STACKELBERG GAME APPROACH FOR PRESERVATION OF MULTI-ITEMS INVENTORY SYSTEM FOR TRENDED-DEMAND WITH MAXIMUM LIFETIME AND ALLOWABLE CREDIT PERIOD

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ABSTRACT
In this paper, an inventory system having trended demand comprising of a single manufacturer and single retailer for multi-items is considered. The manufacturer offers a credit period to the retailer to boost the demand in the market. The units in inventory are deteriorating and have a maximum lifetime. To increase the life of units in the model manufacturer invests in preservation technology. Our objective is to minimize the total cost of the players involved in a supply chain using the Stackelberg game approach. Numerical examples show how a joint decision is beneficiary to reducing the total cost of the supply chain. Managerial insights are provided using sensitivity analysis.

KEYWORDS: Multi-Item Inventory Model, Deterioration, Maximum Life-time, Preservation Technology Investment, Trended Demand

MSC: 90B05

RESUMEN
En este documento, se considera un sistema de inventario que tiene una demanda en tendencia que consta de un solo fabricante y un solo minorista para artículos múltiples. El fabricante ofrece un período de crédito al minorista para impulsar la demanda en el mercado. Las unidades en inventario se están deteriorando y tienen una vida útil máxima. Para aumentar la vida útil de las unidades en el modelo, el fabricante invierte en tecnología de conservación. Nuestro objetivo es minimizar el costo total de los jugadores involucrados en la cadena de suministro utilizando el enfoque del juego Stackelberg. Los ejemplos numéricos muestran cómo la decisión conjunta se beneficia para reducir el costo total de la cadena de suministro. Los conocimientos gerenciales se proporcionan mediante análisis de sensibilidad.

PALABRAS CLAVE: Modelo de inventario de artículos múltiples, deterioro, vida útil máxima, inversión en tecnología de conservación, demanda tendencial

1. INTRODUCTION

In today's world, every firm wants to maximize its profit and for that one of the key factors is the minimization of cost. Cost may be minimized by using different low-cost materials, minimizing operating expenses, having mass production, sometimes importing goods and branding or finishing it and then selling it. Here it is attempted by taking a joint decision of supply chain players into the consideration and using the Stackelberg game approach. Chen and Zadrozny (2002) obtained an inventory model which incurs continuous feedback solution for an infinite-horizon, linear-quadratic, dynamic, Stackelberg game. Mukaidani (2007) considered the computation of the linear closed-loop Stackelberg strategies of the SPS. Chang *et al*. (2009) determined the optimal strategy for an integrated vendor–buyer inventory system under the condition of trade credit linked to the order quantity, where the demand rate is considered to be a decreasing function of the retail price. Yang (2010) developed an integrated inventory model with crashing cost which was determined by the length of lead time is polynomial. Hoque (2013) developed a vendor–buyer integrated production–inventory model following normal distribution of lead time. Wang

