

# AN IMPROVED CLASS OF RATIO ESTIMATORS UNDER SYSTEMATIC RANDOM SAMPLING IN PRESENCE OF NON-RESPONSE USING NON- CONVENTIONAL LOCATION PARAMETERS

Zakir Hussain Wani,

Department of Computer Science, Government Degree College Women Sopore, Jammu and Kashmir-193201, India

## ABSTRACT:

In systematic random sampling, the issue of non-response on study variables is examined in order to estimate the population mean using a single auxiliary variable. In the presence of non-response, a generalised ratio estimator is proposed for the population mean. The special instances of the generalised estimator are also investigated by varying the constant values. The MSEs for proposed estimators are derived in order to show that proposed estimators are efficient. The proposed estimators are compared with the other existing estimators empirically to validate the efficiency of the proposed estimators over existing estimators.

**KEYWORDS:** Systematic random sampling, Mean square error, Non-response, Percent relative efficiency, Study variable, Auxiliary variable

**MSC:** 62D05

## RESUMEN:

En el muestreo aleatorio sistemático, se examina el problema de la no respuesta en las variables de estudio con el fin de estimar la media poblacional utilizando una sola variable auxiliar. En presencia de no respuesta, se propone un estimador de razón generalizado para la media poblacional. También se investigan los casos especiales del estimador generalizado variando los valores constantes. Se derivan los errores cuadráticos medios (ECM) de los estimadores propuestos para demostrar que dichos estimadores son eficientes. Los estimadores propuestos se comparan empíricamente con otros estimadores existentes para validar su eficiencia frente a los estimadores ya conocidos.

**PALABRAS CLAVE:** Muestreo aleatorio sistemático, Error cuadrático medio, No respuesta, Eficiencia relativa porcentual, Variable de estudio, Variable auxiliar.

## 1. INTRODUCTION:

The simplest method of random sampling is systematic sampling. It allows us to select the appropriate number of units from the entire population with a single random start, without having to grasp any further sampling techniques or mathematical work. This selection can also be done in the field without the assistance of a statistician. As a result, systematic random sampling is simple to apply in the field and is a common sampling approach for any census or survey field activities. Due to the advantages of this technique, systematic random sampling is utilised in most national surveys and censuses with a single random start for unit selection. Auxiliary information refers to supplementary or prior information. It's normally available from past records, or it can be obtained from a first-phase sample, which is easier to obtain and costs less. Laplace [18] used auxiliary information to estimate the population of France. Cochran [5], amongst many others Murthy [20], Neyman [22], and Robson [23] also employed supplementary data for population mean estimate. Non-response is a critical issue in every census or survey, and it must be addressed in order to obtain reliable results. Non-response can be due to a variety of factors, including not being at home, migration for any cause, non-cooperation, indifference, fear of taxes, fear of lawsuit, and sensitive matters such as a poll on alcoholic use or any sickness. The statisticians must deal with the challenge of non-response using several tactics, such as imputation, referring back to blank questionnaires, and seeking assistance from official records, among others. It's also a difficult assignment for statisticians to come up with accurate and acceptable estimates in the face of non-response. Ahmad et al. [2], Chaudhry et al. [4], Ismail et al. [11], Khare et al. [15], Kumar and Bhogal [16], Singh and Kumar [25,26], and Verma et al. [31,32] developed many estimators in the presence of non-response which are available in the literature. Consider a finite population of size  $N$  from which a simple random sample of size  $n$  is selected without replacement. Because of the problem of non-response, which arises due to lack of interest, not being at home, refusal, and lack of understanding about the surveys, information on all units is often not gathered for estimating the population mean by sample values. Hansen and Hurwitz were the first to tackle the problem of non-response in (1946), they suggested a sub-sampling scheme for estimating population mean comprising the following steps.

- (a) a simple random sample of size  $n$  is drawn and the questionnaire is mailed to the sample units

(b) a sub sample of size  $r = (n_2/T)$ , ( $T \geq 1$ ) from the  $n_2$  non responding units in the initial step attempt is contacted through personal interviews.

