

COMPARISON OF FIXED EFFECTS AND RANDOM EFFECTS PANEL MODELS FOR THE ESTIMATION OF ACCOUNTING BETA COEFFICIENT.

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ABSTRACT

The accounting beta coefficient is a financial indicator used to measure the volatility of a company in the market, which allows incorporating the idiosyncrasies of closely held companies. The following paper proposes the application of a market risk measurement methodology based on the use of the accounting beta coefficient. In order to meet the objective, the research was carried out from a quantitative approach, using an exploratory research design. The study analyses a total of 2351 unlisted companies in the Colombian service sector. The results of the measurements show that the leverage of the companies is a determining element in the level of risk of the companies analysed. It can be concluded that accounting beta measured through ROE is the measure that best captures the risk assumed by shareholders and/or investors.

KEYWORDS: leverage, deleveraging, correlation, performance.

MSC: 62H20, 62M10, 62P20.

RESUMEN

El coeficiente beta contable es un indicador financiero que se utiliza para medir la volatilidad de una compañía en el mercado, el cual permite incorporar la idiosincrasia de las empresas de capital cerrado. El siguiente trabajo propone la aplicación de una metodología de medición del riesgo de mercado basado en el uso del coeficiente beta contable. Para cumplir el objetivo, se efectuó la investigación desde un enfoque cuantitativo, mediante un diseño de investigación exploratorio. En el estudio se analizan un total de 2351 empresas del sector servicio colombiano que no cotizan en bolsa de valores. Los resultados de las mediciones muestran que el apalancamiento de las empresas es un elemento determinante dentro del nivel de riesgo de las empresas analizadas. Se puede concluir que el beta contable medido a través del ROE es la medida que mejor captura el riesgo asumido por los accionistas y/o inversionistas.

PALABRAS CLAVE: apalancamiento, desapalancamiento, correlación, rendimiento.

1. INTRODUCTION

The notion of risk in the field of finance is considered a crucial anchor point, as its estimation directly affects the performance of company [14]. Financial risk is related to various economic factors, including credit, liquidity, and the market [51] [25] [32]. Authors such as [10] [5] [22] define credit risk as the potential loss caused by a debtor or counterparty failing to meet their obligations in a financial transaction. Regarding liquidity risk [20] [2] define it as the probability of an economic loss resulting from a shortage of funds that prevents the organization from meeting its financial obligations as agreed. Market risk is defined as the possibility that economic conditions (represented through price) of an object do not match the expected values [47], resulting from market movements and volatility [27].

Since risks affect the entire market rather than a specific company or industry, it is crucial to monitor and manage them. Therefore, effective risk management and informed decision-making regarding asset allocation are essential for privately held companies. For this reason, risk management has become a discipline that has attracted the attention of accounting and finance professionals and researchers [12]. From the perspective of modern financial theory, there are several indicators that allow for the quick and easy quantification of market risk. These indicators

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include the Capital Asset Pricing Model (CAPM), Sharpe ratio [45] Treynor ratio [49] and Jensen's alpha [23] [24]. The indicators focus on the use of exogenous variables to measure the relationship between market risk and performance. In these indicators (except for the Sharpe ratio), the beta factor is used as a central component, expressing the dependence relationship between the returns of asset "i" and the market portfolio [8].

The beta coefficient represents a measure of systematic risk of an asset compared to the market. Its calculation is based on the relationship between the covariance of asset returns and market returns, divided by the variance of market returns.

$$\beta_c = \frac{COV(R_E, R_{Mm})}{VAR(R_{Mm})}$$

Where R_E represents the asset returns, and R_{Mm} represents the market returns. The results of the beta coefficient indicate that: a) a beta coefficient equal to 1 indicates that the asset has the same volatility as the market, while a beta greater than 1 indicates that the asset is riskier than the market, and a beta less than 1 indicates that the asset is less risky than the market [33]. The beta coefficient allows measuring the degree of sensitivity of the asset to market fluctuations, based on stock market information. Therefore, the beta coefficient is primarily used as an indicator for publicly traded companies. However, the financial discipline has recognized the use of accounting beta as a variant (of the original beta) for privately held companies [19].

Authors like [18] acknowledge that the estimation of the accounting beta is done similarly to a market-based beta, assuming that accounting returns are generated by a stochastic process that is structurally similar to the one that generates returns in a stock market. Several studies confirm the relevance of using the accounting beta as a significant measure for estimating the systematic risk of non-listed companies. Some notable studies include those conducted by [37] [38] [48] [43] [41] [49] [36] [40].

The use of the accounting beta coefficient allows identifying the relationship between uncontrollable variables and the systematic risk of privately held companies [39]. Its calculation is estimated through linear regression between the company's accounting measure (ROA and ROE) and the average of the same accounting measure related to all market companies [14]. Although this type of accounting beta represents an important indicator for estimating the relative risk of companies [6], authors such as [46] [29] recommend using leveraged beta. Therefore, this research proposes to include the effects of leverage on the accounting beta by applying the leverage factor proposed by [7].

$$\beta_{CA} = \beta_c * \left[1 + (1 - t) * \left(\frac{\kappa}{\lambda} \right) \right]$$

Where t represents the corporate income tax in the accounting period, κ represents the monetary value of liabilities, and λ represents the monetary value of equity. Considering that the proposal focuses on calculating the leveraged accounting beta, it is necessary to define the calculation structure of the financial indicators used as the basis for performance calculation.

The first financial indicator is the Return on Assets (ROA), which measures the asset's ability to generate profits regardless of how it is financed [34]. The calculation structure of ROA is defined as the net income divided by the book value of total assets at the end of the previous accounting period [1]. It is common for this financial indicator to be calculated using the formula. However, authors such as [44] [32] consider that assets arise from the investment made by the company, and operating profits are the result of the efficiency of that investment.

