APPROXIMATION OF TEMPERATURE DISTRIBUTION IN THE TISSUE OF THE HUMAN BODY BY FINITE DIFFERENCE METHOD

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ABSTRACT
Tissue is a group of cells with a similar structure that performs a function and biochemical processes are temperature dependent. Hence, studying heat transfer plays a very important role in living systems. The temperature distribution in the tissue of the human body is studied by developing a one-dimensional steady-state mathematical model in Cartesian coordinates based on Penne's bio-heat transfer equation. Finite Difference Method solutions are found for one Dirichlet-type condition, one Neumann-type condition, and two mixed-type conditions at the boundary of the tissue in the human body. The values of Physical and Physiological parameters are taken from the literature. Results are drawn as graphs for Finite Difference Method solutions under four different boundary conditions using MATLAB 2015a. Also, results are compared with both the experimental data and analytic method solutions obtained by researchers.

KEYWORDS: Penne's Bioheat equation, physical and physiological parameters, and Boundary conditions

MSC: 92B05, 35G15, 34B05, 65N06, 65M06

RESUMEN
El tejido es un grupo de células con una estructura similar que realiza una función y los procesos bioquímicos son dependientes de la temperatura. Por lo tanto, el estudio de la transferencia de calor juega un papel muy importante en los sistemas vivos. La distribución de temperatura en el tejido del cuerpo humano se estudia mediante el desarrollo de un modelo matemático unidimensional de estado estacionario en coordenadas cartesianas basado en la ecuación de transferencia de biocalor de Penne. Las soluciones del método de diferencias finitas se encuentran para una condición de tipo Dirichlet, una condición de tipo Neumann y dos condiciones de tipo mixto en el límite del tejido en el cuerpo humano. Los valores de los parámetros físicos y fisiológicos se toman de la literatura. Los resultados se dibujan como gráficos para las soluciones del método de diferencias finitas bajo cuatro condiciones de contorno diferentes usando MATLAB 2015a. Además, los resultados se comparan con los datos experimentales y las soluciones de métodos analíticos obtenidos por los investigadores.

PALABRAS CLAVE: Ecuación de biocalor de Penne, parámetros físicos y fisiológicos, y condiciones de contorno

1. INTRODUCTION

The human body is organized into six levels. They are chemicals, cells, tissues, organs, organ systems, and organisms. The human body is often described as a large vessel containing chemicals that are constantly reacting with one another. Cells are the smallest functional units in the body. The human body consists of trillions of cells. Tissue is a group of cells with a similar structure that performs a function. Since biochemical processes are temperature dependent, the tissue is not homogeneous, isotropic, having uniform blood flow and metabolic heat generation, thus the study of heat transfer play's important role in living systems and the accurate thermal investigation of tissue is demanding. Bioheat Transfer is the study of the transport of thermal energy in living systems.

