APPLICATION OF NEUTROSOPHIC AHP IN THE ANALYSIS OF SHARED CUSTODY: A STUDY OF FAMILY LAW

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ABSTRACT
The study addresses a critical issue in family law: the evaluation of shared custody decisions using a systematic and quantifiable framework. Traditional approaches to custody determinations cannot often incorporate multiple, conflicting criteria, such as the emotional well-being of the child, the financial stability of the parents, and the logistical feasibility of shared arrangements. This research aims to bridge this gap by applying a Neutrosophic Analytic Hierarchy Process (AHP) model, which allows for the incorporation of uncertainty and subjectivity inherent in custody decisions. By introducing mathematical rigor to a field often dominated by qualitative assessments, the study offers a pathway to more balanced and transparent decision-making. The results demonstrate the utility of the Neutrosophic AHP method in handling complex, multi-criteria evaluations in family law. Key findings include the identification of weighting vectors that prioritize the child's welfare while addressing practical considerations for both parents. The integration of linguistic terms with neutrosophic numbers, such as truth (T), indeterminacy (I), and falsity (F), provides a structured mechanism to capture the nuances of judicial discretion. This innovative approach contributes not only to the theoretical advancement of decision-making methodologies in legal contexts but also offers practical implications for policymakers and legal practitioners seeking to enhance fairness and clarity in shared custody rulings.
KEYWORDS: Neutrosophic AHP, shared custody, family law, multi-criteria analysis, decision-making, uncertainty.

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RESUMEN
El estudio aborda un tema crítico en el derecho de familia: la evaluación de las decisiones de custodia compartida utilizando un marco sistemático y cuantificable. Los enfoques tradicionales de las determinaciones de custodia a menudo carecen de la capacidad de incorporar criterios múltiples y conflictivos, como el bienestar emocional del niño, la estabilidad financiera de los padres y la viabilidad logística de los arreglos compartidos. Esta investigación tiene como objetivo cerrar esta brecha aplicando un modelo de proceso de jerarquía analítica neutrosófica (AHP), que permite la incorporación de la incertidumbre y la subjetividad inherentes a las decisiones de custodia. Al introducir el rigor matemático en un campo a menudo dominado por evaluaciones cualitativas, el estudio ofrece una vía de toma de decisiones más equilibrada y transparente. Los resultados demuestran la utilidad del método AHP neutrosófico en el manejo del manejo de evaluaciones complejas de criterios múltiples en el derecho de familia. Los hallazgos clave incluyen la identificación de vectores de ponderación que priorizan el bienestar del niño al tiempo que abordan consideraciones prácticas para ambos padres. La integración de los términos lingüísticos con números neutrosóficos, como la verdad (t), la indeterminación (I) y la falsedad (F), proporciona un mecanismo estructurado para capturar los matices de la discreción judicial. Este enfoque innovador contribuye no solo al avance teórico de las metodologías de toma de decisiones en contextos legales, sino que también ofrece implicaciones prácticas para los responsables políticos y profesionales legales que buscan mejorar la equidad y la claridad en las decisiones de custodia compartida.
PALABRAS CLAVE: AHP neutrosófico, custodia compartida, derecho de familia, análisis de criterios múltiples, toma de decisiones, incertidumbre.

1. INTRODUCTION

Shared custody has emerged as a crucial yet underexplored concept in the realm of family law. It involves the joint responsibility of parents to ensure the proper upbringing of minors, focusing on equitable sharing of time, financial obligations, and emotional care. While shared custody aligns with the principles of equality and co-responsibility enshrined in Ecuador's Constitution [13], its implementation faces significant challenges, particularly due to the absence of clear legal guidelines and the limited capacity of judicial systems to enforce agreements effectively. Historically, shared custody has been conceptualized as a modern response to evolving family dynamics. Originating in Sweden, this legal institution gained momentum across Europe, with France introducing more comprehensive legislation by 2002 that allowed children to maintain dual residences with both parents [6]. In Latin America, countries like Chile, Argentina, and Mexico have incorporated shared custody into their legal

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frameworks, recognizing its potential to mitigate the emotional and psychological impact of parental separation on children [7]. Ecuador, however, remains at a crossroads, with shared custody only enforceable through parental agreement and judicial approval, leaving significant gaps in its practical application. The core problem addressed in this study is the limited implementation of shared custody in Ecuador despite its alignment with the constitutional principle of the child's best interest. How can a framework be developed to ensure equitable co-responsibility while respecting the individual rights of all family members? This question drives the research, seeking to uncover the legal, social, and institutional barriers to shared custody and propose actionable solutions to address them.

