhu, et al., 2021; Olatayo, et al., 2021; King, et al., 2020).

The outline of this paper contains the following Sections; and a simple review for each Section is: the justifications of the studied problem and the objectives of this paper are given in Section 2. Section 3 is allocated for the literature

of the subject. Modelling the temperature data using the trigonometric models, the technique of estimating the proposed model and the results of Global Warning using the trigonometric models are discussed in Sections 4 and 5 respectively. Sections 6 and 7 are for the results discussion, limitations, and future directions.

2. THE JUSTIFICATIONS AND OBJECTIVES

This paper will contribute to national and international studies by studying the problem of Global Warming in Iraq with a reliable scientific methodology that has formed the basis for previous studies and includes studying the dimensions of the problem and its impact in a comprehensive manner. In addition, the paper will study the level of Iraq's impact on this phenomenon and how it is modeled for the purpose of study and prediction, and, through the reliance of the research on the data of a long time period of 50 years.
The modelling of Global Warming is an internationally recognized problem; and the model is developed by estimating and studying its features, studying its suitability for those data, and its accuracy in predicting the phenomenon in the three largest governorates.
It is necessary to clarify that there are many motives for studying the Global Warming phenomenon. One of the first is to shed light for the purpose of increasing popular interest in it, and addressing its comprehensive effects on the sectors of society in order to mitigate its effects. Second, we did not notice the existence of systematic and comprehensive studies of this phenomenon in Iraq, meaning that the second motivation is to provide some studies on this problem, i.e. to close the gap of national and international information related to this problem in Iraq, knowing that Iraq and its neighboring countries have begun to suffer from high temperature.
Building an effective model without mathematical complications and difficulties, to make accurate predictions of this problem is the third motivation. The expectation of this problem is a very important issue, in order to understand the potential effects of the phenomenon on the environment, the economy, agriculture, humans,... etc. In fact, accurate and long predictions on this problem are not available in Iraq and neighboring countries.
It is noteworthy that one of the most widely cited models for understanding scientific reasoning is induction and deduction. As is well known, induction is a process generalization of specific examples. Moreover, there are many climate models available and currently applied in studying the Global Warming, but most of them are complex, and all of them include many variables. Some of these variables are well measured, and some are not measured, which will greatly affect the estimation of the model parameters and make the model building process of Global Warming inaccurate (see, Oreskes, 2005). The above reasons explain why we are developing different models but simple and accurate ones.
As a final note regarding the justifications, a distinction must be made between the modeling objective of the daily maximum temperatures of a particular city or country, and the modeling of maximum and minimum temperatures of the same city or country for the purposes of identifying the characteristics of Global Warming, and predictions related to Global Warming, which will be done in the methodology of this paper. The available periods of the maximum/minimum temperatures will be divided into two equal periods, and the methodology will be applied into each period to find out the amount and significance of the changes taking place in it.
The outline of this paper contains the following objectives [to]:

a) review the problem of climate changes/global warming and its impact;
b) discuss the published works in modelling the climate changes/global warming;
c) review, discuss and apply the method of trigonometric regression models to three big cities for data of a 50-year period;
d) develop heavy statistical analysis to the data, estimators of parameters and results;
e) highlight the importance of applying the Trigonometric Regression Model, and the corresponding results and findings;
f) give and review the results of modelling the data of Global Warning;
g) discuss the results of fitting the models of Global Warning;
h) draw some conclusions, remarks, limitations of the problem and future direction.

3. REVIEW OF LITERATURE

While reviewing the sources of temperature (Maximum/ Minimum) modeling, or Global Warming, we will find a huge amount of relevant papers and studies. In addition, we will find a large number of papers dealing with the relationship between man and Global Warming. We will also find a large number of papers related to the use of mixed distributions in modeling temperature or Global Warming. We will also find a good set of research related to modeling temperature (Maximum/ Minimum) or Global Warming using the trigonometric models.

