INTEGRATED SUPPLY CHAIN MODEL WITH INFLATION INDUCED DEMAND AND PARTIALLY BACKLOGGED SHORTAGE

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ABSTRACT
Generally, inventory control policies for deteriorating items are very sensitive to different marketing policies especially in chemical, food and pharmaceutical industries. Realizing the importance of such inventory policies in practice, an integrated production-inventory-marketing model is developed. Inflation induced demand is considered which is accelerated with frequency of advertisement. The vendor-buyer integrated inventory model is subjected to partial backlogging. Mathematical model is developed and solved analytically to find the optimal production period, shortage period and average total cost of the integrated supply chain model. Empirical investigation is carried out and sensitivity analysis is performed to check the stability of the system. This paper can assist the inventory manager in determining the optimal total cost of the integrated inventory system where inflation has significant effect on demand and hence total cost.

KEYWORDS: Integrated supply chain, Deterioration, Inflation, Partial Backlogging

MSC : 90B05

RESUMEN
Generalmente las política de control de inventarios para ítems-deteriorables son y sensibles a las diferentes políticas de marketing, en especial en las industrias química, alimentaria y la farmacéutica. Tomando en cuenta la importancia de las políticas de inventario en la práctica, una integración de los inventarios de producción y mercadeo es desarrollada. La . Inflación inducida por la demanda se considera es acelerada con la frecuencia por la publicidad. El modelo vendedor-comprador integrado es sujeto a un parcial-backlogging. El modelo matemático es desarrollado y resuelto analíticamente para hallar los periodos optimales de producción de las carencias , y el average del costo total del modelo de la cadena de abastecimiento integrado . Empíricamente se desarrolla una investigación y un análisis de sensibilidad es desarrollado para checar la estabilidad del sistema. Este paper puede ayudar al encargado de los inventarios en determinar los costos y el optimo total del sistema de inventario integrado donde la inflación posee un significante efecto sobre la demanda y por tanto en el costo total.

PALABRAS CLAVE : Cadena de Suministro Integrada, Deterioración, Inflación, Partial-Backlogging

1. INTRODUCTION

An integrated supply chain consists of different business players like supplier, manufacturer, distributor, retailer and customer who work together to attain more sustainability. Based on mutual relationship, the vendor and buyer usually plan for a long term setup. The idea to optimize the total cost function for vendor and buyer was initiated by Goyal (1976). Banerjee(1986) optimized the ordering policy for both parties to avail benefit. Goyal and Gunasekaran(1995) extended that model for deteriorating items. Huang(2004) considered a single-vendor single-buyer integrated production inventory model and discussed an optimal policy with consideration of unreliability. Hoque(2009) determined a solution technique to find the optimality for single-vendor single-buyer production inventory model. Tayal et al. (2016) allowed credit period in his integrated vendor buyer inventory model. Mishra and Talati (2017) introduced advertisement frequency and quantity discount to accelerate demand.

In today's competitive market the consumer's preferences change rapidly. Customers are often fickle and less loyal resulting in partial backlogging. Only a fraction of customers are waiting for the product till they arrive. Hence in realistic world of management, partially backlogged shortage is a more practical assumption for better business performance. Wee (1993), Abad (1996), Dye et al. (2006), Tripathy and Pradhan (2011), Pandey et al. (2017), Bag et al. (2017), Rastogi et al. (2017) are some researchers who developed the inventory models subject to deterioration and partially backlogged shortage.

In today's esoteric economy, inflation is a crucial attribute that curbs the purchasing power of money. Many countries experience high inflation rate that influence the demand rate for certain products. The increasing rate

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of inflation erodes the future worth of savings. As a result more spending on luxurious items takes place that influence the demand for certain products. So it would be unethical if the effect of inflation is ignored. Buzacott (1975) first discussed EOQ model with inflationary effect subject to different pricing policies. Chang et al. (2010) discussed the effect of inflation on the inventory model for deteriorating items in consideration with partial backlogging. Other researchers like Jaggi et al.(2006), Chern et al.(2008), Jaggi et al.(2016), Thangam and Uthayakumar(2010), Yang et al.(2010), Tripathy et al.(2016) contribute their valuable efforts in developing the inventory models under inflationary effect.