Ecuador's current legal framework presents both opportunities and obstacles. The recognition of parental co-responsibility and the right of children to maintain relationships with both parents are foundational principles [18]. Yet, the judicial system lacks the authority to unilaterally impose shared custody without a mutual parental agreement, which can hinder its broader adoption [9]. Additionally, societal perceptions and logistical challenges further complicate its implementation, emphasizing the need for a systematic analysis to bridge these gaps. This research leverages the Neutrosophic Analytic Hierarchy Process (AHP) to provide a structured methodology for evaluating shared custody. By incorporating linguistic terms and mathematical tools, the study captures the complexity of factors influencing custody decisions. These include parental cooperation, the psychological well-being of children, and the capacity of judicial systems to enforce equitable arrangements. This innovative approach allows for the simultaneous consideration of conflicting criteria, offering a balanced framework for decision-making.

Preliminary findings reveal that shared custody has the potential to positively impact children's development when parents maintain amicable relationships [15.21]. However, the lack of comprehensive legislation and judicial discretion often results in outcomes that fail to align with the best interest of the child. By addressing these shortcomings, the study aims to contribute to the development of a more robust legal framework that ensures equitable parental co-responsibility while prioritizing the child's welfare. The objectives of this research are twofold: first, to identify the key factors that influence the successful implementation of shared custody in Ecuador; and second, to propose a decision-making model that integrates the principles of family law with mathematical rigor. This model not only offers a pathway for fairer custody decisions but also establishes a foundation for future legal reforms. Ultimately, this study seeks to bridge the gap between theory and practice in family law. Applying a Neutrosophic AHP framework, it provides a novel approach to understanding and addressing the complexities of shared custody. The results have the potential to inform policymakers, legal practitioners, and families, contributing to a more equitable and effective legal system that aligns with the best interests of children and adolescents.

2. PRELIMINARIES

2.1 Saaty's Analytic Hierarchy Process (AHP)

It is one of the most widespread methods for solving multi-criteria decision-making problems. This technique models the problem leading to the formation of a hierarchy representative of the associated decision-making scheme. This hierarchy presents at the upper level the objective pursued in solving the problem, and at the lower level, it includes the different alternatives from which a decision must be made. The intermediate levels detail the set of criteria and attributes considered [20, 21].

The AHP is theory-oriented toward the decision-maker and serves to identify the best alternative according to the allocated resources. This method can be applied to situations involving technical, economic, political, social, and cultural factors. That is, it aims to be a scientific tool to address those aspects that are difficult to quantify but sometimes require a unit of measurement.

Some authors suggest that the AHP has not been well understood, as it goes beyond being a simple methodology for choice situations. It is then proposed that the best way to understand the method is by describing its three basic functions: structuring complexity, measuring on a scale, and synthesizing. The hierarchy is constructed so that the elements are of the same order of magnitude and can be related to some of the following levels [19]. The steps are:

1. Prioritization of the elements of the hierarchical model.
2. Binary comparison of the elements.
3. Evaluation of the elements by assigning weights.
4. Ranking of the alternatives according to the given weights.
5. Synthesis and sensitivity analysis.

The AHP, proposed by Thomas Saaty in 1980, is one of the most widespread methods for solving multi-criteria decision-making problems. This technique outlines the process for building a hierarchical structure that reflects the decision-making framework. At the top level, the hierarchy defines the main objective to be achieved in addressing the problem. The bottom level contains the various alternatives among which a decision must be made.

The intermediate levels serve to organize the criteria and attributes that influence the decision-making process. To fully explain the method, it is essential to introduce the following definitions.

Definition 1: The Neutrosophic set *N* is characterized by three membership functions, which are the truth-membership function *TA*, indeterminacy-membership function *IA*, and falsehood-membership function *FA*, where *U* is the Universe of Discourse and $\forall x \in U$, $TA(x), IA(x), FA(x) \subseteq]-0, 1+[$, and

$$-0 \leq inf\, TA(x) + inf\, IA(x) + inf\, FA(x) \leq sup\, TA(x) + sup\, IA(x) + sup\, FA(x) \leq 3+.$$

Notice that, according to the definition, $TA(x)$, $IA(x)$, and $FA(x)$ are real standard or non-standard subsets $]-0, 1+[$ and hence, $TA(x)$, $IA(x)$ and $FA(x)$ can be subintervals of [0, 1].