As we explained in the previous two Sections one of the objectives of this research is not to use complex models, the application of which requires specialists. Therefore, in this Section we will present a very brief survey of the research on this topic, focusing on papers related to modeling of trigonometric functions, with fast and simplify referring to research related to other methods as much as possible.
Searching for the references to this topic, we find a consensus by a large number of researchers that human industrial or construction work has the greatest role in climate change and Global Warming. This is of great importance and impact on society. In addition, if we find some articles that doubt this fact, they have political motives, and negatively affect the ability to tackle the problem of Global Warming and the resulting emerging problems. Some of the references on this point are: William, et al., 2010; Butler, 2018; Cianconi, et al., 2021; Lynas, et al., 2021 and Soldatenko, et al., 2021.
Moreover, we find a huge number of sources that apply mixed distributions to temperature and Global Warming, and the associated methods for finding estimates (MLE and the EM algorithm) of the parameters of those distributions. There is no doubt about the accuracy of these methods, but their applications are surrounded by many mathematical problems. Some of the references which are related to the modelling of the data of the Global Warming using the mixed distributions are: McLachlan et al., 2004; Hunter and Lange, 2004; McLachlan, et. al., 2004; Hunter, et al., 2007; Chen, et al., 2008;; Prates, et al., 2013; McLachlan, et al., 2017; Aleksandrova, et al., 2018; Geissen, et. al., 2018; Chauveau, et al., 2019; Negarestani, et al., 2019; Ochoa, et al., 2019; Geissen,, et al., 2019; Sahabi-Abed, S., 2022 and Kourat, et. al., 2022.
It has also been noted that there are many engineering and environmental applications of trigonometric regression models, and we will list some of them as follows: Zielinski, et al., 2014 modelling the circadian data using trigonometric functions. ÇELİK, 2016 applied the trigonometric functions in modelling the relative humidity. Akinnubi and Adeniyi, 2017 studied the diurnal pattern of air temperature using trigonometric functions. Olatayo, et al., 2018 studied the effect of sinusoidal movement on rainfall by the trigonometric functions in western Nigeria. King, et al., 2020 studied the problem of solar energy in western Michigan and modelled its data using Trigonometric function. Zhu, et al., 2021 studied and applied different trigonometric function of water vapor in China.
It may be worthwhile mentioning that several researchers have obviated the problems of the data of climate change and global warming data by proposing the trigonometric regression models to model the data of these phenomenon, and estimated the parameters using suitable techniques, and so have forecast the future behavior of these phenomena. A good example regarding this problem is the data of the phenomenon "Atmospheric Carbon Dioxide" (CO_2) trapped in some long-existing bubbles in polar ice published by NOAA (https://gml.noaa.gov/ccgg/trends/) had been plotted and it was fitted by two different Sine functions, where the results of the graphs of Sine functions were fairly acceptable in representing the data of this phenomenon (https://ibmathsresources.com/tag/climate-change/).
Moreover, some other examples exist, and two additional examples are the pattern of the "relative change in solar output" and the "change in heat content of oceans" (Wang, and Chameides, 2007, pages 4-5), and the "ocean heat transports" (Schmidt, et al., 2014, page 174 and Kelley, et al., 2020, page 26) which are closer to trigonometric functions. In addition, several papers have utilized the idea of applying the trigonometric regression models in studying different problems of climate change and Global Warming and modeled the data using the trigonometric functions. Examples may be observed in Lando and Lando, 1977; Lorentzen, 2008; and Sofyan, et al., 2020.

4. MODELLING THE TEMPERATURE DATA USING THE TRIGONOMETRIC REGRESSION MODELS

In this Section, the problem of modelling the temperature data of three cities using the trigonometric regression models is to be studied comprehensively. In addition, the linearization method and the Least Squares Method (LSM) to be developed, in order to obtained the Least Squares Estimators (LSEs).
The trigonometric regression models are six models, and very well-known and they are not complicated mathematical models and can be defined as ratios of the sides (opposite, adjacent and hypotenuse) of right-angled triangles. The Sine and Cosine regression models have used a plethora of methods in modelling the climate temperature or climate change (see, Lando and Lando, 1977; Lorentzen, 2008 and Sofyan, et al., 2020).
It may be worthwhile to mention that the National Institute of Water & Atmospheric Research (NIWA) of New Zealand has modelled and studied the climate change of New Zealand specifically in some sensitive places like Wellington Airport and Kapiti Coast and published several reports on the topic of modelling the climate change (see, Baldi, et al., 2007; Bell, et al., 2016; Pearce, et al., 2019). The general equation of Sine regression model may be written as,

$$y_i = A\,Sin\left(\frac{2\pi}{B}(x_i - C)\right) + D \quad , i = 1,2,\dots,n, \tag{1}$$

where A is stands for the amplitude, B= π/6, C is stands for the horizontal shift, and D stands for the vertical shift. The models of modelling the climate change/global warming by the trigonometric regression models are manly based on Sine and Cos regression models with different constants; it may be worthwhile mentioning that the one of earlier study of the Wellington Airport monthly temperatures based on data from 1971 to 2000 is modeled using two trigonometric regression models (https://niwa.co.nz/education-and-training/schools/resources/climate/modelling), these models are defined by:

$$y = 4.2x \cos((X - 1)\,\pi/6) + 13.7, \text{ and } y = 4.2x \sin((X + 1)\,\pi/6) + 13.7.$$

It may be mentioned here, that the trigonometric regression models are have been applied to the modeling of some climate studies/problems as illustrated in the literature in a simple manner, with simple data, and were neither followed by any investigation of data properties or estimators
properties, nor by tests of the level of suitability fitting of the proposed model to the data.