Therefore, the authors propose the following calculation formula for the financial indicator ROA:

$$ROA = \frac{\text{Earnings from operating activities}}{\text{Total assets}_{t-1}}$$

Assuming that financial information of companies does not exhibit linear behavior and is subject to (i) changes in the environment, (ii) operating conditions, and (iii) accounting results obtained during the accounting period, the present research proposes the logarithmic transformation of the ROA equation as a technique to normalize data series that may experience nonlinear behavior. This technique has been used by authors such as [50] [42] [48] to reduce the complexity and trends of financial information, thereby improving its analysis and understanding. For this reason, the new calculation structure will be referred to as Adjusted Return on Assets (ROA_a) and is presented as follows:

$$ROA_a = \ln \left(1 + \left(\frac{\text{Earnings from operating activities}}{\text{Total assets}_{t-1}} \right) \right)$$

As for the second financial indicator, the return on equity (ROE) will be analyzed. This technique measures the return earned on the investment of common shareholders in the company [16]. Its calculation structure is represented by the relationship between the net income for the accounting period divided by the equity value from the previous period [26] [35].

Although the previously described calculation structure can be considered the most widely used and accepted [24] proposes calculating the return on equity as the ratio of operating income to the equity value from the previous

period, without considering the effects of financing or debt of the company (if any). The calculation structure proposed by [24] is shown below:

$$ROE = \frac{\text{Earnings from operating activities}}{\text{Equity Value}_{t-1}}$$

Taking as a reference the above mentioned about the linear behavior of financial information, for this research, it is proposed to use the ROE ratio with the definition proposed by [41], including the logarithmic transformation. Therefore, the present study acknowledges the term Adjusted Return on Equity (ROEa) with the following calculation structure.

$$ROE_a = \ln \left(1 + \left(\frac{\text{Earnings from operating activities}}{\text{Equity Value}_{t-1}} \right) \right)$$

In order to present a proposal for the calculation of the leveraged accounting beta ratio for closely held companies, the following calculation structure is presented:

Variante 1. Leveraged Accounting Beta Coefficient based on ROAa:

$$\beta_{c_{ROAa}} = \left(\frac{COV(ROA_a, \overline{ROA_M})}{VAR(\sigma_{ROA_M}^2)} \right) * \left[1 + (1 - T) * \left(\frac{\kappa}{\lambda} \right) \right]$$

Variante 2. Leveraged Accounting Beta Coefficient based on ROEa:

$$\beta_{c_{ROEa}} = \left(\frac{COV(ROE_a, \overline{ROE_M})}{VAR(\sigma_{ROE_M}^2)} \right) * \left[1 + (1 - T) * \left(\frac{\kappa}{\lambda} \right) \right]$$

In order to determine the significance of the ratios in the accounting beta, the use of panel data methodology is proposed. Panel data methodology is a tool in the fields of statistics and econometrics that allows researchers to analyze data over multiple time periods (temporal component) and for various cross-sectional units (spatial component), combining the strengths of both approaches and providing a more comprehensive view of the investigated phenomenon [9]. Regression analysis with panel data combines time series and cross-sectional data, addressing three main models: Common Effects (CE), Fixed Effects (FE), and Random Effects (RE). Each model, with its specific characteristics and applications, is estimated using different techniques. To select the most suitable model, statistical validation techniques such as Chow, Hausman, and the Lagrange Multiplier are employed [53]. The popularity of panel data studies is attributed to the availability of data, their ability to model the complexity of variable behaviors, and the associated methodological challenges. The study of panel data offers greater precision in parameter inference and captures dynamic relationships, controlling for the impact of omitted variables [21].

In recent years, the use of panel data methodology has been employed in accounting research, with notable studies conducted by [4], examining the relevance of the value of financial variables for publicly traded companies in Tunisia. The research by [31] analyzes the relationship between the stock beta coefficient and the financial information of Spanish companies listed on the stock exchange. The study conducted by [11] examines the association between accounting earnings management and the manipulation of real activities in a weaker regulatory environment within Tunisian public companies. Another study utilizing panel data methodology is carried out by [14], focusing on the use of the accounting beta as a risk measurement indicator in listed companies on the Casablanca Stock Exchange. Recently, the research developed by [13] focuses on empirically verifying whether the Economic Value Added (EVA) measure and EVA calculated with accounting beta are better indicators than traditional metrics in explaining Market Value Added.

The use of panel data methodology in Accounting and Finance is increasingly growing and significant [15]. As acknowledged by [28], the primary advantage of using panel data models is related to controlling individual heterogeneity. This implies that the impact produced by variations between observations in different periods can be analyzed individually, as well as closely monitoring the progress of relevant variables for a specific entity over time [15]. Analysis using panel data increases the number of observations, and its effectiveness allows for examining cause-and-effect relationships by observing before and after. While cross-sectional analyses are effective for examining causal relationships based on theoretical models, they lack the essential temporal dimension to determine causality. Panel data also enables the examination of the stability of relationships between variables over time, unlike cross-sectional analyses that are limited to a single time point [52].

2. METHODS

The research process was conducted using a quantitative approach, through an exploratory research design that seeks solutions to problems and/or proposes new potential approaches or ideas (hypotheses) related to the study subject. The results of the empirical test are obtained using a panel data methodology, where information from multiple individuals at a given point in time is combined over several time periods [17]. The application of panel data methodology validates its relevance using a random effects model and a fixed effects model. Both models

were validated using the Hausman test. To identify the companies under study, the selection criterion was defined as companies that recurrently report financial information in the Integrated System of Corporate Information of the Superintendence of Companies of Colombia (SIIS) during the period 2017-2021.

Based on the gathered information, the sample for the present study consisted of companies in the Services sector of Colombia. According to the [3] the dynamics of this sector in recent decades have experienced sustained growth in the current millennium. To examine the associations between the variables defined for the calculation of leveraged accounting betas, a correlational analysis is proposed. For this purpose, the following correlation matrices are defined.