In addition to inflationary effect, the demand may also be affected by the frequency of advertisement. The proposed model is developed for an integrated production system where demand depends on frequency of advertisement and inflation subject to partial backlogging. The model is optimized to minimize the total cost of the system.

The rest of the chapter is developed as follows. Notations and assumptions are placed in the next section. Mathematical formulation with solution procedure is established next. In the third section empirical investigation is carried out. In the fourth section sensitivity analysis is performed with respect to major parameters. The conclusion and future research scope is demonstrated in the last section.

2. NOTATIONS AND ASSUMPTIONS

Notations

$I_1(t)$: Inventory at time 't' for the manufacturer.

$I_2(t)$: Inventory at any time 't' for the retailer.

β: The time up to which production occurs.

ν: The time at which the inventory level for the retailer becomes zero.

T: Complete planning horizon.

k: Deterioration rate parameter, $k< 1$

λ: Production rate parameter, $\lambda > 1$

B: Cost of advertisement.

η: Frequency of advertisement.

α: Constant rate of inflation.

θ: Backlogging rate parameter.

m: Production cost per unit.

c: Purchasing cost per unit for retailer.

h_v: Holding cost per unit for vendor.

h_b: Holding cost per unit for buyer.

d_v: Deteriorating cost per unit for the vendor.

d_b: Deteriorating cost per unit for buyer.

p_1: Setup cost for the vendor per production run.

p_2: Ordering cost per order.

s: Shortage cost per unit.

l: Lost sale cost per unit.

Q_1: Initial inventory level.

Q_2: Backordered quantity.

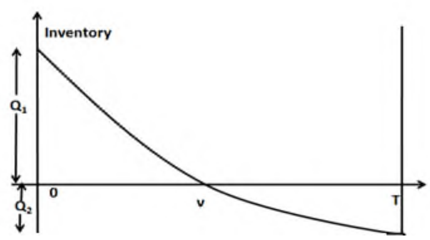
Assumptions

1. The products considered in this model are deteriorating in nature.
2. Demand rate is a function of frequency of advertisement and inflation.

   Demand = $\alpha e^{\propto t} B^{\eta}$
3. The deterioration rate is a function of time.
4. The production rate depends on rate of demand.
5. Shortage is allowed for retailer only.
6. The shortage is partially backlogged with time dependent backlogging rate.

3 . MATHEMATICAL MODEL

The present chapter is developed with an inventory model for both vendor and retailer. The production of the vendor starts at $t = 0$. The production process goes on up to time β. The inventory level decreases during time $[\beta, T]$ and it becomes zero at $t = T$. The retailer has initial inventory level Q_1 and it depletes to zero at $t = v$. During time $[v, T]$ the shortage occurs and is partially backlogged.



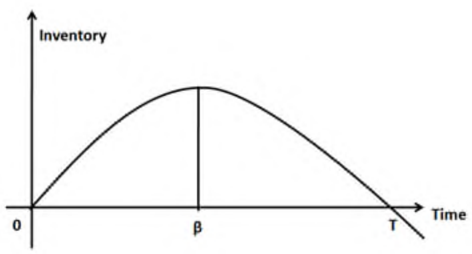


Figure 1 (Inventory level for vendor) **Figure 2 (Inventory level for retailer)**

Mathematical Model for vendor

In the figure of inventory level for vendor (fig.1), the rate of change of inventory when time 't' lies within the production period 'β' is increasing function of production and a decreasing function of demand and deterioration. So

$$\frac{dI_1(t)}{dt} = \frac{\lambda a e^{\alpha t} B^{\eta}}{T}(T-t) - \frac{a e^{\alpha t} B^{\eta}}{T}(T-t) - ktI_1(t) , \qquad 0 \le t \le \beta \tag{1}$$

$$\frac{dI_1(t)}{dt} = \frac{a e^{\alpha t} B^{\eta}}{T}(T-t)(\lambda-1) - ktI_1(t) ,$$

When time 't' lies between 'β' and cycle time 'T', there is no production. So rate of change of inventory is only a decreasing function of demand and deterioration.

$$\frac{dI_1(t)}{dt} = \frac{-a e^{\alpha t} B^{\eta}}{T}(T-t) - ktI_1(t) \qquad \beta \le t \le T \tag{2}$$