In this paper the Sine regression model (equation 1) is to be applied for modelling the data of temperature and Global Warning of the biggest three cities for a 50-year period; and to be analyzed. The parameters of the proposed trigonometric regression models are unknown and need to be estimated. In order to estimate the unknown parameters of the proposed Sine regression model, the least squares method is one of the best choices for this purpose. It is well-known that the linearization method may be developed first for equation 1, in order to apply the least squares method.

In addition, in this paper several tests for each parameter estimator, the overall estimated model of the whole period and for each period/each city, and for Max, and Min temperatures are developed. Also, fitting the Sine model for each city/period and for Max, and Min temperatures, estimating the parameters by the least squares principle, and testing each model by ANOVA(analysis of variance) are also developed for getting accurate fitting.

Assume the initial values of the unknown parameters are given respectively by (A_0, B_0, C_0, D_0) the linearization method gives,

$$y_i = f|\psi = \psi_0 + \left(\frac{\partial f}{\partial A}|\psi = \psi_0\right)(\Delta A) + \left(\frac{\partial f}{\partial B}|\psi = \psi_0\right)(\Delta B) + \left(\frac{\partial f}{\partial C}|\psi = \psi_0\right)(\Delta C) + \left(\frac{\partial f}{\partial D}|\psi = \psi_0\right)(\Delta D), \tag{2}$$

where $f|\psi = \psi_0 = A_0 Sin\left(\frac{2\pi}{B_0}(i - C_0)\right) + D_0$, $\frac{\partial f}{\partial A}|\psi = \psi_0 = Sin\left(\frac{2\pi}{B_0}(i - C_0)\right)$,

$$\frac{\partial f}{\partial B}|\psi = \psi_0 = \frac{2\pi}{B_0^2}(i - C_0)A_0 Cos\left(\frac{2\pi}{B_0}(i - C_0)\right) + D_0, \frac{\partial f}{\partial C}|\psi = \psi_0 = \frac{2\pi}{B_0}A_0 Cos\left(\frac{2\pi}{B_0}(i - C_0)\right),$$

and $\frac{\partial f}{\partial D}|\psi = \psi_0 = 1$.

ΔA, ΔB , ΔC and ΔD are called correction values of the parameters A, B , C and D. Now, using the least squares method, the correction values (ΔA , ΔB , ΔC , ΔD) in equation 2 above are to be estimated and to be added to the initial values (A_0 , B_0 , C_0 , D_0) to get,

$$A = A_0 + \Delta A, B = B_0 + \Delta B, C = C_0 + \Delta C, \text{ and } D = D_0 + \Delta D, \tag{3}$$

where A_0 is the wave amplitude and equals to the average range of the two extreme values of the daily averages of temperature extremes, i.e. $A_0 = [\bar{y}_{(\max)} - \bar{y}_{(\min)}]/2$, B_0 stands for the wave cycle length which equals to the number of days in an a year (=360), C_0 stand for x-shift and equals to the sequence of the days whose temperature are equal to the mean temperature of the year, D_0 denotes the y-shift which is the average of the daily temperature for the whole year $(\bar{y})$.

It may be mentioned here that for the purpose of extending the Sine regression model using the linearization method (Taylor method), assumed (initial) values for the unknown parameters are to be used in order to estimate the parameters and obtain new values through several iterations. The estimation process and the iterations are to be terminated when the difference between the estimates in n^{th} stage and $(n-1)^{th}$ stage approaches zero, so the estimation of the n^{th} stage of approximation is considered as final estimates of the parameters.

It is worth mentioning that the process of choosing the initial values for the unknown parameters is very important, and it must be chosen very carefully because it may sometimes lead to slow or non-convergence of estimates from the iterations' stages or wide oscillation. In addition, the experimenter's previous experience in the problem of study is very important and it should be used in choosing the initial values.

In this paper, the above model is applied to the big collected data set of temperature and Global Warning of three cities in Iraq (Baghdad, Mosul and Basra) for the period 1970 to 2019; and the obtained results of applying the

above model are divers and related to the estimation of the model parameters, and have been calculated and applied to several statistical measures and tests, like the standard error, t-value, p-value, lower confidence limit, upper confidence limit, ANOVA Tables, graphs of minimum temperatures and max temperatures, and the scatter diagram of the regression models of each period of data. These results are to be discussed in the following Section.