Definition 2: The Single-Valued Neutrosophic Set (SVNS) *N* over *U* is $A = \{\langle x; TA(x), IA(x), FA(x)\rangle : x \in U\}$, where $TA: U \rightarrow [0, 1]$, $IA: U \rightarrow [0, 1]$, and $FA: U \rightarrow [0, 1]$,

$$0 \leq TA(x) + IA(x) + FA(x) \leq 3.$$

The Single-Valued Neutrosophic Number (SVNN) is represented by $N = (t, I, f)$, such that $0 \leq t, I, f \leq 1$ and

$$0 \leq t + I + f \leq 3.$$

Definition 3: The single-valued trapezoidal neutrosophic number, $\tilde{a} = \langle (a_1, a_2. a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy, and falsehood membership functions are defined as follows, respectively

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a\_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, & a_2 \leq x \leq a_3 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \beta_{\tilde{a}}(x-a_1)\right)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, & a_2 \leq x \leq a_3 \\ \frac{\left(x - a_2 + \beta_{\tilde{a}}(a_3-x)\right)}{a_3-a_2}, & a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \gamma_{\tilde{a}}(x-a_1)\right)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, & a_2 \leq x \leq a_3 \\ \frac{\left(x - a_2 + \gamma_{\tilde{a}}(a_3-x)\right)}{a_3-a_2}, & a_3 \leq x \leq a_4 \\ 1, & otherwise \end{cases} \quad (3)$$

where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$, $a_1, a_2, a_3, a_4 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3 \leq a_4$.

Definition 4: Given $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued trapezoidal neutrosophic numbers and λ any non-null number in the real line. Then, the following operations are defined:

Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$

Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$ (4)

Inversion: $\tilde{a}^{-1} = \langle ({a_4}^{-1}, {a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, *where* $a_1, a_2, a_3, a_4 \neq 0$.

Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{Bmatrix} \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{Bmatrix}$$

Definitions 3 and 4 pertain to single-valued triangular neutrosophic numbers under the condition $a_2=a_3$. To streamline the process, the linguistic scale of triangular neutrosophic numbers, as illustrated in Table 1, is employed and compared to the predefined scale therein. The importance levels or weights assigned to the criteria are calculated through paired comparisons, allowing for a systematic estimation of their relative significance.

$$S = \left\{\frac{1}{9}, \frac{1}{7}, \frac{1}{5}, \frac{1}{3}, 1,3,5,7,9\right\} \quad (5)$$

Through the use of the theory of AHP technique in a neutrosophic framework (Neutrosophic AHP, or NAHP for short), the indeterminacy of decision-making can be modeled.

$$\tilde{A} = \begin{bmatrix} \tilde{1} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{1} \end{bmatrix} \quad (6)$$

Matrix $\tilde{A}$ must satisfy condition $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1}$, based on the inversion operator of Definition 4.

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (7)$$

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (8)$$

To convert neutrosophic triangular numbers into crisp numbers, there are two indexes defined in, they are the so-called score and accuracy indexes, respectively, see Equations 7 and 8:

Saaty's scale	Definition	Neutrosophic Triangular Scale
1	Equally influential	$\tilde{1} = \langle(1,1,1); 0.50, 0.50, 0.50\rangle$
3	Slightly influential	$\tilde{3} = \langle(2,3,4); 0.30, 0.75, 0.70\rangle$
5	Strongly influential	$\tilde{5} = \langle(4,5,6); 0.80, 0.15, 0.20\rangle$
7	Very strongly influential	$\tilde{7} = \langle(6,7,8); 0.90, 0.10, 0.10\rangle$
9	Absolutely influential	$\tilde{9} = \langle(9,9,9); 1.00, 1.00, 1.00\rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle(1,2,3); 0.40, 0.65, 0.60\rangle$ $\tilde{4} = \langle(3,4,5); 0.60, 0.35, 0.40\rangle$ $\tilde{6} = \langle(5,6,7); 0.70, 0.25, 0.30\rangle$ $\tilde{8} = \langle(7,8,9); 0.85, 0.10, 0.15\rangle$

Table 1. Saaty's scale translated to a neutrosophic triangular scale. Source: [13]

3. METHODS

The study began with a comprehensive bibliographic review, employing a mixed qualitative-quantitative methodology [11, 12]. A detailed literature search was conducted to deepen the understanding of shared custody, focusing on its challenges and potential improvements. Additionally, input was gathered from 20 specialists with extensive experience in family law, who were asked to identify the most common causes threatening the proper functioning of shared custody. Their expertise allowed them to provide insights aimed at safeguarding the well-being of minors and enhancing the effectiveness of shared custody arrangements.

To analyze the identified causes of non-compliance, a Pareto diagram was employed, highlighting the most significant issues contributing to shared custody challenges. The proposed solutions, generated through expert consensus, were then evaluated using the Analytic Hierarchy Process (AHP) method in its neutrosophic version, which accounts for uncertainty and subjectivity in decision-making [13, 19]. The methodology followed in this research is detailed in the subsequent sections.