With boundary conditions $I_1(0) = I_1(T) = 0$, the solutions of equations (1) and (2) are

$$I_1(t) = \frac{aB^{\eta}(\lambda-1)}{T}\left[Tt + \left(\frac{\alpha T}{2} - 1\right)t^2 + \left(\frac{kT}{6} - \frac{\alpha}{3}\right)t^3 + \frac{1}{8}kt^4\right]e^{\frac{-kt^2}{2}} \tag{3}$$

$$I_1(t) = \frac{aB^{\eta}}{T}\left[\frac{T^2}{2} + \frac{1}{6}\alpha T^3 + \frac{1}{24}kT^4 - Tt + \left(1 - \frac{\alpha}{2}\right)t^2 + \left(\frac{\alpha}{3} - \frac{kT}{6}\right)t^3 + \frac{1}{8}kt^4\right]e^{\frac{-kt^2}{2}} \tag{4}$$

Mathematical Model for retailer

In the figure of inventory level for retailer (fig.2), the rate of change of inventory level when time 't' lies within the period of the level up to which no shortage occurs(v) is a decreasing function of demand and deterioration.

$$\frac{dI_2(t)}{dt} = \frac{-a e^{\alpha t} B^{\eta}}{T}(T-t) - ktI_2(t) \qquad 0 \le t \le v \tag{5}$$

When 't' is in between 'v' and cycle time 'T', only demand is there and the inventory level is a decreasing function of demand only.

$$\frac{dI_2(t)}{dt} = -a e^{\alpha t} B^{\eta} \qquad v \le t \le T \tag{6}$$

With boundary condition $I_2(v) = 0$, the solutions of equations (5) and (6) are

$$I_2(t) = \frac{aB^{\eta}}{T}\left[T(v-t) + \frac{\alpha T}{2}(v^2 - t^2) + \frac{kT}{6}(v^3 - t^3) - \frac{1}{2}(v^2 - t^2) - \frac{1}{8}k(v^4 - t^4) - \frac{1}{3}\alpha(v^3 - t^3)\right]e^{\frac{-kt^2}{2}} \tag{7}$$

$$I_2(t) = \frac{aB^\eta}{\alpha}(e^{\alpha v} - e^{\alpha t}) \tag{8}$$

At $t = 0$, equation (7) becomes

$$I_2(0) = Q_1 = \frac{aB^\eta}{T}\left[Tv + \frac{\alpha T}{2}v^2 + \frac{kT}{6}v^3 - \frac{1}{2}v^2 - \frac{1}{8}kv^4 - \frac{1}{3}\alpha v^3\right] \tag{9}$$

The backorder quantity due to shortage will be

$$Q_2 = \int_v^T (ae^{\alpha t}B^\eta)e^{-\theta(T-t)}dt$$

$$= aB^\eta\left[(T-v)(1-\theta T) + \left(\frac{\theta}{2} + \frac{\alpha}{2}\right)(T^2 - v^2)\right] \tag{10}$$

Manufacturing Cost for the vendor

Total manufacturing cost of the vendor

$$\text{M.C.} = m\int_0^\beta \frac{aB^\eta e^{\alpha t}}{T}(T-t)\lambda dt$$

$$= \frac{maB^\eta\lambda}{T}\left[T\beta - \frac{\beta^2}{2} + \alpha\left(\frac{T\beta^2}{2} - \frac{\beta^3}{3}\right)\right] \tag{11}$$

Purchasing Cost for the retailer

Total purchasing cost for the retailer is calculated as

P.C. = $c(Q_1+Q_2)$ (12)

Holding Cost

The holding cost for both vendor and retailer is a function of inventory level at time t and it can be calculated as follows.

For the vendor:

Total holding cost for the vendor is

$$\text{H.C.}_v = h_v\left(\int_0^\beta I_1(t)dt + \int_\beta^T I_1(t)dt\right)$$

$$= h_v\frac{aB^\eta}{T}\left[\begin{array}{l}\lambda\left(T\frac{\beta^2}{2} + \alpha T\frac{\beta^3}{6} + kT\frac{\beta^4}{24} - \frac{\beta^3}{6} - \frac{\alpha\beta^4}{12} + \frac{k\beta^5}{40}\right) + (1-T)\alpha\frac{\beta^3}{6} - \frac{k}{20}\beta^5 \\ -\left(\frac{T^2}{2} + \frac{\alpha}{6}T^3 + \frac{k}{24}T^4\right)\beta + \frac{\alpha T^4}{4} + (1-\alpha)\frac{T^3}{6} + \frac{kT^5}{40}\end{array}\right] \tag{13}$$

