A MATHEMATICAL MODEL FOR PRODUCT LIFE CYCLE FOR DETERIORATING ITEMS WITH EXPONENTIAL DEMAND - IN THIRD ORDER EQUATION

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ABSTRACT
Product life cycle is the progression of product at various stages of life and ends with the withdrawal from the market of both the product and its support. In general, it is considered that most of the products have five phases in their life, although some of the products may not see all the phases of their life. In this paper, a mathematical mode, linking the exponentially increasing demand function and four stages of product life cycle that is introduction, growth, maturity and decline are considered. Triangular inequalities view is considered for developing the mathematical models. The objective is to derive the cycle time and optimal production lot size that minimizes total costs of the product life cycle. The relevant model is built, solved. Illustrative examples are provided and numerically verified. Sensitivity analysis is performed to show how the optimal values of the policy variables in the model change as various model parameters are changed. The validation of result in this model was coded in Microsoft Visual Basic 6.0.

KEYWORDS: Mathematical models, Product Life Cycle, Maturity stage, Growth stage, Exponential Demand and Production

RESUMEN
El ciclo de vida del producto es la progresión del producto en varias etapas de su vida y termina con la retirada del mercado, tanto del producto como de su soporte. En general, se considera que la mayoría de los productos tienen cinco fases en su vida, aunque algunos de los productos pueden no pasar por todas las fases de su vida. En este artículo, se presenta un modelo matemático que vincula la función de aumento exponencial de la demanda y cuatro etapas del ciclo de vida del producto, que son la introducción, el crecimiento, la madurez y el declive. La visión obtenida a partir de desigualdades triangulares se utiliza para el desarrollo de los modelos matemáticos. El objetivo es derivar el tiempo de ciclo y el tamaño óptimo del lote de producción que minimice los costos totales del ciclo de vida del producto. El modelo relevante se construye y se resuelve. Se proporcionan ejemplos ilustrativos y se verifican numéricamente. El análisis de sensibilidad se realiza para mostrar cómo cambian los valores óptimos de las variables de política en el modelo a medida que se modifican varios parámetros del modelo. La validación del resultado en este modelo se codificó en Microsoft Visual Basic 6.0.

PALABRAS CLAVE: Modelos matemáticos, Ciclo de vida del producto, Etapa de madurez, Etapa de crecimiento, Demanda y producción exponencial

1. INTRODUCTION

A product life cycle is the life span of a product which the period begins with the initial product specification and ends with the withdrawal from the market of both the product and its support. A new product is first developed and then introduced to the market. Once the introduction gets successful, a growth period follows with wider awareness of the product and increases sales. The product enters maturity when sales stop growing and demand stabilizes. Eventually, sales may decline until the product is finally withdrawn from the market or re-developed. A product life cycle can be divided into several stages characterized by the revenue generated by the product. The product life cycle concept may apply to brand or to a category of the product. Its duration may be too short to few months for a faded item or a century or more for product categories. When the product is introduced, sales will be low until customers become aware of the product and its benefits.
In this paper, introduction, growth, maturity and decline stages of product life cycle is considered and also demand function is developed as a exponentially increasing function and constant deteriorative items is considered. The objective of this paper is to find the optimum production quantity in the cycle period with minimum overall total cost. In this model, mathematical derivation is provided, illustrative example is analyzed and sensitivity analysis is developed. The remainder of the paper is organized as follows. Section 2 presents review of literature, Section 3 is

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given as the assumptions and notations. Section 4 is for formulation of mathematical model with exponential demand function and numerical examples. Finally, the paper summarizes and concludes in section 5.

2. REVIEW OF LITERATURE:

The introduction of the idea of a product life cycle (PLC) almost 30 years ago, a great deal has been written on the subject and several empirical studies have appeared. Numerous managerial-oriented articles and books have discussed the PLC. Researchers have focused almost exclusively on validating the existence of the product life cycle concept. Non-durable consumer goods have represented the primary products studied. Limited studies are available in the mathematical model of the product life cycle.

The concept of a Product Life Cycle (PLC) has occupied a prominent position in the marketing literature as both a forecasting instrument by Kovac et al. (1972) and a guideline for corporate marketing strategy by Levitt (1965) and it has been discussed widely in research (see the over view by Kotler, 2003). In the theory, atleast two conflicting definitions about the PLC can be derived.   The first refers to the progress of a product from raw material, through which the production and use, to its final disposal. The second definition of the PLC describes the evolution of a product measured by its sales over time as seen in figure. Kotler(1967 and 2003) present the product life cycle concept as a marketing management tool for consumer branded products, i.e. (i) Introduction – the product is introduced in the market, and its awareness and acceptance are minimal. (ii) Growth- the product begins to make rapid sales and gains because of the cumulative effects of introductory promotion, distribution and word-of mouth influence. (iii) Maturity-growth of sales continuous. Sales reach and remain on a plateau marked by the level of replacement demand. (iv) Decline – Sales begin to diminish absolutely as the product is gradually edged out by better products or by its substitutes.