Figure 1: ROA Correlation Matrix.

$$M_{\beta_{caROA}} = \begin{pmatrix} 1 & m_{12}(x_{1et}, x_{2et}) & m_{13}(x_{1et}, x_{3et}) & m_{14}(x_{1et}, x_{4et}) & m_{15}(x_{1et}, x_{5et}) \\ m_{21}(x_{2et}, x_{1et}) & 1 & m_{23}(x_{2et}, x_{3et}) & m_{24}(x_{2et}, x_{4et}) & m_{25}(x_{2et}, x_{5et}) \\ m_{31}(x_{3et}, x_{1et}) & m_{32}(x_{3et}, x_{2et}) & 1 & m_{34}(x_{3et}, x_{4et}) & m_{35}(x_{3et}, x_{5et}) \\ m_{41}(x_{4et}, x_{1et}) & m_{42}(x_{4et}, x_{2et}) & m_{43}(x_{4et}, x_{3et}) & 1 & m_{45}(x_{4et}, x_{5et}) \\ m_{51}(x_{5et}, x_{1et}) & m_{52}(x_{5et}, x_{2et}) & m_{53}(x_{5et}, x_{3et}) & m_{54}(x_{5et}, x_{4et}) & 1 \end{pmatrix}$$

Where:

$M_{\beta_{caROA}}$: Leveraged Accounting Beta (ROA) Correlation Matrix.

$\frac{v_{et}}{Y_{t-1e}}$: Measures the proportion of company e sales in year t relative to the assets of period t-1.

$\frac{\varphi_{et}}{Y_{t-1et}}$: Measures the proportion of company e cost of sales in year t relative to the assets of period t-1.

$\frac{\phi_{et}}{Y_{t-1et}}$: Measures the proportion of company e administrative expenses in year t relative to the assets of period t-1.

$\frac{\psi_{et}}{Y_{t-1et}}$: Measures the proportion of company e selling expenses in year t relative to the assets of period t-1.

ω_{et} : Leverage factor of company e in year t.

Figure 2: ROE Correlation Matrix.

$$M_{\beta_{caROE}} = \begin{pmatrix} 1 & m_{12}(x_{1et}, x_{2et}) & m_{13}(x_{1et}, x_{3et}) & m_{14}(x_{1et}, x_{4et}) & m_{15}(x_{1et}, x_{5et}) \\ m_{21}(x_{2et}, x_{1et}) & 1 & m_{23}(x_{2et}, x_{3et}) & m_{24}(x_{2et}, x_{4et}) & m_{25}(x_{2et}, x_{5et}) \\ m_{31}(x_{3et}, x_{1et}) & m_{32}(x_{3et}, x_{2et}) & 1 & m_{34}(x_{3et}, x_{4et}) & m_{35}(x_{3et}, x_{5et}) \\ m_{41}(x_{4et}, x_{1et}) & m_{42}(x_{4et}, x_{2et}) & m_{43}(x_{4et}, x_{3et}) & 1 & m_{45}(x_{4et}, x_{5et}) \\ m_{51}(x_{5et}, x_{1et}) & m_{52}(x_{5et}, x_{2et}) & m_{53}(x_{4et}, x_{3et}) & m_{54}(x_{5et}, x_{4et}) & 1 \end{pmatrix}$$

Where:

$M_{\beta_{caROE}}$: Leveraged Accounting Beta (ROE) Correlation Matrix.

$\frac{v_{et}}{Y_{t-1e}}$: Measures the proportion of company e sales in year t relative to the assets of period t-1.

$\frac{\varphi_{et}}{Y_{t-1et}}$: Measures the proportion of company e cost of sales in year t relative to the assets of period t-1.

$\frac{\phi_{et}}{Y_{t-1et}}$: Measures the proportion of company e administrative expenses in year t relative to the assets of period t-1.

$\frac{\psi_{et}}{Y_{t-1et}}$: Measures the proportion of company e selling expenses in year t relative to the assets of period t-1.

ω_{et} : Leverage factor of company e in year t.

The correlation between variables is analyzed using the scale of relationship between variables defined by [30].

- If the range is between 0 to 0.25, the relationship is Poor.
- If the range is between 0.26 to 0.50, the relationship is Weak.
- If the range is between 0.51 to 0.75, the relationship is Moderate.
- If the range is between 0.76 to 1, the relationship is Strong.

Considering that panel data methodology utilizes two regression models, namely fixed effects model and random effects model, below is the calculation structure for the random effects model.

$$B_{et} = b_0 + b_1x_{1et} + b_2x_{2et} + b_3x_{3et} + b_4x_{4et} + b_5x_{5et} + \tau_e + \varepsilon_{et}$$

Where:

B_{et} : Leveraged accounting beta of firm e in period t.

b_0 : Model constant.

The variables x_{1et} , x_{2et} , x_{3et} , x_{4et} , x_{5et} are defined in the correlation matrices for each measurement indicator.

$\tau_e \sim N(0, \sigma_\tau^2)$

$\varepsilon_{et} \sim N(0, \sigma^2)$

Once the calculation structure for the random effects model has been presented, below is the calculation structure for the fixed effects model.

$$B_{et} = b_0 + b_1x_{1et} + b_2x_{2et} + b_3x_{3et} + b_4x_{4et} + b_5x_{5et} + \varepsilon_{et}$$

Where:

B_{et} : Leveraged accounting beta of firm e in period t.

b_0 : Model constant.

The variables x_{1et} , x_{2et} , x_{3et} , x_{4et} , x_{5et} are defined in the correlation matrices for each measurement indicator.

$\varepsilon_{et} \sim N(0, \sigma^2)$

Considering that fixed and random effects models allow capturing unobserved heterogeneities among cross-sectional units that could be correlated with independent variables [19]. The aforementioned is crucial in contexts such as the Colombian business environment, where assuming homogeneity (as in a pooled regression) could be inappropriate for the proposed analysis.

In order to measure whether the differences between the proposed models for calculating the leveraged accounting beta are systematic and significant, the use of the Hausman test is proposed as a statistical validation technique.

This test is established as follows:

$$h = (\hat{\beta}_a - \hat{\beta}_f)' \left(\sum \hat{\beta}_f - \sum \hat{\beta}_a \right)^{-1} (\hat{\beta}_a - \hat{\beta}_f)$$

Where:

$\hat{\beta}_a$ = Estimators of the random effects model.