Theoretical methods

✓ Analytical Synthetic Method: The analytical method allows the decomposition of the whole into specific aspects to understand and comprehend the structure; it facilitates observance to better understand the components. In this context, this method implies synthesis, that is, the union of dispersed elements to form a total component. This is put into practice through the review of the bibliography, which emphasizes what is necessary to argue the research and the topic of analysis.

✓ Inductive Deductive Method: This research method allows logical reasoning. While the inductive method starts from specific premises to reach general aspects, the deductive method is the opposite, as it starts from general to reach particular aspects. However, both methods are essential in the construction of knowledge. For this research, these methods allowed to understand the problem and propose possible solutions.

✓ Historical Logical Method: These methods allow the construction of research based on historical elements, to understand the essential elements of the same and its historical evolution.

Methods for Information Processing

✓ Pareto Diagram: it was used for the selection of criteria. It was introduced by J.M. Jurán in his Quality Control Handbook based on what was described in 1909 by V. Pareto under the principle of "the vital few and the trivial many." This diagram is based on problem analysis and is used to present data, drawing attention to the causes of greatest incidence in the problem in question [14,20]. Its objective is to determine 20% of the causes that provoke 80% of the problems.

Its main advantages are:

✓ It allows focusing on the aspects whose improvement will have the most impact, thus optimizing efforts.

✓ It provides a simple and quick view of the relative importance of problems.

✓ It helps prevent some causes from worsening while trying to solve others that are less significant.

✓ Its graphical view of the analysis is easy to understand and stimulates the team to continue with the improvement.

For its development, the following algorithm is executed:

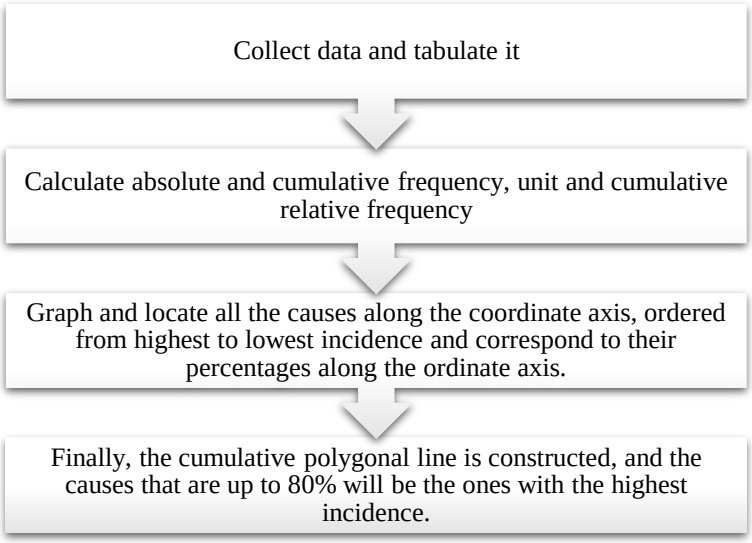

Figure 1. Algorithm for executing the Pareto Diagram. Source: own elaboration.

Neutrosophic AHP

Step 1. Select a group of experts.

Step 2. Structure the neutrosophic pair-wise comparison matrix of factors, sub-factors, and strategies, through the linguistic terms shown in Table 1.

The neutrosophic scale is attained according to expert opinions. The neutrosophic pair-wise comparison matrix of factors, sub-factors, and strategies is described in Equation 6.

Step 3. Check the consistency of experts' judgments.

If the pair-wise comparison matrix has a transitive relation, ie, $a_{ik} = a_{ij}a_{jk}$ for all i,j and k, then the comparison matrix is consistent, focusing only on the lower, median, and upper values of the triangular neutrosophic number of the comparison matrix.

Step 4. Calculate the weight of the factors from the neutrosophic pair-wise comparison matrix, by transforming it to a deterministic matrix using Equations 9 and 10. To get the score and the accuracy degree of $\tilde{a}_{ji}$ the following equations are used:

$$S(\tilde{a}_{ji}) = {}^{1}\Big/_{S(\tilde{a}_{ij})} \tag{9}$$

$$A(\tilde{a}_{ji}) = {}^{1}\Big/_{A(\tilde{a}_{ij})} \tag{10}$$

With compensation by the accuracy degree of each triangular neutrosophic number in the neutrosophic pair-wise comparison matrix, we derive the following deterministic matrix:

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} \tag{11}$$

Determine the ranking of priorities, namely the Eigen Vector X, from the previous matrix:

Note that Step 3 refers to considering the use of the calculation of the Consistency Index (CI) when applying this technique, which is a function depending on λ_{max}, the maximum eigenvalue of the matrix. Saaty establishes that the consistency of the evaluations can be determined by equation:

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{12}$$

where n is the order of the matrix. In addition, the Consistency Ratio (CR) is defined by equation:

$$CR = \frac{CI}{RI} \tag{13}$$

RI is given in [13]. If $CR \leq 0.1$, it indicates that the experts' evaluation is sufficiently consistent, allowing for the application of NAHP. This procedure is then applied to matrix "A" in Equation 12.