Figure – 2.1 Product Life Cycle

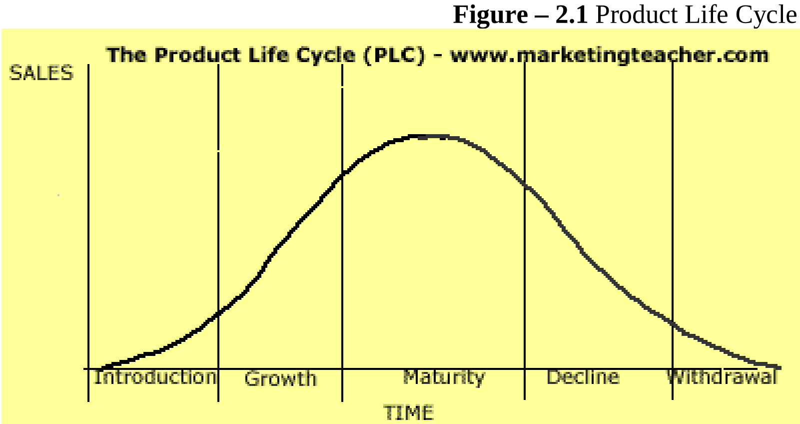

Figure – 2.2 Six Types of Product Life Cycle for Ethical Drugs

Robert D. Buzzell (1972) – the introductory period is characterized by heavy promotion aimed to buildup primary demands; price is relatively unimportant. During this growth phase, more competition appears and there is an increasing pressure on price. Promotional expenditures decline in relation to sales; there is a shift to competition on the basis of brands and specific features. As the product enters maturity, there is increasing product brand competition, promotional expenditures and prices tend to stabilize, manufacturers begin efforts to extend life cycles and new brands may appear. Finally, in the decline phase, further declines in price and promotional expenditures can be expected. Tellisand Crawford (1981) presents the product life cycle as modeled on the fixed cycle of birth-growth-maturity-death through which higher living organisms pass. The PLC can be analyzed on different levels from the main product type (product class) down to different product models. Steven and Kleppper(1996) developed a model and use it to analyse this whole process in the industry and highlighted two types of innovation, the product innovation and process innovation. The characteristics of the life cycle and its effects on the reversed supply chain have been discussed by Tibben-Lembke (2002) presents although it lacks a discussion on its effects on remanufacturing operation.   When the historical sales data is known, this data can be used as a basis for forecasting when these products are likely to be returned. Umeda el. al (2005) present a model based on empirical data from return rates for remanufacturing of a single use camera and a photo copier. In this model, a simple normal distribution function has been shown to sufficient results in predicting returns when using average life as an indicator for timing of returns. The distribution of disposed products S(t) is calculated as the historical sales Data D(t) over a limited time frame $D(\tau)\Delta\tau$, distributed as a normal distribution function with a standard deviation (σ) after an average usage time μ. Seo et al. (2007) have studied an approximate method of providing the preliminary life cycle cost. Learning algorithms trained to use the known characteristics of existing products can perhaps allow the life cycle cost of new products to be approximated quickly during the conceptual design phase without the overheads of defining new Product life cycle cost models. Huang and Treng (2008), the authors proposed a forecast methodology for predicting both product life time and non-line product life cycle based upon a two-stage, fuzzy, piecewise regression analysis model. In different to traditional time-based forecast methodology, a generation-based approach was applied, which predicts product life cycle by deriving the annual fuzzy regression lines, based upon the annual shipments of earlier generation products. Alexandru and Voda (2008) considered a model regarding the product life cycle from a reliability theory view point and modified the transfer curve in a probability density function which allows the application of statistical inferential procedures. Che-Fu Hsuch (2010) investigates inventory control policies in a manufacturing system during the product life cycle, the closed-form formulas of optimal production in lot size, reorder point and safety stock in each phase of product life cycle are derived. Ostlinet al. (2009) have studied strategies to balance supply and demand for its remanufactured product life cycle; that does not present a clear inventory control policy. Li and Chen (2011) designed a mechanism for the competition and simulate to coincide with the past regularities of product life cycle. This model has a generous structure with discrete periods that can be applied to specific industry.C. Krishnamoorthi (2012) developed an inventory model for product life cycle with defective items single manufacturing system which consists of introduction, growth, maturity and decline stages and the defective rate is considered as a variable of known proportions. C. Krishnamoorthi (2012) developed an inventory model for product life cycle with defective items single manufacturing system which consists of introduction, growth, maturity and decline stages and the defective rate is considered as a variable of known proportions and also considered as shortages in this paper. Milton Borsato (2014) presented research proposes an ontology that relates sustainability terms to product and process data entities through semantic ties. Wu et al. (2016) This study takes the perspective of the foreign competitor and investigates the conditions that influence the foreign competitor's decision of whether to conduct or abstain from an anti-dumping rebuttal. The results of a path analysis show that the potential value created from an anti-dumping rebuttal and the target product's stage within the product life cycle, through perceived benefits and competitive rivalry, respectively, jointly influence the foreign competitor's reputation for toughness, which determines whether or not the foreign competitor pursues an anti-dumping rebuttal. Fuzzy set/qualitative comparative analysis (fsQCA) offers additional evidence for the predicted relationships. These findings broaden the theoretical understanding of the regulative, normative, and cognitive elements of institutions in the context of anti-dumping rebuttals. Aytun, U., & Kılıçaslan, Y. (2017). The aim of this study, by assuming that life cycle stage of a product represents its level of technology intensity, is to measure the innovative capabilities of selected benchmark and MENA countries by developing a maturity index and then to see how MENA countries adapt themselves to relative maturity changes of products at the global level. Empirical findings using COMTRADE bilateral trade data for the period 1996-2013 showed that most of MENA countries' –especially in Algeria and Turkey- adaptation performance fall in high- and low-tech industries. Halstenberg et al. (2017) a list of Input-Output matching tools was analysed regarding data sources which are currently used for input-output Matching. Specifications of by-products in the DPPM industry were reviewed in order to identify a list of requirements for data sources. Shortcomings of the currently existing input-output matching tools were identified and suggestions for additional data sources used for input-output matching in IS in DPPM were given. Results show that datasets currently used do not include organisational data sources such as Product Data Management (PDM) systems, Enterprise Resource Planning (ERP) systems, Supply Chain Management (SCM) systems, and or Manufacturing Execution Systems (MES). Pinna et al. (2018) introduced and test three propositions: (i) the implementation of a PLM solution is positively related to firm's process management capability, thus improves NPD performances; (2) the implementation of a PLM solution is positively related to firm's coordination capability, thus improves NPD performances; and (3) the usefulness of PLM functionalities differs for each NPD stage. Sarbjit Singh Oberoi (2019) formulated a model for the products having only three phases of the life cycle and having a very short life span and mathematical model considered here has only three phases of life cycle which matches with the life cycles of the electronics products whose demand increases rapidly during the growth period and declines exponentially during the decline phase.

3. ASSUMPTIONS AND NOTATIONS

3.1 Assumptions

1) The demand rate is time dependent demand D = De^{RT} , 2**)**Items are produced and added to the inventory, 3**)**The item is a single product; it does not interact with any other inventory items, 4**)**The production rate is always greater than or equal to the sum of the demand rate, 5) The introduction time (T_1) the time for growth stage (T_2) and the time for maturity stage (T_3) are calculated based on triangular inequality, 6) P_1, P_2, P_3 - rate of production during introduction, growth and maturity period respectively. 7**)** The other assumptions are in classical Production Inventory model.

3.2 Notations:

1) P – Production rate in units per unit time,
2) D – Demand rate in units per unit time,
3) T – optimal cycle time,
4) T_1 - time during introduction of the product,
5) T_2 - time during growth stage,
6) T_3 - time during maturity stage period,
7) Q - optimal quantity,
8**)** Q_1 – on hand inventory during introduction time T_1 ,
9**)** Q_2 -on hand inventory level at time T_2,
10) Q_3 - on hand inventory level at time T_3,
11**)** C_p – Production Cost per unit,
12**)** C_h -Holding cost per unit/ per unit time,
13**)** C_0 – Setup cost per setup ,
14**)** TC - Total cost,
15) θ - Rate of Deteriorative items.

Computational Algorithm

Step 1 : Assign values to the parameters with proper units.
Step 2: To find the two variables T and Q in model 1 and model 2. Here two variables T1 and T has to be calculated so the partial differential equation is used.
Step 3: The partial differential equation for optimality is as follows
1. $\frac{\partial TC(T)}{\partial T_2} = 0$ and $\frac{\partial^2 TC(T)}{\partial T_2^2} > 0$
2. $\frac{\partial TC(T)}{\partial T} = 0$ and $\frac{\partial^2 TC(T)}{\partial T^2} > 0$

Step 4: The cubic equation can be solved using the following algorithm.
1. Let the cubic equation be a$y^3 + by^2 + cy + d = 0$
2. Let us consider an example, $y^3 - 0.7660y^2 + 0.1345y - 0.0058 = 0$
   Where A = 1, B = - 0.7660, C=0.1345 and D=-0.0058
3. The cubic equations have to be solved in several steps:
4. Define a variable " f_2 ". Therefore , $f_2 = \frac{1}{3}\left[\frac{3C}{A} - \frac{B^2}{A^2}\right] = -0.06105$
5. Define a variable " g_2 ". Therefore , $g_2 = \frac{1}{27}\left[\frac{2B^3}{A^3} - \frac{9BC}{A^2} + \frac{27D}{A}\right] = -0.00474$
6. Define a variable " h_2 ". Therefore , $h_2 = \frac{g^2}{4} + \frac{f^3}{27} = -0.0000028$
7. Define a variable " i ". Therefore , $i = \left[\frac{g_2^2}{4} - h_2\right]^{\frac{1}{2}} = 0.0029$
8. Define a variable " j ". Therefore , $j = [i]^{\frac{1}{3}} = 0.14266$
9. Define a variable " k ". Therefore , $k = arccosin\left[-\frac{g}{2i}\right] = 0.6147$

10. Define a variable " L". Therefore , $L = -j = -0.14266$
11. Define a variable " M". Therefore , $M = cos(k/3) = 0.9791$
12. Define a variable " N". Therefore , $N = \sqrt{3} sin(k/3) = 0.3524$
13. Define a variable " P". Therefore , $P = \frac{-B}{3A} = 0.2553$
Therefore the roots of cubic equation are as follows:

$$y_1 = 2jcos(k/3) - B/3A = 0.5347$$

$y_2 = L(M + N) + P$=0.0654
$y_3 = L(M - N) + P$=0.1659.
From above, all roots are real.
Step 5: All data's are programmed and generated from Visual Basic 6.0 software.

