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ROBUST SELECTION OF A COMMON ROCK-WOOL INSULATION THICKNESS FOR OIL PIPELINES OPERATING BETWEEN 50 AND 70 °C

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ABSTRACT

Operators insulating a pipeline network often must standardise a single insulation thickness across the whole diameter range, because purchasing and installation logistics do not accommodate one thickness per branch. Published thermo-economic procedures return an optimum per diameter and do not answer that question. This work formulates the choice of a common thickness as a robust decision problem over the eighteen states given by six nominal diameters and three flow temperatures. A four-layer cylindrical resistance model on real ASME B36.10M geometry, and a life-cycle cost model with capital recovery and a heating-system efficiency, give the total annualised cost of every state-thickness pair for rock wool on pipes from 0.08 to 0.30 m at 50, 60 and 70 °C. The relative thermo-economic regret is then minimised in the maximum over all states, restricted to commercially available thicknesses. Under the base scenario the global minimax thickness is 0.100 m, with a worst-case regret of 3.10 % attained at the smallest diameter and the lowest temperature; per-temperature minimax values range from 0.090 to 0.120 m. The global value therefore coincides with the 60 °C minimax and departs from the other two, so it is a compromise across the interval rather than the best choice for a network whose operating temperature is known. A Latin-Hypercube analysis over ten factors places the selected thickness between 0.060 and 0.120 m at the 5th and 95th percentiles. An alternative compensatory formulation (CRITIC-TOPSIS over four alternatives) is examined rather than compared: its binary coding makes the two extreme coverage indicators complementary and therefore symmetric, so the identity alone cannot favour either; the asymmetry that gives one of them the largest weight comes from its correlation with a margin criterion introduced by the analyst. The recommendation is conditional on the declared scenario.

KEYWORDS: economic insulation thickness; minimax regret; robust decision making; life-cycle cost; global sensitivity analysis; pipeline standardisation.

MSC: 90C31, 90B50, 80A20.

RESUMEN

Un operador que aísla una red de tuberías necesita con frecuencia estandarizar un solo espesor de aislante para todo el rango de diámetros, porque la logística de compra e instalación no admite un espesor por ramal. Los procedimientos termo-económicos publicados entregan un óptimo por diámetro y no responden a esa pregunta. Este trabajo formula la elección de un espesor común como un problema de decisión robusta sobre los dieciocho estados que resultan de seis diámetros nominales y tres temperaturas de flujo. Un modelo de resistencias cilíndricas de cuatro capas sobre geometría real ASME B36.10M, y un modelo de costo de ciclo de vida con recuperación del capital y rendimiento del sistema de calentamiento, proporcionan el costo total anualizado de cada par estado-espesor para lana de roca en tuberías de 0,08 a 0,30 m a 50, 60 y 70 °C. Se minimiza después el arrepentimiento termo-económico relativo en su máximo sobre todos los estados, restringido a espesores comercialmente disponibles. Bajo el escenario base el espesor minimax global es 0,100 m, con un arrepentimiento de peor caso de 3,10 % que se alcanza en el diámetro menor y la temperatura más baja; los minimax por temperatura van de 0,090 a 0,120 m, de modo que el valor global coincide con el de 60 °C y se aparta de los otros dos: es un compromiso sobre el intervalo, no la mejor elección para una temperatura de operación conocida. Un análisis de hipercubo latino sobre diez factores sitúa el espesor seleccionado entre 0,060 y 0,120 m en los percentiles 5 y 95. Se examina además una formulación compensatoria alternativa (CRITIC-TOPSIS sobre cuatro alternativas) —no se compara con ella, porque no comparten espacio de decisión—: su codificación binaria hace complementarios, y por tanto simétricos, a los dos indicadores de cobertura extremos, de modo que la identidad por sí sola no puede favorecer a ninguno; la asimetría que otorga a uno el peso mayor procede de su relación con un criterio de margen incorporado por el analista. La recomendación es condicional al escenario declarado.

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PALABRAS CLAVE: espesor económico de aislamiento; arrepentimiento minimax; decisión robusta; costo de ciclo de vida; análisis global de sensibilidad; estandarización en tuberías.

1. INTRODUCTION

Oil transport by pipeline is a central activity of the energy system, and the thermal behaviour of pipelines governs their design and operation. In systems without steam tracing, thermal insulation selection is the main way to limit heat loss and keep the crude's temperature within the range where its viscosity allows efficient pumping [21].

The problem of determining the economic insulation thickness is well established. The classical formulation balances the cost of lost energy, which decreases with thickness, against the annualized cost of the material, which increases with it; their sum has an interior minimum. Keçebaş, Alkan and Bayhan [12] solve that formulation for district-heating networks through life-cycle cost analysis, and Aristizábal, Schäfer and Barrera [2] quantify the avoidable losses in poorly insulated steam networks. In the heavy-crude domain, Gilbert [9] and Gilbert et al. [10] solve the thickness selection for steam-traced pipelines, and the line of work by Laurencio and collaborators [13, 14, 15, 16, 17] characterizes the rheology and temperature drop of Cuban crude CM-650.

All of these antecedents share a structural feature: they solve the problem diameter by diameter and therefore return a different optimal thickness for each pipe. That answer is the correct one when each section can be insulated with its own thickness. But an operator insulating a whole network often faces a constraint the literature does not model: purchasing, storage and installation logistics favour — and sometimes impose — a single thickness across the whole diameter range. Which single thickness to choose when the optima do not coincide is a different question, and it is a decision problem under uncertainty.

This work's contribution is twofold. Methodologically, thickness standardization is formulated as minimizing the thermo-economic regret over a set of states that includes both diameter and flow temperature, and it is argued why a compensatory multicriteria aggregation over binary indicators does not adequately represent this problem. In application, the recommended common thickness is delivered for rock wool on oil pipelines from 0.08 to 0.30 m between 50 and 70 °C, restricted to commercial thicknesses, with its cost-overrun guarantee by state and the distribution that thickness takes under the joint uncertainty of ten factors.

The analysis is reproducible from the declared inputs: parameters come from cited public sources or are explicitly declared as scenario, and all random sampling uses a fixed seed.

2. LITERATURE REVIEW

The determination of the economic insulation thickness is approached from three angles. The first is the classical thermo-economic one, which minimizes the total life-cycle cost over a single degree of freedom: Keçebaş et al. [12] apply it to five pipe sizes and four fuels and show that the optimum depends strongly on the price of the energy source; Aristizábal et al. [2] and Pacherres and Chucuya [20] document the economic impact of deficient insulation. The second is the thermo-fluid angle, which characterizes the fluid to feed the thermal model: Laurencio and Delgado [13] and Laurencio et al. [15] model the apparent viscosity of 11° API crude, Falconi et al. [8] the pressure drop, Laurencio et al. [17] the temperature profile, and Cabrera-Escobar et al. [5] solve the heavy-crude transport under Ecuadorian Oriente conditions by computational fluid dynamics. The third is the energy efficiency of thermal installations, where Retirado-Mediaceja et al. [24, 25], Camaraza-Medina et al. [6] and Torres Tamayo et al. [28] establish transfer coefficients and performance indicators in cylindrical geometries.

None of these angles addresses the common-thickness constraint. Faced with decisions with conflicting criteria, the usual engineering recourse is multicriteria analysis: TOPSIS [11] with objective weights from the CRITIC method [7] is one of the most widespread combinations, with applications in model and classifier selection [18, 22]. This work shows that, for this problem's particular structure, the compensatory variant based on binary indicators does not adequately represent it, and adopts instead the decision-under-uncertainty formulation of Wald [29] and Savage [27], whose minimax regret criterion offers a per-state guarantee. Latin hypercube sampling [19] is used for uncertainty propagation.