For the retailer:

Total holding cost for the retailer is

$$\text{H.C.}_b = h_b\int_0^v I_2(t)dt$$

$$= h_b\frac{aB^\eta}{T}\left[\frac{v^2T}{2} + \frac{\alpha T}{3}v^3 + \frac{kTv^4}{8} - \frac{v^3}{3} - \frac{kv^5}{10} - \frac{\alpha v^4}{4}\right] \tag{14}$$

Deterioration Cost for vendor:

The deterioration cost for the vendor is given by

$$\text{D.C.}_v = d_v\frac{aB^\eta}{T}\left[\int_0^\beta e^{\alpha t}(T-t)\lambda dt - \int_0^T e^{\alpha t}(T-t)dt\right]$$

$$= d_v\frac{aB^\eta}{T}\left[T\lambda\beta + \left(\frac{\alpha T\lambda}{2} - \frac{\lambda}{2}\right)\beta^2 - \frac{\alpha\lambda\beta^3}{3} - \frac{T^2}{2} - \frac{\alpha T^3}{6}\right] \tag{15}$$

For retailer:

The deterioration cost for the retailer is given by

$$\text{D.C.}_b = d_b\left[I_2(0) - \int_0^v \frac{aB^{\eta}}{T} e^{\alpha t}(T-t)dt\right] = d_b \frac{aB^{\eta}}{T}\left[\frac{kTv^3}{6} - \frac{kv^4}{8}\right] \tag{16}$$

Set up Cost for the vendor

The ordering cost for the vendor (S.U.C.) = p_1 (17)

Ordering Cost for the buyer

The ordering cost for the retailer (O.C.) = p_2 (18)

Shortage Cost

Due to stock out condition the shortage arises at $t = v$. The shortage cost in the time interval $[v, T]$ is calculated by

$$\text{S.C.} = s\int_v^T aB^{\eta} e^{\alpha t} dt = saB^{\eta}\left[T + \frac{\alpha T^2}{2} - v - \frac{\alpha v^2}{2}\right] \tag{19}$$

Lost Sale Cost

Some customers are impatience in nature and they seek alternative supply when stock out condition occurs. The cost associated with losing of customers gives rise to lost sale cost and it is calculated as follows.

$$\text{L.S.C.} = l\int_v^T aB^{\eta} e^{\alpha t}(1 - e^{-\theta(T-v)})dt = laB^{\eta}\theta(T-v)^2 \tag{20}$$

Total Average Cost

The total average cost of the integrated inventory system can be found out by summing the total average cost for both the vendor and the retailer.

$$\phi(\beta, v) = \phi_v(\beta, v) + \phi_b(\beta, v) = \frac{1}{T}\left(M.C. + P.C. + H.C._v + H.C._b + D.C._v + D.C._b + S.U.C. + O.C. + S.C. + L.S.C.\right) \tag{21}$$

$$= \frac{1}{T}\left[\begin{array}{l} \frac{maB^{\eta}\lambda}{T}\left(T\beta - \frac{\beta^2}{2} + \alpha\left(\frac{T\beta^2}{2} - \frac{\beta^3}{3}\right)\right) + c\left(\begin{array}{l} \frac{aB^{\eta}}{T}\left(Tv + \frac{\alpha T}{2}v^2 + \frac{kT}{6}v^3 - \frac{v^2}{2} - \frac{kv^4}{8} - \frac{\alpha v^3}{3}\right) \\ + aB^{\eta}\left((T-v)(1-\theta T) + \left(\frac{\theta}{2} + \frac{\alpha}{2}\right)(T^2 - v^2)\right) \end{array}\right) \\ + h_v \frac{aB^{\eta}}{T}\left(\begin{array}{l} \lambda\left(\frac{T\beta^2}{2} + \alpha T\frac{\beta^3}{6} + \frac{kT\beta^4}{24} - \frac{\beta^3}{6} - \frac{\alpha\beta^4}{12} + \frac{k\beta^5}{40}\right) + (1-t)\frac{\alpha\beta^3}{6} - \frac{k\beta^5}{20} \\ - \beta\left(\frac{T^2}{2} + \frac{\alpha T^3}{6} + \frac{kT^4}{24}\right) + \frac{\alpha T^4}{4} + (1-\alpha)\frac{T^3}{6} + \frac{kT^5}{40} \end{array}\right) \\ + h_b \frac{aB^{\eta}}{T}\left(\frac{v^2 T}{2} + \frac{\alpha T}{3}v^3 + \frac{kTv^4}{8} - \frac{v^3}{3} - \frac{kv^5}{10} - \frac{\alpha v^4}{4}\right) \\ + (d_v + d_b)\frac{aB^{\eta}}{T}\left(T\lambda\beta + (\alpha T - 1)\frac{\lambda}{2}\beta^2 - \frac{\alpha\lambda\beta^3}{3} - \frac{T^2}{2} - \frac{\alpha T^3}{6} + \frac{kTv^3}{6} - \frac{kv^4}{8}\right) + p_1 + p_2 \\ + saB^{\eta}\left(T + \frac{\alpha T^2}{2} - v - \frac{\alpha v^2}{2}\right) + laB^{\eta}\theta(T-v)^2 \end{array}\right] \tag{22}$$