4. MATHEMATICAL MODELS - A MATHEMATICAL MODEL FOR PRODUCT LIFE CYCLE FOR DETERIORATIVE ITEMS WITH EXPONENTIAL DEMAND RATE

In introduction stage, products have to be carefully monitored to ensure that they start to grow. Substantial research and development costs may be high in order to test the market, undergo launch promotion and setup distribution channels. The cycle start at t =0. In this stage, inventory is increasing at the rate of P and simultaneously decreasing at the rate of D. Thus inventory accumulates at the rate of P - D units. Therefore, the maximum inventory level shall be equal to $(P - D)t_1$. In growth stage, more customers become aware of the product and its benefits and additional market segments are targeted. The growth stage is characterized by rapid growth in sales and profits. Profits arise due to an increase in output and possibly better prices. When the product enters growth stage at T_1, Production and Demand increases at the rate of "m" time of P-D i.e. m(P-D) where "m" is a constant. In maturity stage, sales growth continuous and a company has achieved its market share goals enjoys that most profitable period. Production and Demand increases at the rate of "n" time of P-D i.e. n(P-D) where "n" is a constant. In decline stage, the market is shrinking, reducing overall amount of profit that can be shared amongst the remaining competitors. The product becomes technically obsolete or customer taste changes. Care should be taken to control the amount of stocks of the product. The inventory level starts to decrease due to demand at a rate Dand the deteriorative items up to time T_3. Time T needed to consume all units Q at demand rate. The process is repeated. The variation of the underlying inventory system for one cycle is shown in the figure 2.

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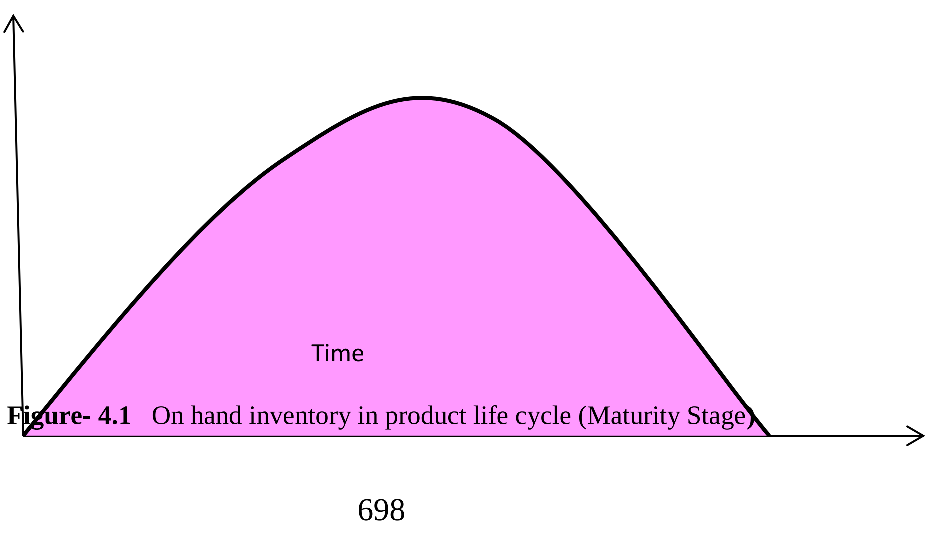

Figure- 4.1 On hand inventory in product life cycle (Maturity Stage)

The production rate of good items is always greater than or equal tothe demand rate. So, we must have P $\geq$ D. Let I(t) denote the inventory level of the system at time T. The differential equation describing the system in the interval (0,T) are given by

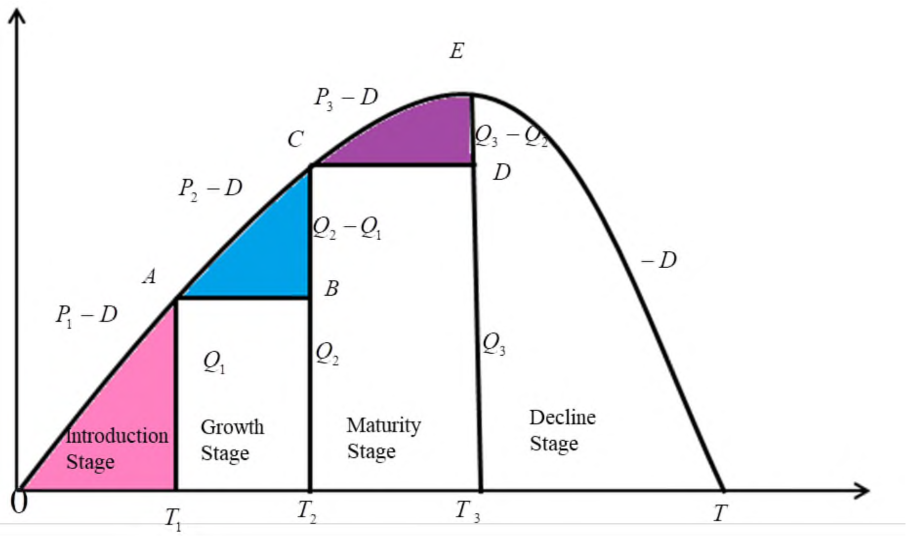

Figure- 4.2 On hand inventory in product life cycle (Maturity Stage)

Let I(t) denote the inventory level of the system at time t. The differential equation describing the system in the interval (0,T) are given by

$$\frac{dI(t)}{dt} + \theta I(t) = P_1 - De^{Rt}; \quad 0 \le t \le T_1 \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = P_2 - De^{Rt}; \; T_1 \le t \le T_2$$

(2)

$$\frac{dI(t)}{dt} + \theta I(t) = P_3 - De^{Rt}; \; T_2 \le t \le T_3 \qquad (3)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -De^{Rt}; \; T_3 \le t \le T \qquad (4)$$

The boundary conditions are

I(0) = 0; I(T_1) = Q_1; I(T $_2$) = Q_2, $I(T_3) = Q_3$ and I(T) =0 (5)

The solutions of the above equations are

From the equation (1), $I(t) = \frac{P_1}{\theta}\left(1 - e^{-\theta t}\right) + \frac{D}{R+\theta}\left(e^{-\theta t} - e^{Rt}\right)$ (6)

From the equation (2), $I(t) = \frac{P_2}{\theta}\left(1 - e^{-\theta t}\right) + \frac{D}{R+\theta}\left(e^{-\theta t} - e^{Rt}\right)$ (7)

From the equation (3), $I(t) = \frac{P_3}{\theta}\left(1 - e^{-\theta t}\right) + \frac{D}{R+\theta}\left(e^{-\theta t} - e^{Rt}\right)$ (8)