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et al. (2016) formulated joint optimization of PFA planning and supply chain configuration as a Stackelberg game. Basu (1995) developed a model in a duopoly framework and introduced Stackelberg solution in his model.
Jha and Shanker (2014) established an integrated inventory model with transportation for a single-vendor and multi-buyer using coordinated two-phase iterative approach. Ishii et al. (1988) incorporated a 'pull type' ordering system consisting of manufacturer, wholesaler, and retailer. Haq *et al.* (1991) presented the application of an integrated production inventory distribution model involving set-up time and cost, lead times. Goyal and Nebebe (2000) determined an economic production shipment policy for a system of a single supplier and single buyer and achieved a lower-cost inventory policy. Woo *et al*. (2001) gave an inventory model in which vendors and all buyers are willing to invest in reducing ordering costs by establishing an electronic data interchanges based inventory control system for a single vendor and multiple buyers. Rau *et al.* (2003) developed a multi-echelon inventory model having deteriorating items with optimal joint total cost. Zhou and Li (2007) established a coordinated quantity decision problem in a supply chain contract having random demand and showed that the profit of retailer and supply chain is increased. Shah *et al.* (2009) demonstrated that an integrated attitude decreases the total joint cost for quadratic demand when units in inventory are subject to deterioration. Shah and Shukla (2010) analyzed ordering and pricing policies for two levels of trade credit for a declining market. Liu and Cruz (2012) focused to offer an analytical framework for the investigation of how financial risk shakes the supply chain. Soni (2013) extended the model of Min *et al*. (2010) in which he imposed a terminal condition of zero ending inventory which is extended to nonzero ending inventory. Jiangtao *et al.* (2014) considered a multi-items inventory model for deteriorating items for stock-dependent demand and maximized the total profit. Cárdenas-Barrón and Sana (2014) explored the issues of coordination for a two-echelon supply chain having one retailer and one manufacturer. Cárdenas-Barrón and Sana (2015) proposed an inventory model for a two-stage supply chain with multiple items and demand depends partly on the promotional activity. Shah and Chaudhari (2015) studied the supply chain having three players - manufacturer, distributor, and retailer in which demand is depending on the credit period offered by the distributor to the retailer. Shah and Jani (2016) developed an inventory model for the retailer's ordering policies for units in inventory subject to deterioration and having a fixed lifetime.
In reality, most of the items are deteriorating with respect to time like milk products, vegetables, drugs, medical products, etc. Some of the items are deteriorating and it is usable up to a certain time which is called a fixed lifetime. Only a few researchers have acknowledged the deteriorating items with fixed lifetime. Ghare and Schrader (1963) developed an EOQ model for deteriorating items with linear demand and variable deterioration rates. Raffat (1991), Shah and Shah (2000), Goyal and Giri (2001), and Bakker *et al.* (2012) developed inventory models throwing light on part of the deterioration. Sarkar (2012) worked on the EOQ model where demand and deterioration rate both are depending on time. Chung and Cárdenas-Barrón (2013) simplified and improved the solution procedure used in Min *et al.* (2010). Some of the motivating and inspirational articles are by Ouyang *et al.* (2013), Sarkar *et al.* (2014), Chung *et al.* (2014), Wu *et al.* (2014), Shah *et al.* (2015), Shah *et al.* (2017) and their mentions. Moreover, Shah and Barrón (2015) considered an inventory model in which the supplier offers a cash deduction or a stable credit period to the retailer and the retailer passes it to the customer. Shah *et al.* (2014) gave an inventory model incorporating optimal preservation technology investment for two-level trade credit financing. Shah *et al.* (2016) developed an inventory model that suggests when a retailer needs to purchase additional stock to take advantage of the current lower price or purchase at a new price.
In this research article, the demand is trended and the manufacturer gives a credit period to the retailer to improve his demand. The units in inventory are subject to deterioration and have a maximum lifetime. Manufacturer invests in preservation technology to increase the life of units. To minimize the total cost of both the players, two policies are tested viz independent decision (retailer's decision) and joint decision. The best outcome is analyzed and discussed for the optimum total cost of the supply chain with consideration of cycle time and preservation technology investment.
The rest of the research article is planned as follows. Section 2 is about the notations and assumptions used throughout the article. Section 3 is the development of the inventory mathematical model. Section 4 is to identify sensitive parameters using sensitivity analysis and numerical examples to illustrate the proposed inventory model. This section also discusses managerial insights. As a final point, section 5 gives the conclusion and future direction for further research.

2. NOTATIONS AND ASSUMPTIONS

Following notations and assumptions shall be used to develop the mathematical model of the problem.

2.1 Notations

Retailer's Parameters:

A_r	Ordering cost per order
s_i	Selling price per unit of i^{th} item
$\theta_i(t)$	Time-varying deterioration rate at any time t for i^{th} item where $0 \le \theta_i(t) < 1$
m_i	Maximum lifetime (in years) for i^{th} item
u_i	Preservation technology investment per unit time to diminish deterioration rate for i^{th} item (a decision variable)
$f(u_i)$	Proportion of reduced deterioration rate where $0 \le f(u_i) < 1$. The reduced deterioration rate because of implementing suitable preservation technology as $\mathrm{f}(u_i) = 1 - \dfrac{1}{1+\mu u_i}, \mu > 0$
$I_{ri}(t)$	Level of inventory for the retailer of i^{th} item at any time $t,\ 0 \le t \le T$
T	Cycle time (a decision variable)
$t_i = \beta_i \cdot T$	Production Run time of i^{th} item at the manufacturer
Q	Retailer's procurement quantity per cycle
h_{ri}	Holding cost per unit per annum for i^{th} item
i_{er}	Rate of Interest earned for retailer for i^{th} item / \$ / year
i_{cr}	Rate of Interest charged for retailer for i^{th} item / \$ / year, $I_{er} \le I_{cr}$
M	Permissible delay period offered by the supplier to the retailer in years
δ_i	Percentage of the good item out of total production for i^{th} item
TC_r	Total cost of the retailer

Manufacturer's Parameters:

p_i	Total production of i^{th} item
P_i	Good production of i^{th} item, $P_i = \delta_i \cdot p_i;\ 0 < \delta_i \le 1$
A_m	Manufacturer's ordering cost per order
w_i	Wholesale price for i^{th} item
C_i	Purchasing cost of raw materials by the manufacturer for i^{th} item, $C_i < w_i < s_i$
h_{mi}	Holding cost for manufacturer per unit per annum for i^{th} item
e_i	Idle time cost per unit for i^{th} item
i_{em}	Rate of Interest earned for manufacturer for i^{th} item / \$ / year
i_{cm}	Rate of Interest charged for manufacturer for i^{th} item / \$ / year, $i_{em} \le i_{cm}$
TC_m	Total cost of the manufacturer
TC_J	Total cost of the supply chain for the joint decision

2.2. Assumptions

1. The supply chain consists of a single manufacturer and a single retailer for multi- items.
2. Demand rate is $R_i(t) = a_i(1+b_i t)$; where $a > 0$ is scale demand and $0 \le b < 1$ denotes the rate of change of demand.
3. The inventory system under study deals with deteriorating items having an expiry rate. The deterioration rate tends to 1 when time tends to maximum life-time *m*. Following Sarkar (2012), Chen and Teng (2014), and Wang *et al*. (2013), the functional form for deterioration rate is

$$\theta_i(t) = \frac{1}{1+m_i-t}; \; 0 \le t \le T; \quad 0 \le \theta_i \le 1$$

There is no repair or replacement of deteriorated items during the cycle time.