2. OVERVIEW OF LITERATURE

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In 1948, Harry H. Pennes, an American Physician and Clinical Researcher introduced the Mathematical Model for the rate of heat production by tissues with the effects of blood perfusion and metabolism. Also, Pennes (1948) analyzed tissue and arterial blood temperature in the resting human forearm with his mathematical model. Saxena V. P. (1983) investigated Temperature distribution in human skin and subdermal tissues. Arya and Saxena (1986) examined Temperature variation in skin and subcutaneous layers under different environmental conditions in a two-dimensions. Saxena et all (1991) analysed the effect of the dermal tumor on temperature distribution in the skin with variable blood flow. Pardasani and Adlakha (1995, 1986) studied two-dimensional heat distribution problem in dermal regions of human limbs using Coaxial circular sector elements and exact solution to a heat flow problem in peripheral tissue layers with a solid tumor in the dermis. Wissler (1998) validated Penne's Mathematical Model by comparing it with the results of earlier models published. Yue Kai Zahang and Fan You Xinxin (2004) found an Analytic Solution of the one-dimensional steady-state Penne's Bioheat Transfer Equation in cylindrical coordinates and analyzed the effects of physical and physiological parameters. Temperature Distribution in Biological Tissue by Analytical Solutions of Pennes' equation was studied by Zhou Minhua and Chen Qian (2009).
Agrawal et all (2010, 2011, 2014, 2015, 2016) investigated heat flow in dermal regions of elliptical and tapered shape human limbs, the thermal effect of uniformly perfused tumors in dermal layers of an elliptical-shaped human limb, steady state temperature distribution in dermal regions of an irregular tapered shaped human limb with variable eccentricity, the thermal effect of tumors in dermal regions of irregular tapered shaped human limbs and temperature distribution in the skin and deep tissues of human limbs by three-dimensional finite element model. Gurung et all (2012) developed a two-dimensional temperature distribution model in the human dermal region exposed at low ambient temperatures with airflow. The explicit formula of the finite difference method to estimate human peripheral tissue temperatures during exposure to severe cold stress was modeled by Khanday and Hussain (2015). An analytic approach is used to investigate the effect of various parameters on temperature distribution in the human body by Luitel, Kabita et all (2018). Pennes Mathematical Model is also called Pennes Bioheat Equation and it became the foundation for hundreds of papers and influential articles in the field of bioheat transfer. Bioheat transfer gained a lot of interest from scientists since the beginning of medical research in the last century because Bioheat transfer models are applicable for diagnostic and therapeutic applications involving either heat transfer or mass transfer.
The literature review concluded that no researcher attempted to find a finite difference method solution of Penne's Mathematical Model in the Cartesian coordinate system in one direction only.

3. OBJECTIVES

- The temperature distribution in the tissue of the human body is studied by developing a one-dimensional steady-state mathematical model in Cartesian coordinates based on Penne's bio-heat transfer equation. The values of Physical and Physiological parameters are taken from the literature.
- Finite Difference Method solutions are found for one Dirichlet-type condition, one Neumann-type condition, and two mixed-type conditions at the boundary of the tissue in the human body.
- Results are drawn as graphs for Finite Difference Method solutions under four different boundary conditions using MATLAB 2015a.
- The results are compared with both Penne's experimental data and Analytic Method solutions obtained by Zhou Minhua and Chen Qian (2009).

4. MATHEMATICAL MODEL

Penne's Bioheat Transfer equation is $\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + w_b c_b (T_a - T) + Q_m$ (1)

Where ρ, c, k, w_b, c_b, T_a, Q_m and T are density, specific heat, thermal conductivity, blood perfusion rate, the specific heat of blood, temperature of arterial blood, metabolic heat generation and temperature in the tissue respectively.
The mathematical model is developed based on Penne's Bioheat Transfer equation with the following assumptions. Considered that the tissue is of thickness H, at a steady state, as a homogenous medium with constant properties,

having a heat transfer due to metabolism, conduction, and blood perfusion and having heat transfer mainly in the direction normal to the skin surface.
Hence, a one-dimensional steady-state mathematical model in cartesian coordinates for the temperature distribution in the tissue of the human body based on Penne's bioheat transfer equation (1) is $\frac{d^2T}{dx^2} - \alpha T + \beta = 0$ (2)

Where $\alpha = {(w_b c_b)}/{k}$ and $\beta = {(w_b c_b T_a + Q_m)}/{k}$.