$\hat{\beta}_f$ = Estimators of the fixed effects model.

$\sum \hat{\beta}_f$ = Variance-Covariance matrix of the fixed effects model.

$\sum \hat{\beta}_a$ = Variance-Covariance matrix of the random effects model.

$h \sim X_k^2$

For the validation of the test, the following hypothesis test is presented:

- ✓ H_0 = The random effects model (P-value > 0.05) adequately reflects the behavior of the data. Therefore, the unobservable effect is not correlated with the explanatory variables.
- ✓ H_1 = The fixed effects model (P-value < 0.05) adequately reflects the behavior of the data.

Taking into account that the Breusch-Pagan test assesses whether the variance associated with each individual is significantly different from zero, while the Hausman test compares the estimates of fixed effects and random effects models to determine the presence of systematic differences [17]. Therefore, it is considered that the Hausman test is sufficient to validate the specification of the models, as it is the research objective, making the use of the Breusch-Pagan test unnecessary in this context.

3. RESULTS

The empirical analysis of calculating leveraged accounting betas is presented based on the results of applied descriptive statistics, correlation tests, the calculation of random effects and fixed effects models, and finally, the result of the Hausman test.

Table 1: Descriptive statistics of leveraged accounting beta using adjusted ROA.

Variable		Mean	Std. Dev.	Min	Max	Observations
Beta ROAa	overall	3,12	87,78	-1.323,00	7.281,13	N = 11.755
	between		45,70	-398,45	1.501,30	n = 2.351
	within		74,96	-1.495,40	5.782,95	T = 5
$\frac{\text{Sales}}{\text{Total assets}_{t-1}}$	overall	0,78	1,32	-14,77	18,04	N = 11.755
	between		1,25	-6,39	12,77	n = 2.351
	within		0,41	-7,60	11,11	T = 5
$\frac{\text{Cost of sales}}{\text{Total assets}_{t-1}}$	overall	0,45	1,03	0,00	17,05	N = 11.755
	between		1,00	0,00	12,13	n = 2.351
	within		0,27	-3,82	7,42	T = 5
$\frac{\text{Administration costs}}{\text{Total assets}_{t-1}}$	overall	0,19	0,40	-0,16	10,51	N = 11.755
	between		0,37	-0,04	5,55	n = 2.351
	within		0,15	-4,06	6,26	T = 5

$\frac{\text{Selling expenses}}{\text{Total assets}_{t-1}}$	overall	0,08	0,30	0,00	6,32	N = 11.755
	between		0,29	0,00	5,62	n = 2.351
	within		0,10	-1,61	2,04	T = 5
Leverage Factor	overall	4,58	176,99	-2.489,20	17.889,68	N = 11.755
	between		85,75	-493,04	3.604,80	n = 2.351
	within		154,83	-3.597,22	14.289,47	T = 5

The average result of the leveraged accounting beta, estimated through adjusted ROA, is greater than one, suggesting that the risk is high for companies in the services sector. The same behavior is observed for the rest of the variables since they are the same inputs in both models used. From a financial standpoint, it can be concluded that with this beta estimation methodology, there may be greater sensitivity in the estimation, which increases the average.

Table 2: Correlation Matrix of Leveraged Accounting Beta using Adjusted ROA.

Variables	Beta ROAa	$\frac{\text{Sales}}{\text{Total assets}_{t-1}}$	$\frac{\text{Cost of sales}}{\text{Total assets}_{t-1}}$	$\frac{\text{Administration costs}}{\text{Total assets}_{t-1}}$	$\frac{\text{Selling expenses}}{\text{Total assets}_{t-1}}$	Leverage Factor
Beta ROAa	1,00					
$\frac{\text{Sales}}{\text{Total assets}_{t-1}}$	0,04	1,00				
$\frac{\text{Cost of sales}}{\text{Total assets}_{t-1}}$	0,02	0,88	1,00			
$\frac{\text{Administration costs}}{\text{Total assets}_{t-1}}$	0,08	0,50	0,18	1,00		
$\frac{\text{Selling expenses}}{\text{Total assets}_{t-1}}$	0,00	0,31	0,05	0,07	1,00	
Leverage Factor	0,75	0,00	0,00	0,00	0,00	1,00

Table 3: Random Effects Regression Model Leveraged Accounting Beta using adjusted ROA.

Random-effects GLS regression		Number of obs =		11.755		
Group variable: Empresa		Number of groups =		2.351		
within = 0,63		min =		5		
between = 0,39		Obs per group: avg =		5		
overall = 0,56		max =		5		
R-sq:		Wald chi2 (5)		17.314,45		
corr(u_i, X) = 0 (assumed)		Prob > chi2		0,00		
Beta ROAa	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
$\frac{\text{Sales}}{\text{Total assets}_{t-1}}$	-3,94	2,36	-1,67	0,10	-8,56	0,68
$\frac{\text{Cost of sales}}{\text{Total assets}_{t-1}}$	4,98	2,54	1,96	0,05	-0,01	9,96
$\frac{\text{Administration costs}}{\text{Total assets}_{t-1}}$	20,27	3,20	6,34	0,00	14,01	26,53
$\frac{\text{Selling expenses}}{\text{Total assets}_{t-1}}$	3,18	3,36	0,95	0,34	-3,40	9,75
Leverage Factor	0,38	0,00	131,20	0,00	0,37	0,38
_cons	-1,81	0,85	-2,12	0,03	-3,48	-0,13
sigma_u	27,33	(Fraction of variance due to u_i)				
sigma_e	51,10					
rho	0,22					

It is evident that the leverage factor has a moderate relationship with risk. For the rest of the variables, the relationship is weak. From this, it can be concluded that one of the characteristics of companies with high levels of risk may be the level of leverage.

The random effects model reveals that the only statistically significant variables are administration expenses over assets and leverage factor. The variable cost of sales over assets from the previous period is statistically significant, but with a p-value of 0.10. On the other hand, we have sales over assets, which is on the threshold (p-value of 0.10). In this regression model, sales expenses are not statistically significant.