From the equation (4), $I(t) = \frac{D}{R+\theta}\left(e^{(R+\theta)T - \theta t} - e^{Rt}\right)$ (9)

To find T_1, T_2, T_3, Q_1, Q_2 and Q_3

From the right triangular inequality OAT1 and ABC

$\frac{P_1 - D}{P_2 - D} = \frac{T_1 - 0}{T_2 - T_1}$ that is , $T_1 = \frac{(P_1 - D)T_2}{P_2 + P_2 - 2D}$ and

From triangular inequality OAT1 and CDE

$\frac{P_1 - D}{P_3 - D} = \frac{T_1 - 0}{T_3 - T_2}$ that is, $T_2 = T_3 - \frac{(P_3 - D)T_2}{P_1 + P_2 - 2D}$, that is, $T_2 = \frac{(P_1 + P_2 - 2D)T_3}{P_1 + P_2 + P_3 - 3D}$

Therefore, the value of the time T_1 and T_2 using triangular inequality are as follows:

$$T_1 = \frac{(P_1 - D)T_3}{P_1 + P_2 + P_3 - 3D} \text{ and } T_2 = \frac{(P_1 + P_2 - 2D)T_3}{P_1 + P_2 + P_3 - 3D} \qquad (10)$$

Maximum inventory Q_1 : The maximum inventory (Q_1) during time T_1 is calculated from equations (5) and (6), Therefore, $Q_1 = (P_1 - D)T_1$ (11)

Maximum inventory Q_2 : The maximum inventory (Q_2) during time T_2 is calculated from the equations (5) and (7), Therefore,

$$Q_2 = (P_2 - a)T_2 \qquad (12)$$

Maximum inventory Q_3 : The maximum inventory (Q_3) during time T_2 is calculated from the equations (5) and (8), Therefore,

$$Q_3 = (P_2 - a)T_3 \qquad (13)$$

Total Cost: The total cost comprise of the sum of the Production cost, Ordering cost, holding cost and Deteriorating cost. They are grouped together after evaluating the above cost individually.

1. Ordering Cost per unit time = $\frac{C_0}{T}$ (14)

2. Production cost = DC_P (15)

3. Holding Cost per unit time :

$$= \frac{C_h}{T}\left[\begin{array}{l} \int_0^{T_1}\left[\frac{P_1}{\theta}(1-e^{-\theta t}) + \frac{D}{R+\theta}(e^{-\theta t} - e^{Rt})\right]dt + \int_{T_1}^{T_2}\left[\frac{P_2}{\theta}(1-e^{-\theta t}) + \frac{D}{R+\theta}(e^{-\theta t} - e^{Rt}\right]dt \\ + \int_{T_2}^{T_3}\left[\frac{P_3}{\theta}(1-e^{-\theta t}) + \frac{D}{R+\theta}(e^{-\theta t} - e^{Rt}\right]dt + \int_{T_3}^{T}\left[\frac{D}{R+\theta}(e^{(R+\theta)T-\theta t} - e^{Rt}\right]dt \end{array}\right]$$

$$= \frac{C_h}{T}\left[\begin{array}{l} \frac{P_1}{\theta^2}\left(T_1\theta + e^{-\theta T_1} - 1\right) + \frac{P_2}{\theta^2}\left(\theta T_2 + e^{-\theta T_2} - \theta T_1 - e^{-\theta T_1}\right) + \frac{P_3}{\theta^2}\left(\theta T_3 + e^{-\theta T_3} - \theta T_2 - e^{-\theta T_2}\right) \\ - \frac{D}{R\theta(R+\theta)}\left(\mathrm{Re}^{-\theta T_1} + \theta e^{RT_1} - R - \theta\right) - \frac{D}{R\theta(R+\theta)}\left(\mathrm{Re}^{-\theta T_2} + \theta e^{RT_2} - \mathrm{Re}^{-\theta T_1} - \theta e^{RT_1}\right) \\ - \frac{D}{R\theta(R+\theta)}\left(\theta e^{RT_3} + \mathrm{Re}^{-\theta T_3} - \theta e^{RT_2} - \mathrm{Re}^{-\theta T_2}\right) - \frac{D}{R\theta(R+\theta)}\left(\begin{array}{l} \mathrm{Re}^{RT} + \theta e^{RT} \\ - \mathrm{Re}^{(R+\theta)T - \theta T_3} - \theta e^{RT_3} \end{array}\right) \end{array}\right]$$

$$= \frac{C_h}{T}\left[\begin{array}{l} \frac{P_1}{\theta^2}\left(T_1\theta + e^{-\theta T_1} - 1\right) + \frac{P_2}{\theta^2}\left(\theta T_2 + e^{-\theta T_2} - \theta T_1 - e^{-\theta T_1}\right) + \frac{P_3}{\theta^2}\left(\theta T_3 + e^{-\theta T_3} - \theta T_2 - e^{-\theta T_2}\right) \\ - \frac{D}{R\theta(R+\theta)}\left((R+\theta)(e^{RT} - 1) + \mathrm{Re}^{-\theta T_3}(1 - e^{(R+\theta)T}\right) \end{array}\right]$$

Substitute the value of T_1 in the above equation and simplify,

$$= \frac{C_h}{T}\left[\begin{array}{l} \frac{P_1}{\theta^2}\left(\frac{\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}+e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}-1\right) \\ +\frac{P_2}{\theta^2}\left(\frac{\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}+e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}-\frac{\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}-e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}\right) \\ +\frac{P_3}{\theta^2}\left(\theta T_3+e^{-\theta T_3}-\frac{\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}-e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}\right) \\ -\frac{D}{R\theta(R+\theta)}\left[(R+\theta)(e^{RT}-1)+R(1-e^{(R+\theta)T})e^{-\theta T_3}\right] \end{array}\right]$$

(16)

4.Deteriorating Cost per unit time: Deteriorating cost

$$= \frac{\theta C_d}{T}\left[\begin{array}{l} \frac{P_1}{\theta^2}\left(\frac{\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}+e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}-1\right) \\ +\frac{P_2}{\theta^2}\left(\frac{\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}+e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}-\frac{\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}-e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}\right) \\ +\frac{P_3}{\theta^2}\left(\theta T_3+e^{-\theta T_3}-\frac{\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}-e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}\right) \\ -\frac{D}{R\theta(R+\theta)}\left[(R+\theta)(e^{RT}-1)+R(1-e^{(R+\theta)T})e^{-\theta T_3}\right] \end{array}\right]$$

(17)

TC= Purchase Cost + Ordering Cost + Holding Cost + Deteriorating Cost

$$\frac{C_0}{T}+DC_P+\frac{C_h+\theta C_d}{T}$$

$$=\left[\begin{array}{l}\frac{P_1}{\theta^2}\left(\frac{\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}+e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}-1\right)\\+\frac{P_2}{\theta^2}\left(\frac{\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}+e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}-\frac{\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}-e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}\right)\\+\frac{P_3}{\theta^2}\left(\theta T_3+e^{-\theta T_3}-\frac{\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}-e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}\right)\\-\frac{D}{R\theta(R+\theta)}\left[(R+\theta)(e^{RT}-1)+R(1-e^{(R+\theta)T})e^{-\theta T_3}\right]\end{array}\right] \quad (18)$$