4. RESULTS

Among the reasons identified for the non-compliance with the functioning of shared custody as argued by consulted family lawyers and judges are the following:

1. Failure to comply with and irregularity in the routines and life schedule of the minor.
2. Parental alienation syndrome.
3. Instability in fulfilling responsibilities granted by either parent.
4. Wrongful retention of children.
5. Lack of follow-up by the assigned team of social workers and prosecution.
6. Difficulties in communication between parents regarding the established dynamics and functioning that generate conflicts and high tension.
7. Resistance or refusal by one of the parents to accept shared custody.
8. Emotional or psychological stress that affects the ability to comply with custody.
9. Logistic problems such as lack of an adequate location for the custody of the minor or respect for their privacy.
10. Significant differences in parental skills.
11. Presence of alcohol and drug use among the parents.
12. Disregard of the children's opinions.
13. Lack of social support or resources to maintain shared custody by one of the parents.
14. Failure to comply with prior agreements due to resentment or desires for revenge.
15. Interference from relatives or third parties in the relationship between the child and one of the parents.

These causes were subjected to further analysis using the Pareto diagram, and the results obtained are reflected in the following figure:

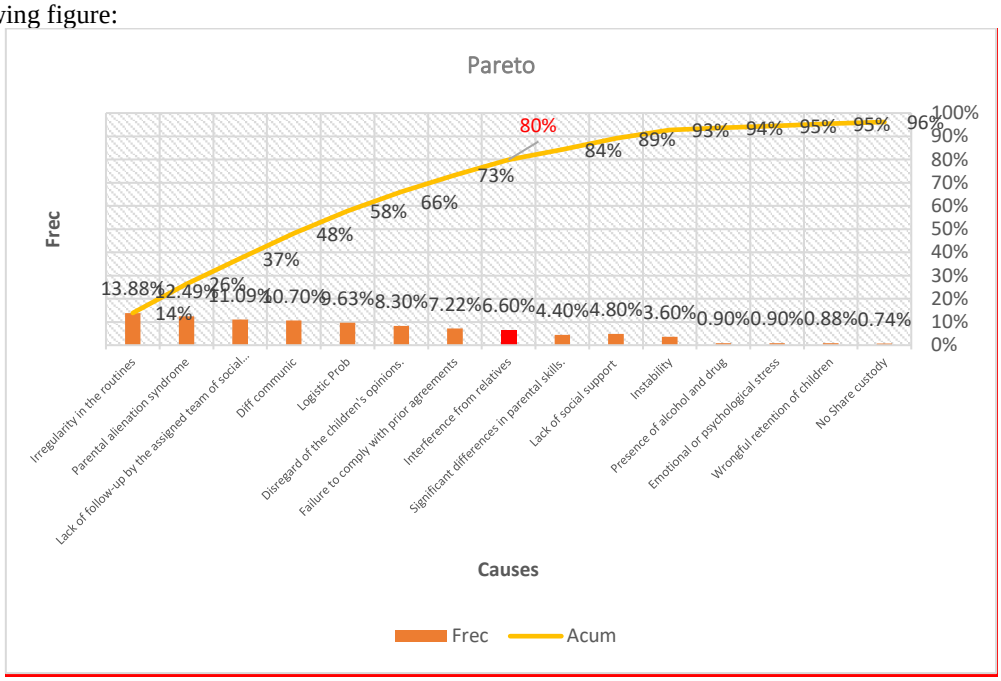

Figure 2. Pareto diagram. Source: consultations with family lawyers and judges.
Source: own elaboration.

From the causes mentioned earlier, according to the Pareto diagram analysis, those that frequently impact the problem of non-compliance with shared custody measures are:

1. Failure to comply with and irregularity in the routines and life schedule of the minor.
2. Parental alienation syndrome.
3. Lack of follow-up by the assigned team of social workers and the prosecutor's office.
4. Communication difficulties among parents regarding the established dynamics and operation that generate conflicts and high tension.
5. Logistic problems such as lack of an adequate location for the custody of the minor or respect for their privacy.
6. Disregard of the children's opinions.
7. Failure to comply with prior agreements due to resentment or desires for revenge.