The necessary conditions to minimize the total average cost of the inventory system and to find the optimal values of production period and shortage period are

$$\frac{\partial\phi(\beta, v)}{\partial\beta} = 0 \text{ and } \frac{\partial\phi(\beta, v)}{\partial v} = 0 \tag{23}$$

For convexity of the cost function the following sufficient conditions should be satisfied.

$$\frac{\partial^2\phi(\beta,v)}{\partial\beta^2}>0 \quad , \quad \frac{\partial^2\phi(\beta,v)}{\partial v^2}>0 \text{ and } \left(\frac{\partial^2\phi(\beta,v)}{\partial\beta^2}\right)\left(\frac{\partial^2\phi(\beta,v)}{\partial v^2}\right)-\left(\frac{\partial^2\phi(\beta,v)}{\partial\beta\partial v}\right)>0$$

Equation (23) is equivalent to

$$\left[\begin{array}{l} \dfrac{maB^{\eta}\lambda}{T}\left[T-\beta+\alpha\left(T\beta-\beta^2\right)\right]+h_v\dfrac{aB^{\eta}}{T}\left[\begin{array}{l}\lambda\left(T\beta+\alpha T\dfrac{\beta^2}{2}+\dfrac{kT\beta^3}{6}-\dfrac{\beta^2}{2}-\dfrac{\alpha\beta^3}{3}+\dfrac{k\beta^4}{8}\right)\\ +(1-T)\alpha\dfrac{\beta^2}{2}-\dfrac{k\beta^4}{4}-\left(\dfrac{T^2}{2}+\dfrac{\alpha T^3}{6}+\dfrac{kT^4}{24}\right)\end{array}\right] \\ +\left(d_v+d_b\right)\dfrac{aB^{\eta}}{T}\left[T\lambda+\left(\alpha T-1\right)\lambda\beta-\alpha\lambda\beta^2\right] \end{array}\right]=0$$

and

$$\left[\begin{array}{l} c\left\{\dfrac{aB^{\eta}}{T}\left(T+\alpha Tv+\dfrac{kTv^2}{2}-v-\dfrac{kv^3}{2}-\alpha v^2\right)+aB^{\eta}\left(\left(\theta T-1\right)+\left(\dfrac{\theta}{2}+\dfrac{\alpha}{2}\right)(-2v)\right)\right\} \\ +h_b\dfrac{aB^{\eta}}{T}\left(vT+\alpha Tv^2+\dfrac{kTv^3}{2}-v^2-\dfrac{kv^4}{2}-\alpha v^3\right)+\left(d_v+d_b\right)\dfrac{aB^{\eta}}{T}\left(\dfrac{kTv^2}{2}-\dfrac{kv^3}{2}\right) \\ +p_1+p_2+saB^{\eta}\left(-1-\alpha v\right)-2laB^{\eta}\theta\left(T-v\right) \end{array}\right]=0 \qquad (24)$$

The solutions for optimal production period β and shortage period v are determined with the help of MATHEMATICA- 5.1 software and the convexity conditions are also checked.

Solution procedure

Step 1: The parameters in inventory system are assigned with values.
Step2: The simultaneous equations (24) are solved with MATHEMATICA 5.1.
Step 3: The sufficiency conditions for convexity are tested.
Step 4: The total cost of the inventory system are found out by equation (22).