Hansen and Hurwitz (1946) suggested an unbiased estimator for the population mean  $\bar{Y}$  is given by

$$\bar{y}^* = w_1 \bar{y}_1 + w_2 \bar{y}_{2r}$$

where  $w_1 = n_1/n$  and  $w_2 = n_2/n$

Here mean of  $n_1$  respondents is  $\bar{y}_1$ , mean of sub-sample of  $n_2$  units out of non-respondents is  $\bar{y}_{2r}$ , whereas the  $\bar{x}$  and  $\bar{y}^*$  are the usual unbiased estimators of  $\bar{X}$  and  $\bar{Y}$  respectively.

The variance of auxiliary variable is:

$$Var(\bar{x}) = \lambda[1 + (n - 1)\rho_x]S_x^2$$

and the variance of  $\bar{y}^*$  is given by

$$Var(\bar{y}^*) = \lambda[1 + (n - 1)\rho_y]S_y^2 + \frac{W_2(T-1)}{n}S_{y(2)}^2 \quad (1)$$

where  $\lambda = \frac{1-f}{n}$  and  $f = n/N$ ,  $\rho_y$  and  $\rho_x$  are intra-class correlation coefficients,  $S_y^2$  and  $S_x^2$  are variances for Y and X variables respectively, while variance of non-respondents is  $S_{y(2)}^2$ , the  $W_2$  is non-response rate and T is the sub-sample rate.

To obtain the expressions for the bias and MSE of the proposed estimators, we use the following error terms

$$\xi_y^* = \frac{\bar{y}^* - \bar{Y}}{\bar{Y}} \quad \text{and} \quad \xi_x = \frac{\bar{x} - \bar{X}}{\bar{X}}$$

Such that  $E(\xi_y^*) = E(\xi_x) = 0$  and

$$E(\xi_y^{*2}) = \lambda[1 + (n - 1)\rho_y]C_y^2 + \frac{W_2(T-1)}{n}C_{y(2)}^2, \\ E(\xi_x^2) = \lambda[1 + (n - 1)\rho_x]C_x^2$$

$$E(\xi_y^* \xi_x) = \lambda(1 + (n - 1)\rho_y)^{\frac{1}{2}}(1 + (n - 1)\rho_x)^{\frac{1}{2}}\rho C_y C_x$$

where,

$$\lambda = \frac{1}{n} - \frac{1}{N}, \quad \rho_x = \frac{E(x_{ij} - \bar{X})(x_{ij} - \bar{X})}{E(x_{ij} - \bar{X})^2}, \quad \rho_y = \frac{E(y_{ij} - \bar{Y})(y_{ij} - \bar{Y})}{E(y_{ij} - \bar{Y})^2}, \quad \rho = \frac{E(x_{ij} - \bar{X})(y_{ij} - \bar{Y})}{(E(x_{ij} - \bar{X})^2(x_{ij} - \bar{X})^2)^{\frac{1}{2}}}$$

$$\rho^* = \frac{(1 + (n - 1)\rho_y)^{\frac{1}{2}}}{(1 + (n - 1)\rho_x)^{\frac{1}{2}}}, \quad B = \frac{S_{xy}}{S_x^2}, \quad \beta_{2(x)} = \frac{\mu_4}{\mu_2^2}, \quad K = \rho \frac{C_y}{C_x}, \quad W_2 = \frac{n_2}{N}, \quad T = \frac{n_2}{h_2}$$

$$HL = \text{median} \left[ \frac{(X_i + X_k)}{2}, 1 \leq j \leq k \leq N \right] \quad \text{Hodges - Lehman estimator}$$

$$MR = \frac{(X_{(1)} + X_{(N)})}{2} \quad \text{Population mid range}$$

$$TM = \frac{Q_1 + 2Q_2 + Q_3}{4}$$

Sing et al. [28] assumed that intra-class correlation is the same for both the variables Y and X, e.g.  $\rho = \rho_x = \rho_y$