8. Interferences from relatives or third parties in the relationship between the child and one of the parents.

For these situations that frequently occur and threaten the proper functioning of shared custody, the following measures or alternatives are proposed:

1. Continuous monitoring. (Maintain regular monitoring of the case to ensure compliance with agreements and take action in cases of non-compliance) (C1)
2. Education on co-parenting. (Provide resources and guidance to parents on how to develop a healthy co-parenting relationship, which can reduce conflicts and non-compliance) (C2)
3. Psychological counseling. (Through therapy, help parents understand how non-compliance can emotionally affect their children and convey knowledge on how to deal with the situation they face) (C3)
4. Flexible negotiation. (In some cases, it may be necessary to review and adjust the terms of the agreement to accommodate the changing needs of the parents, as long as this does not affect the well-being of the minor) (C4)
5. Request modification. (If circumstances change significantly for one of the parents, the agreement can be modified) (C5)
6. Formal notification. (In case of persistent non-compliance, the lawyer can send a formal letter to the non-compliant parent reminding them of their obligations and the possible legal consequences in case of further non-compliance) (C6)
7. Suggest mediation. (This can be used as a possible way to resolve disputes. A neutral mediator could help conflicting parents reach beneficial solutions for both parties and fundamentally for the minor) (C7)
8. Enforcement of sanctions. (If the parent continues to not comply, the lawyer can request the enforcement of sanctions such as fines or compensatory time for the affected parent) (C8)

From the analysis through the multi-criteria decision method, the following results were obtained, as stated in the tables below.

Criteria	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
C1	Equally influential	⟨(6,7,8); .90,.10,.10⟩	⟨(4,5,6); .80,.15,.20⟩	⟨(6,7,8); .90,.10,.10⟩	⟨(6,7,8); .90,.10,.10⟩	⟨(4,5,6); .80,.15,.20⟩	⟨(4,5,6); .80,.15,.20⟩	⟨(6,7,8); .90,.10,.10⟩
C2	1/⟨(6,7,8); . 90, .10, .10⟩	Equally influential	⟨(2,3,4); .30,.75,.70⟩	⟨(4,5,6); .80,.15,.20⟩	⟨(4,5,6); .80,.15,.20⟩	⟨(2,3,4); .30,.75,.70⟩	⟨(4,5,6); .80,.15,.20⟩	⟨(4,5,6); .80,.15,.20⟩
C3	1/(4,5,6); .80,.15,.20⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	Equally influential	⟨(2,3,4); .30,.75,.70⟩	⟨(2,3,4); .30,.75,.70⟩	⟨(4,5,6); .80,.15,.20⟩	⟨(4,5,6); .80,.15,.20⟩	⟨(4,5,6); .80,.15,.20⟩
C4	1/ ⟨(6,7,8); .90,.10,.10⟩	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	Equally influential	⟨(1,1,1); .50,.50,.50⟩	⟨(2,3,4); .30,.75,.70⟩	⟨(2,3,4); .30,.75,.70⟩	⟨(2,3,4); .30,.75,.70⟩
C5	1/ ⟨(6,7,8); .90,.10,.10⟩	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	1/ ⟨(1,1,1); .50,.50,.50⟩	Equally influential	⟨(1,1,1); .50,.50,.50⟩	⟨(2,3,4); .30,.75,.70⟩	⟨(2,3,4); .30,.75,.70⟩
C6	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	1/ ⟨(1,1,1); .50,.50,.50⟩	Equally influential	⟨(1,1,1); .50,.50,.50⟩	⟨(1,1,1); .50,.50,.50⟩
C7	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	1/ ⟨(1,1,1); .50,.50,.50⟩	Equally influential	⟨(1,1,1); .50,.50,.50⟩
C8	1/ ⟨(6,7,8); .90,.10,.10⟩	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(4,5,6); .80,.15,.20⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	1/ ⟨(2,3,4); .30,.75,.70⟩	1/ ⟨(1,1,1); .50,.50,.50⟩	1/ ⟨(1,1,1); .50,.50,.50⟩	Equally influential

Table 2. Saaty's scale translated to a neutrosophic triangular scale. Source: [13]

Criteria	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**Weight**
C1	0.49	0.76	0.50	0.40	0.38	0.25	0.21	0.27	0.41
C2	0.07	0.11	0.30	0.28	0.27	0.15	0.21	0.19	0.20
C3	0.10	0.04	0.10	0.17	0.16	0.25	0.21	0.19	0.15
C4	0.07	0.02	0.03	0.06	0.05	0.15	0.13	0.12	0.08
C5	0.07	0.02	0.03	0.06	0.05	0.05	0.13	0.12	0.07
C6	0.10	0.04	0.02	0.02	0.05	0.05	0.04	0.04	0.04
C7	0.10	0.02	0.02	0.02	0.02	0.05	0.04	0.05	0.04
C8	0.07	0.02	0.02	0.02	0.02	0.05	0.04	0.04	0.03

Table 3. Weights of the criteria using the Neutrosophic AHP method. Source: own elaboration.