5. THE ESTIMATED MODEL AND THE RESULTS OF GLOBAL WARNING USING THE TRIGONOMETRIC REGRESSION MODEL

It may be highlighted here that a comprehensive and heavy statistical analysis is developed in order to model the data of three cities for a 50-year period, which is dominated by all the pervious applications of climate changes. In details, the linearization technique and the Least Squares Method are applied to the parameters of the Sine regression model, and several statistical tests for the parameters of each model of Baghdad, Basra and Mosul cities. In addition, it is really interesting to observe that, the significant values of ANOVA tests for the Sine regression model and for the whole daily extreme temperatures (Minimum), and for the periods 1970-2019, 1970-1994 and 1995-2019, and for the three cities are highly significant.

Some of the statistical results are given in graphs 1-6, and Tables 1-5 in order to show the significant tests for LSE of the parameters , and the significance values of the T-test (Sig.) for each parameter of Sine Regression Models of $\bar{z}_j$ (Min. and Max.), and for the three cities' estimated models, and finally the results of ANOVA of the estimated Sine Models of $\bar{z}_j$ (Min. and Max.), and for the three cities are significant.

In addition, the effect of Global Warming, and how to know that the degree of impact of Global Warming on the three cities can be deduced. The results of this research have shown that the changes occurring in the climate in the three cities are more severe than the changes occurring in the past and in other countries of the Middle East region. This means that the three cities are affected by Global Warming and further means then that the whole country is under high risk.

In this Section, the data of Global Warning is to be modelled and studied by the Sine regression model. In addition, for the purpose of studying Global Warning the period of the data collected (Ps) is for 50 years and is to be divided into two equal secondary periods for the purpose of studying the phenomenon of Global Warming in the three cities of Iraq (Baghdad, Mosul and Basra), and, also, in order to study the level of increases in the temperatures during the periods of the study, and whether the trend of the lowest temperatures is increasing more than the trend of the increase in maximum temperatures or not.

It may be remarked here that the first period of the data is about 25 years (P1), starting from January 1, 1970, to December 31, 1994, and the second period (P2) is also about 25 years, starting from January 1, 1995 to December 31, 2019. In addition, the behavioral pattern of each period is to be analyzed using several measures and indicators.

In this Section, we will discuss the results of developing the Sine regression model. The parameters A, B, C, D are estimated for the whole data period, for each period and for Min and Max temperatures, standard error; t-values; significance values; lower confidence limit and upper confidence limit; ANOVA tests and annual increase rate.

The estimators of the unknown parameters A, B, C, and D of the Sine regression model and for the three cities are given in the previously mentioned equation; standard error; t-values; significance value; the lower confidence limit and the upper confidence limit are calculated using STATISTICA. In order to save space, only few results are given in Table 1 below. Table 1 shows that the value of D) represents the overall average of daily temperatures for the years of the study (50 years).

Table 1: The LSE of the parameters (P) of Sine Model, and the significance values of the T-test (Sig.) for the parameters (P) for Baghdad, Basra and Mosul cities.

Model	Period	P	Baghdad		Basra		Mosul	
			Estimate	Sig.	Estimate	Sig.	Estimate	Sig.
Min.	1970-2019	A	12.1353	0.00	12.1353	0.00	12.2124	0.00
		B	379.0796	0.00	379.0796	0.00	355.1111	0.00
		C	110.0980	0.00	110.0980	0.00	118.6657	0.00
		D	17.6529	0.00	17.6529	0.00	13.8674	0.00
	1970-1994	A	11.6392	0.00	11.6392	0.00	11.7029	0.00
		B	389.3552	0.00	389.3552	0.00	352.5329	0.00
		C	104.3323	0.00	104.3323	0.00	117.2412	0.00
		D	17.1333	0.00	17.1333	0.00	13.2891	0.00