## 2. EXISTING ESTIMATORS:

**2.1.** The classical ratio estimator for population mean of study variable Y having non-response in systematic random sampling, with the assumption that  $\bar{X}$  is known, can be defined as:

$$\bar{y}_R^* = \frac{\bar{y}^*}{\bar{x}} \bar{X}$$

with mean square error

$$MSE(\bar{y}_R^*) = \lambda \bar{Y}^2 [1 + (n - 1)\rho_x] [\rho^{*2} C_y^2 + (1 - 2K\rho^*) C_x^2] + \frac{W_2(T-1)}{n} S_{y(2)}^2 \quad (2)$$

**2.2.** Javaid et.al [12] suggested the following estimators.

$$\bar{y}_1 = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{\bar{x}_{sys}} \bar{X}, \quad \bar{y}_2 = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys} + C_x)} (\bar{X} + C_x)$$

$$\bar{y}_3 = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys} + \beta_2(x))}(\bar{X} + \beta_2(x)), \quad \bar{y}_4 = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}\beta_2(x) + C_x)}(\bar{X}\beta_2(x) + C_x)$$

$$\bar{y}_5 = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}C_x + \beta_2(x))}(\bar{X}C_x + \beta_2(x))$$

The MSE of  $\bar{y}_i, i = 1, 2, \dots, 5$  is given as

$$\text{MSE}(\bar{y}_i) = \lambda[1 + (n-1)\rho_x]S_x^2 \left\{ [R_i + B]^2 - 2[R_i + B]\rho^*B + \rho^{*2} \frac{S_y^2}{S_x^2} \right\} + \frac{W_2(T-1)}{n} S_{y(2)}^2 \quad (3)$$

$$\text{where } R_1 = \frac{\bar{Y}}{\bar{X}} = R, \quad R_2 = \frac{\bar{Y}}{\bar{X} + C_x}, R_3 = \frac{\bar{Y}}{\bar{X} + \beta_2(x)}, R_4 = \frac{\bar{Y}}{\bar{X}\beta_2(x) + C_x}, R_5 = \frac{\bar{Y}}{\bar{X}C_x + \beta_2(x)}$$

### 3. PROPOSED ESTIMATOR.

Motivated from Abid et.al [1] we proposed the following ratio estimators for the population mean  $\bar{Y}$  in systematic random sampling under non-response using non-conventional location parameters as auxiliary information when non-response occurs only on the study variable

$$\bar{y}_{PG} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}\eta + \psi)}(\bar{X}\eta + \psi) = Z_{PG}\bar{X}_G$$

where  $Z_{PG} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}\eta + \psi)}$ ,  $\bar{x}_G = (\bar{x}_{sys}\eta + \psi)$ ,  $\bar{X}_G = (\bar{X}\eta + \psi)$ ,  $\eta$  and  $\psi$  are the known constants of auxiliary variable such as

*Family of estimators are:*

Estimators	$\eta$	$\psi$
$\bar{y}_{P1} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{\bar{x}_{sys}S_k + TM}(\bar{X}S_k + TM)$	$S_k$	$TM$
$\bar{y}_{P2} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}C_x + TM)}(\bar{X}C_x + TM)$	$C_x$	$TM$
$\bar{y}_{P3} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}\rho + TM)}(\bar{X}\rho + TM)$	$\rho$	$TM$
$\bar{y}_{P4} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys} + MR)}(\bar{X} + MR)$	1	$MR$
$\bar{y}_{P5} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}C_x + MR)}(\bar{X}C_x + MR)$	$C_x$	$MR$
$\bar{y}_{P6} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}\rho + MR)}(\bar{X}\rho + MR)$	$\rho$	$MR$
$\bar{y}_{P7} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys} + HL)}(\bar{X} + HL)$	1	$HL$
$\bar{y}_{P8} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}C_x + HL)}(\bar{X}C_x + HL)$	$C_x$	$HL$
$\bar{y}_{P9} = \frac{\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}\rho + HL)}(\bar{X}\rho + HL)$	$\rho$	$HL$