3. PROBLEM FORMULATION

3.1 Problem states

The recommendation is requested for a range of flow temperatures, not for a single temperature. A thickness that is optimal at 60 °C need not be optimal at 50 or 70 °C, so temperature, just like diameter, is a dimension over which the decision must be guaranteed. The set of states is therefore the Cartesian product

$\mathcal{S} = \mathcal{D} \times \mathcal{T}$, with $\mathcal{D}$ = {0.08; 0.10; 0.15; 0.20; 0.25; 0.30} m and $\mathcal{T}$ = {50; 60; 70} °C,

which gives 18 states. Each state has its own thermo-economic optimum, and the common thickness imposes on each one a cost overrun relative to that optimum.

3.2 Continuous domain and commercial thicknesses

Two sets that the literature tends to conflate are distinguished here. The first, Ω, is the continuous domain over which the physical optimization is solved: here [0.010; 0.250] m with a step of 0.001 m. Its extent was chosen by checking that no optimum falls at the domain's edge, a check the code verifies and that is reported in Section 7.

The second, Δ, is the discrete set of thicknesses that can be purchased. The final decision must belong to Δ, not to Ω: recommending a thickness that is not manufactured is not a recommendation. The manufacturer's datasheet [26] documents a thickness range of 20 to 200 mm and inner diameters of 22 to 915 mm, but does not publish the table of thicknesses available for each diameter. The industry's preferred series bounded to that documented range is accordingly adopted, and it is declared as an assumption, not as data: the actual availability by diameter remains information pending confirmation with the distributor. Note that this distinction is not a formality: the series does not contain the value 0.095 m, so any procedure returning it is recommending a product that does not exist.

3.3 The decision

Choosing a single element of Δ for all states of $\mathcal{S}$ imposes a cost overrun on every state whose optimum does not coincide with it. The two defensible ways to distribute it are minimizing the aggregate overrun over the network's actual composition, or minimizing the maximum overrun suffered by any state. The first requires knowing the installed meters of each diameter; the second does not, and is therefore adopted as the main criterion, with the first as a contrast.

4. THERMO-ECONOMIC MODEL

4.1 Geometry

The nominal diameter of a pipe is neither its outer diameter nor its inner diameter. Taking it as either introduces a systematic error in the radii that govern the resistances in equation (1). Table 1 tabulates, for each pipe, the designation, the real outer diameter, the wall thickness of the Schedule 40 series and the resulting inner diameter, all from the ASME B36.10M standard [3]; the model's inner and outer radii follow from these.

Designation	DN (m)	OD (mm)	Sch 40 wall (mm)	ID (mm)	r_1 (m)	r_2 (m)
NPS 3 (DN 80)	0.08	88.90	5.49	77.92	0.03896	0.04445
NPS 4 (DN 100)	0.10	114.30	6.02	102.26	0.05113	0.05715
NPS 6 (DN 150)	0.15	168.28	7.11	154.06	0.07703	0.08414
NPS 8 (DN 200)	0.20	219.08	8.18	202.72	0.10136	0.10954
NPS 10 (DN 250)	0.25	273.05	9.27	254.51	0.12726	0.13653
NPS 12 (DN 300)	0.30	323.85	10.31	303.23	0.15161	0.16193

Table 1. Geometry of the six pipes according to ASME B36.10M, Schedule 40 series. The nominal diameter is kept as a label; the model's radii are obtained from the real inner and outer diameters.

4.2 Thermal resistance and heat loss

The pipe is modelled as four concentric layers —the crude's inner film, the steel wall, the rock-wool insulation and the protective sheet— plus the outer air film. The thermal resistance per unit length is the sum of the five resistances

in series, equation (1). Each layer carries its own conductivity: failing to do so cancels out the insulation's effect, whose conductivity is three orders of magnitude lower than that of steel.

$$R_{\mathrm{L}} = \frac{1}{r_1 h_{\mathrm{p}}} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{k_{\mathrm{A}}} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_{\mathrm{B}}} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k_{\mathrm{C}}} + \frac{1}{r_4 h_{\mathrm{a}}} \tag{1}$$

where:

r_1 inner radius of the pipe; (m).

r_2 outer radius of the pipe; (m).

r_3 outer radius of the insulation; (m).

r_4 outer radius of the protective sheet; (m).

h_{p} convection coefficient of the heavy crude; (W/(m²·K)).

h_{a} convection coefficient of the outer air; (W/(m²·K)).

k_{A} thermal conductivity of the pipe's steel; (W/(m·K)).

k_{B} thermal conductivity of the insulation; (W/(m·K)).

k_{C} thermal conductivity of the protective sheet; (W/(m·K)).

The heat loss per unit length is obtained from equation (2).

$$q_{\mathrm{s}} = \frac{2\pi(T_{\mathrm{i}} - T_{\mathrm{e}})}{R_{\mathrm{L}}} \tag{2}$$

The conductivity of the rock wool depends on the insulation's mean temperature, which in turn depends on the heat flux. The system formed by (1) and (2) is solved by fixed point, interpolating the conductivity table declared by the manufacturer and tested according to ASTM C335 [4, 26]; convergence is reached in fewer than ten iterations in all cases.

4.3 Life-cycle cost

The heat that escapes through the wall is not lost for free: it must be replaced. In a system with crude heating, that replacement is done by equipment with an efficiency η, so the energy that is paid for is greater than the heat lost. Equation (3) incorporates that efficiency, and E denotes the price of the energy feeding the heating system —in the base scenario, electric energy at the national average tariff [1]—.

$$C_{\mathrm{E}}(D, T, \delta) = \frac{q_{\mathrm{s}}(D,T,\delta) \cdot E \cdot Z}{\eta \cdot 1000} \tag{3}$$

The physical boundary of this equation must be stated precisely, because it determines whether the term is an actual disbursement or an accounting convention. Three situations are possible. With distributed heating —electric or steam tracing along the pipeline— the heat that escapes is replaced at the same point where it is lost, and equation (3) is a direct disbursement. With upstream heating —a heater at the pumping station— the replacement is not local: the crude leaves hotter and cools along the route, so equation (3) approximates the disbursement only if the delivery temperature is fixed by process and the heater effectively compensates the accumulated loss. And if there is no equipment at all replacing the loss, the term is not a disbursement but an equivalent energy cost of the thermal loss, and should be read as such.

This work adopts the second assumption, usual in pipelines without tracing, and declares it as such. In the third case the real penalty shows up through another channel —crude cooling, increased viscosity and higher pumping power— , a thermo-hydraulic coupling this model does not incorporate. It is not claimed here in which direction that omission biases the result: insulation simultaneously reduces heat loss and pumping penalty, which would suggest a larger benefit, but pumping cost depends on the crude's rheology, the flow rate and the temperature profile in a way that is not monotonic in thickness, and without solving that coupling the direction of the bias is not established. It is declared as an uncertainty, not as a bound.