4. The manufacturer offers a credit period M to the retailer.
5. The retailer generates revenue by selling items and earns interest during $[0, M]$ at rate i_{er}.
6. When $M \le T$, the retailer pays interest during $[M, T]$ at the rate i_{cr} for unsold stock.
7. The planning horizon is infinite.
8. Lead time is negligible.
9.

3. MATHEMATICAL MODEL

In the proposed model, the retailer receives several items from the supplier to fulfill the demand of its customers. Here retailer is offered a permissible delay M by the supplier. During this period, retailers may earn interest on the accumulated amount. After the credit period retailer has to pay interest for the unsold products.

As shown in figure 1, at the start of production, the manufacturer gets a lot size $R_i T$ of raw materials and the inventory level goes up with rate p_i over time t_i. Here t_i is called production run time. Therefore, the retailer receives a lot size $R_i T$ at the end of the manufacturer's cycle length t_i.

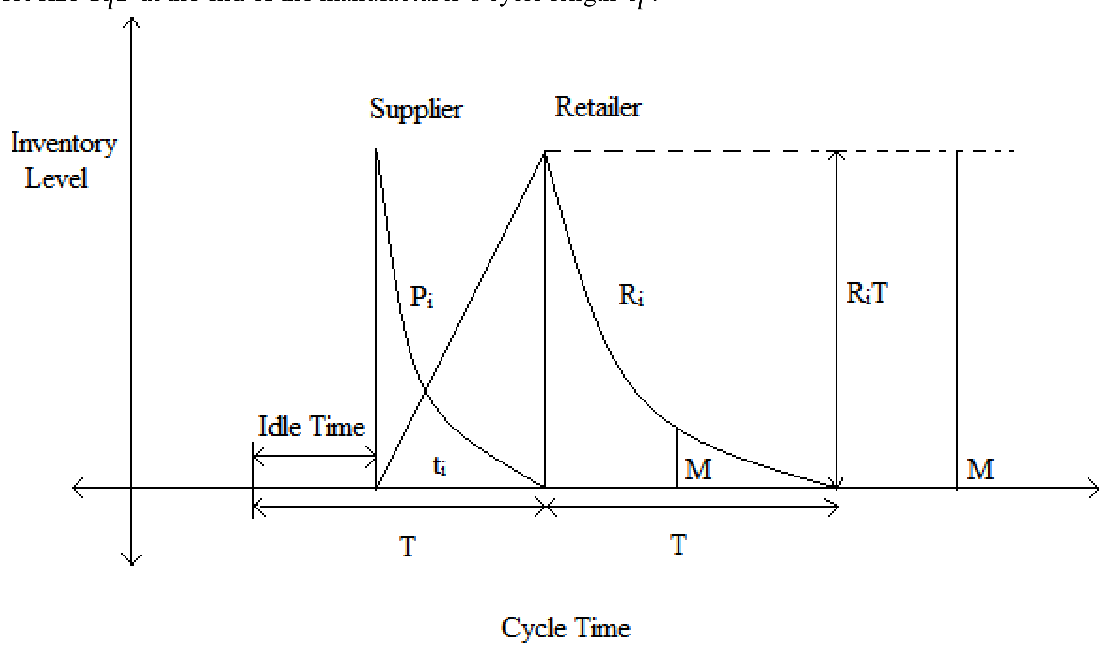

Fig. 1: Inventory Levels

3.1 Retailer's Individual Decision Perspective

The inventory level of the retailer at any instant of time t is governed by the differential equation.

$$\frac{dI_{ri}(t)}{dt} = -R_i(t) - \theta_i(t)\left(1-\mathrm{f}(u_i)\right)I_{ri}(t), \quad 0 \le t \le T$$

$$\text{where, } \theta_i(t) = \frac{1}{1+m_i-t}; \quad 0 \le \theta \le 1, \quad 0 \le t \le m_i \le T,$$

with $I_{ri}(T) = 0$.

The solution of the differential equation is

$$I_{ri}(T) = (1+m_i-t)^{\frac{1}{\mu u_i+1}}\left(\frac{(1+m_i-t)^{1-\frac{1}{\mu u_i+1}}(\mu u_i+1)(b_i m_i \mu u_i + b_i \mu t u_i + b_i \mu u_i + b_i m_i + 2\mu u_i + b_i + 1)a_i}{\mu u_i(2\mu u_i+1)} - \frac{(1+m_i-T)^{1-\frac{1}{\mu u_i+1}}(\mu u_i+1)(Tb_i \mu u_i + b_i m_i \mu u_i + b_i \mu u_i + b_i m_i + 2\mu u_i + b_i + 1)a_i}{\mu u_i(2\mu u_i+1)}\right)$$

Consequently, the retailer's order quantity is

$$Q_i = I_{ri}(0) = I(0)$$

$$= (1+m_i)^{\frac{1}{\mu u_i+1}}\left(\frac{(1+m_i)^{1-\frac{1}{\mu u_i+1}}(\mu u_i+1)(b_i m_i \mu u_i + b_i \mu t u_i + b_i \mu u_i + b_i m_i + 2\mu u_i + b_i + 1)a_i}{\mu u_i(2\mu u_i+1)} - \frac{(1+m_i-T)^{1-\frac{1}{\mu u_i+1}}(\mu u_i+1)(Tb_i \mu u_i + b_i m_i \mu u_i + b_i \mu u_i + b_i m_i + 2\mu u_i + b_i + 1)a_i}{\mu u_i(2\mu u_i+1)}\right)$$

The various costs associated with the retailer are as follows.