Partially differentiate the total cost (18) with respect to T_3,

$$\left[\begin{array}{l}\frac{P_1}{\theta^2}\left(\frac{\theta(P_1-D)}{P_1+P_2+P_3-3D}-\frac{\theta(P_1-D)}{P_1+P_2+P_3-3D}e^{\frac{-\theta(P_1-D)T_2}{P_1+P_2+P_3-3D}}\right)\\+\frac{P_2}{\theta^2}\left(\begin{array}{l}\frac{\theta(P_1+P_2-2D)}{P_1+P_2+P_3-3D}-\frac{\theta(P_1+P_2-2D)}{P_1+P_2+P_3-3D}e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}\\-\frac{\theta(P_1-D)}{P_1+P_2+P_3-3D}+\frac{\theta(P_1-D)}{P_1+P_2+P_3-3D}e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}\end{array}\right)\\+\frac{P_3}{\theta^2}\left(\theta-\theta e^{-\theta T_3}-\frac{\theta(P_1+P_2-2D)}{P_1+P_2+P_3-3D}+\frac{\theta(P_1+P_2-2D)}{P_1+P_2+P_3-3D}e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}\right)\\-\frac{D}{R\theta(R+\theta)}\left[-R\theta e^{-\theta T_3}(1-e^{(T+\theta)T})\right]\end{array}\right]=0$$

On simplifications,

$$T_3=\frac{D(P_1+P_2+P_3-3D)^2T}{P_3(P_1+P_2+P_3-3D)^2-(P_2-P_1)(P_1-D)^2-(P_3-P_2)(P_1+P_2-2D)^2} \quad (19)$$

Partially differentiate the total cost (19) with respect to T,

$$\left[\begin{array}{l} -\dfrac{P_1}{\theta^2}\left(\dfrac{\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}+e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}-1\right) \\ -\dfrac{P_2}{\theta^2}\left(\dfrac{\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}+e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}-\dfrac{\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}-e^{\frac{-\theta(P_1-D)T_3}{P_1+P_2+P_3-3D}}\right) \\ -\dfrac{P_3}{\theta^2}\left(\theta T_3+e^{-\theta T_3}-\dfrac{\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}-e^{\frac{-\theta(P_1+P_2-2D)T_3}{P_1+P_2+P_3-3D}}\right) \\ -\dfrac{D}{R\theta(R+\theta)}\left[\begin{array}{l} R(R+\theta)Te^{RT}-(R+\theta)(e^{RT}-1) \\ -R(R+\theta)Te^{-\theta T_3}e^{(R+\theta)T}-\mathrm{Re}^{-\theta T_3}(1-e^{(R+\theta)T}) \end{array}\right] \end{array}\right]=0$$

Substitute the value of T_3 in the above equation then the above equation which is the optimum solution for T. For our convenience, the above equation is reduced to fourth order equation and the analysis is made based on third order equation. Expanding the above equation in the exponential series and then the reduced fourth order equation is

$$\left[\begin{array}{l} \left(\dfrac{(P_2-P_1)(P_1-D)^2+(P_3-P_2)(P_1+P_2-2D)^2-P_3(P_1+P_2+P_3-3D)^2}{2(P_1+P_2+P_3-3D)^2}\right)T_3^2+\dfrac{DT^2}{2} \\ -\left(\dfrac{(P_2-P_1)\theta(P_1-D)^3+(P_3-P_2)\theta(P_1+P_2-2D)^3-P_3\theta(P_1+P_2+P_3-3D)^3}{6(P_1+P_2+P_3-3D)^3}\right)T_3^3 \\ +\dfrac{2DRT^3}{4}+\dfrac{D\theta T^3}{3}-\dfrac{D(R+\theta)T^2T_3}{2}=\dfrac{C_0}{C_h+\theta C_d} \end{array}\right]$$

$$\left[\begin{array}{l} \left(\dfrac{-D^2(P_1+P_2+P_3-3D)^2T^2}{2\left[P_3(P_1+P_2+P_3-3D)^2-(P_2-P_1)(P_1-D)^2-(P_3-P_2)(P_1+P_2-2D)^2\right]}\right)+\dfrac{DT^2}{2} \\ -D^3(P_2+P_2+P_3-3D)^3\left(\dfrac{\begin{array}{l}(P_2-P_1)\theta(P_1-D)^3+(P_3-P_2)\theta(P_1+P_2-2D)^3\\-P_3\theta(P_1+P_2+P_3-3D)^3\end{array}}{6\left[\begin{array}{l}P_3(P_1+P_2+P_3-3D)^2-(P_2-P_1)(P_1-D)^2\\-(P_3-P_2)(P_1+P_2-2D)^2\end{array}\right]^3}\right)T^3 \\ +\dfrac{2DRT^3}{4}+\dfrac{D\theta T^3}{3}-\dfrac{D^2(R+\theta)(P_1+P_2+P_3-3D)^2T^3}{\begin{array}{l}P_3(P_1+P_2+P_3-3D)^2-(P_2-P_1)(P_1-D)^2\\-(P_3-P_2)(P_1+P_2-2D)^2\end{array}}=\dfrac{C_0}{C_h+\theta C_d} \end{array}\right]$$

Substitute the value of T_3 in the above equation. The reduced equation is

$$\left[\left[\left(\frac{D^3(P_1+P_2+P_3-3D)^3\begin{pmatrix}P_3\theta(P_1+P_2+P_3-3D)^3-(P_2-P_1)\theta(P_1-D)^3\\-(P_3-P_2)\theta(P_1+P_2-2D)^3\end{pmatrix}}{\left(P_3(P_1+P_2+P_3-3D)^2-(P_2-P_1)(P_1-D)^2-(P_3-P_2)(P_1+P_2-2D)^2\right)^3}\right)T^3 \right.\right.$$

$$\left.+2(2R+\theta)-\frac{3D^2(R+\theta)(P_1+P_2+P_3-3D)^2}{\begin{pmatrix}P_3(P_1+P_2+P_3-3D)^2-(P_2-P_1)(P_1-D)^2\\-(P_3-P_2)(P_1+P_2-2D)^2\end{pmatrix}}\right]$$

$$\left.+3D\left(\frac{\begin{matrix}(P_3-D)(P_1+P_2+P_3-3D)^2-(P_2-P_1)(P_1-D)^2\\-(P_3-P_2)(P_1+P_2-2D)^2\end{matrix}}{\begin{matrix}P_3(P_1+P_2+P_3-3D)^2-(P_2-P_1)(P_1-D)^2\\-(P_3-P_2)(P_1+P_2-2D)^2\end{matrix}}\right)T^2=\frac{6C_0}{(C_h+\theta C_d)}\right] \quad (20)$$

which is optimum solution for T in third order equation.

Numerical Example, In order to better understand the problem and also to illustrate the proposed three rates of production inventory models , the numerical example, problem have been considered with the dates as P_1 = 5000 units, P_2 = 5500, P_3 = 6000, D = 4500 units, C_h = 10, C_p = 100, C_d = 100, θ = 0.01 ,. R = 0.1, C_0 =100

Optimum solution: The cubic equation is $-1113.06.89T^3-3135.07T^2+54.54=0$

T = 0.1351, Q* = 608.34, T_1 = 0.0172, T_2 = 0.0518, T_3 = 0.1037, Q_1 = 8.64, Q_2 = 51.89, Q_3 = 155.68, Production cost = 450000,Setup cost = 739.71, Holding cost =706.37, Deteriorating cost = 70.64, Total cost = 451516.72.