The insulation cost is annualized over the volume of material actually installed per meter of pipe, equation (4). The price appearing in it is that of the material; the installation multiplier brings it to installed cost. Note that this term depends on thickness, which is what allows an interior minimum to exist.

$$C_{\mathrm{A}}(D,\delta) \;=\; \pi({r_3}^2 \;-\; {r_2}^2) \;\cdot\; p_{\mathrm{mat}} \;\cdot\; f_{\mathrm{inst}} \;\cdot\; (\mathrm{FRC} \;+\; f_{\mathrm{mant}}) \tag{4}$$

The capital recovery factor in equation (5) converts the investment into an equivalent annuity over the useful life, at the declared discount rate; with the values in Table 3 it equals 0.11746 year^{-1}. No salvage value is considered, out of prudence.

$$\mathrm{FRC} \;=\; \frac{i_{\mathrm{d}}(1+i_{\mathrm{d}})^{n}}{(1+i_{\mathrm{d}})^{n}-1} \tag{5}$$

The total annualized cost is the sum of both components, equation (6).

$$C_{\mathrm{T}}(D,T,\delta) \;=\; C_{\mathrm{E}}(D,T,\delta) \;+\; C_{\mathrm{A}}(D,\delta) \tag{6}$$

5. ROBUST DECISION MODEL

The regret of adopting the common thickness δ in state (D, T) is the relative cost overrun with respect to that state's continuous optimum, equation (7). Normalizing by the minimum cost makes the quantity dimensionless and comparable across states, which is indispensable because the absolute cost grows with diameter and temperature, and an unnormalized difference would automatically overweight the more expensive states.

$$\mathrm{Arr}(D,T,\delta) \;=\; \frac{C_{\mathrm{T}}(D,T,\delta) \;-\; \mathrm{mín}_{w\,\in\,\Omega}\, C_{\mathrm{T}}(D,T,w)}{\mathrm{mín}_{w\,\in\,\Omega}\, C_{\mathrm{T}}(D,T,w)} \tag{7}$$

The main criterion is minimax regret, equation (8): among the commercial thicknesses, the one whose worst cost overrun over the eighteen states is the smallest possible is chosen. The criterion does not require knowing the network's composition or the distribution of operating temperatures, and offers a per-state guarantee rather than an average guarantee.

$$\delta^{*} \;=\; \mathrm{argmín}_{\delta\,\in\,\Delta}\; \mathrm{máx}_{(D,T)\,\in\,S}\; \mathrm{Arr}(D,T,\delta) \tag{8}$$

Its trade-off should be stated: minimax is a conservative criterion, governed by the worst-served state, and can sacrifice average performance to protect it. For that reason, the thickness that minimizes the total expected cost over a declared network composition is also computed, equation (9). The weights are joint over the eighteen states: each scenario declares a distribution of installed meters by diameter and an operating-temperature profile, and the weight of a state is the product of both. Summing only over diameter would leave temperature free and the quantity undefined. Since neither the composition nor the profile is known, six scenarios are reported with their two explicit vectors instead of a single number.

What that factorization assumes should be stated. Writing the joint weight as the product of a diameter weight and a temperature profile amounts to assuming that both distributions are INDEPENDENT: that the operating temperature does not depend on the section's diameter. It is a scenario adopted for lack of joint data, not a property of the problem, and it may not hold: in a real network the larger-diameter sections tend to be closer to the pumping station and operate hotter, which would induce an association between the two variables. An application with data should use the state's empirical weights directly, without factorizing them; the formulation in equation (9) admits them as they are, because it sums over states and not over the two factors separately.

$$\delta^{\mathrm{esp}} \;=\; \mathrm{argmín}_{\delta\,\in\,\Delta}\; \textstyle\sum_{(D,T)\,\in\,S} p_{D,T} \;\cdot\; C_{\mathrm{T}}(D,T,\delta) \tag{9}$$

Against a compensatory aggregation, this formulation preserves the economic magnitude instead of reducing it to binary indicators, does not allow slack in one state to offset the overrun in another, and needs no weightings that the analyst must justify.

6. DATA AND SCENARIOS

Table 2 gathers the physical parameters and Table 3 the economic ones, with their value, unit, source and range. Two categories are distinguished. Parameters with a cited public source —conductivities, inner coefficient, energy price— are fixed at the source's value. Those that depend on the specific installation and are not found in any accessible source are explicitly declared as scenario and their effect is examined in Section 8. It is worth giving the exact figure and delimiting it well. Tables 2 and 3 contain fourteen SCALAR parameters; of these, two —the inner convection coefficient and the energy price— come from a verifiable public source and the remaining twelve are declared scenarios. The model also uses two inputs that are not scalar parameters and that are documented: the insulation's conductivity curve as a function of temperature, taken from the manufacturer's datasheet [4, 26], and the geometry of the six pipes in Table 1, taken from the standard [3]. The exact statement is, therefore, that twelve of the fourteen scalar parameters are declared scenarios, not that the whole model rests on only two public sources. The results are accordingly reported as belonging to the declared base scenario and not as properties of the physical system. Section 7.1 further shows that three of those twelve —the two metallic conductivities and the sheet thickness— are numerically negligible within the examined ranges, and Section 8.2 that a fourth, the outer coefficient, is the least influential of the ten factors analysed.

Symbol	Definition	Value	Unit	Source	Range
k_A	Thermal conductivity of the carbon-steel pipe	50	W/(m·K)	DECLARED SCENARIO — not reported in the study (negligible contribution, see §7.1)	45 - 55
k_C	Thermal conductivity of the galvanized protective sheet	50	W/(m·K)	DECLARED SCENARIO — not reported in the study (negligible contribution, see §7.1)	45 - 55
h_p	Inner convection coefficient of heavy crude oil	52	W/(m²·K)	Cabrera-Escobar et al. [5]	30 - 500
h_a	Outer convection coefficient (natural plus radiation)	10	W/(m²·K)	DECLARED SCENARIO — not reported in the study ($\eta^2 = 0.010$, the least influential factor)	8 - 12
T_e	Outdoor air temperature	20	°C	DECLARED SCENARIO — not reported in the study	10 - 30
e_{prot}	Thickness of the protective sheet	0.0006	m	DECLARED SCENARIO — not reported in the study	0.0005 - 0.0008

Table 2. Physical parameters of the thermal model.

Symbol	Definition	Value	Unit	Source	Range
E	Unit energy price of the heating system	0.1061	USD/kWh	ARCONEL, Electricity Public Service Tariff Schedule 2025 (national average price)	0.061 - 0.130
η	Efficiency of the system that replaces the heat loss	0.85	-	DECLARED SCENARIO — not reported in the study	0.60 - 1.00
Z	Annual operating time	8000	h/year	DECLARED SCENARIO — not reported in the study	6000 - 8760
p_{mat}	Price of the insulating MATERIAL per unit volume	220	USD/m³	DECLARED SCENARIO — not reported in the study	150 - 350
f_{inst}	Installation multiplier on the material price	2	-	DECLARED SCENARIO — not reported in the study	1.5 - 2.5
f_{mant}	Annual maintenance as a fraction of the investment	0.02	1/year	DECLARED SCENARIO — not reported in the study	0.01 - 0.04
i_d	Discount rate	0.1	1/year	DECLARED SCENARIO — not reported in the study	0.06 - 0.15
V_U	Service life of the insulation	20	year	DECLARED SCENARIO — not reported in the study	15 - 25

Table 3. Economic parameters. «DECLARED SCENARIO» marks the values that do not come from a source and whose joint effect is quantified in Section 8.2.

The rock wool's conductivity is not a single value but a function of the insulation's mean temperature, taken from the manufacturer's table: 0.036 W/(m·K) at 38 °C and 0.042 W/(m·K) at 93 °C, with linear interpolation between points

[26]. The set Δ of commercial thicknesses considered is {20, 25, 30, 40, 50, 60, 70, 80, 90, 100, 120, 140, 160, 180, 200} mm.