	1995-2019	A	12.7250	0.00	12.7250	0.00	12.7329	0.00
		B	371.2696	0.00	371.2696	0.00	357.2534	0.00
		C	114.8962	0.00	114.8962	0.00	120.0429	0.00
		D	18.1095	0.00	18.1095	0.00	14.4547	0.00
Max.	1970-2019	A	14.5712	0.00	14.5712	0.00	15.5648	0.00
		B	386.9642	0.00	386.9642	0.00	363.9038	0.00
		C	106.4607	0.00	106.4607	0.00	114.3254	0.00
		D	31.3467	0.00	31.3467	0.00	27.7064	0.00
	1970-1994	A	14.0540	0.00	14.0540	0.00	15.6160	0.00
		B	399.4728	0.00	399.4728	0.00	364.5352	0.00
		C	103.1741	0.00	103.1741	0.00	114.6826	0.00
		D	30.6690	0.00	30.6690	0.00	27.3432	0.00
	1995-2019	A	15.1526	0.00	15.1526	0.00	15.5132	0.00
		B	376.8578	0.00	376.8578	0.00	363.1615	0.00
		C	109.1008	0.00	109.1008	0.00	113.9920	0.00
		D	31.9657	0.00	31.9657	0.00	28.0740	0.00

It was noted that the overall averages of maximum temperatures of the three cities (Baghdad/Mosul/Basra) are 30.53/27.7064/31.3467, 29.86/27.3432/30.6690 and 31.21/28.0740/31.9657 for the total period, first period and second period respectively; and it is shown in these statistics that the maximum temperature of the second period is greater than the first period, i.e. with an increment of 1.35/0.708/1.1967 degrees in the three cities respectively. It is also noted that the minimum temperatures (D) are 15.58/13.8674/17.6529, 14.47/13.2891/17.1333 and 16.68/14.4547/18.1095 for the total period, first period and second period respectively; and it is shown by these statistics that the minimum temperature of the second period is greater than the first period by 2.21/1.16565/1.762 which are large amounts and greater
than the amount of the increment of the maximum temperatures.

The above result means that the biggest cities of Iraq (Baghdad/Mosul/Basra) and then the country in the near future will witness a noticeable rise in temperatures, and this means that the country is affected by Global Warming.

The estimated Sine regression models ($\widehat{y_{ij}}$) of each city and period can be written on the basis of equation 1 (Section 4) and the estimators of each equation for each city as given in Table 1. In order to know the level of significance of the estimated Sin models ($\widehat{y_{ij}}$), the ANOVA tests for the models of the Min. and Max. temperatures and for the three cities are developed. Again, for space consideration, some of the results of these tests are given in Tables 2 and 3 respectively below. The results of Tables 2 and 3 show that the significance value of all tests is 0.00, which means that the Sine models of the whole data, and the data of each city and each part are very significant.

Table 2: Shows the results of ANOVA of the Sine Models for the daily Extreme temperatures (Minimum), for the periods 1970-2019, 1970-1994 and 1995-2019, and for the three cities.

Period	S.V	Baghdad		Basra		Mosul	
		M.S.	F and Sig.	M.S.	F and Sig.	M.S.	F and Sig.
1970-2019	Regression	29784.58	87288.08 0.000	36381.04	89846.05 0.000	23713.46	48809.86 0.000
	Residual	0.34		0.40		0.49	
	Total						
	Corrected Total (CT)						
	Regression vs. CT	29784.58	404.69 0.000	36381.04	504.71 0.000	23713.46	309.53 0.000
1970-1994	Regression	26334.25	59123.37 0.000	34952.25	71323.76 0.000	21620.82	35154.82 0.000
	Residual	0.45		0.49		0.62	
	Total						
	Corrected Total (CT)						
	Regression vs. CT	26334.25	401.07 0.000	34952.25	542.90 0.000	21620.82	304.96 0.000
	Regression	33469.40	60881.29	37900.00	55779.30	25921.21	40570.34

1995-2019	Residual	0.55	0.000	0.68	0.000	0.64	(0.000
	Total						
	Corrected Total (CT)						
	Regression vs. CT	33469.40	405.44 0.000	37900.00	467.71 0.000	23713.46	312.18 (0.000)

Table 3: Shows the results of ANOVA of the Sine Models for the daily Extreme temperatures (Maximum), for the periods 1970-2019, 1970-1994 and 1995-2019, and for the three cities.