Factors	F1	F2	F3	F4	F5	F6	F7	F8

Approximate eigenvalues	10.404963	9.82341749	8.73584877	8.30751768	8.5296233	8.70069562	7.99317717	8.46685733

Table 4. Analysis of the consistency of the paired matrix. Source: own elaboration.

The consistency of the exercise was smaller than or equal to 0.10, with an eigenvalue of 8.87026, with CI= 0.12 and CR= 0.09, demonstrating effectiveness in decisions. As a result, the most successful decision alternative according to experts was number 1, followed by the others at subsequent levels. The consulted family lawyers and judges considered that proper follow-up on cases assigned under shared custody would prevent non-compliance. In case of non-compliance, the remaining alternatives are available, provided that the welfare of the child and the right of the parents to provide them with a safe and stable life from the economic, emotional, and emotional point of view is taken into account.

1. Continuous monitoring. (Maintain regular monitoring of the case to ensure compliance with agreements and take action in cases of non-compliance)
2. Education on co-parenting. (Provide resources and guidance to parents on how to develop a healthy co-parenting relationship, which can reduce conflicts and non-compliance)
3. Psychological counseling. (Through therapy, help parents understand how non-compliance can emotionally affect their children and convey knowledge on how to deal with the situation they face)

4. CONCLUSIONS

This study demonstrates that the application of the neutrosophic AHP method effectively supports decision-making in complex family law scenarios, particularly in cases of shared custody. The results indicate a consistency ratio (CR) of 0.09 and an eigenvalue of 8.87026, reflecting a robust and reliable evaluation process. The most effective decision alternative identified by experts was continuous monitoring, followed closely by education on co-parenting and psychological counseling. These findings underscore the importance of proactive measures to ensure compliance and prioritize the well-being of children. From a practical perspective, these results provide valuable insights for family lawyers, judges, and policymakers. Continuous monitoring offers a mechanism to address non-compliance promptly, while education on co-parenting equips parents with tools to foster healthier relationships and reduce conflict. Psychological counseling further complements these measures by addressing the emotional dimensions of shared custody, ensuring a comprehensive approach that safeguards the interests of children and supports parents in their roles. The study's contributions lie in its integration of neutrosophic logic with the AHP framework, a novel methodological approach in the context of family law. This combination allows for nuanced decision-making that accounts for uncertainty and subjectivity, advancing the theoretical understanding of shared custody while offering actionable recommendations. By bridging the gap between qualitative legal principles and quantitative decision-making models, the research contributes to more effective and equitable custody arrangements. However, the study is not without limitations. The reliance on expert opinions, while valuable, introduces potential bias and restricts generalizability across different jurisdictions or cultural contexts. Additionally, the focus on a limited set of alternatives may overlook other innovative solutions that could address compliance challenges. Expanding the analysis to include a broader range of options and incorporating input from diverse stakeholders could enhance the study's applicability.

Future research should explore additional methods, such as fuzzy logic or machine learning, to further refine the evaluation process. Investigating the long-term outcomes of implementing these alternatives, particularly in varying socio-economic contexts, would provide deeper insights into their effectiveness. Moreover, expanding the scope to include input from families directly affected by shared custody decisions could enrich the understanding of practical challenges and potential solutions. In conclusion, this research highlights the critical role of structured decision-making frameworks in addressing the complexities of shared custody. By prioritizing continuous monitoring, co-parenting education, and psychological counseling, the study emphasizes the need for a holistic approach that balances the rights of parents with the paramount interest of the child. These findings lay the groundwork for future studies and practical applications that aim to enhance fairness and stability in family law.

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REFERENCES

[1] ALHASSAN, A. (2024). Analyzing the application of mixed method methodology in medical education: a qualitative study. **BMC Medical Education**, 24(1), 1–12.