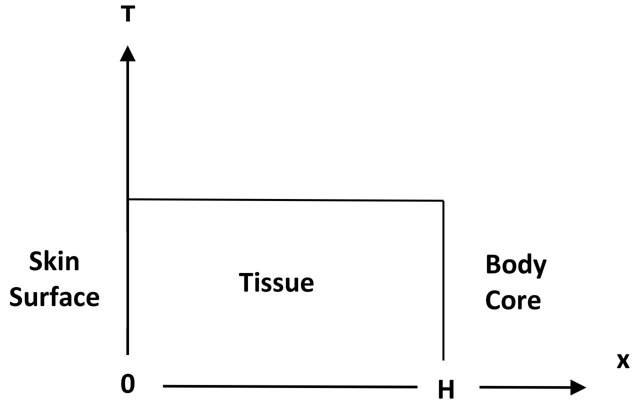


Figure 1: Tissue is fixed in Cartesian coordinate system

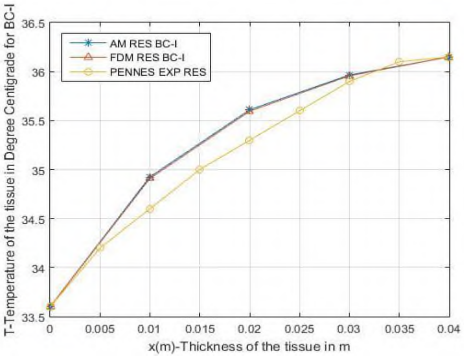
All possible suitable boundary conditions are framed as follows.

Dirichlet-type boundary condition: - The skin surface and the body core temperatures are assumed as T_s and T_c respectively. The boundary condition can be written as $T = \begin{cases} T_s, & x = 0 \\ T_c, & x = H \end{cases}$

First Mixed-type boundary condition: - The skin surface and body core temperatures are assumed as T_s and thermally insulated respectively. The boundary condition can be written as $T = \begin{cases} T_s, & x = 0 \\ -k\frac{dT}{dx} = 0, & x = H \end{cases}$

Second Mixed type boundary condition: - The body core temperature is assumed as T_c . Heat transfer in the tissue at the skin surface is assumed as due to convection and radiation. The effect of evaporation is ignored due to normal temperature. The boundary condition can be written as $T = \begin{cases} k\frac{dT}{dx} = h_0(T_s - T_e), & x = 0 \\ T_c, & x = H \end{cases}$

Neumann-type boundary condition: - Heat transfer in the tissue at the skin surface is assumed as due to convection and radiation. The body core temperature is assumed as thermally insulated. The boundary condition can be written as $T = \begin{cases} k\frac{dT}{dx} = h_0(T_s - T_e), & x = 0 \\ -k\frac{dT}{dx} = 0, & x = H \end{cases}$



5. SOLUTION METHODOLOGY

A central difference scheme is applied to the equation (2) at $x=x_i$, i=0 to 4 with and $T(x_4)=T_c$. The backward difference is applied at the body core in the First Mixed type boundary condition and the Neumann type boundary condition. The forward difference is applied to the body skin in the Second Mixed type boundary condition and Neumann type boundary condition. Later, equation (2) and the conditions expressed in finite differences will be reduced into a nonhomogeneous system AX=B for four different boundary conditions separately. The nonhomogeneous system AX=B is solved with suitable physical and physiological parameters by the Gauss elimination method using MATLAB 2015a for four different boundary conditions separately.

6. RESULTS

The values of Physical and Physiological Parameters are assumed as in Table 1 to find solutions for all boundary conditions and draw graphs using MATLAB 2015a.

Parameter	Value	Parameter	Value
Thermal conductivity (k)	0.5 W/m^0C	Metabolic Heat (Q_m)	418.6 W/m^3
Blood Perfusion Rate (w_b)	0.52 $Kg/s.m^3$	Blood Specific Heat (c_b)	4186 $Kg/s.m^3$
Tissue Thickness (H)	0.04 m	Arterial Temperature (T_a)	36.15^{0}C
Heat Transfer Coefficient (h_0)	10 $W/m^{2\,0}C$	Environmental Temperature (T_e)	26.6^{0}C
Core Temperature (T_c)	36.15^{0}C	Skin Temperature (T_s)	33.6^{0}C

Table 1: The values of Physical and Physiological Parameters

From Fig. 2 results of three methods are coincided at the boundaries and results of methods are very close to the experiment in the interior of tissue. Results of three methods are coincided nearly at the skin surface, in the interior of the tissue and slightly differed at the body core in the Fig. 3. In Fig. 4, results of finite difference method are agreed with the penne's experimental results at the boundaries and not the same in the interior of the tissue.