4. EMPIRICAL INVESTIGATION

Numerical illustration-1: Let m = Rs 40 per unit, a = 400 units, $\lambda=1.1$, T = 20 months, c = Rs 45 per unit, k = 0.04, $\theta=0.01$, $h_v=$ Rs 0.35 per unit, $h_b=$ Rs 0.4 per unit, $d_v=$ Rs 31 per unit, $d_b=$ Rs 40 per unit, $p_1=$ Rs 600, p_2 = Rs 500, s = Rs 30, l = 28, $\eta=$ 1.3, B = 2, α = 0.1

Result: $\beta=$19.8253, $v=$ 13.9925, Total cost = Rs 187955

Numerical illustration-2: Let m = Rs 50 per unit, a = 400 units , λ =1.3 , T = 70 days, c = Rs 45 per unit, k = 0.04, $\theta=0.01$, $h_v=$ Rs 0.35 per unit, $h_b=$ Rs 0.4 per unit, $d_v=$ Rs 31 per unit, $d_b=$ Rs 40 per unit, $p_1=$ Rs 600, p_2 = Rs 500, s = Rs 30, l = 28, $\eta=$ 1.3, B = 2, α = 0.1

Result: $\beta=$ 9.73289, $v=$ 4.90244, Total cost = Rs 378798

Numerical illustration-3: Let m = Rs 15 per unit, a = 400 units , $\lambda=1.1$, T = 70 days, c = Rs 45 per unit, k = 0.004, $\theta=0.008$, $h_v=$ Rs 0.35 per unit, $h_b=$ Rs 0.4 per unit, $d_v=$ Rs 31 per unit, $d_b=$ Rs 56 per unit, $p_1=$ Rs 600, p_2 = Rs 700, s = Rs 27, l = 28, $\eta=$ 2.3, B = 2, α = 0.05

Result: $\beta=$66.0172, $v=$ 18.0083, Total cost = Rs 447699

Numerical illustration-4: Let m = Rs 25 per unit, a = 300 units , $\lambda=12$, T = 80 days, c = Rs 20 per unit, k = 0.03, $\theta=0.05$, $h_v=$ Rs 0.45 per unit, $h_b=$ Rs 0.8 per unit, $d_v=$ Rs 30 per unit, $d_b=$ Rs 45 per unit, $p_1=$ Rs 450, $p_2=$ Rs 550, s = Rs 30, l = 25, $\eta=$ 1.4, B = 2, α = 0.1

Result: β = 46.0232, $v=$ 10.7332, Total cost = Rs 305845

Numerical illustration-5: Let m = Rs 60 per unit, a = 500 units , $\lambda = 12$, T = 90 days, c = Rs 15 per unit, k = 0.06, $\theta = 0.01$, $h_v =$ Rs 0.35 per unit, $h_b =$ Rs 0.4 per unit, $d_v =$ Rs 28 per unit, $d_p =$ Rs 42 per unit, $p_1 =$ Rs 600, p_2 = Rs 500, s = Rs 30, l = 24, $\eta =$ 1.5, B = 3, α = 0.1

Result: $\beta =$ 47.0223, $\nu =$ 5.41182, Total cost = Rs 1158110

5. SENSITIVITY ANALYSIS

The sensitivity analysis for different system parameters of the integrated supply chain model by changing one parameter, keeping others unchanged is carried out as follows.