Purchase Cost: $$PC_r = \sum_{i=1}^{n} Q_i \cdot w_i$$

Ordering Cost: $$OC_r = \frac{A_r}{T}$$

Inventory Holding Cost: $$HC_r = \sum_{i=1}^{n} \frac{h_{ri}}{T}\int_0^T I_{ri}(t)\,dt$$

Preservation Technology Cost: $$PTI_r = \sum_{i=1}^{n} u_i \cdot T$$

Case-1 $M \le T$

The retailer earns interest IE_r at the rate i_{er} during $[0, M]$ as

$$IE_r = \sum_{i=1}^{n} \frac{i_{er} s_i}{T}\int_0^M t \cdot R_i(t)\,dt$$

and pays interest IC_r to supplier at the rate i_{cr} per annum during $[M, T]$ on unsold stock as

$$IC_r = \sum_{i=1}^{n} \frac{i_{cr} w_i}{T} \int_M^T I_{ri}(t)\, dt$$

Therefore, the retailer's total cost for n items is

$$TC_{r1}(u_i, T) = \mathrm{PC}_r + \mathrm{HC}_r - \mathrm{IE}_r + IC_r + \mathrm{PTI}_r + \mathrm{OC}_r$$

Case-2 $M > T$

The retailer earns interest IE_r at the rate i_{er} during $[0, M]$ as

$$IE_r = \sum_{i=1}^{n} \frac{i_{er} s_i}{T} \left[\int_0^T t\, R_i(t)\, dt - Q_i (M - T) \right]$$

Therefore, the retailer's total cost for n items is

$$TC_{r2}(u_i, T) = \mathrm{PC}_r + \mathrm{HC}_r - \mathrm{IE}_r + \mathrm{PTI}_r + \mathrm{OC}_r$$

Therefore, the total cost of the retailer is

$$TC_r(u_i, T) = \begin{cases} TC_{r1}(u_i, T), & M \le T \\ TC_{r2}(u_i, T), & M > T \end{cases}$$

3.2 Manufacturer's Perspective

Production run time is always less than or equal to the cycle time of the retailer's inventory because shortages at any stage are not permitted.
The various costs associated with the retailer are as follows.

Production Cost:

$$\mathrm{PrC}_m = \sum_{i=1}^{n} \frac{C_i}{\beta_i T} \int_0^{\beta_i T} P_i(t)\, dt$$

Ordering set up cost of the manufacturer is

$$OC_m = \sum_{i=1}^{n} \frac{A_m}{\beta_i T}$$

For the raw materials, the inventory holding cost per unit is

$$HC_m = \sum_{i=1}^{n} \frac{h_{mi}}{\beta_i T} \int_0^{\beta_i T} t\, P_i\, dt$$

Preservation Technology Cost:

$$PTI_m = \sum_{i=1}^{n} u_i \cdot T$$

As a manufacturer receives the raw material at the starting of the production with full payment to the third party, the average cost of the idle time $(T - t_i)$ is

$$ITC_m = \sum_{i=1}^{n} e_i \left(\frac{P_i}{\int_0^T R_i(t)\,dt} - 1 \right)$$

Idle Time Cost:
This idle time is considered at the start of the cycle of manufacturer to avoid extra inventory cost of the whole lot $R_i \cdot T$ during the idle time $(T - t_i)$ if production starts at very beginning of the cycle T.

While production, not the 100% products are of good quality. Out of produced items p_i, the good items P_i are obtained as follows.

Good Production $P_i = \delta_i \cdot p_i;\ 0 < \delta_i \le 1$

Case-1: $M \le T$

The manufacturer earns interest from the retailer IE_m at the rate i_{em} during $[M, T]$ as

$$IE_m = \sum_{i=1}^{n} \frac{i_{em} w_i}{T} \int_0^{T-M} t\, P_i\, dt$$

and interest charged IC_m at the rate i_{cm} per annum during $[0, M]$ on raw material as

$$IC_m = \sum_{i=1}^{n} \frac{i_{cm} C_i}{\beta_i T} \int_0^{M} I_{ri}(T)\, dt$$

Therefore, the manufacturer's total cost for n items is

$$TC_{m1}(u_i, T) = \mathrm{PrC}_m + \mathrm{HC}_m - \mathrm{IE}_m + \mathrm{IC}_m + \mathrm{PTI}_m + ITC_m + \mathrm{OC}_m$$