MSE of this generalized class of estimators can be found using Taylor series method defined as

$$h(\bar{x}, \bar{y}) \cong h(\bar{X}, \bar{Y}) + \frac{\partial h(c, d)}{\partial c} \Big|_{\bar{X}, \bar{Y}} (\bar{x} - \bar{X}) + \frac{\partial h(c, d)}{\partial d} \Big|_{\bar{X}, \bar{Y}} (\bar{y} - \bar{Y})$$

where  $h(\bar{x}, \bar{y}) = Z_{PG}$  and  $h(\bar{X}, \bar{Y}) = R$

$$\begin{aligned} Z_{PG} &\cong R + \frac{\partial \{[\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})]/(\bar{x}_{sys}\eta + \psi)\}}{\partial \bar{x}} \Big|_{\bar{X}, \bar{Y}} (\bar{x} - \bar{X}) \\ &\quad + \frac{\partial \{[\bar{y}_{sys}^* + b(\bar{X} - \bar{x}_{sys})]/(\bar{x}_{sys}\eta + \psi)\}}{\partial \bar{y}} \Big|_{\bar{X}, \bar{Y}} (\bar{y}_{sys}^* - \bar{Y}) \\ Z_{PG} - R &\cong \frac{\partial}{\partial \bar{x}} \left( \frac{\bar{y}_{sys}^*}{(\bar{x}_{sys}\eta + \psi)} + \frac{b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}\eta + \psi)} \right) \Big|_{\bar{X}, \bar{Y}} (\bar{x} - \bar{X}) + \frac{\partial}{\partial \bar{y}} \left( \frac{\bar{y}_{sys}^*}{(\bar{x}_{sys}\eta + \psi)} + \frac{b(\bar{X} - \bar{x}_{sys})}{(\bar{x}_{sys}\eta + \psi)} \right) \Big|_{\bar{X}, \bar{Y}} (\bar{y}_{sys}^* - \bar{Y}) \\ E(Z_{PG} - R)^2 &\cong \left[ \frac{\bar{Y} + B(\bar{X}\eta + \psi)}{(\bar{X}\eta + \psi)^2} \right]^2 V(\bar{x}_{sys}) - 2 \frac{\bar{Y} + B(\bar{X}\eta + \psi)}{(\bar{X}\eta + \psi)^3} Cov(\bar{x}_{sys}, \bar{y}_{sys}^*) + \frac{1}{(\bar{X}\eta + \psi)^2} V(\bar{y}_{sys}^*) \\ \text{MSE}(\bar{y}_{PG}) &\cong (\bar{X}\eta + \psi)^2 E(Z_{PG} - R)^2 \\ \text{MSE}(\bar{y}_{PG}) &\cong \frac{(\bar{Y} + B(\bar{X}\eta + \psi))^2}{(\bar{X}\eta + \psi)^2} V(\bar{x}_{sys}) + V(\bar{y}_{sys}^*) - 2 \frac{\bar{Y} + B(\bar{X}\eta + \psi)}{(\bar{x}_{sys}\eta + \psi)} Cov(\bar{x}_{sys}, \bar{y}_{sys}^*) \end{aligned}$$

Using the value of  $V(\bar{x}_{sys})$  and  $V(\bar{y}_{sys}^*)$  we get the value of the proposed estimators

$$\begin{aligned} \text{MSE}(\bar{y}_{PG}) &\cong \lambda [1 + (n - 1)\rho_x] S_x^2 \left\{ \left[ \frac{\bar{Y}}{\bar{X}\eta + \psi} + B \right]^2 - 2 \left[ \frac{\bar{Y}}{\bar{X}\eta + \psi} + B \right] \rho^* B + \rho^{*2} \frac{S_y^2}{S_x^2} \right\} + \frac{W_2(T - 1)}{n} S_{y(2)}^2 \\ \text{MSE}(\bar{y}_{PG}) &\cong \lambda [1 + (n - 1)\rho_x] S_x^2 \left\{ [R_{Pj} + B]^2 - 2[R_{Pj} + B]\rho^* B + \rho^{*2} \frac{S_y^2}{S_x^2} \right\} + \frac{W_2(T - 1)}{n} S_{y(2)}^2 \quad ; j \\ &= 1, 2, 3, \dots, 9 \end{aligned} \quad (4)$$