7. RESULTS

7.1 Continuous optimum by state

Figure 1 shows the heat loss as a function of thickness: it decreases monotonically and with diminishing returns, as corresponds to a logarithmic resistance in the radius. In the analysed range the critical-radius phenomenon does not appear, whose value —the ratio between the insulation's conductivity and the outer coefficient— is on the order of 3.6 mm, far below all the pipe outer radii considered.

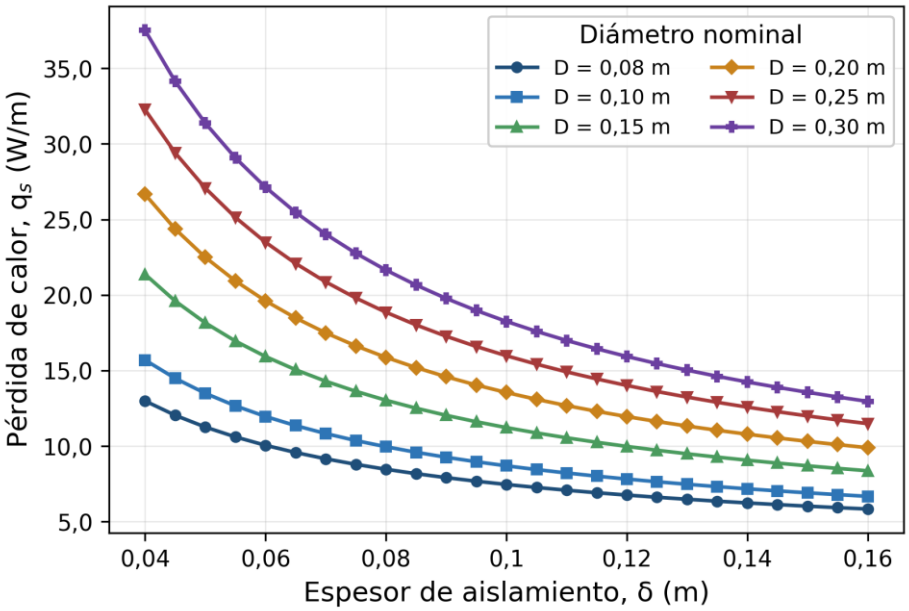


Figure 1. Heat loss per unit length against insulation thickness, for the six nominal diameters, at Ti = 60 °C.

The distribution of thermal resistance among the five layers in series explains why several parameters of the model, despite being declared as scenario, cannot affect the result. At the optimum of each diameter at 60 °C, the insulation contributes at least 96.07 % of the total resistance; the steel wall and the protective sheet together do not reach 0.009 %, and the two convection films sum the rest. A ±10 % uncertainty in the conductivity of steel or of the sheet therefore translates into a variation of the total resistance below one thousandth of a percentage point: the value of those conductivities is irrelevant to the decision, and that is why declaring them as scenario is not a limitation.

The distribution between the two convection films also has a physical reading worth noting. The inner film's contribution decreases with diameter, from 1.56 % in the smallest pipe to 0.83 % in the largest, because its resistance goes as the inverse of the inner radius. The outer one, by contrast, stays practically constant around 2.36 %, because the radius it acts on is that of the insulation, and this grows with the optimal thickness, offsetting the diameter's growth. This is why the outer convection coefficient appears in Section 8.2 as the least influential of the ten factors: its effect acts on a small and stable fraction of the total resistance.

Figure 2 decomposes the total cost into its two components for a representative diameter. The energy cost decreases and the insulation cost increases, both monotonically; only their sum has an interior minimum. The position of that minimum is what Table 4 tabulates for the eighteen states.

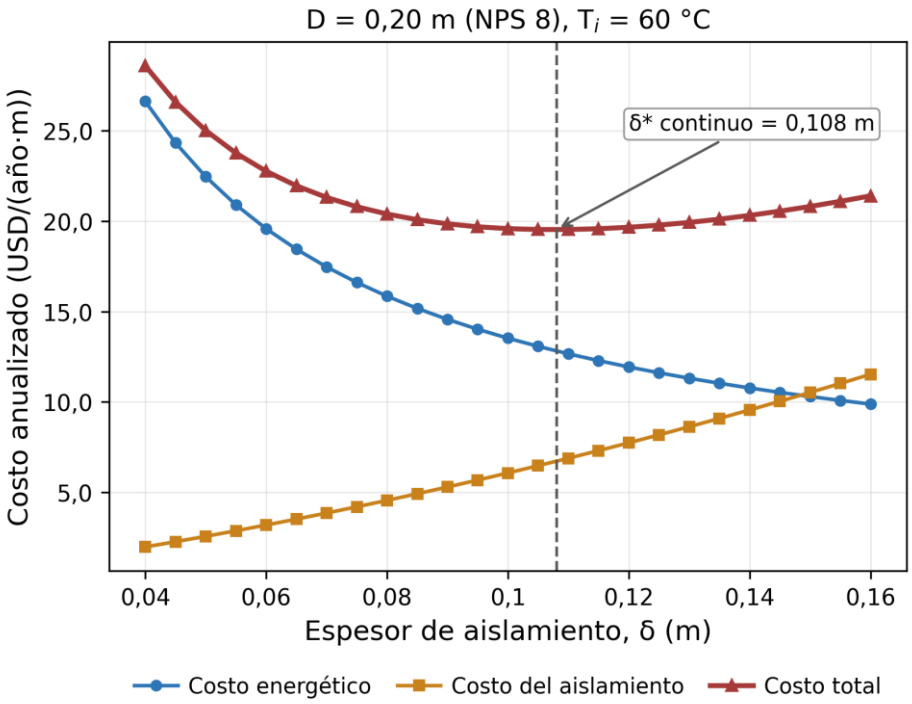


Figure 2. Decomposition of the total annualized cost and location of the continuous optimum, for pipe NPS 8 at Ti = 60 °C.

Table 4 gathers the continuous optimum for each state. The last column verifies that no optimum falls at the edge of domain Ω, a condition without which the result would be censored by the domain's extent rather than by the problem's physics; the check runs automatically in the code and halts the run if it fails.

D (m)	T_i (°C)	δ* (m)	q_s (W/m)	C_E	C_A	C_T	Inner
0.08	50	0.079	6.35	6.34	2.52	8.86	yes
0.10	50	0.083	7.25	7.24	3.11	10.35	yes
0.15	50	0.090	8.96	8.95	4.42	13.37	yes
0.20	50	0.095	10.46	10.44	5.67	16.11	yes
0.25	50	0.098	12.08	12.06	6.91	18.97	yes
0.30	50	0.101	13.49	13.47	8.15	21.62	yes
0.08	60	0.089	7.96	7.95	3.01	10.96	yes
0.10	60	0.094	9.02	9.00	3.72	12.72	yes
0.15	60	0.102	11.07	11.06	5.24	16.30	yes
0.20	60	0.108	12.84	12.82	6.71	19.53	yes
0.25	60	0.112	14.72	14.69	8.19	22.89	yes
0.30	60	0.115	16.44	16.41	9.59	26.00	yes
0.08	70	0.098	9.56	9.55	3.48	13.03	yes
0.10	70	0.104	10.77	10.75	4.31	15.07	yes
0.15	70	0.113	13.15	13.13	6.04	19.17	yes
0.20	70	0.119	15.24	15.22	7.64	22.86	yes
0.25	70	0.124	17.36	17.34	9.36	26.69	yes
0.30	70	0.128	19.27	19.25	10.99	30.23	yes

Table 4. Continuous optimum over Ω for each of the eighteen states. Costs in USD/(year·m).