Case-2: $M > T$

The manufacturer's interest charged IC_m at the rate i_{cm} during $[0, M]$ on raw material is

$$IC_m = \frac{i_{cm} C_i}{\beta_i T} \left[\int_0^{M} I_{ri}(T)\, dt \right]$$

Therefore, the manufacturer's total cost for n items is

$$TC_{m2}(u_i, T) = \mathrm{PrC}_m + \mathrm{HC}_m + \mathrm{IC}_m + \mathrm{PTI}_m + ITC_m + \mathrm{OC}_m$$

Therefore, the total cost of the manufacturer is

$$TC_m(u_i, T) = \begin{cases} TC_{m1}(u_i, T), & M \le T \\ TC_{m2}(u_i, T), & M > T \end{cases}$$

Independent decision policy

In an independent decision policy, the retailer is the decision maker of the whole supply chain. The retailer will set the selling prices of two items and the time to order and policy is followed by the manufacturer. With these decisions, the manufacturer will deduce his profit.

Joint decision policy

In the joint policy, the joint total cost of the supply chain for case -1 and case -2 are

$$TC_J = \begin{cases} TC_{r1} + TC_{m1}, & M \le T \\ TC_{r2} + TC_{m2}, & M > T \end{cases}$$

Here, the decision variables will be obtained by setting partial derivatives of the objective function to be zero. For obtained values of cycle time and preservation technology investment, the joint profit of the supply chain will be computed.

The objective is to minimize the joint total cost TC_j per unit time with respect to cycle time T, preservation technology investments u_1 and u_2 for items. The objective function is a non-linear and continuous function of three variables. The necessary conditions for the existence of the solution are $\frac{\partial TC_j}{\partial T}=0,\ \frac{\partial TC_j}{\partial u_1}=0$ and $\frac{\partial TC_j}{\partial u_2}=0,$ if

$$\begin{vmatrix} \frac{\partial^2 TC_j}{\partial T^2} & \frac{\partial^2 TC_j}{\partial T\partial u_1} & \frac{\partial^2 TC_j}{\partial T\partial u_2} \\ \frac{\partial^2 TC_j}{\partial u_1\partial T} & \frac{\partial^2 TC_j}{\partial u_1^2} & \frac{\partial^2 TC_j}{\partial u_1\partial u_2} \\ \frac{\partial^2 TC_j}{\partial u_2\partial T} & \frac{\partial^2 TC_j}{\partial u_2\partial u_1} & \frac{\partial^2 TC_j}{\partial u_2^2} \end{vmatrix} > 0.$$

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1 Numerical Examples

Example-1: $\left(\text{Case-1:} M \le T\right)$

We have considered values of parameters of two types of items as follows. $p_1=400$ units, $p_2=420$ units, $a_1=150$ units, $b_1=0.5,\ a_2=170$ units, $b_2=0.2,\ m=1,\ w_1=\$35$ per unit, $w_2=\$40$ per unit, $A_r=150$ per order, $h_r=\$3$ per unit per annum, $M=\frac{30}{365}$ days, $i_{er}=0.09,\ i_{cr}=0.11,$ $S_1=\$40$ per unit, $S_2=\$50$ per unit, $C_1=\$20$ per unit, $C_2=\$15$ per unit, $A_m=\$30$ per order, $h_m=\$0.8$ per unit per annum, $e_1=\$8$ per unit, $e_2=\$6$ per unit, $i_{em}=0.05,$ $i_{cm}=0.06,\ \beta_1=0.4,\ \beta_2=0.3,\ \delta_1=0.9,\ \delta_2=0.95,\ \mu=0.5$. Then the optimal solution for individual decision of retailer for this case is $T=0.11$ year, $u_1=\$15.28,\ u_2=\$17.42,\ TC_r=\$2735.31$, $TC_m=\$\,15219.13,\ TC_J=\$\,17954.45$ and the optimal solution for joint decision of both retailer and manufacturer is $T=0.16$ year, $u_1=\$13.63,\ u_2=\$15.45,\ TC_r=\$3017.93,\ TC_m=\$\,14513.22$ and $TC_J=\$\,17531.15$.

The above result shows that individual decision is more cost savings for the retailer than the joint decision. But if we focus on the whole supply chain then it is clear from table 1 and figure 2 that a joint decision is more cost-saving for the whole supply chain as compared to an independent decision. The cost of the whole supply chain is reduced by 2.36% if players follow a joint decision.