Table 4: Fixed Effects Regression Model of Leveraged Accounting Beta using Adjusted ROA.

Fixed-effects (within) regression		Number of obs =	11.755			
Group variable: Empresa		Number of groups =	2.351			
	within = 0,63	min =	5			
R-sq:	between = 0,39	Obs per group: avg =	5			
	overall = 0,56	max =	5			
		F(5,9399)	3.178,87			
corr(u_i, Xb) = -0.0676		Prob > chi2	0,00			
Beta ROA	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
$\frac{\text{Sales}}{\text{Total assets}_{t-1}}$	-1,67	2,99	-0,56	0,58	-7,54	4,19
$\frac{\text{Cost of sales}}{\text{Total assets}_{t-1}}$	4,31	3,70	1,17	0,24	-2,94	11,56
$\frac{\text{Administration costs}}{\text{Total assets}_{t-1}}$	16,33	4,76	3,43	0,00	7,00	25,66
$\frac{\text{Selling expenses}}{\text{Total assets}_{t-1}}$	2,11	5,70	0,37	0,71	-9,05	13,28
Leverage Factor	0,38	0,00	125,98	0,00	0,38	0,39
_cons	-2,49	1,10	-2,27	0,02	-4,64	-0,34
sigma_u	36,02	(Fraction of variance due to u_i)				
sigma_e	51,10					
rho	0,33					
F test that all u_i=0: F(2350, 9399) = 2.46		Prob > F = 0.0000				

In the fixed effects regression model, the statistical significance of the same variables as in the random effects model is evident. This allows us to conclude that for this panel of data, under both regression models, administration expenses and leverage factor would explain the risk amplification (beta) of companies in the services sector.

Table 5: Hausman test based on Accounting Beta leveraged by adjusted ROA.

Variable	Coefficients			
	(b)	(B)	(b-B)	sqrt(diag(V_b-V_B))
	fe1	re1	Difference	S.E.
$\frac{\text{Sales}}{\text{Total assets}_{t-1}}$	-1,67	-3,94	2,26	1,84
$\frac{\text{Cost of sales}}{\text{Total assets}_{t-1}}$	4,31	4,98	-0,67	2,69
$\frac{\text{Administration costs}}{\text{Total assets}_{t-1}}$	16,33	20,27	-3,94	3,53
$\frac{\text{Selling expenses}}{\text{Total assets}_{t-1}}$	2,11	3,18	-1,06	4,60
Leverage Factor	0,38	0,38	0,01	0,00
b = consistent under Ho and Ha; obtained from xtreg				
B = inconsistent under Ha, efficient under Ho; obtained from xtreg				
Test: Ho: difference in coefficients not systematic				
chi2(5)	= (b-B)'[(V_b-V_B)^(-1)](b-B)			
	= 44,98			
Prob>chi2	= 0,0000			

According to the results of the Hausman test, it can be concluded that the model that best represents the behaviour of the data is the fixed effects model.

Table 6: Descriptive statistics of leveraged accounting beta using adjusted ROE.

Variable		Mean	Std. Dev.	Min	Max	Observations	
Beta ROE	overall	16,93	1.938,85	-54.986,59	201.714,30	N	11.755
	between		869,10	-10.923,21	40.408,85	n	2.351
	within		1.733,22	-44.046,45	161.322,40	T	5
$\frac{\text{Sales}}{\text{Equity Value}_{t-1}}$	overall	3,34	28,70	-1.708,20	1.209,42	N	11.755
	between		11,46	-116,53	197,29	n	2.351
	within		26,31	-1.588,32	1.108,91	T	5
$\frac{\text{Cost of sales}}{\text{Equity Value}_{t-1}}$	overall	1,94	17,73	-778,59	1.139,51	N	11.755
	between		7,57	-55,49	155,86	n	2.351
	within		16,04	-721,16	1.007,45	T	5
$\frac{\text{Administration costs}}{\text{Equity Value}_{t-1}}$	overall	0,77	10,51	-783,53	472,86	N	11.755
	between		3,66	-49,87	91,84	n	2.351
	within		9,85	-732,89	523,50	T	5
$\frac{\text{Selling expenses}}{\text{Equity Value}_{t-1}}$	overall	0,35	5,63	-60,56	454,18	N	11.755
	between		2,87	-25,04	89,90	n	2.351
	within		4,84	-99,32	364,62	T	5
Leverage Factor	overall	4,58	176,99	-2.489,20	17.889,68	N	11.755
	between		85,75	-493,04	3.604,80	n	2.351
	within		154,83	-3.597,22	14.289,47	T	5

The average leveraged accounting beta in the service sector companies, measured through adjusted return on equity (ROE), increases compared to the value estimated with adjusted return on assets (ROA). In this sense, it can be concluded that the risk in equity is higher compared to the risk estimated in assets. Additionally, the data dispersion also increases.

Table 7: Correlation Matrix of Leveraged Accounting Beta using Adjusted ROE.

Variable	Beta ROE	$\frac{\text{Sales}}{\text{Equity Value}_{t-1}}$	$\frac{\text{Cost of sales}}{\text{Equity Value}_{t-1}}$	$\frac{\text{Administration costs}}{\text{Equity Value}_{t-1}}$	$\frac{\text{Selling expenses}}{\text{Equity Value}_{t-1}}$	Leverage Factor
Beta ROE	1,00					
$\frac{\text{Sales}}{\text{Equity Value}_{t-1}}$	0,02	1,00				
	0,02					
$\frac{\text{Cost of sales}}{\text{Equity Value}_{t-1}}$	0,02	0,91	1,00			
	0,03	0,00				
$\frac{\text{Administration costs}}{\text{Equity Value}_{t-1}}$	0,02	0,79	0,54	1,00		
	0,07	0,00	0,00			
$\frac{\text{Selling expenses}}{\text{Equity Value}_{t-1}}$	0,01	0,43	0,26	0,18	1,00	
	0,48	0,00	0,00	0,00		
Leverage Factor	0,92	0,01	0,01	0,01	0,00	1,00
	0,00	0,20	0,23	0,24	0,83	

A strong correlation is observed between the leverage factor and the accounting beta measured by ROE, confirming the relationship between a company's financial leverage and its risk in this estimation. For the rest of the variables, the relationship is weak.