Period	S.V	Baghdad		Basra		Mosul	
		M.S.	F and Sig.	M.S.	F and Sig.	M.S.	F and Sig.
1970-2019	Regression	99087.83	81741.71 0.000	103708.3	88897.76 0.000	23713.46	79514.63 0.000
	Residual	1.21		1.2		0.49	
	Total						
	Corrected Total (CT)						
	Regression vs. CT	99087.83	898.67 0.000	103708.3	1014.89 0.000	23713.46	658.26 0.000
1970-1994	Regression	95685.12	69971.44 0.000	101252.1	70582.21 0.000	21620.82	60776.83 0.000
	Residual	1.37		1.4		0.62	
	Total						
	Corrected Total (CT)						
	Regression vs. CT	95685.12	886.09 0.000	101252.1	1101.55 0.000	21620.82	640.07 0.000
1995-2019	Regression	102551.87	72694.75 0.000	106238.9	76457.14 0.000	25921.21	73657.57 0.000
	Residual	1.41		1.4		0.64	
	Total						
	Corrected Total (CT)						
	Regression vs. CT	102551.87	908.15 (0.000)	106238.9	934.95 (0.000)	23713.46	674.65 (0.000)

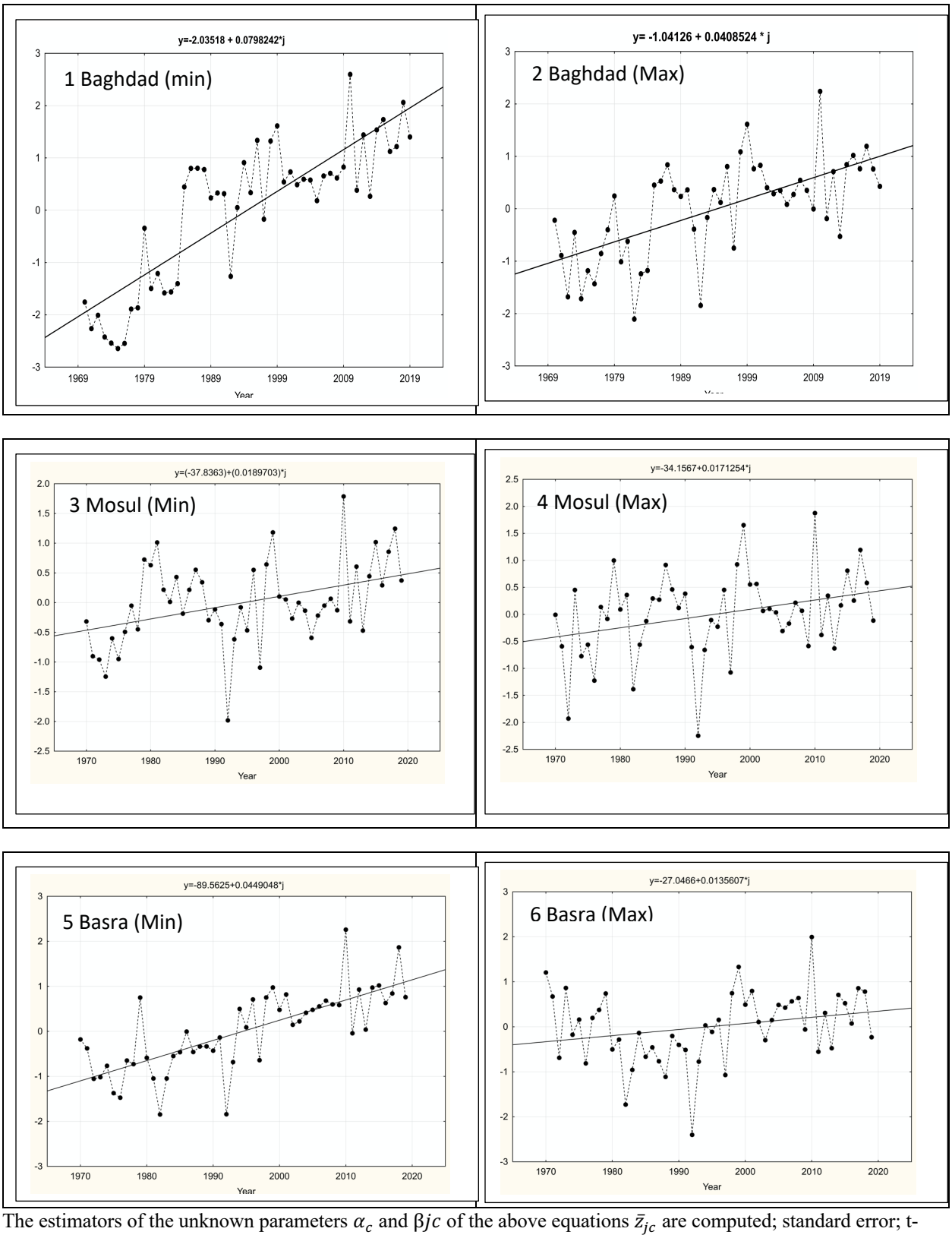
For Global Warming studies purposes and evaluation, it is very important to estimate the annual rate of increase in the temperatures. In addition, the annual rate of increase is calculated by STATISTICA and applied to the different equations for each city, where $k = 1$ stands for Baghdad; $k = 2$ stands for Mosul; and $k = 3$ stands for Basra,

$$z_{ijc} - \widehat{y_{ijc}} = 0, i = 1,2, \dots ,365 , j = 1,2, \dots ,50, \quad \bar{z}_j = \frac{\sum_{i=1}^{365} z_{ij}}{365}, \; j = 1,2, \dots ,50, \; \mathrm{k} = 1,2,3. \tag{4}$$