where,  $R_{Pj} = \frac{\bar{Y}}{\bar{X}\eta + \psi}$

$$\begin{aligned} R_{P1} &= \frac{\bar{Y}}{\bar{X}S_k + TM} \quad R_{P2} = \frac{\bar{Y}}{\bar{X}C_x + TM} R_{P3} = \frac{\bar{Y}}{\bar{X}\rho + TM} R_{P4} = \frac{\bar{Y}}{\bar{X} + MR} R_{P5} = \frac{\bar{Y}}{\bar{X}C_x + MR} \\ R_{P6} &= \frac{\bar{Y}}{\bar{X}\rho + MR} R_{P7} = \frac{\bar{Y}}{\bar{X} + HL} R_{P8} = \frac{\bar{Y}}{\bar{X}C_x + HL} R_{P9} = \frac{\bar{Y}}{\bar{X}\rho + HL} \end{aligned}$$

#### 4. EFFICIENCY COMPARISON:

1.  $\bar{y}_{pj}$  (i.e.;  $i=1, 2, \dots, 9$ ) perform better than  $\bar{y}^*$  if:

$$\text{MSE}(\bar{y}_{pj}) < \text{MSE}(\bar{y}^*)$$

$$[1 + (n - 1)\rho_x] \left\{ [R_{Pj} + B]^2 - 2[R_{Pj} + B]\rho^* B + \rho^{*2} \frac{S_y^2}{S_x^2} \right\} S_x^2 - [1 + (n - 1)\rho_y] S_y^2 < 0$$

2.  $\bar{y}_{pj}$  (i.e;  $j=1, 2, \dots, 9$ ) perform better than  $\bar{y}_i^*$ ,  $i = 1, 2, \dots, 5$  if

$$\text{MSE}(\bar{y}_{pj}) < \text{MSE}(\bar{y}_R^*)$$

$$S_x^2 \left\{ [R_{Pj} + B]^2 - 2[R_{Pj} + B]\rho^* B + \rho^{*2} \frac{S_y^2}{S_x^2} \right\} - \{\rho^{*2} S_y^2 + R(1 - 2K\rho^*) S_x^2\}$$

3.  $\bar{y}_{pj}$  (i.e;  $j=1,2,\dots,9$ ) perform better than  $\bar{y}_i^*, i = 1, 2, \dots, 5$  if

$$MSE(\bar{y}_{pj}) < MSE(\bar{y}_i^*)$$

$$(R_{pj} - R_i)(R_{pj} + R_i + 2B - 2\rho^*B) < 0$$

## 5. EMPIRICAL STUDY:

In order to see the performance of the proposed estimators empirically in comparison to the existing estimators, we have taken the data set from Murthy (1967) in which Y denotes the number of workers and X denotes the output. We may apply this data for numerical solution of mathematical conditions. For solution of mathematical conditions, the value of  $\rho^* = 1$  i.e.,  $\rho = \rho_x = \rho_y$

Descriptive statistics are:

N=80	N=20	$\rho = 0.915$	$\bar{Y} = 285.125$
$\bar{X} = 5182.638$	$S_y = 270.429$	$S_x = 1835.659$	$\beta_{1(x)} = 0.133$
$\beta_{2(x)} = -0.702$	HL=5091	MR=5213	TM=5172.063
B=0.1348	$\lambda = 0.0375$		
<b>When <math>W_2 = 20\% \text{ Non - response}</math></b>			
$S_{y(2)} = 10.01$			
<b>When <math>W_2 = 30\% \text{ Non - response}</math></b>			
$S_{y(2)} = 19.174$			