Two regularities in Table 4 are worth noting. The first is that the optimum grows monotonically with diameter at fixed temperature, and with temperature at fixed diameter: the transfer area and the thermal gradient both push in the same direction, so the most demanding state is the largest diameter at the highest temperature, and the least demanding the smallest diameter at the lowest. The second is that the optimum's span within the set of states is wide —from 0.079 to 0.128 m, that is, nearly five centimetres—, and that span is exactly the magnitude that standardization must absorb with a single value.

Note also that the energy cost dominates the insulation cost in every state, in a proportion that goes from two to one in small pipes to somewhat less in large ones. This is why the optimum falls at high thicknesses: as long as the marginal energy saving keeps exceeding the material's marginal cost, it pays to keep insulating. This proportion depends entirely on the base scenario's price ratio and is the mechanism that explains the sensitivity results of Section 8.2.
Figure 3 places these optima and compares them with the curve from the previous procedure for the same problem.

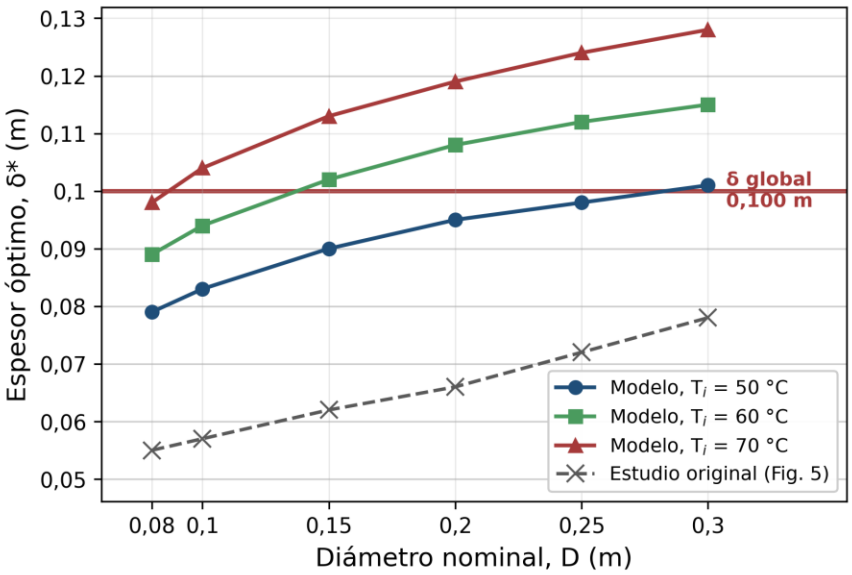


Figure 3. Continuous optimal thickness against diameter for the three temperatures, with the commercial series in grey and the recommended common thickness in red.

7.2 From the continuous optimum to the commercial thickness

The continuous optimum almost never coincides with a thickness that can be purchased. Table 5 bounds each optimum between the immediately lower and higher commercial thickness, and reports the cost penalty of adopting the better of the two. That penalty is 0.09 % on average and 0.28 % at most: small compared with the standardization overruns, but not zero, and it must be declared because it is part of the decision's real cost.

D (m)	T_i (°C)	Continuous δ* (m)	Lower Δ (m)	Upper Δ (m)	Chosen Δ (m)	Penalty (%)
0.08	50	0.079	0.070	0.080	0.080	0.01
0.08	60	0.089	0.080	0.090	0.090	0.01
0.08	70	0.098	0.090	0.100	0.100	0.02
0.10	50	0.083	0.080	0.090	0.080	0.08
0.10	60	0.094	0.090	0.100	0.090	0.10
0.10	70	0.104	0.100	0.120	0.100	0.07
0.15	50	0.090	0.090	0.090	0.090	0.00
0.15	60	0.102	0.100	0.120	0.100	0.02
0.15	70	0.113	0.100	0.120	0.120	0.21
0.20	50	0.095	0.090	0.100	0.090	0.14
0.20	60	0.108	0.100	0.120	0.100	0.28
0.20	70	0.119	0.100	0.120	0.120	0.01
0.25	50	0.098	0.090	0.100	0.100	0.01
0.25	60	0.112	0.100	0.120	0.120	0.27
0.25	70	0.124	0.120	0.140	0.120	0.05
0.30	50	0.101	0.100	0.120	0.100	0.01
0.30	60	0.115	0.100	0.120	0.120	0.09
0.30	70	0.128	0.120	0.140	0.120	0.20

Table 5. Rounding of the continuous optimum to the most convenient commercial thickness, and associated cost penalty.

7.3 Common thickness: minimax by temperature and global minimax

Solving the minimax within each temperature separately gives three different answers: 0.090 m at 50 °C, 0.100 m at 60 °C and 0.120 m at 70 °C, with maximum regrets of 0.92 %, 1.05 % and 2.22 % respectively. It is worth being

precise about what this means. The global value coincides with the 60 °C minimax, the interval's central temperature, and differs from those of 50 and 70 °C. It is therefore not unrelated to any temperature: it is the one that solves the problem at the intermediate temperature and the one that best bounds the worst case when all three must be covered at once. What is true is that it does not coincide with the continuous thermo-economic optimum of any individual state —none of the eighteen has its optimum exactly there— and that if the network operates stably near one end of the interval, that temperature's minimax is a better recommendation than the global one.

Over the eighteen states of $\mathcal{S}$, the thickness that minimizes the maximum regret is $\delta = 0.100$ m, with a worst-case overrun of 3.10 %. That worst case is reached at state D = 0.08 m at a flow temperature of 50 °C, that is, at the smallest diameter at the lowest temperature, which is the state whose optimum lies furthest from the common thickness. Figure 4 presents the complete matrix.

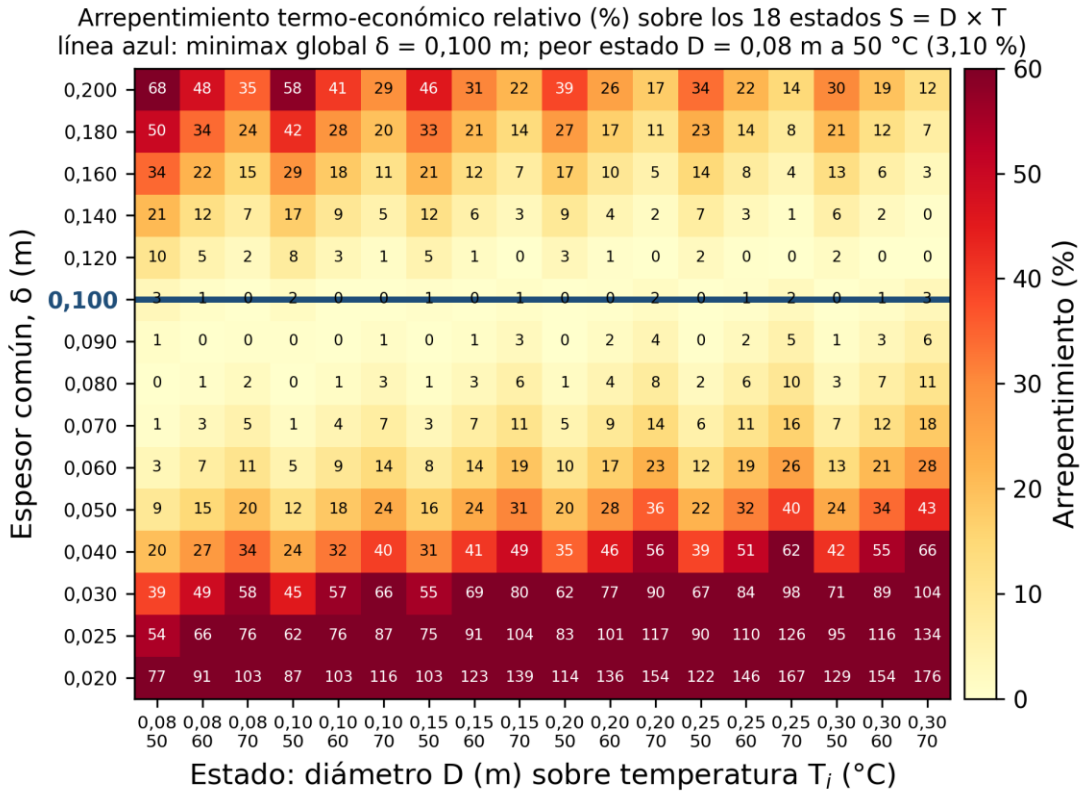


Figure 4. Relative thermo-economic regret of each commercial thickness against the eighteen states. The line marks the global minimax solution.