Table 1: Sensitivity analysis of numerical illustration-1

Parameters	% change	Value of the parameter	β	ν	Total Cost
m	-10	36	19.8187	13.9925	184344
	-5	38	19.722	13.9925	186149
	5	42	19.6284	13.9925	189760
	10	44	19.4313	13.9925	191566
a	-10	360	19.8313	14.001	164483
	-5	380	19.8313	13.9965	173621
	5	420	19.8313	13.9888	191897
	10	440	19.8313	13.9855	201035
T	-10	18	17.8518	10.1978	172087
	-5	19	18.8382	12.4494	179572
	5	21	20.813	15.3558	197028
	10	22	21.8018	16.6286	206719
k	-10	0.36	19.8522	12.6825	201566
	-5	0.38	19.8517	13.4365	204024
	5	0.042	19.841	14.4367	209665
	10	0.044	19.8206	14.8065	212744
θ	-10	0.009	19.8313	13.997	206734
	-5	0.0095	19.8313	13.9947	206739
	5	0.0105	19.8313	13.9902	206750
	10	0.011	19.8313	13.9879	206756
$\propto$	-10	0.09	19.8603	14.2729	181335
	-5	0.095	19.8453	13.6966	184615
	5	0.105	19.8183	13.3816	191355
	10	0.11	19.8061	13.3816	194817
h_v	-10	0.3150	19.8313	13.9925	206624
	-5	0.3325	19.8313	13.9925	206684
	5	0.3675	19.8313	13.9925	206806
	10	0.385	19.8313	13.9925	206866
h_b	-10	0.36	19.8482	13.9184	206362
	-5	0.38	19.8398	13.9557	206553
	5	0.42	19.8229	14.0287	206938
	10	0.44	19.8145	14.0645	207132
η	-10	1.17	19.8313	13.999	171764
	-5	1.235	19.8313	13.996	179677
	5	1.365	19.8313	13.9891	196614
	10	1.43	19.8313	13.9859	205673
d_v	-10	27.90	19.8267	14.7262	186414
	-5	29.45	19.829	14.6634	187178
	5	32.55	19.8336	14.1144	188743
	10	34.1	19.8358	14.0299	189543
d_b	-10	36	19.8187	16.6424	185976
	-5	38	19.822	15.8245	186955
	5	42	19.8284	15.1485	188974
	10	44	19.8313	14.2942	190012

Table 2: Sensitivity analysis of numerical illustration-2

Parameters	% change	Value of the parameter	β	ν	Total Cost
m	-10	45	10.8369	4.90244	378008
	-5	47.5	10.2711	4.90244	378419
	5	52.5	9.22081	4.90244	379147
	10	55	8.73325	4.90244	379470
a	-10	360	9.73289	4.8953	340920
	-5	380	9.73289	4.89906	359859
	5	420	9.73289	4.90549	397737

	10	440	9.73289	4.90826	416676
T	-10	63	3.76664	4.91512	323267
	-5	66.5	6.52985	4.9072	350615
	5	73.5	13.1629	4.9003	407575
	10	77	17.4353	4.9003	437100
k	-10	0.036	11.63383	5.23435	368319
	-5	0.038	10.67147	5.26076	373621
	5	0.042	9.8154	5.75731	383849
	10	0.044	8.9155	5.82367	388770
θ	-10	0.009	9.73289	4.8611	378470
	-5	0.0095	9.73289	4.88182	378634
	5	0.0105	9.73289	4.92295	378961
	10	0.011	9.73289	4.94335	379125
$\propto$	-10	0.09	10.808	4.91504	356834
	-5	0.095	10.2362	4.85854	367822
	5	0.105	9.28696	4.84674	389764
	10	0.11	8.88949	4.8146	400722
h_v	-10	0.3150	9.73289	4.90244	365649
	-5	0.3325	9.73289	4.90244	372223
	5	0.3675	9.73289	4.90244	385372
	10	0.385	9.73289	4.90244	391947
h_b	-10	0.36	7.24927	4.92318	378080
	-5	0.38	8.4763	4.91278	378318
	5	0.42	11.0151	4.89214	379524
	10	0.44	12.3175	4.88189	380497
η	-10	1.17	9.73289	4.89638	346159
	-5	1.235	9.73289	4.89348	362111
	5	1.365	9.73289	4.89266	396254
	10	1.43	9.73289	4.78456	414514
d_v	-10	27.90	10.4043	4.98386	383378
	-5	29.45	10.0634	4.94268	381094
	5	32.55	9.41242	4.86309	376490
	10	34.1	9.10162	4.82461	374171
d_b	-10	36	10.6072	5.00821	384698
	-5	38	10.1613	4.95454	381758
	5	42	9.32121	4.85183	375818
	10	44	8.92544	4.80265	372820