5 SENSITIVITY ANALYSIS:

A sensitivity analysis is performed to study the effects of change in the system parameters, rate of deteriorative items (θ), ordering cost per order (C_0), holding cost per unit per year (C_h), production cost per unit (C_P), deteriorating rate per unit on optimal cycle time (T), optimal quantity (Q), time during first level of production (T_1), time during second level of production (T_2), maximum inventory during first level of production (Q_1), maximum inventory during second level of production (Q_2), setup cost, holding cost, deteriorative cost and total cost. The sensitivity analysis is performed by changing (increasing or decreasing) the parameter taking at a time, keeping the remaining parameters at their original values.

5.1 Sensitivity Analysis with respect to Rate of Deteriorative items:

The result of sensitivity analysis with respect to rate of deteriorative items is given in the table 1. It is observed that from the table, that there increase in the rate of deteriorative items with the increase in the optimum values of setup cost, deteriorative cost, and total cost then there is positive relationship between them. Also, it is observed that there is increase in rate of deteriorative items with the decrease in the optimum values of Optimum Time (T), Optimum quantity (Q), Production Time (T_1, T_2, T_3), the maximum inventory (Q_1, Q_2, Q_3), and holding cost decreases then there is negative relationship between them.

Table 5.1 The result of sensitivity analysis with respect to Rate of Deteriorative items

θ	T / T_1	T_2/T_3	Q/Q_1	Q_2/Q_3	Setup cost	Holding cost	DC	Total cost

0.01	0.1351 0.0172	0.0518 0.1037	608.34 8.64	51.89 155.68	739.71	706.37	70.64	451516.72
0.02	0.1295 0.0165	0.0497 0.0994	582.79 8.28	49.72 149.15	772.13	676.71	135.34	451584.18
0.03	0.1245 0.0159	0.0478 0.0955	560.27 7.96	47.79 143.38	803.18	650.55	195.16	451648.90
0.04	0.1200 0.0153	0.0461 0.0921	540.21 7.68	46.08 138.25	832.99	627.26	250.90	451711.17
0.05	0.1160 0.0148	0.0445 0.0891	522.21 7.42	44.54 133.64	861.72	606.35	303.17	451771.25
0.06	0.1124 0.0143	0.0431 0.0863	505.92 7.19	43.15 129.47	889.46	587.44	352.46	451829.37
0.07	0.1091 0.0139	0.0418 0.0837	491.10 6.98	41.89 125.68	916.30	570.24	399.16	451885.71
0.08	0.1061 0.1035	0.0407 0.0814	477.54 6.79	40.74 122.21	942.32	554.49	443.59	451940.41
0.09	0.1033 0.0132	0.0396 0.0793	465.06 6.61	39.67 119.02	967.59	540.01	486.01	451993.61
0.10	0.1007 0.0129	0.0386 0.0773	453.54 6.44	38.69 116.07	992.17	526.65	526.63	452045.44

The graphical reprehensive between rate of deteriorative items and total cost is given below. It is observed that it is in the upward straight line.

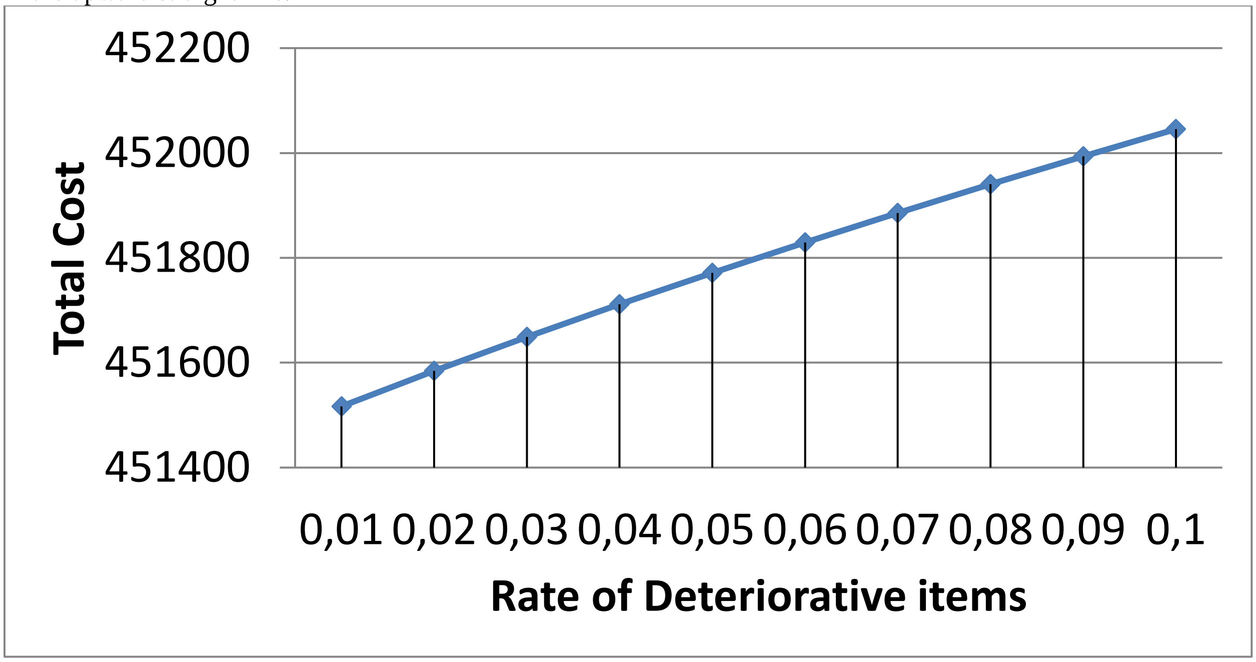

Figure 5.1 Relationship between Rate of Deteriorative items with total cost

5.2 Sensitivity Analysis with respect to Rate of Growth in Demand:

The result of sensitivity analysis with respect to rate of growth of demand is given in the table 2. It is observed that from the table, that there increase in the growth of demand with the increase in the optimum values of Cycle time, Optimum quantity, Production time (T_1, T_2, T_3), holding cost. Negligible increase in decline time (T_1), the maximum inventory ($Q_1, Q_{2,}, Q_3$) . Then, there is positive relationship between them. And also, it is observed that there is increase in the rate of growth in demand with the decrease in optimum values of Setup cost then there is negative relationship between them.

Table 5.2 Results of sensitivity analysis with respect to rate of growth in demand

R	T / T_1	T_2 / T_3	Q / Q_1	Q_2 / Q_3	Setup cost	Holding cost	DC	Total cost
0.05	0.1336 0.0171	0.0512 0.1025	601.23 8.54	51.29 153.87	748.45	698.11	69.81	451516.38
0.09	0.1348 0.0172	0.0517 0.1035	606.88 8.63	51.77 155.31	741.48	704.67	70.46	451516.63
0.1	0.1351 0.0172	0.0518 0.1037	608.34 8.64	51.89 155.68	739.71	706.37	70.64	451516.72
0.2	0.1386 0.0177	0.0532 0.1064	623.96 8.87	53.29 159.68	721.19	724.50	72.45	451518.15
0.3	0.1426 0.0182	0.0547 0.1095	641.90 9.12	54.76 164.28	701.03	745.34	74.53	451520.91
0.4	0.1473 0.0188	0.0565 0.1131	662.92 9.42	56.55 169.66	678.77	769.79	76.97	451525.54
0.5	0.1529 0.0195	0.0587 0.1174	688.41 9.78	58.72 176.18	653.67	799.34	79.94	451532.95
0.6	0.1601 0.0204	0.0614 0.1229	720.60 10.24	61.47 184.42	624.47	836.72	83.67	451544.86
0.7	0.1699 0.0217	0.0652 0.1304	764.56 10.87	65.22 195.67	588.57	887.76	88.77	451565.11
0.8	0.1857 0.0237	0.0713 0.1426	835.87 11.88	71.30 213.92	538.35	970.57	97.05	451605.98

The graphical reprehensive between rate of growth of demand and total cost is given below. It is observed that it is in the upward straight line.