Example-2: $\left(\text{Case-2:} M > T\right)$

We have considered values of parameters of two types of items in as follows $p_1=400$ units, $p_2=420$ units, $a_1=150$ units, $b_1=0.5,\ a_2=170$ units, $b_2=0.2,\ m=1,\ w_1=\$35$ per unit, $w_2=\$40$ per unit,

$A_r = 150$ per order, $h_r = \$3$ per unit per annum, $M = \frac{75}{365}$ days, $i_{er} = 0.09$, $i_{cr} = 0.11$, $S_1 = \$40$ per unit, $S_2 = \$50$ per unit, $C_1 = \$20$ per unit, $C_2 = \$15$ per unit, $A_m = \$30$ per order, $h_m = \$0.8$ per unit per annum, $e_1 = \$8$ per unit, $e_2 = \$6$ per unit, $i_{em} = 0.05$, $i_{cm} = 0.06$, $\beta_1 = 0.4$, $\beta_2 = 0.3$, $\delta_1 = 0.9$, $\delta_2 = 0.95$, $\mu = 0.5$. Then the optimal solution for individual decision of retailer for this case is $T = 0.12$ year, $u_1 = \$19.36$, $u_2 = \$22.30$, $TC_r = \$2825.21$, $TC_m = \$\,15004.66$, $TC_J = \$\,17829.87$ and the optimal solution for joint decision is $T = 0.17$ year, $u_1 = \$14.30$, $u_2 = \$16.19$, $TC_r = \$3028.22$, $TC_m = \$\,14488.57$, $TC_J = \$\,17516.80$.

The above result shows that individual decision is more cost savings for the retailer than the joint decision. But if we focus on the whole supply chain then it is clear from table 1 and figure 2 that a joint decision is more cost-saving for the whole supply chain as compared to an independent decision. The cost of the whole supply chain is reduced by 1.76% if players follow a joint decision.

The total cost for the above two different cases can be described by the following graph in figure 2. The optimum solution is exhibited in Table 1.

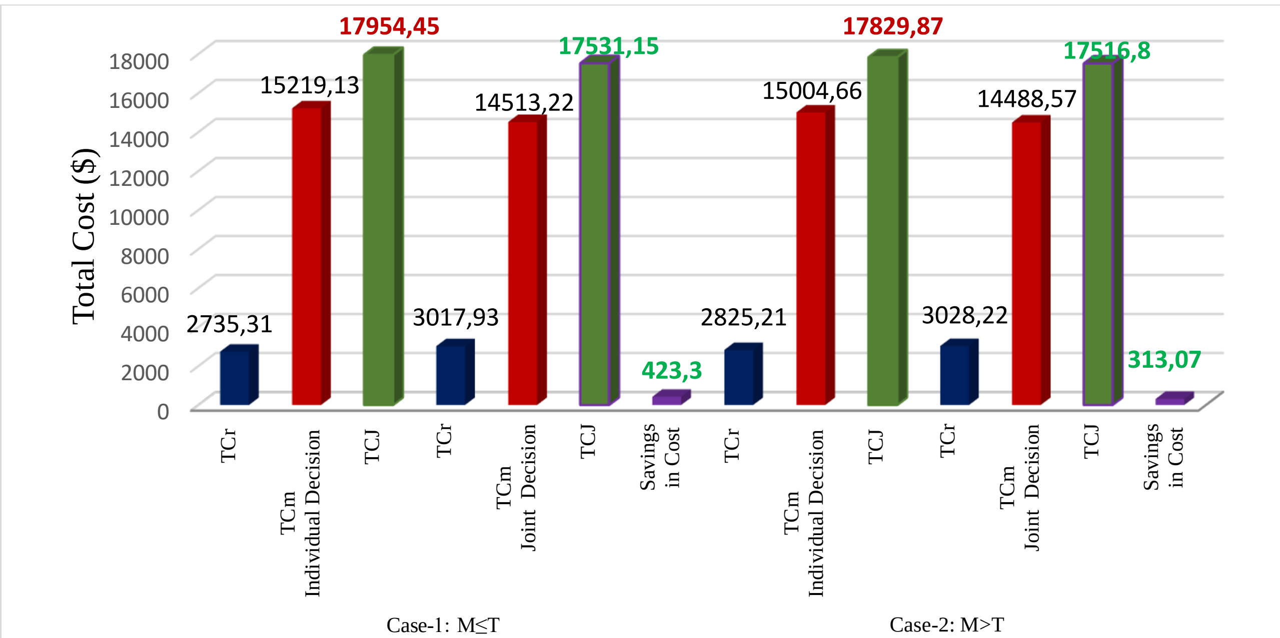

Fig.2: Comparison of costs of players for Independent Decision and Joint Decision

Table 1: Optimal Solution for Joint Decision and Independent Decision

Case	**Strategy**	**Decision Variables**	**Cost of the Player ($)**	**Total Cost of the Supply Chain ($)**	**Cost Reduction in Joint Decision**	**%**
$M \le T$	Individual	$T = 0.11$ year $u_1 = \$15.28$ $u_2 = \$17.42$	$TC_r = \$\,2735.31$ $TC_m = \$\,15219.13$	$TC_J = \$\,17954.45$		

	Joint	$T=0.16$ year $u_1=\$13.63$ $u_2=\$15.45$	$TC_r=\$3017.93$ $TC_m=\$\,14513.22$	$TC_J=\$\,17531.15$	\$ 423.294	2.36
$M>T$	Individual	$T=0.12$ year $u_1=\$19.36$ $u_2=\$22.30$	$TC_r=\$2825.21$ $TC_m=\$\,15004.66$	$TC_J=\$\,17829.87$		
	Joint	$T=0.17$ year $u_1=\$14.30$ $u_2=\$16.19$	$TC_r=\$3028.22$ $TC_m=\$\,14488.57$	$TC_J=\$\,17516.80$	\$ 313.074	1.76
	Adjusted		$TC_r=\$2671$ $TC_m=\$\,14860$	$TC_J=\$\,17531$		

In table 1, independent and joint decisions are compared for two different cases.
It is observed from the table that in a joint decision, the retailer is looser and the supplier is beneficial. To attract retailers for the joint decision, Goyal (1976) reallocated costs as follows for case 1.