### Percent Relative efficiency of existing & proposed estimators with respect to $\bar{y}^*$

<b>When <math>W_2 = 0.2</math></b>					
<b>Estimators</b>	<b>T=4</b>	<b>T=5</b>	<b>T=6</b>	<b>T=7</b>	<b>T=8</b>
$\bar{y}^*$	100	100	100	100	100
$\bar{y}_R^*$	219.5628	219.5576	219.5524	219.5472	219.542
$\bar{y}_1$	330.8558	330.8407	330.8255	330.8103	330.7951
$\bar{y}_2$	330.8767	330.8615	330.8463	330.8312	330.816
$\bar{y}_3$	330.8145	330.7993	330.7841	330.769	330.7538
$\bar{y}_4$	224.3034	224.2979	224.2923	224.2868	224.2812
$\bar{y}_5$	78.42018	78.42052	78.42086	78.42119	78.42153
$\bar{y}_{p1}$	367.8893	367.8697	367.8502	367.8306	367.811
$\bar{y}_{p2}$	418.2915	418.2651	418.2386	418.2122	418.1857
$\bar{y}_{p3}$	497.7522	497.7128	497.6735	497.6342	497.5949
$\bar{y}_{p4}$	<b>506.4442</b>	506.4033	506.3624	506.3215	506.2806
$\bar{y}_{p5}$	419.8428	419.8161	419.7895	419.7628	419.7361
$\bar{y}_{p6}$	498.5283	498.4888	498.4494	498.4099	498.3704
$\bar{y}_{p7}$	504.3292	5047.412	5042.454	5037.507	5032.57
$\bar{y}_{p8}$	415.1731	415.1471	415.1211	415.0951	415.0691
$\bar{y}_{p9}$	496.194	496.1549	496.1159	496.0768	496.0378

When $W_2 = 0.3$					
Estimators	T=4	T=5	T=6	T=7	T=8
$\bar{y}^*$	100	100	100	100	100
$\bar{y}_R^*$	219.4924	219.4637	219.4351	219.4064	219.3778
$\bar{y}_1$	330.6509	330.5676	330.4843	330.401	330.3179
$\bar{y}_2$	330.6718	330.5884	330.5051	330.4218	330.3387
$\bar{y}_3$	330.6097	330.5263	330.4431	330.3598	330.2767
$\bar{y}_4$	224.2286	224.1981	224.1677	224.1372	224.1069
$\bar{y}_5$	78.42473	78.42657	78.42843	78.43025	78.43213
$\bar{y}_{p1}$	367.625	367.5174	367.41	367.3026	367.1954
$\bar{y}_{p2}$	417.9345	417.7892	417.6442	417.4991	417.3544
$\bar{y}_{p3}$	497.2213	497.0055	496.79	496.5745	496.3595
$\bar{y}_{p4}$	<b>505.8923</b>	505.6678	505.4437	505.2197	504.9963
$\bar{y}_{p5}$	419.4827	419.3362	419.1899	419.0437	418.8977
$\bar{y}_{p6}$	497.9956	497.7789	497.5627	497.3464	497.1307
$\bar{y}_{p7}$	503.7824	503.5601	503.3382	503.1162	502.8949
$\bar{y}_{p8}$	414.8222	414.6794	414.5369	414.3943	414.2521
$\bar{y}_{p9}$	495.6669	495.4525	495.2385	495.0245	494.8111

## 6. CONCLUSION:

In systematic sampling, a modified estimators are proposed for estimating the population mean under non-response using an auxiliary variable. For various values of T and non-response rate of 20% and 30%, the proposed estimator's mean square error is compared to that of other existing estimators. It has been found that the proposed estimators outperform existing estimators. We can see that with the increase in the value of T the efficiency decreases and also  $\bar{y}_{p4}$  outperforms among all of the other estimators. As a result, we conclude that the proposed estimators are suitable for application in practice.

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