USE OF A NETWORK DEA MODEL TO ASSESS COUNTRIES PARTICIPATING IN THE RIO 2016 OLYMPIC GAMES

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ABSTRACT
There are several studies that evaluate countries in the Olympics with DEA, but few with NDEA and, as far as we know, they do not resemble the hybrid orientation model used in this study. This study uses a Network DEA model to assess the efficiency of the countries that participated in the Rio 2016 Olympic Games. Population and GDP values were used as inputs, the number of athletes were used as an intermediate variable and the number of medals won by each country were used as outputs. A two-dimensional representation of the efficiency frontier was also developed to facilitate the interpretation of the results. Granada was the most efficient country, while it was possible to observe, in general, the poor performance of countries that are well ranked in the lexicographic method.

KEYWORD: Network DEA, Efficiency, Olympic Games, Two-Dimensional Representation

MSC: 90C99

RESUMEN
Hay varios estudios que evalúan países en los Juegos Olímpicos con DEA, pero pocos con NDEA y, hasta donde sabemos, no se asemejan al modelo de orientación híbrido utilizado en este estudio. Este estudio utiliza un modelo Network DEA para evaluar la eficiencia de países que participaron de los Juegos Olímpicos de Río 2016. Los valores de población y PIB se utilizaron como inputs, el número de atletas como variable intermedia y el número de medallas ganadas por cada país como outputs. También se desarrolló una representación bidimensional de la frontera de eficiencia para facilitar la interpretación de los resultados. Granada fue el país más eficiente, también podemos notar, en general, el pobre desempeño de países que están bien clasificados en el método lexicográfico.

PALABRAS CLAVE: Network DEA, Eficiencia, Juegos Olímpicos, Representación Bidimensional

1. INTRODUCTION

The Olympic Games are one of the biggest sports competitions in the world. They are the scene of intense disputes, not only between athletes but also between countries, which want to demonstrate power and influence based on their achievements. Historically, several examples demonstrate that the Games were used for this purpose. For example, Hitler used the 1936 edition of the Olympic Games in Berlin to try to demonstrate the superiority of the Nazi regime [22]. During the Cold War, the United States and the Soviet Union also tried to demonstrate strength through their Olympic achievements [10]. More recently, the Chinese hosted the Beijing 2008 Olympics and invested a lot to make their country the biggest gold medal winner, thus showing the strength of the Communist State [2].

Even with this strong bias of competition between countries, the International Olympic Committee (IOC), responsible for organizing the Olympic Games, never released an official ranking of participating nations. However, the media uses a lexicographic method to evaluate the performance of each country in each edition of the Games. The method ranks countries according to the highest number of gold medals obtained. In case of a tie, silver medals are considered and, finally, bronze medals [18].

Several studies criticized the lexicographic model used and proposed alternative rankings and methodologies based on Multicriteria Decision Support [9; 23; 24], economics concepts [27], bibliometric indexes [21; 28] and Data Envelopment Analysis (DEA) to evaluate countries' performance in the Olympics. DEA is a non-parametric method, developed by Charnes et al. [3], who assesses multidimensional efficiency by preparing a production frontier.

For example, Lins et al. [18] proposed a ranking to examine the results of the Sydney 2000 Olympics. As input, they used the variables population and Gross Domestic Product (GDP) and as output, the gold, silver, and titbronze medals won by each country. The DEA zero-sum gains model was proposed (ZSG-DEA) in this study since the number of medals must remain constant.

Several other authors proposed Olympic rankings based on DEA [8; 12; 14; 15; 16; 17; 19; 25; 30; 31; 32; 33; 34; 35]. Of these,

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studies by Jablonsky [12] and Li et al. [15; 17] used Network DEA (NDEA) to evaluate countries at the Olympic Games. They used as inputs the values of GDP or GDP per capita, population, and tradition in the modality (used only by Jablonsky [12]), as an intermediate variable the number of athletes from the country, and as outputs the number of medals obtained. Li et al. [15] proposed an output-oriented NDEA BCC model, Jablonsky [12] proposed an output-oriented NDEA model, while the model by Li et al. [17] is a fixed-sum output output-oriented NDEA model.

However, as far as we know, there are no models that resemble the hybrid orientation model proposed by Despotis et al. [7] which will be used in this study. Thus, the present study proposes the use of the Composition approach of the Network DEA method, proposed by Despotis et al. [7] to assess the performance of the countries participating in the Rio 2016 Summer Olympics. As it has a hybrid orientation, the model proposed by Despotis et al. [7] is difficult to apply, which means that there are not many studies with it. However, the characteristics of the proposed problem fit the required orientations of the model.

Often the results of the DEA and NDEA models are difficult to interpret. Part of this difficulty comes from the fact that it is a mathematical model and, therefore, those who are not familiar with linear programming have difficulty in its interpretation. As we will be using an advanced NDEA model, we propose a graphical representation to facilitate the interpretation of the results obtained. This graphical representation presents visual information to the manager or decision-maker, regarding the proximity of the Decision Making Units (DMUs) to the efficient frontier and how much the inputs must be reduced, or the outputs increased for a DMU to become efficient. In addition, with the graphical representation, it is possible to analyze a DMU in comparison with all the others in a more simplified way.

The proposed representation intends to use the values calculated by the model of exogenous virtual inputs and exogenous virtual outputs for the elaboration of the graph. This graph will provide simultaneous information on the efficiency of the overall system, the first stage, and the second stage. In addition, we will also divide the DMUs into two groups, according to their characteristics.