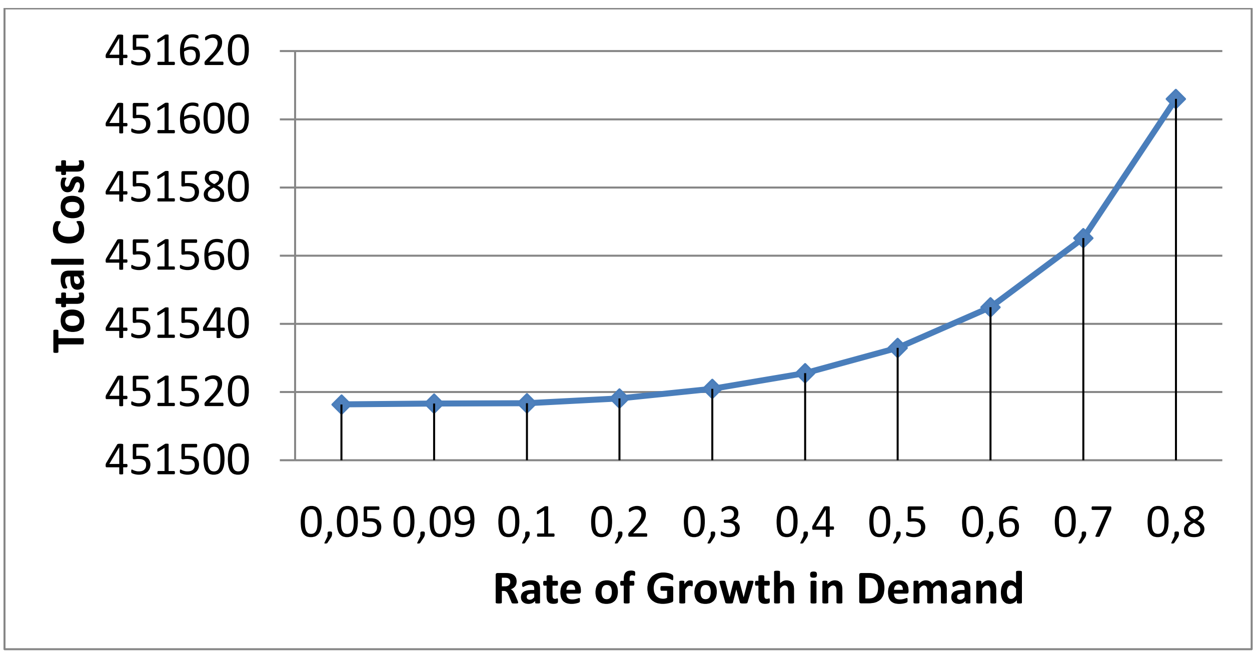

Figure 5.4 Relationship between total cost and rate of growth of demand

5.3 Sensitivity Analysis with respect to setup cost per set

The result of sensitivity analysis with respect to setup cost per set is given in the table 3. It is observed that from the table, that there is a increase in setup cost per set with the increase in the Optimum Time (T), Optimum quantity

(Q), The production Time (T_1, T_2, T_3), Setup cost, holding cost, deteriorative cost and total cost and the increase in the maximum inventory ($Q_1, Q_{2,}, Q_3$), There is positive relationship between them.

Table 5.3 Results of sensitivity analysis with respect to setup cost per set

Setup/ per set	T / T_1	T_2 / T_3	Q / Q_1	Q_2 / Q_3	Setup cost	Holding cost	DC	Total cost
80	0.1205 0.0154	0.0462 0.0925	542.64 7.71	46.29 138.87	663.47	630.08	63.01	451356.51
90	0.1280 0.0163	0.0491 0.0983	576.36 8.19	49.16 147.50	702.68	669.23	66.92	451438.84
100	0.1351 0.0172	0.0518 0.1037	608.34 8.64	51.89 155.68	739.71	706.37	70.64	451516.72
110	0.1419 0.0181	0.0544 0.1089	638.84 9.08	54.49 163.49	774.83	741.78	74.17	451590.80
120	0.1484 0.0189	0.0569 0.1139	668.06 9.49	56.99 170.97	808.31	775.71	77.57	451661.59
130	0.1547 0.0197	0.0594 0.1187	696.15 9.89	59.38 178.16	840.32	808.33	80.83	451729.49
140	0.1607 0.0205	0.0616 0.1233	723.25 10.28	61.69 185.09	871.06	839.79	83.97	451794.84
150	0.1665 0.0213	0.0639 0.1278	749.46 10.65	63.93 191.80	900.64	870.22	87.02	451857.89
160	0.1721 0.0220	0.0661 0.1322	774.85 11.01	66.10 198.30	929.19	899.72	89.97	451918.89
170	0.1776 0.0227	0.0682 0.1364	799.54 11.36	68.20 204.62	956.79	928.37	92.83	451978.01

The graphical reprehensive between rate of deteriorative items and total cost is given below. It is observed that it is in the upward straight line.