Table 8: Random Effects Regression Model Leveraged Accounting Beta using adjusted ROE.

Random-effects GLS regression		Number of obs =	11.755
Group variable: Empresa		Number of groups =	2.351
	within =	0,87	min = 5
R-sq:	Between=	0,76	Obs per group: avg = 5
	overall =	0,85	max = 5
		Wald chi2 (5)	71.259,51
corr(u _i , X) = 0 (assumed)		Prob > chi2	0,00
Beta ROE	Coef.	Std. Err.	z
			P> z
			[95% Conf. Interval]

$\frac{\text{Sales}}{\text{Equity Value}_{t-1}}$	4,01	3,09	1,30	0,20	-2,05	10,07
$\frac{\text{Cost of sales}}{\text{Equity Value}_{t-1}}$	-3,52	3,22	-1,09	0,28	-9,84	2,80
$\frac{\text{Administration costs}}{\text{Equity Value}_{t-1}}$	-3,42	3,57	-0,96	0,34	-10,40	3,57
$\frac{\text{Selling expenses}}{\text{Equity Value}_{t-1}}$	-2,64	3,23	-0,82	0,41	-8,97	3,69
Leverage Factor	10,24	0,04	266,88	0,00	10,16	10,31
_cons	-33,03	8,81	-3,75	0,00	-50,29	-15,76
sigma_u	286,70	(fraction of variance due to u_i)				
sigma_e	687,15					
rho	0,15					

In the regression using the random effects model for the leveraged accounting beta estimated by ROE, it is evident that the only statistically significant variable is the leverage factor. Considering this, it can be observed that in this method, the measurement of leverage risk plays an important role.

Table 9: Fixed Effects Regression Model of Leveraged Accounting Beta using Adjusted ROE.

Fixed-effects (within) regression		Number of obs =	11.755			
Group variable: Empresa		Number of groups =	2.351			
within =	0,87	min =	5			
R-sq: between =	0,76	Obs per group: avg =	5			
overall =	0,85	max =	5			
		F (5,9399)	13.076,56			
corr(u_i, Xb) = -0.1517		Prob > chi2	0,00			
Beta ROE	Coef.	Std. Err.	t	P> t 	[95% Conf. Interval]	
$\frac{\text{Sales}}{\text{Equity Value}_{t-1}}$	-3,59	3,44	-1,04	0,30	-10,35	3,16
$\frac{\text{Cost of sales}}{\text{Equity Value}_{t-1}}$	3,95	3,57	1,10	0,27	-3,06	10,95
$\frac{\text{Administration costs}}{\text{Equity Value}_{t-1}}$	6,11	3,96	1,54	0,12	-1,65	13,86
$\frac{\text{Selling expenses}}{\text{Equity Value}_{t-1}}$	5,33	3,55	1,50	0,13	-1,63	12,30
Leverage Factor	10,47	0,04	255,66	0,00	10,39	10,55
_cons	-33,23	6,41	-5,18	0,00	-45,80	-20,65
sigma_u	446,50	(fraction of variance due to u_i) Prob > F = 0.0000				
sigma_e	687,15					
rho	0,30					
F test that all u_i=0: F(2350, 9399) = 2,05						

Similarly to the random effects model, in the fixed effects model, it is evident that the only statistically significant variable is the leverage factor. This confirms, from a financial perspective, the importance of this variable in estimating the risk of companies.

Table 10: Hausman test based on Accounting Beta leveraged by adjusted ROE.

Variable	Coefficients			
	(b)	(B)	(b-B)	sqrt(diag(V_b-V_B))
	fe1	re1	Difference	S.E.
$\frac{\text{Sales}}{\text{Equity Value}_{t-1}}$	-3,59	4,01	-7,60	1,52
$\frac{\text{Cost of sales}}{\text{Equity Value}_{t-1}}$	3,95	-3,52	7,47	1,54
$\frac{\text{Administration costs}}{\text{Equity Value}_{t-1}}$	6,11	-3,42	9,53	1,72
$\frac{\text{Selling expenses}}{\text{Equity Value}_{t-1}}$	5,33	-2,64	7,97	1,48
Leverage Factor	10,47	10,24	0,23	0,01

b = consistent under Ho and Ha; obtained from xtreg

B = inconsistent under Ha, efficient under Ho; obtained from xtreg	
Test: Ho: difference in coefficients not systematic	
chi2(5)	= (b-B)'[(V_b-V_B)^(-1)](b-B)
	0= 308,24
Prob>chi2	= 0,0000

Based on the results of the Hausman test, it can be concluded that for this methodology of regression on the leveraged accounting beta, the fixed effects model is the one that best represents the data behavior.

4. CONCLUSIONS

The use of accounting beta allows for an accurate measurement of risk in non-publicly traded organizations (a high percentage in Colombia), focusing on the intrinsic risk of the company. This coefficient is useful as it calculates the risk using available accounting information, enabling the identification of the specific relationship between company risk and systematic risk.

The application of the proposed procedure enables the calculation of performance in closed-capital service sector companies. Its practical methodology helps overcome theoretical and technical limitations in estimating the beta coefficient in emerging markets such as Colombia, which is characterized by low market liquidity, few issuers, and low transactional levels.

The statistical results indicate that leverage plays a determining role in a company's risk. Correlations revealed moderate and strong values, suggesting that the level of leverage will be one of the characteristics of higher-risk companies.

Considering that the fixed-effects model best represents the data behavior, it can be concluded that the average risk level in the service sector is more associated with the heterogeneity of the companies rather than variations in indicators over time.

The accounting beta, measured through return on equity (ROE), is higher than the one estimated through return on assets (ROA), reflecting the shareholders' assumption of higher risk compared to the overall company risk.