The scatter diagrams and the estimated models of z_{ijc} for Min and Max temperatures have been fitted by the model given below and given in Graphs 1, 2,3, …, 6,

$$\bar{z}_{jc} = \alpha_c + \beta jc + \varepsilon_{jc}, \;\; j = 1,2, \dots 50, c = 1,2,3. \tag{5}$$

Graphs 1-6: Shows the estimated Sine Models of $\bar{z}_j$ (Min. and Max.), and for the three Cities



The estimators of the unknown parameters α_c and βjc of the above equations $\bar{z}_{jc}$ are computed; standard error; t-values; significance value; the lower confidence limit and the upper confidence limit are calculated, and only the values of t-tests and the values of significance are given in Table 4 below in order to save space.

Table (4) LSs of the parameters , and the significance values of the T-test (Sig.) for the parameters (P) Sin Models of $\bar{z}_j$ (Min. and Max.), and for the three Cities

	P	Baghdad		Basra		Mosul	
		Estimate	Sig.	Estimate	Sig.	Estimate	Sig.
Min.	α	-2.04	0.00	-37.836	0.0045	-89.56	0.000
	β	0.08	0.00	0.019	0.0045	0.05	0.000
Max	α	-1.04	0.00	-34.156	0.024	-27.05	0.028
	β	0.04	0.00	0.017	0.024	0.01	0.028

The results of computations show that the annual rate of increase in minimum temperatures is 0.08 degrees and ranges between 0.06 to 0.09 with a probability of 95% while the average annual increase in maximum temperatures is 0.04 degrees and ranges between 0.03 to 0.06, with a probability of 95%. In addition, the tests for the Min and Max data of $\bar{z}_{jc}$ are developed and given in Table 5 (below) and show high significance. Moreover, the values of t-tests for both models (Min and Max temperatures) are highly significant, i.e. the data of the daily temperatures is fitted by the Sine models with highly significant values.

Table 5: Shows the results of ANOVA of the estimated Sin Models of $\bar{z}_j$ (Min. and Max.), and for the three Cities

	S.V.	Baghdad		Mosul		Basra	
		M.S.	F and Sig.	M.S.	F and Sig.	M.S.	F and Sig.
Min	Regression	33.17	57.54 0.000	10.498	29.468 0.000	2.873590	6.8149 0.0119
	Residual	0.58		0.356		0.421669	
	Total						
	Corrected Total						
	Regression vs. Corrected Total	33.17	17.29 0.000	10.498	13.503 0.000	2.873590	5.8700 0.018
Max	Regression	8.69	16.48 0.000	1.957	3.9376 0.0212	2.526890	5.4223 0.0177
	Residual	0.53		0.497		0.46019	
	Total						
	Corrected Total						
	Regression vs. Corrected Total	8.69	9.97 0.000	1.957	3.1369 0.0326	2.526890	4.0959 0.0194

6. DISCUSSION, AND CONCLUDING REMARKS

In this paper, the Global Warming problems for the biggest three cities: Baghdad, Mosul and Basra in Iraq are studied using the trigonometric regression models which are modeled and studied. The Sine regression model is estimated, the estimators of the unknown parameters A, B, C, and D of the model are calculated. In addition, the ANOVA tests for models of Min. and Max. temperatures data are developed to assure the model estimation. The data of maximum/minimum temperatures of 50 years is collected and divided into two equal secondary periods in order to study the effect of Global Warning.

Regarding the problem of studying the phenomenon of Global Warming in the three cities of Iraq, and in order to study the level of increase in the temperatures during the period of the data, and to see whether the trend of the lowest temperatures is increasing more than the trend of the increase in maximum temperatures or not, and to see which of the above cities of Iraq is more affected by this phenomenon, the collected data is divided into two secondary periods, the first (P1) is 25 years, starting from January 1st , 1970 to December 31st , 1994, and the second period (P2) is also 25 years, starting from January 1st , 1995 to December 31st , 2019. The increments of the temperatures of the three cities (Table 1) show that increments of the three cities for the minimum periods of the second period is greater than the increments of the first period; and greater than the increment of the same cities of the maximum temperatures; i.e. the biggest cities (Baghdad/Mosul/Basra) and then the country in the near future will witness a noticeable rise in temperatures, and this means that the country is affected by the Global Warming.