7.4 Pareto frontier

Figure 5 presents the trade-off between cost and thermal loss. Both axes are weighted averages over the eighteen states with the joint weights of the first scenario in Table 6 —equiprobable diameters and uniform temperature—, not sums or totals of a real network, whose length is unknown. No temperature is left implicit: the axes average all three. The non-dominated front's knee is located with a reproducible criterion —maximum perpendicular distance to the chord joining its ends, on normalized axes— and falls at $\delta = 0.140$ m, which does NOT coincide with the minimax solution of 0.100 m. The discrepancy is to be expected and is worth stating explicitly: the knee is a trade-off between two aggregated objectives, while minimax protects the worst-served state; they are different criteria and need not give the same point. The choice between the two depends on what guarantee the operator needs, not on which figure seems preferable: the knee is appropriate when the network behaves, in practice, like the aggregated scenario that averages the eighteen states, and what matters is the average trade-off between cost and thermal loss; minimax is appropriate when the operator needs a guaranteed cost-overrun bound for the worst-served diameter and temperature, regardless

of how much network length operates under those conditions. Section 8 further shows that minimax is insensitive to the network's declared composition, while the knee is not by construction, since it depends on the aggregated scenario's weights.

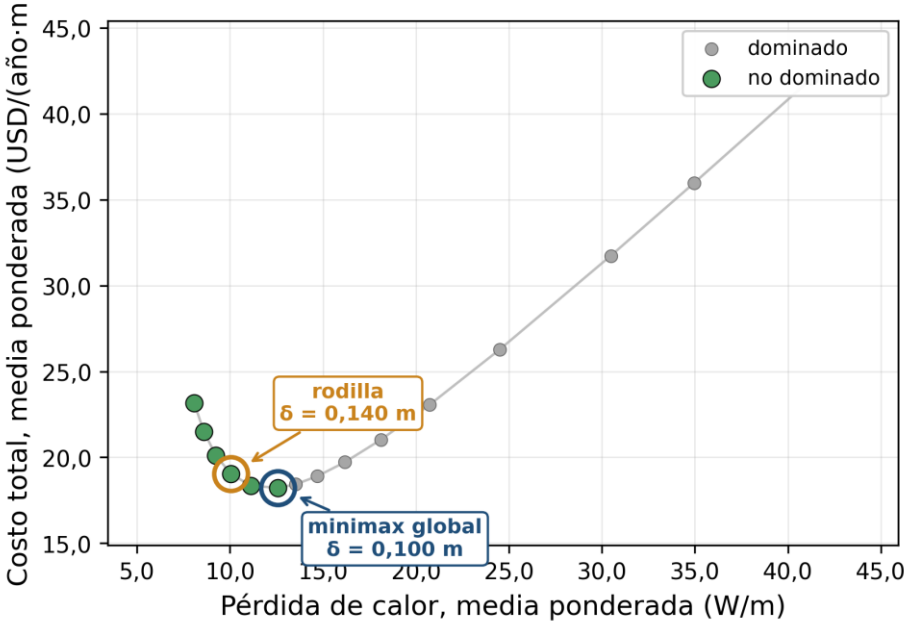


Figure 5. Pareto frontier between cost and heat loss, both as a weighted average over the eighteen states with joint weights p(D,T) = w(D)·u(T) of the equiprobable scenario with uniform temperature. The detected knee and the global minimax solution are marked.

8. SENSITIVITY AND UNCERTAINTY

8.1 Network composition

Table 6 gathers the six scenarios with their two explicit vectors: installed meters by diameter and operating-temperature profile. The expected-cost criterion does depend on them and shifts, spanning the interval {0.100, 0.120} m, in the expected direction: toward larger thicknesses when large diameters predominate.

Scenario	w(0.08)	w(0.10)	w(0.15)	w(0.20)	w(0.25)	w(0.30)	u(50)	u(60)	u(70)	Expected δ (m)
Equiprobable uniform T	0.167	0.167	0.167	0.167	0.167	0.167	0.333	0.333	0.333	0.100
Mostly small uniform T	0.333	0.333	0.083	0.083	0.083	0.083	0.333	0.333	0.333	0.100
Mostly medium uniform T	0.083	0.083	0.333	0.333	0.083	0.083	0.333	0.333	0.333	0.100
Mostly large uniform T	0.083	0.083	0.083	0.083	0.333	0.333	0.333	0.333	0.333	0.120
Equiprobable T skewed to 50 °C	0.167	0.167	0.167	0.167	0.167	0.167	0.600	0.200	0.200	0.100
Equiprobable T skewed to 70 °C	0.167	0.167	0.167	0.167	0.167	0.167	0.200	0.200	0.600	0.120

Table 6. Declared scenarios, with the weight vector by diameter (w), the operating-temperature profile (u) and the thickness that minimizes the expected cost over the eighteen states. The joint weight of a state is p(D,T) = w(D)·u(T). Weights are shown rounded to three decimals for readability; the calculation uses the exact fractions of the normalized vectors, which sum to one.

It is worth being precise about the scope of this result. The minimax criterion returns the same thickness in all six scenarios, but that is not empirical evidence of robustness: minimax does not use the mixture's weights anywhere in its definition, so its invariance to them is an algebraic property of the criterion and not a finding about the problem.

What does inform about the problem is that the expected-cost criterion, which does use the weights, moves by only one commercial step between scenarios.

8.2 Joint uncertainty

An analysis that perturbs one parameter at a time cannot detect interactions and tends to underestimate uncertainty. It is replaced by a global Latin hypercube sample [19] of N = 1024 realizations, with seed 20260905, over ten jointly varied factors: outdoor temperature, energy price, material price, installation factor, heating-system efficiency, outer convection coefficient, discount rate, useful life, operating hours and insulation conductivity. For each realization the complete minimax problem over the eighteen states is solved. Table 7 details the full sampling design and Figure 6 presents the resulting distribution.

Factor	Distribution	Minimum	Maximum
T_e	uniform	10	30
E	uniform	0.061	0.13
p_{mat}	uniform	150	350
f_{inst}	uniform	1.5	2.5
η	uniform	0.6	1
h_a	uniform	8	12
i_d	uniform	0.06	0.15
V_U	uniform	15	25
Z	uniform	6000	8760
Insulation conductivity	uniform (common multiplier)	0.90	1.10

Table 7. Sampling design. The ten factors are sampled with a uniform distribution between the bounds in Tables 2 and 3, independently and without imposing any correlation among them. The method is Latin hypercube [19] via scipy.stats.qmc.LatinHypercube with seed 20260905 and N = 1024 realizations.