Therefore, it can be affirmed that this measure captures the probability that company owners face adverse events. The objective of the present research is to provide tools for financial analysts, chief financial officers, and decision-makers in companies through the application of fixed and random effects models that allow identifying which variables affect accounting beta as a measure of systematic risk. This study demonstrates that company leverage plays a determining role in the level of risk, offering a methodology for measuring systematic risk in non-publicly traded companies. Accounting beta, especially when measured through ROE, emerges as an effective indicator reflecting the risk assumed by shareholders, facilitating informed decision-making in emerging market environments such as Colombia. For future lines of research, it would be interesting to explore the application of these models in different economic sectors and geographic regions, as well as to integrate macroeconomic variables to enrich the analysis of systematic risk in non-publicly traded companies.

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REFERENCES

- [1] AGYEMANG-MINTAH, P. (2015). The nomination committee and firm performance: An empirical investigation of UK financial institutions during the pre/post financial crisis. **Corporate Board: Role, Duties & Composition**, 11, 176-190.
- [2] ARIAS, G. A. C., GUERRERO, F. A. C., & PRIET, A. B. T. (2021). Indicadores financieros y rentabilidad en bancos grandes y medianos ecuatorianos, periodo: 2016-2019. **INNOVA Research Journal**, 6, 225-239.
- [3] ASOCIACIÓN BANCARIA y DE ENTIDADES FINANCIERAS de COLOMBIA. (2017). El sector servicios: ¿vamos por buen camino? **Semana Económica (1115)**, 1-11.
- [4] BEN N.S. & GOAIED, M. (2004). The value relevance of accounting and financial information: panel data evidence. **Applied Financial Economics**, 14, 1219-1224.
- [5] CAICEDO, E., CLARAMUNT, M. & CASANOVAS, M. (2011). Medición del riesgo de crédito mediante modelos estructurales: Una aplicación al mercado colombiano. **Cuadernos de Administración**, 24, 73-100.

- [6] COELHO JUNIOR, L. M., FONSECA, A. J. D. S., CASTRO, R., MELLO, J. C. D. O., SANTOS, V. H. R. D., PINHEIRO, R. B., ... & RAMOS, D. S. (2022). Empirical Evidence of the Cost of Capital under Risk Conditions for Thermoelectric Power Plants in Brazil. *Energies*, 15, 1-12. <https://doi.org/10.3390/en15124313>
- [7] DAMODARAN, A. (2012). **Investment valuation: Tools and techniques for determining the value of any asset**. Hoboken, NJ: John Wiley & Sons, N. York.
- [8] DE la OLIVA, F. (2016). **Gestión del riesgo financiero internacional**. Félix Varela.
- [9] DEVČIĆ, K., TONKOVIĆ PRAŽIĆ, I., BELJO, I. (2022). Panel modeli – primjena u ekonomskim istraživanjima. *Zbornik radova Veleučilišta u Šibeniku*, 16, 211-220.
- [10] ELIZONDO, A., & ALTMAN, E. I. (2004). **Medición integral del riesgo de crédito**. Editorial Limusa, Madrid.
- [11] ELLEUCH HAMZA, S., & KORTAS, N. (2019). The interaction between accounting and real earnings management using simultaneous equation model with panel data. *Review of Quantitative Finance and Accounting*, 53, 1195-1227.
- [12] ERIN, O., BAMIGBOYE, O., & ARUMONA, J. (2020). Risk governance and financial performance: an empirical analysis. *Verslas: Teorija ir praktika/Business: Theory and Practice*, 21, 758-768.
- [13] FAITEH, A., & AASRI, M. R. (2023). Economic value added: The best indicator for measuring value creation or just an illusion. *Investment Management and Financial Innovations*, 20, 138-150.
- [14] FAITEH, A., & AASRI, M. R. (2022). Accounting Beta as an Indicator of Risk Measurement: The Case of the Casablanca Stock Exchange. *Risks*, 10, 149.
- [15] FÁVERO, L. P. L. (2013). Panel data in accounting and finance: theory and application. *Brazilian Business Review*, 10, 127-149.
- [16] GITMAN, L., & CHAD, Z. (2012). **Principles of Managerial Finance (13th Ed)**. Pearson Education, N. York.
- [17] Granados, R. M. (2011). *Efectos fijos o aleatorios: test de especificación*. Documentos de Trabajo en Economía Aplicada, Universidad de Granada. España.
- [18] Hill, N. C., & Bernell K. S. (1980). Accounting Betas, Systematic Operating Risk, and Financial Leverage: A Risk-Composition Approach to the Determinants of Systematic Risk. *The Journal of Financial and Quantitative Analysis*, 15, 595–637.
- [19] Hiestand, T. (2005). Using pooled model, random model and fixed model multiple regression to measure foreign direct investment in Taiwan. *International Business & Economics Research Journal (IBER)*, 4(12), 37-52.
- [20] Horcher, K. A. (2011). *Essentials of financial risk management*. John Wiley & Sons.
- [21] Hsiao, C. (2007). Panel data analysis—advantages and challenges. *Test*, 16(1), 1-22.
- [22] Jaramillo L. y Pazmiño, J. E. (2021). Administración de la cobranza y su impacto en la cartera de crédito: un enfoque a las instituciones bancarias privadas ecuatorianas. *Revista Contribuciones a la Economía*, 19(2), 66-75. <https://doi.org/10.51896/contrieconomia/RUDH3842>
- [23] Jensen, M. C. (1968). The performance of mutual funds in the period 1945–1964. *Journal of Finance*, 23(2), 389–416.
- [24] Jensen, M. C. (1972). Optimal utilization of market forecasts and the evaluation of investment performance. *Mathematical methods of investment and finance*, 310–335.
- [25] Jiménez, V. J. (2009). Aplicación de una metodología de medición del riesgo financiero de mercado en empresas del sector real. *Heurística*, 16, 13-32.
- [26] Kalia, D., & Aggarwal, D. (2023). Examining impact of ESG score on financial performance of healthcare companies. *Journal of Global Responsibility*, 14(1), 155-176.
- [27] Kasztelnik, K. (2021). Innovative bank market risk measurement strategies using a modern machine learning approach: A novel agglomerative clustering model analysis. *Journal of Business and Accounting*, 14(1), 1-13.
- [28] Marqués, L.D. (2000). Modelos dinámicos con datos de panel: revisión de la literatura. *Centro de Estudios y Pronósticos Macroeconómicos, Facultad de Economía de Oporto*, 30, 37, 1-80.
- [29] Martínez, I., Batistela, G. C., & Simões, D. (2021). Management of economic and financial risk of investments in assets for extraction of non-metallic mineral extracts. *INGENIARE-Revista Chilena de Ingeniería*, 29(4), 691-699.
- [30] Martínez, R. M., Tuya, L. C., Martínez, M., Pérez, A., & Cánovas, A. M. (2009). El coeficiente de correlación de los rangos de Spearman caracterización. *Revista Habanera de Ciencias Médicas*, 8(2), 1-19.
- [31] Menéndez-Plans, C., Orgaz, N., & Prior, D. (2012). Does stock market beta correlate with disclosed accounting information? *Academia Revista Latinoamericana de Administración*. 49, 1-16.
- [32] Merlo, M.; Scarfó, E.; Vélez, I.; Sandoval, J.; Castilla, P. & Ortiz, D. (2022). *Análisis Financiero Integral: Teoría y práctica*. Alphaeditorial.
- [33] Montenegro, E., Tinajero, F. y Pacheco, I. (2014). Estimación del riesgo de acciones a través de un modelo financiero y de modelos de heteroscedasticidad condicional autorregresiva. *UTCiencia*, 1(2), 61-71.