[2] ALTAN, E., & Z. IŞIK. (2023). Digital twins in lean construction: a neutrosophic AHP – BOCR analysis approach. **Engineering, Construction and Architectural Management**. https://doi.org/10.1108/ECAM-11-2022-1115

[3] BOURNE, P., et al. (2022). The Disintegration of Jamaican Families: Using Divorce Data, 2001-2020. **International Journal of Transformation in English & Education**, 7(1), 17–36.
[4] CROSS, T., et al. (2020). The Relationship of Needs Assessed at Entry into Out-of-Home Care to Children and Youth's Later Emotional and Behavioral Problems in Care. **Children and Youth Services Review**, 122, 105896.
[5] DEL ROCÍO, M. E. M., C. V. M. JOSÉ, C. O. L. XIMENA, & Y. V. C. SALOMÓN (2023). The Well-being of Children; and, Adolescents and Shared Tenure. **Russian Law Journal**, 11(13S), 246–252.
[6] GLUCKLICH, T. (2023). Dispute intensity, parent-child relationships and child functioning: Analysis of social workers' reports on families with high-intensity parental disputes. **Child Abuse & Neglect,** 144(October), 106342.
[7] GONZÁLEZ, I. A., A. J. R. FERNÁNDEZ & J. E. RICARDO (2021). Violación del derecho a la salud: Caso Albán Cornejo Vs Ecuador. **Universidad y Sociedad**, 13(S2), 60–65.
[8] GREINER, M. V., A. DUNCAN, K. NAUSE, K. FLINCHUM & S. J. BEAL (2022). Child welfare experiences with automated medical data sharing for children in protective custody. **Children and Youth Services Review**, 136(May), 106453.
[9] KAMIŃSKA, K. (2021). The evolution of parental care with emphasis on joint physical custody (joint physical custody). **Miscellanea Historico-Iuridica**, 19(1), 89–120.
[10] MA, C., & C. QIRUI (2023). **Spatial-temporal evolution pattern and optimization path of family education policy: An LDA thematic model approach.** Heliyon, 9(7), e17460.
[11] MATOVIC, N., & K. OVESNI (2021). Interaction of quantitative and qualitative methodology in mixed methods research: Integration and/or combination. **International Journal of Social Research Methodology**, 26(1), 51–65.
[12] MITE REYES, P. F., & C.F. MORALES VERA (2024). Estudio del impacto adverso de las barreras al aprendizaje en la calidad de la educación mediante el análisis de estadísticas plitogénicas. **Neutrosophic Computing and Machine Learning**, 33, 16–27. https://doi.org/10.5281/zenodo.13799901
[13] PRADHAN, A. (2020). Evolution of Child Custody Laws from 'Parens Patriae' to the 'Welfare of the Child'. **Indian JL & Just**., 11(1 Part II), 158–166.
[14] PUPO, I. P., P. Y. P. PÉREZ, R. G. VACACELA, R. BELLO, O. SANTOS, & M. Y. L. VÁZQUEZ, (2018). **Extensions to linguistic summaries indicators based on neutrosophic theory, applications in project management decisions**. Infinite Study.
[15] RANA, A. A. (2022). The right of custody of minor: A comparative study of Sharī'ah and Pakistani legal system. **International Journal of Human Rights and Constitutional Studies**, 9(4), 350–368.
[16] RICARDO, J. E., A. J. FERNÁNDEZ, & M. Y. L. VÁZQUEZ, (2022). Compensatory Fuzzy Logic with Single Valued Neutrosophic Numbers in the Analysis of University Strategic Management. **International Journal of Neutrosophic Science** (IJNS), 18(4).
[17] RICARDO, J. E., M. E. L. POMA, A. M. ARGÜELLO, A. D. A. N. PAZMIÑO, L. M. ESTÉVEZ, & N. BATISTA (2019). Neutrosophic model to determine the degree of comprehension of higher education students in Ecuador. Neutrosophic Sets and Systems, 26, 54–61.
[18] RICARDO, J. E., Z. M. M. ROSADOE. K. CASTRO PATARON & V. Y. V. VARGAS, (2021). Measuring Legal and Socioeconomic Effect of the Declared Debtors Using The AHP Technique in a Neutrosophic Framework**. Neutrosophic Sets and Systems**, 44(1), 356–366.
[19] SHI, H., et al. (2023). Influence of the living Pareto chart and data transparency on patient outcomes in neurosurgery. **Journal of Neurosurgery: Pediatrics**, 31(5), 1–8.
[20] SMARANDACHE, F. (2022). Extensión de Soft Set a Hypersoft Set, y luego a Plithogenic Hypersoft Set. **Neutrosophic Computing and Machine Learning**, 25, 103–106. https://doi.org/10.5281/zenodo.7519268
[21] TORRES, C., D. VALLEJO-HUANGA & X. R. OCAÑA (2021). Quantitative evaluation of family functions and changes in a postmodern context. **Heliyon**, 7(6).