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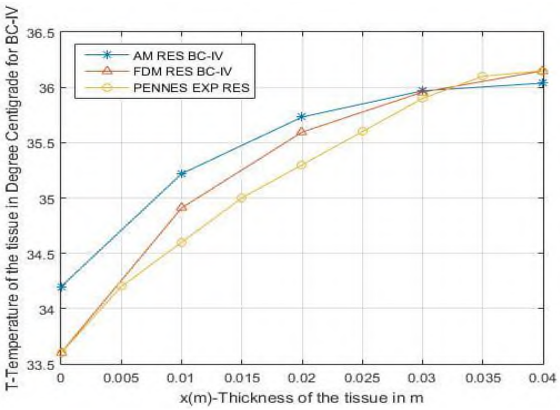

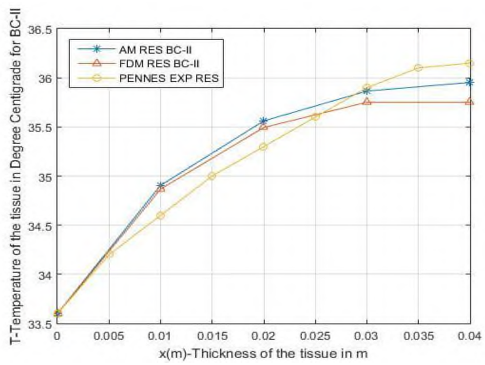

Figure 3: Tissue temperatures for first mixed condition Cartesian coordinate system

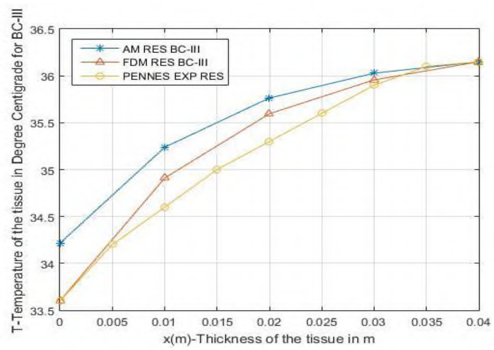

Figure 4: Tissue temperatures for first mixed condition Cartesian coordinate system

Fig. 5 depicts that results of finite difference method are well agreed with the penne's experimental results at the skin surface as well as the body core and not the same in the interior of the tissue

7. CONCLUSIONS AND DISCUSSION

The one-dimensional steady-state Bioheat transfer equation for the temperature distribution of the tissue is developed based on Penne's Bioheat transfer model and solved for the numerical solution for all the different boundary conditions by the finite difference method. Graphs of Temperature distribution in the tissue with suitable physical and physiological parameters taken from the literature are drawn using MATLAB 2015a for all the different boundary conditions. Results are validated by comparing with Pennes experimental data and analytical method solutions from the literature. The results delivered a worthy understanding of the thermal manners of biological tissue, which is valuable for the measurement of thermal parameters to know the thermal diagnostic disease and the reconstruction of the temperature field to treat the disease.

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REFERENCES

[1]. AGRAWAL, M. and PARDASANI, K. R. (2016): Finite element model to study temperature distribution in skin and deep tissues of human limbs. **Journal of thermal biology**, 62, 98-105.