- [34] Ortiz Anaya, H. (2018). *Análisis financieros aplicado, bajo NIIF (16a ed.)*. Universidad Externado de Colombia.
- [35] Pennacchi, G.G. & Santos, J. A.C. (2021). Why do banks target ROE? *Journal of Financial Stability*, 54, 1-19. <https://doi.org/10.1016/j.jfs.2021.100856>
- [36] Reyes-Clavijo, M. A., Pinos-Luzuriaga, L. G., Orellana-Osorio, I. F. y Tonon-Ordóñez, L. B. (2023). Modelo de Valoración de Activos Financieros (CAPM) aplicado al sector empresarial de Ecuador. *Retos Revista de Ciencias de la Administración y Economía*, 13(25), 123-136. <https://doi.org/10.17163/ret.n25.2023.08>
- [37] Roque, D. I., Álvarez, A. N. M., Rodríguez, J. H. E., & de Con, F. D. L. O. (2021). The use of accounting beta as a risk assessment method for unlisted companies in Colombia. *Universidad y Sociedad*, 13(2), 23-30.
- [38] Roque, D. I., & Caicedo Carrero, A. C. (2021). Uso del valor económico agregado en empresas no cotizantes en el mercado de valores de Colombia. *Universidad y Sociedad*, 13(S3), 592-602.
- [39] Roque, D. I., & Caicedo Carrero, A. (2021). Relación entre la creación de valor económico y la insolvencia financiera en empresas no cotizantes en el mercado de valores de Colombia (2016-2019). *Semestre Económico*, 24(57), 76-97.
- [40] Roque, D. I., Carrero, A. C., & De Con, F. D. L. O. (2023). Medición de los factores que determinan la creación de valor en los sectores económicos colombianos: periodo 2016-2020. *Revista Finanzas y Política Económica*, 15(1), 213-244
- [41] Rutkowska-Ziarko, A. (2022). Market and Accounting Measures of Risk: The Case of the Frankfurt Stock Exchange. *Risks*, 10(14).
- [42] Sarmiento-Sabogal, J., & Sadeghi, M. (2015). Estimating the cost of equity for private firms using accounting fundamentals. *Applied Economics*, 47(3), 288-301. <https://doi.org/10.1080/00036846.2014.969826>
- [43] Sarmiento, J., Sadeghi, M., Sandoval, J. S., & Cayon, E. (2021). The application of proxy methods for estimating the cost of equity for unlisted companies: evidence from listed firms. *Review of Quantitative Finance and Accounting*, 57(3), 1009-1031.
- [44] Shahrier, N. A., Ho, J. S. Y., & Gaur, S. S. (2020). Ownership concentration, board characteristics and firm performance among Shariah-compliant companies. *Journal of Management and Governance*, 24, 365-388.
- [45] Sharpe, W. F. (1966). Mutual Fund Performance. *Journal of Business*, 39, 119-138.
- [46] Siahkali, H., & Mahabadi, D. (2018). Application of Valuation Model in Iranian Electricity Distribution Companies. *Signal Processing and Renewable Energy*, 2(1), 41-47.
- [47] Spanagel, F. F. (2021). Identification and classification of risks of entrepreneurs in the Digital Economy. *European Proceedings of Social and Behavioural Sciences*, 106, 1-10. <https://doi.org/10.15405/epsbs.2021.04.02.74>
- [48] Teixeira, I. C., Maia, V. M., & Teixeira, R. F. A. P. (2020). A risk study of Brazilian credit unions based on accounting beta. *Revista Catarinense da Ciência Contábil*, 19, 1-20.
- [49] Treynor, J. L. (1966) How to rate management investment funds. *Harvard Business Review*, 43, 63-75.
- [50] Vuolteenaho, T. (2002). What drives firm-level stock returns? *The Journal of Finance*, 57(1), 233-264.
- [51] Woods, M., & Dowd, K. (2008). *Financial risk management for management accountants*. Management Accounting Guideline, London: CIMA.
- [52] Xu, H., Hwan Lee, S., & Ho Eom, T. (2007). *Introduction to Panel Data Analysis*. In Miller/Handbook of Research Methods in Public Administration, 571-590.
- [53] Zulfikar, R. (2019). *Estimation Model and Selection Method of Panel Data Regression: An overview of Common Effect, Fixed Effect, and Random Effect Model*. Universitas Islam Kalimantan MAB Banjarmasin, 1-10.