Two design decisions require explanation. The first: the insulation's conductivity is not a scalar but a function of mean temperature, interpolated from the manufacturer's table. Perturbing it point by point would amount to sampling a function, not a parameter. Instead, each realization draws a SINGLE uniform multiplier in [0.90; 1.10] and applies it to ALL values of the conductivity curve: the uncertainty is represented as a proportional shift of the whole curve, not as independent noise at each temperature. Conductivity continues to be evaluated, within each realization, at the mean temperature corresponding to each state; the 45 °C cited is only a representative temperature for interpreting the magnitude of the relative change, not the only point at which conductivity is evaluated.

The second: the support of the selected thickness is discrete —the commercial series—, so a percentile computed by linear interpolation could return a value that is not manufactured. The percentiles reported are empirical quantiles: the smallest commercial thickness whose cumulative frequency reaches the sought probability. In this sample both methods coincide, and this has been verified.

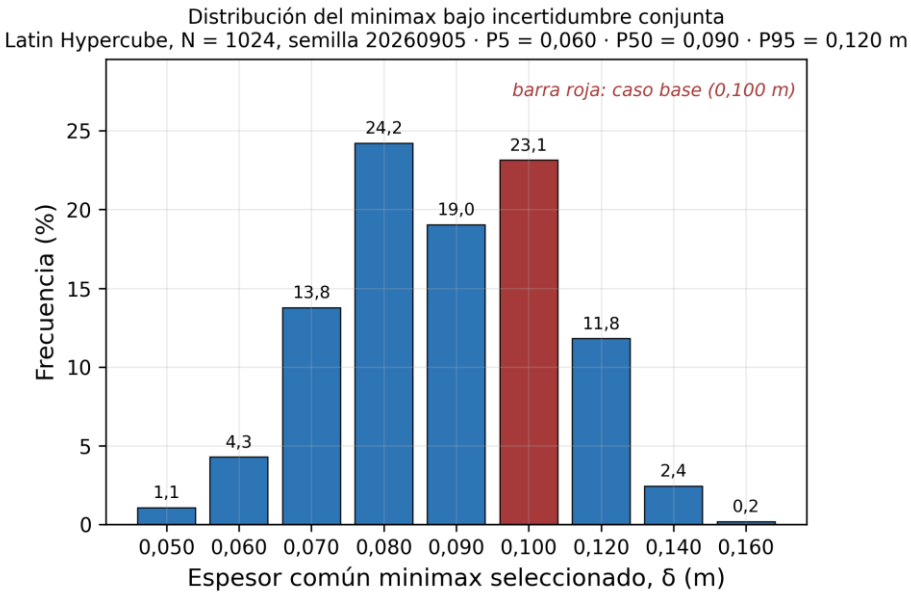


Figure 6. Distribution of the minimax thickness under joint variation of ten factors.

The selected thickness lies between 0.060 and 0.120 m at the 5th and 95th percentiles, with a median of 0.090 m. The most frequent value is 0.080 m, with a frequency of 24.2 %, while that of the base scenario, 0.100 m, is selected in 23.1 % of the realizations. That the mode does not coincide with the base case is a result that must be reported and not smoothed over: it indicates that the base scenario does not occupy the centre of the uncertainty region, and that the recommendation is sensitive to the set of economic assumptions. The maximum regret has a median of 3.85 % and a 95th percentile of 5.36 %, so the cost-overrun guarantee stays at a single-digit percentage throughout the explored space even as the chosen thickness changes.

Figure 7 ranks the factors by two different measures, which should not be conflated. The correlation ratio η^2 measures what fraction of the selected thickness's variance is explained by its mean conditional on the factor, grouped into deciles; it is a signless quantity. Spearman's correlation ρ measures the monotonic association and provides the direction of the effect. By these two measures, material price ($\eta^2 = 0.311$, $\rho = -0.558$) and energy price ($\eta^2 = 0.222$, $\rho = 0.464$) dominate, acting in opposite directions, as corresponds to the two terms competing in equation (6). The outer convection coefficient is the least influential of the ten.

These measures are exploratory associations and not a variance decomposition. In particular, η^2 is not a first-order Sobol index: it is estimated on a single sample, without the two-matrix design that Sobol requires, and the decile discretization introduces an upward bias. For that reason the η^2 values are not summed nor is their complement interpreted as interactions, and additivity is not inferred from a sum close to one: with this estimator the sum can exceed one without that meaning anything. Quantifying the interactions would require a Sobol design, which has not been carried out and remains an open line. What these results support is a ranking of factors by influence, not a partition of variance.

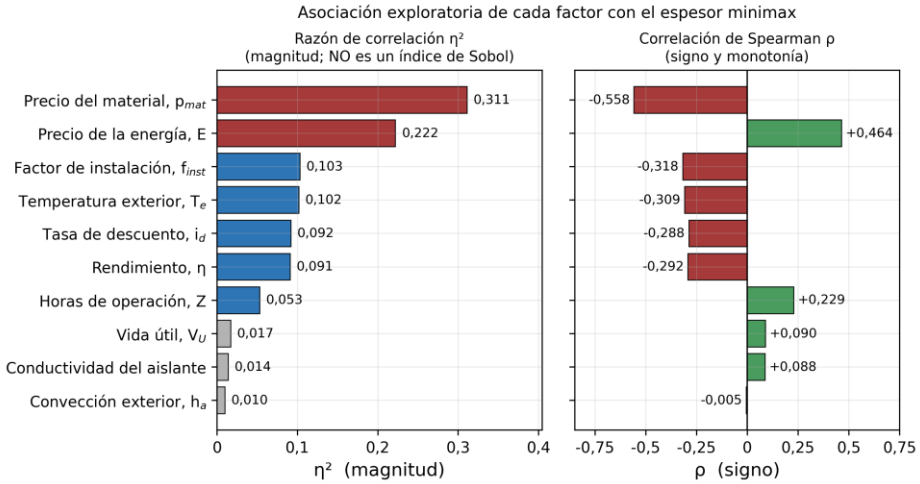


Figure 7. Association of each factor with the minimax thickness: correlation ratio η^2 (magnitude) and Spearman correlation ρ (sign). Both are exploratory measures; η^2 is not a Sobol index.

The inner convection coefficient deserves a separate note. Varying it between 30 and 500 W/(m²·K) —a range covering from heavy crude in laminar regime to very favourable conditions— the optimum of each state shifts by at most one millimetre, because the inner film contributes a negligible fraction of the total resistance compared with the insulation. The uncertainty about that parameter, the worst-known one in the model, does not affect the conclusion.

8.3 Contrast with a compensatory multicriteria aggregation

An alternative compensatory formulation builds a decision matrix of four alternatives and four criteria —three binary indicators of the thickness's membership in the recommended band of each diameter class, plus the relative margin under the range's ceiling—, weights it with CRITIC and aggregates it with TOPSIS. This section is an examination of that formulation, not a comparison of methods: it should be stressed that it operates over four alternatives (0.05, 0.06, 0.07, 0.08 m) and not over the 15 thicknesses of the commercial series used by the minimax, so the two procedures do not share a decision space and their results are not directly comparable. Table 8 gathers the closeness coefficients it produces under two weighting schemes.

δ (m)	CC with CRITIC weights	CC with equal weights
0.05	0.4112	0.5167
0.06	0.4828	0.6000
0.07	0.6214	0.5387
0.08	0.4943	0.3555

Table 8. TOPSIS closeness coefficients of the compensatory formulation under two weighting schemes.