It may be remarked here that several valuable numerical results about the LSE of the parameters (P) of the Sine Regression Model, and the significance values of the T-test (Sig.) for the parameters (P) for Baghdad, Basra and

Mosul cities are given in Table 1(in the appendix). It is observed that the results are for the estimators of the model based on whole data and for each period are highly significant. In addition, it is really interesting to observe that, the significant values of ANOVA tests for the Sine model and for the whole daily extreme temperatures (Minimum), and for the periods 1970-2019, 1970-1994 and 1995-2019, and for the three cities are highly significant, i.e. the data is fitted significantly by the Sine Model (Tables 2 and 3)

Some other results are given in graphs 1-6 (in the appendix) which show the successful fitting of the estimated Sine models of $\bar{z}_j$ (Min. and Max.), and for the three cities, and Tables 4 and 5 (in the appendix) show significant t tests for LSE of the parameters , and the significance values of the T-test (Sig.) for each parameter (P) of Sine Models of $\bar{z}_j$ (Min. and Max.), and for the three cities estimated models, and finally the results of ANOVA of the estimated Sine Models of $\bar{z}_j$ (Min. and Max.), and for the three cities are significant.

One of the results of this paper, is that both proposed models fit the data of maximum and minimum temperatures of the three cities for all periods with highly significant results, i.e., the proposed models (Sine Model) can be used for forecasting the daily temperatures in the three cities, and can be used by the researchers for other cities.

Regarding the effect of Global Warming, and how to know that the degree of impact of Global Warming on the three cities, the results of this research have shown that the changes occurring in the climate in the three cities are more severe than the changes occurring in the past and in other countries of Middle East regions. In addition, the high average temperatures in the three governorates have indicated that Iraq's climate will be hotter and drier in the future.

Thus, the level of impact of the Global Warming is high in all the cities (not only Baghdad, Mosul , Basra); since the three cities are located in the north, middle and south of Iraq respectively. This means that the whole country is affected by Global Warming and is under high risk and needs a comprehensive plan from the government in order to reduce the risk associated with Global Warming. Moreover, the government should support national and international efforts and research to mitigate Global Warming and the gas emissions from the so-called Green House Gases (GHG) and so comply with relevant international agreements.

Finally, our last recommendation will be regarding the usefulness of the proposed model for studying Climate Fluctuations and Global Warming, as it was noted that from the results of this paper that the benefit of applying the trigonometric regression model is high, and the model is not complicated and has succeeded the modeling of the data of this research which is related to Climate Fluctuations and Global Warming of three big cities of Iraq. In general, the results of this paper are stable and strong. In addition, the results of estimating the parameters of the models of the three cities were very clear and reliable. Also, the results of the statistical tests related to this model were highly significant. Thus, we recommend applying this model to study and predict the problems of Climate Fluctuations and Global Warming in the world.

7. LIMITATIONS AND FUTURE RESEARCH DIRECTIONS

The first obstacles that are faced in this research are the absence of any academic studies published in the refereed or indexed journals related to mathematical/statistical modeling about studying the maximum and minimum temperatures in Iraq, and the extent to which level Iraq is affected by Global Warning. Due to this fact, the comparison of the results of this paper with any earlier studies is not possible.

When embarking on this research and taking into account the maximum and minimum temperatures in Iraq, it serves as a comprehensive study, and, to answer the questions: is Iraq affected by the Global Warning, and what is the level of affect of Global Warming? On the other hand, this project is a bridge linking the past reality, i.e., for the past fifty years- and for subsequent studies, and, how to conduct and re-analyze the old and future data for other Iraqi governorates.

One of the most important limitations is the readiness of the row data of the maximum and minimum temperatures for analysis; after obtaining the data of the three governorates, it was noted that this data was not appropriately applicable for the mathematical models proposed in this paper, and, this observation is important for any researcher, and should be taken into account.

With regard to future studies related to modeling the data of minimum and maximum temperatures in Iraq, it is possible to apply many descriptive statistical measures, and to follow some of the published papers that applied regression analysis models and time-series models.

Data availability:

The raw data of this paper is available at Iraqi Meteorological Organization and Seismology http://wikimapia.org/29646461/Iraqi-Meteorological-Organization-and-Seismology

Remark: On behalf of all authors, the corresponding author states that there is no conflict of interest.

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