Table 3: Sensitivity analysis of numerical illustration-3

Parameters	**% change**	**Value of the parameter**	β	ν	**Total Cost**
m	-10	13.5	65.9577	18.0083	444196
	-5	14.2	64.9877	18.0083	445947
	5	15.75	64.0463	18.0083	449451
	10	16.5	63.0749	18.0083	451203
a	-10	360	66.0172	17.9896	402931
	-5	380	66.0172	17.9994	425315
	5	420	66.0172	18.0163	470083
	10	440	66.0172	18.0236	492467
T	-10	63	59.7314	19.4862	414759
	-5	66.5	62.8794	18.6327	431295
	5	73.5	Infeasible	17.5278	Infeasible
	10	77	Infeasible	17.1458	Infeasible
k	-10	0.0036	66.8349	16.0076	445756
	-5	0.0038	65.9256	17.9284	446778
	5	0.0042	64.1097	18.2096	448538
	10	0.0044	63.2031	18.5065	449311
θ	-10	0.0072	66.0172	17.8913	447366
	-5	0.0076	66.0172	17.9499	447533
	5	0.0084	66.0172	18.0665	447865
	10	0.0088	66.0172	181245	448031
$\propto$	-10	0.045	66.341	17.2771	420254
	-5	0.0475	66.1719	17.2352	433982
	5	0.0525	65.8751	17.198	461403
	10	0.055	65.7441	16.8058	475092
h_v	-10	0.3150	66.0172	18.0083	446490
	-5	0.3325	66.0172	18.0083	447094
	5	0.3675	66.0172	18.0083	448304
	10	0.385	66.0172	18.0083	448909
h_b	-10	0.36	66.4149	18.3648	447422
	-5	0.38	66.2162	18.1843	447566
	5	0.42	65.8178	17.8367	447821

	10	0.44	65.618	17.6692	447932
η	-10	1.17	66.0172	17.8079	204572
	-5	1.235	66.0172	17.7242	213999
	5	1.365	66.0172	17.5547	234175
	10	1.43	66.0172	17.4689	244966
d_v	-10	27.90	65.8923	18.4008	446785
	-5	29.45	65.9557	18.2015	447244
	5	32.55	66.0768	18.1209	448151
	10	34.1	66.1346	18.0389	448599
d_b	-10	50.4	65.7855	18.7359	446036
	-5	53.2	65.9047	18.3617	446874
	5	58.8	66.1236	17.6737	448513
	10	61.6	66.2243	17.3563	449315

The sensitivity analysis of the numerical illustrations by changing the parameter values of the inventory system at -10%, -5%, 5% and 10% results the followings.

- Increment in the production cost per unit (*m*) results a decreasing value of production period (β).
- Increased values of deterioration rate '*k*' sensitize the shortage period (ν) to increase.
- Increment in the inflation rate $(\propto)$ results the increment in total cost by reducing the optimal

production period (β) and the time period (ν), when shortage starts.

- Increment in advertisement frequency (η) sensitizes the total cost to increase but shortage period (ν)

decreases simultaneously.

- With increment in the duration of planning horizon (*T*), the total cost and optimal production period

(β) increase.

- The increased values of d_v, d_b, h_v and h_b sensitize the values of shortage period ν to decrease.

The result of sensitivity analysis indicates that to avoid loss for increased value of production cost the retailer should sell the products as early as possible. The optimal production period and shortage period should be reduced to avoid inflationary effects. The planning horizon should be reduced to minimize the system total cost. Due to high deterioration cost and holding cost the shortage period is reduced to enhance the system creditability.
The present paper studies the integrated supply chain management system with inflation induced demand and partially backlogged shortage which is now a complex problem faced by many business organizations. Most existing studies focuses on the supply chain policies with consideration to some of the factors mentioned above. But in real business scenario all the factors considered in the present study are equally important. So this work will be beneficial for the inventory manager to minimize the system total cost.

6. CONCLUSION

A vendor-buyer integrated inventory model subject to partial backlogging is developed for deteriorating items subject to inflation. The study has been conducted to find the optimal production period, shortage period and average total cost of the integrated supply chain model. It is observed that if production cost and deterioration cost are more, it is wise to minimize the production period. Due to inflationary effect the purchasing tendency grows high resulting immediate backlogging. So less production period can also be implemented to minimize the system total cost. Similarly by increasing advertisement frequency the sale becomes more resulting an early backlogging that minimize total cost. This chapter can assist the inventory manager to implement different strategies to minimize the total cost of the integrated inventory system in a complex environment.
This model can also be extended by allowing credit period to retailer and customer. Preservation technology can also be included to reduce deterioration.

RECEIVED: DECEMBER , 2022.
REVISED: APRIL, 2023

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