Retailer's cost $$=17531\times\frac{2735}{2735+15219}=2671$$

Supplier's cost $$=17531\times\frac{15219}{2735+15219}=14860$$

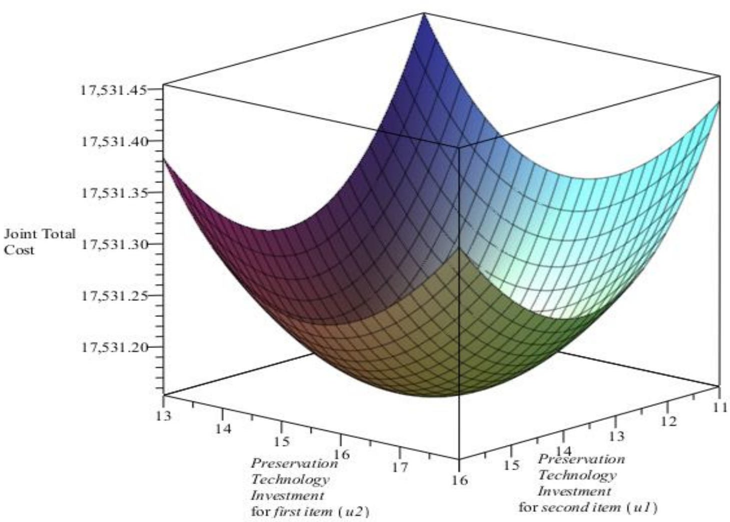

Fig.3: Concavity of joint total cost Vs PTI for the first item (u_1) and PTI for the second item(u_2)

As a final point, from the whole analysis case-1 ($M \le T$) and joint decision policy are the best for this proposed model. So, terms and conditions should be settled so that both the players gladly take the joint decision (for case-1) to have the minimum total cost.

4.2 Sensitivity Analysis

Now, we examine the variations in cycle time T , preservation technology investments for both items u_1 and u_2 by changing inventory parameters as -10%, -5%, 5% and 10%.

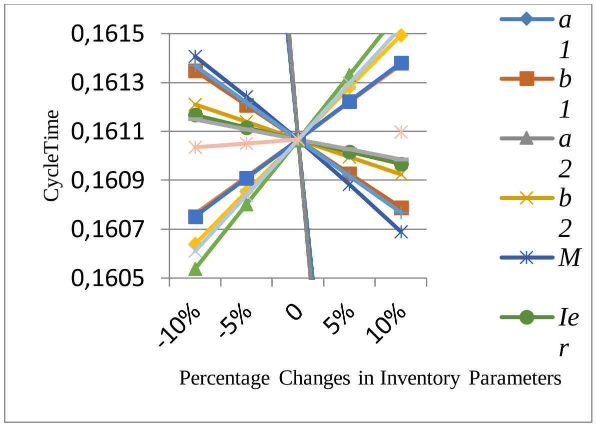

Fig. 4: Sensitivity analysis for cycle time (T)