[2]. AGRAWAL, M., ADLAKHA, N. and PARDASANI, K. R. (2010): Three dimensional finite element model to study heat flow in dermal regions of elliptical and tapered shape human limbs. **Applied Mathematics and Computation**, 217(8), 4129-4140.
[3]. AGRAWAL, M., ADLAKHA, N. and PARDASANI, K. R. (2011): Finite element model to study thermal effect of uniformly perfused tumor in dermal layers of elliptical shaped human limb. **International Journal of Biomathematics**, 4(02), 241-254.
[4]. AGRAWAL, M., PARDASANI, K. R. and ADLAKHA, N. (2014): Steady state temperature distribution in dermal regions of an irregular tapered shaped human limb with variable eccentricity. **Journal of Thermal Biology**, 44, 27-34.
[5]. AGRAWAL, M., PARDASANI, K. R. and ADLAKHA, N. (2015): Finite element model to study the thermal effect of tumors in dermal regions of irregular tapered shaped human limbs. **International journal of thermal sciences**, 98, 287-295.
[6]. ARYA, D. and SAXENA, V. (1986): Temperature variation in skin and subcutaneous layers under different environmental condition – a two dimensional study. **Indian J. pure appl. Math**, 17(1), 84-99.
[7]. BRIAN R. HUNT, RONALD L. LIPSMAN and JONATHAN M. ROSENBERG, (2001): **A Guide to MATLAB for Beginners and Experienced Users**, Cambridge University Press, First edition, UK.
[8]. CENGEL, Y. A. and GHAJAR A. J (2015): **Heat and Mass Transfer; Fundamentals and Applications**, McGraw-Hill 5th Edition, New York, USA.
[9]. GURUNG, D. B. (2012): Two dimensional temperature distribution model in human dermal region exposed at low ambient temperatures with air flow. **Kathmandu University Journal of Science, Engineering and Technology**, 8(2), 11-24.
[10]. GUYTON A. C and HALL J. E (1996): **Text Book of Medical Physiology**, (9th ed.). Philadelphia, PA W.B. Saunders Company.
[11]. KAI VELTEN (2009): **Mathematical Modeling and Simulation Introduction for Scientists and Engineers**, Wiley-VCH Publishers, Weinheim.
[12]. KHANDAY, M. A. and HUSSAIN, F. (2015): Explicit formula of finite difference method to estimate human peripheral tissue temperatures during exposure to severe cold stress. **Journal of thermal biology**, 48, 51-55.
[13]. LUITEL, K., GURUNG, D. B. and UPRETY, K. N. (2018): Effect of various parameters for temperature distribution in human body: An analytic approach. **Advances in Science, Technology and Engineering Systems Journal, Special issue on Recent Advances in Engineering Systems Journal**, 3(5), 421-426.
[14]. MISHRA, V. P. (2002): **A Text Book of Engineering Mathematics-II**, Galgotia Publications, New Delhi, India.
[15]. PARDASANI, K. R. and ADLAKHA, N. (1995): Coaxial circular sector elements to study two-dimensional heat distribution problem in dermal regions of human limbs. **Mathematical and computer modeling**, 22(9), 127-140.
[16]. PARDASANI, K. R. and ADLAKHA, NEERU. (1991): Exact solution to a heat flow problem in peripheral tissue layers with a solid tumor in the dermis. **Ind. J. Pure. Appl. Math**, 22(8), 679-682.
[17]. PENNES H. H. (1948): Analysis of tissue and arterial blood temperatures in the resting human forearm. **Journal of applied physiology**, 1(2), 93-122.
[18]. SAXENA, V. P. (1983): Temperature distribution in human skin and subdermal tissues. **Journal of theoretical biology**, 102(2), 277-286.
[19]. SAXENA, V. P. and PARDASANI, K. R. (1991): Effect of dermal tumours on temperature distribution in skin with variable blood flow. **Bulletin of mathematical biology**, 53(4), 525-536.
[20]. WISSLER, E. H. (1998): Pennes' 1948 paper revisited. **Journal of applied physiology**, 85(1), 35-41.
[21]. YUE KAI ZAHANG and FAN YOU XINXIN, (2004): An Analytic Solution of one-dimensional steady-state Penne's Bioheat Transfer Equation in cylindrical coordinates, **Journal of Thermal Science**, 13(3), 52-54.
[22]. ZHOU, M. and CHEN, Q. (2009, October): Estimation of Temperature Distribution in Biological Tissue by Analytic Solutions of Pennes' Equation. **2nd International Conference on Biomedical Engineering and Informatics. IEEE**, 1-4