The formulation recommends $\delta = 0.07$ m with CRITIC weights and $\delta = 0.06$ m with equal weights: the recommendation reverses when a weighting assumption the problem does not justify is changed. The origin of the effect can be identified precisely. With the coding used, the large-diameter coverage indicator is the exact complement of the small-diameter one, equation (10), because no thickness in the band can belong to both classes.

$$C_3 = 1 - C_1 \implies \rho(C_1, C_3) = -1 \tag{10}$$

A correlation of −1 does not then express an empirical conflict between two measured criteria, but an algebraic identity of the coding. That said, one should not draw more from this than follows. Recoding that indicator as «not small» would literally return the same column, so that check would be tautological and would prove nothing. And the identity alone does not explain the weight either: by making the two indicators complementary it also makes them SYMMETRIC, so it cannot favor one over the other.

The test that is informative is to remove the margin criterion and recompute. The three coverage indicators have identical standard deviation, so among them the weight is decided solely by the conflict term; with C_1, C_2 and C_3 alone

the weights come out at 0.375; 0.250; 0.375, and the small- and large-diameter coverages tie. What breaks the symmetry, and therefore what gives the large-diameter coverage the maximum weight (0.370), is its relationship with the margin criterion: $\rho(C_1, C_4) = +0.894$ versus $\rho(C_3, C_4) = -0.894$. The large-diameter coverage disagrees with the margin and the small-diameter one agrees with it, and that asymmetry contributes 1.894 of conflict against 0.106.

The correct diagnosis is, therefore, more specific than the identity alone would suggest: the alternative formulation's result is decided by the margin criterion, which is an analyst's construction with no demonstrated economic function, acting on a binary coding that made the other two indistinguishable. This is consistent with what Table 8 itself shows: without that criterion the formulation does not discriminate.

Two qualifications to avoid overstating the argument. First: the correlations computed over four alternatives are perfectly well-defined deterministic descriptive associations; what is objected to is not that they are meaningless, but that they are extremely sensitive to the coding and therefore do not support an interpretation as informative conflict between criteria. Second: none of this invalidates TOPSIS or CRITIC in general, nor even their joint use. What this work holds is that the compensatory variant built on binary indicators does not adequately represent a standardization problem, in which failing to cover a state is a technical failure and not a deficit tradeable for slack elsewhere.

9. DISCUSSION

The main result is that, under the base scenario declared in Tables 2 and 3, the common commercial thickness for the full range of diameters and temperatures is 0.100 m, with a maximum penalty of 3.10 % against the state-by-state solution. The figure bounds what standardization costs; whether that cost is offset or not depends on the logistic saving of handling a single thickness, which this work does not quantify and which must be evaluated with each operator's purchasing, storage and installation data.

The breakdown by temperature is as informative as the aggregate. The global value, 0.100 m, is exactly the 60 °C minimax, and those of 50 and 70 °C are 0.090 and 0.120 m; if the network operates stably at one end of the interval, the choice should follow that value rather than the global one. The global minimax is the correct answer when the operating temperature is not guaranteed, not when it is known.

It is worth situating the result against the 0.05 to 0.08 m band that the previous line of work recommended for the same problem. With the model reconstructed here and the base scenario, the continuous optimum falls between 0.079 and 0.128 m, above that band. The discrepancy does not invalidate the earlier recommendation: it situates it. The analysis in Section 8.2 shows that material price and energy price are the two dominant factors, and that a cost ratio less favourable to insulation —more expensive material or subsidized energy— shifts the solution toward smaller thicknesses. The earlier recommendation is compatible with that price regime and ceases to be so once energy is valued at market price.

Methodologically, the case illustrates a pattern that transcends this problem. When an engineering decision is reduced to a small matrix of binary indicators derived from an underlying continuous model, the economic information that model had produced is destroyed, and objective-weighting methods end up operating on identities of the coding. The regret formulation preserves the magnitude, needs no weights and offers a per-state guarantee. This sensitivity-analysis and robust-aggregation perspective on two-coordinate expert evidence coincides with the criteria for multicriteria decision panels formulated by Ramírez Guerra [23].

10. LIMITATIONS

First and principal: this paper's thermo-economic model is a reconstruction, not a reproduction. The parameters that generated the previous procedure's curves are not found in the available documentation and are not recoverable, so the figures presented here come from the declared scenario and do not reproduce those.

Second: twelve of the fourteen parameters are declared scenarios and not measurements of a specific installation. Section 8.2 quantifies that uncertainty's joint effect: the selected thickness spans from 0.060 to 0.120 m between the

5th and 95th percentiles, and the base-scenario value is not the most frequent one. The results correspond to the base scenario and should not be read as a universal recommendation.

Third: the energy term assumes there is equipment that replaces the thermal loss with a declared efficiency. If there is none, that term is an equivalent energy cost and the real penalty shows up as increased viscosity and pumping power, a coupling this model does not incorporate.

Fourth: the model is one-dimensional and steady-state. It does not incorporate thermal bridges at supports and flanges, nor conductivity degradation from moisture or ageing; both increase the effective loss and push the optimum toward larger thicknesses. Different is the case of the thermo-hydraulic coupling with pumping power, for which —as argued in Section 4.3— neither direction nor magnitude is claimed: establishing them would require solving the coupled model, which has not been solved. The reported solution is therefore not described as conservative with respect to that effect.

Fifth: the commercial thickness series is declared from the range documented by the manufacturer, but the actual availability for each inner diameter is not public and must be confirmed with the distributor before committing to a purchase.

Sixth: there is no experimental validation. No figure in this work has been checked against field measurements, and it should not be presented as validated for industrial application without that check.

11. CONCLUSIONS

The selection of a common insulation thickness for an oil-pipeline network was formulated as a robust decision problem over the eighteen states resulting from combining six diameters with three flow temperatures, rather than as a multicriteria aggregation problem.

Under the declared base scenario, the recommended common commercial thickness is 0.100 m by the minimax regret criterion, with a maximum overrun of 3.10 % relative to the optimum of the worst-served state, which turns out to be the 0.08 m diameter at 50 °C. The minimax values computed within each temperature are 0.090, 0.100 and 0.120 m: the global value coincides with that of 60 °C and departs from the other two, so it is a compromise over the interval and not the best choice for a network whose operating temperature is known.

The recommended value belongs to the declared series of commercial thicknesses. This check is not a formality: the continuous optimum and the procedure's intermediate solutions systematically fall on values that are not manufactured, and the penalty for rounding to the most convenient available thickness is 0.09 % on average.

The global uncertainty analysis over ten factors places the selected thickness between 0.060 and 0.120 m at the 5th and 95th percentiles, with material price and energy price as the dominant factors. The cost-overrun guarantee stays at a single-digit percentage throughout the explored space.

An alternative compensatory formulation (CRITIC-TOPSIS over four alternatives rather than the full commercial series) was also examined. It reverses its recommendation when moving from CRITIC weights to equal weights, and the largest weight CRITIC assigns does not come from the coding's algebraic identity —which makes the two extreme indicators symmetric— but from one of them relating to the margin criterion, an analyst's construction: once that criterion is removed, the two indicators tie. The criticism is specific to that binary coding and does not extend to TOPSIS or CRITIC in general.

The recommendation is conditional on the declared scenario and has not been experimentally validated. The minimax formulation offers a cost-overrun guarantee by state, not the lowest total cost of a real network: that would require the installed meters by diameter and the distribution of operating temperatures, the two data this work flags as pending.

The open lines are the thermal model's experimental validation, the incorporation of thermal bridges and of the coupling with pumping power, and the extension of the criterion to a network with known lengths.

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