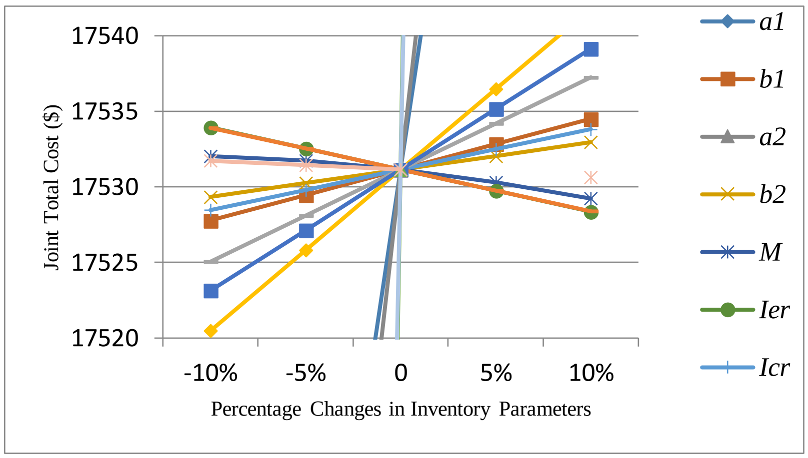

Fig. 5: Sensitivity analysis for Joint Total Cost

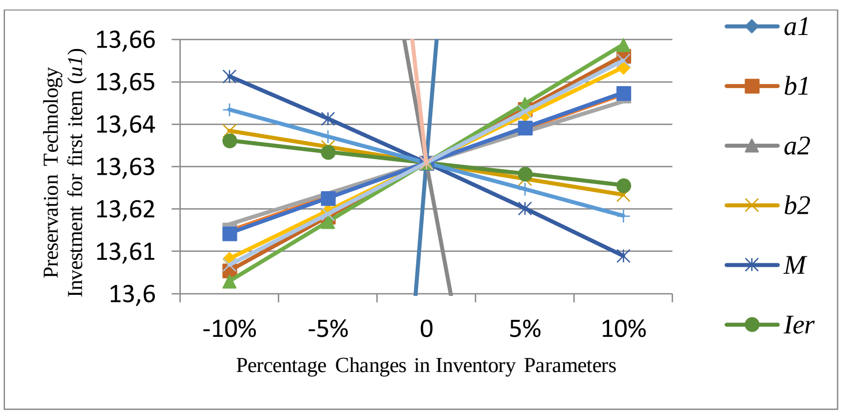

Fig. 6: Sensitivity analysis for PTI for the first item (u_1)

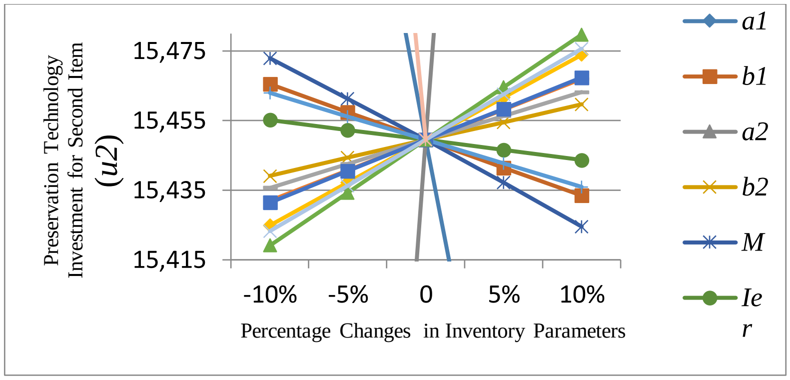

Fig. 7: Sensitivity analysis for PTI for the second item (u_2)

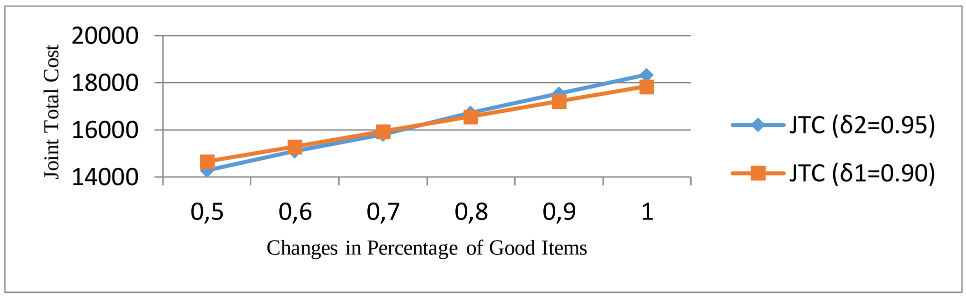

Fig. 8: Sensitivity analysis of δ_1 and δ_2

Table 2: Summary of sensitivity analyses

Inventory Parameters	**Cycle Time T**	**Joint Total Cost**	u_1	u_2
a_1	↓	↑	↑	↓
b_1	↓	↑	↑	↓
a_2	↓	↑	↓	↑
b_2	↓	↑	↓	↑
M	↓	↓	↓	↓
i_{er}	↓	↓	↓	↓
i_{cr}	↓	↑	↓	↓
i_{em}	↑	↓	↑	↑
i_{cm}	↓	↑	↑	↑
e_1	↑	↑	↑	↑
e_2	↑	↑	↑	↑
δ_1	↑	↑	↑	↑
δ_2	↑	↑	↑	↑
m	↑	↓	↓	↓

↑ Shows increasing pattern

↓ Shows decreasing pattern

Table 2 shows the effect of changes in inventory parameters on decision variables. From that, some managerial implications can be derived as follows.

- Total cost increases with an increase in scale demand a_i, rate of change of demand b_i, rates of interest charged i_{cr} and i_{cm}, idle time cost e_i, and percentage of good items δ_i.
- Total cost decreases with an increase in permissible delay period M, maximum lifetime m and rates of interest earned i_{er}, i_{em}.

5. CONCLUSIONS

In this paper, an inventory model consisting single manufacturer and a single retailer is considered. The manufacturer sells several items to the retailer and offers permissible delays in payment. The demand is trended and items in the model are subject to deterioration with a maximum lifetime. To control the deterioration of units in inventory, preservation technology investment is also incorporated. Here, mainly two policies are investigated for the optimum cost. Independent decision in which retailer can take a decision and it is followed by the manufacturer. Individual decision is more cost saving for the retailer than the joint decision. But if we focus on the whole supply chain then a joint decision is more cost-saving as compared to an independent decision. Through the examples it is evident that by joint decision total cost is reduced by 2.36% in case 1 and by 1.76% in case 2. So managers should follow the joint decisions.

Future Work

One can extend this article by allowing shortages in the system, incorporating price-sensitive demand, or considering more players. The retailer can pass the credit period to customers also.

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