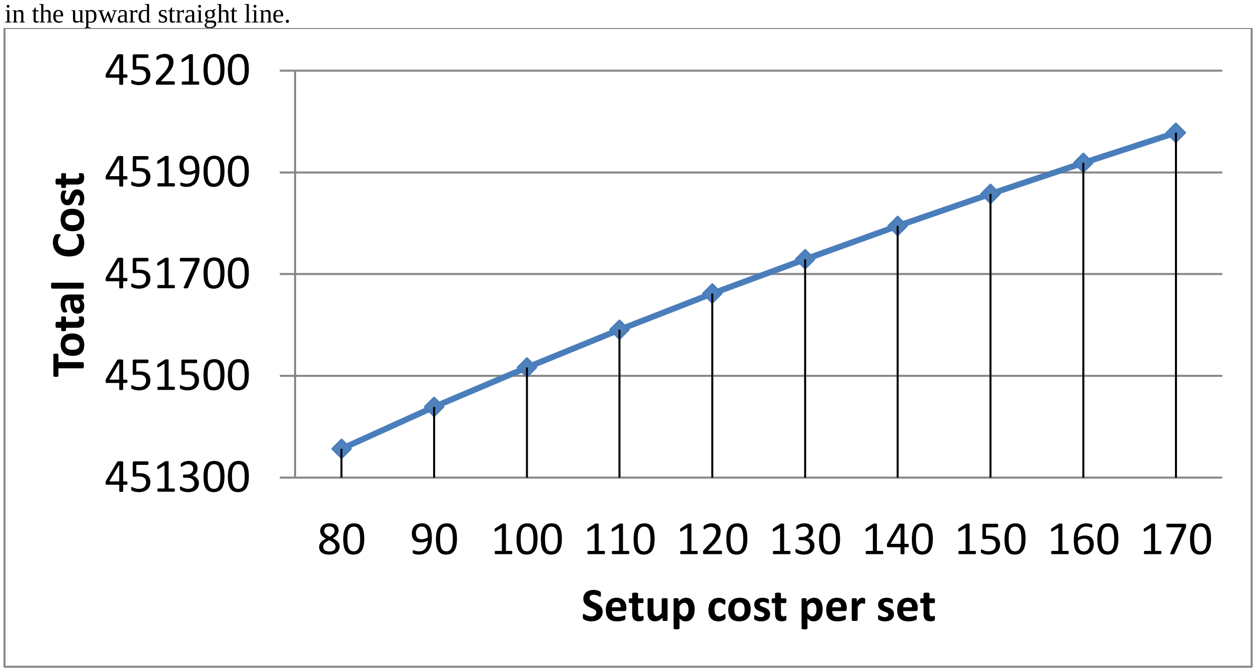

Figure 5 Relationship between total cost with setup cost per set

5.4 Sensitivity Analysis with respect to holding cost per unit per unit time (C_h):

The result of sensitivity analysis with respect to holding cost per unit per unit time is given in the table 4. It is observed that from the table, that there is increase in the holding cost with the increase in setup cost, holding cost and total cost then there is positive relationship between them. And also, it is observed that that is increase in the holding cost per unit per unit time with the decrease in the optimum values of Optimum Time (T), Optimum quantity (Q), Production Time (T_1, T_2, T_3),decline time (T), the maximum inventory ($Q_1, Q_{2,}, Q_3$), deteriorative cost decreases then there is negative relationship between them.

Table 5. 4 Results of sensitivity analysis with respect to holding cost per unit per unit time

HC/ Per unit	T / T_1	T_2 / T_3	Q / Q_1	Q_2 / Q_3	Setup cost	Holding cost	DC	Total cost
7	0.1592 0.0203	0.0611 0.1222	716.56 10.18	61.12 183.38	627.99	582.42	83.20	451293.62
8	0.1498 0.0191	0.0575 0.1150	674.39 9.58	57.53 172.59	667.26	626.45	78.30	451372.02
9	0.1419 0.0181	0.0544 0.1089	638.84 9.08	54.49 163.49	704.39	667.60	74.17	451446.18
10	0.1351 0.0172	0.0518 0.1037	608.34 8.64	51.89 155.68	739.71	706.37	70.64	451516.72
11	0.1292 0.0165	0.0496 0.0992	581.80 8.27	49.63 148.89	773.45	743.11	67.55	451584.12
12	0.1240 0.0158	0.0476 0.0952	558.44 7.93	47.63 142.91	805.81	778.11	64.84	451648.76
13	0.1194 0.0152	0.0458 0.0917	537.67 7.64	45.86 137.60	836.95	811.59	62.43	451710.97
14	0.1153 0.0147	0.0442 0.0885	519.03 7.37	44.27 132.83	866.99	843.74	60.26	451771.00
15	0.1116 0.0142	0.0428 0.0856	502.20 7.14	42.84 128.52	896.04	874.69	58.31	451829.05
16	0.1082 0.0138	0.0415 0.0831	486.90 6.92	41.53 124.61	924.20	904.58	56.53	451885.32

Figure 5,1 Relationship between holding cost per unit/time and total cost

The graphical reprehensive between rate of deteriorative items and total cost is given below. It is observed that it is in the upward straight line.

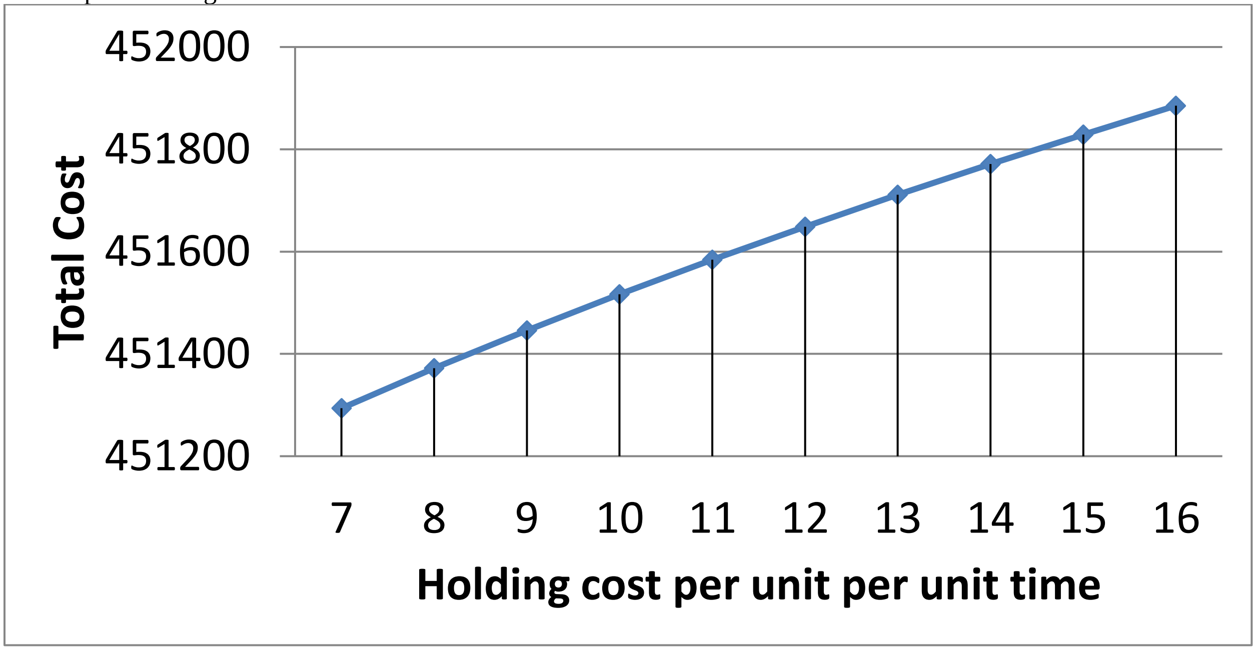

Figure 5.2 Relationship between total cost with holding cost per unit per unit time

7. CONCLUSION

In this paper, a mathematical model for product life cycle for deteriorative items during production with exponential demand is considered. The mathematical model is develop[ed and numerical example is provided. A sensitivity analyse for rate of deteriorative items, growth of demand, holding cost per unit per unit time and setup cost per set is considered. The following points are observed during this research. 1) there is increase in the rate of deteriorative items with the increase in the optimum values of setup cost, deteriorative cost, and total cost then there is positive relationship between them. 2) there is increase in the growth of demand with the increase in the optimum values of Cycle time, Optimum quantity, Production time, holding cost. Then, there is positive relationship between them. 3) there is a increase in setup cost per set with the increase in the Optimum Time, Optimum quantity, The production Time, Setup cost, holding cost, deteriorative cost and total cost and the increase in the maximum inventory, There is positive relationship between them. 4) there is increase in the holding cost with the increase in setup cost, holding cost and total cost then there is positive relationship between them.
Several extensions can be made to this research:

1. The demand in this model is considered as a continuous compound demand. Other extension to this research could be to consider probabilistic demand.
2. The models developed in this research were considered for a single time. One may relax this assumption and consider models with multiple items.
3. A mathematical model with exponential demand is considered in this research and one can may relax this assumption and consider models with linear demand, price dependent demand, stock dependent demand, quadratic demand, etc.
4. 4. In developing the models, only one concept was introduced at a items. One may want to investigate models with combination of several concepts and determine the optimal policies for these cases.

The proposed model can assist the manufacturer and retailer in accurately determining the optimal quantity, cycle time and inventory total cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as food items, fashionable commodities, stationary stores and others.

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