From a practical point of view, it is the first time that the two-dimensional representation of the Despotis et al. [7] model is presented in an article in English and allows, especially with the zoom, to have a good view of the results and analyze the performance of each country, both in the overall system, as well as in the first stage that evaluates the formation of athletes and in the second stage that evaluates the effective performance of these athletes.

The study is divided into sections: the second section presents the Network DEA model used. The third section presents the methodology, the fourth the results, and the fifth the discussion. Finally, the last section presents the study's conclusions.

2. COMPOSITION APPROACH OF NETWORK DEA

In 2014, Despotis and Koronakos [6] showed that the efficiency obtained by the additive NDEA model [4] tends to always favor one of the internal stages of the evaluated DMU. In the same study, they pointed out that the multipliers obtained by the multiplicative relational NDEA model [13] are not unique. In addition, Sotiros et al. [26] showed later that the relational and additive models do not meet the dominance requirement, that is, there may be cases where one DMU is more efficient than the other in all internal stages, however, it is less efficient in the overall system. Thus, Despotis et al. [7] presented a new model to estimate the value of efficiency for basic two-stage structures, to solve the problems of the previously mentioned models.

In this new model, called of NDEA model of overall system efficiency composition, the efficiency values of the internal stages are obtained a priori, and later these values are aggregated in the overall efficiency of the DMU. The model also has the characteristic of being a model with two distinct orientations, one for each internal stage evaluated. The first stage is output-oriented and the second stage is input-oriented. Thus, it is important to highlight that to apply this model, we need a case that follows this logic of orientations.

The model initially proposed in Despotis and Koronakos [6] had only one objective function. Later, Despotis et al. [7] defined the efficiency of each of the two stages by models 1 and 2, to propose a new model. Model 1, which corresponds to the efficiency of the first stage, is output-oriented, while model 2, which corresponds to the efficiency of the second stage, is input-oriented.

In this model, E_0^1 is the efficiency of the first stage and E_0^2 is the efficiency of the second stage, j is the index of the DMUs, v_i, w_d and u_r, are respectively the multipliers of the inputs, the intermediate variables and outputs, x_{ij}, z_{dj}, and y_{rj} are, respectively, the values of the inputs, intermediate variables and outputs and m, d, s are the number of inputs, intermediate variables and outputs, respectively.

$$\begin{aligned} & E_0^1 = \min \frac{\sum_{i=1}^{m} v_i x_{i0}}{\sum_{d=1}^{D} w_d z_{d0}} \\ & \text{s.t.} \ \frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{d=1}^{D} w_d z_{dj}} \geq 1, j = 1,2,\dots,n \end{aligned} \tag{1}$$

$$v_i, w_d \geq 0;\ i = 1,2,\dots,m;\ d = 1,2,\dots,D$$

$$E_0^2 = \max \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{d=1}^{D} w_d z_{d0}}$$
$$\text{s.t.}\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj}} \leq 1, j = 1,2,\dots,n \qquad (2)$$
$$u_r, w_d \geq 0;\ r = 1,2,\dots,s;\ d = 1,2,\dots,D$$

The model proposed by Despotis et al. [7] aggregates models 1 and 2, both the restrictions and the objective functions, creating, therefore, a multiobjective model. Thus, the model intends to simultaneously find a solution that seeks the best efficiency for the first and the second stage, to respect the restrictions that guarantee that the efficiency of each internal stage is equal to or less than one, and considering the same weight for intermediate variables. As the model generated is non-linear, the C-C Transformation can be performed (as in [3]), as the denominators of the two objective functions are identical, and thus, the proposed model is shown in 3.

$$E_0^1 = \min \sum_{i=1}^{m} v_i x_{i0}$$
$$E_0^2 = \max \sum_{r=1}^{s} u_r y_{r0}$$
$$\text{s.t.}\ \sum_{d=1}^{D} w_d z_{d0} = 1 \qquad (3)$$
$$\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1,2,\dots,n$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, j = 1,2,\dots,n$$
$$u_r, v_i, w_d \geq 0, i = 1,2,\dots,m, r = 1,2,\dots,s, d = 1,2,\dots,D$$

Model 3 is multiobjective. A multiobjective problem (MOLP) does not provide a single optimal solution, but a set of non-dominated solutions called the Pareto Frontier. There are some methods for finding one of the optimal solutions within a MOLP [11]. The form chosen by Despotis et al. [7] was by minimizing the maximum deviation of a point from the Pareto Frontier ($\sum_{i=1}^{m} v_i x_{i0}$, $\sum_{r=1}^{s} u_r y_{r0}$) until the ideal point (E_0^1, E_0^2), using the unweighted Tchebycheff distance. The first coordinate of the ideal point to be determined is the situation in which we obtain the best result for the efficiency of the first stage in the Pareto Frontier, while the second coordinate is found in the situation where we obtain the best efficiency value of the second stage in the same frontier.

The model that solves the problem described above is described in 4, where δ is the largest deviation.

$$\min\ \delta$$
$$\text{s.t.}\ \sum_{i=1}^{m} v_i x_{i0} - \delta \leq E_{jo}^1$$
$$\sum_{r=1}^{s} u_r y_{r0} + \delta \geq E_{jo}^2$$
$$\sum_{d=1}^{D} w_d z_{d0} = 1 \qquad (4)$$
$$\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0,\ j = 1, 2, \dots, n$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0,\ j = 1, 2, \dots, n$$
$$u_r, v_i, w_d, \delta \geq 0, i = 1,2,\dots,m, r = 1,2,\dots,s, d = 1,2,\dots,D$$

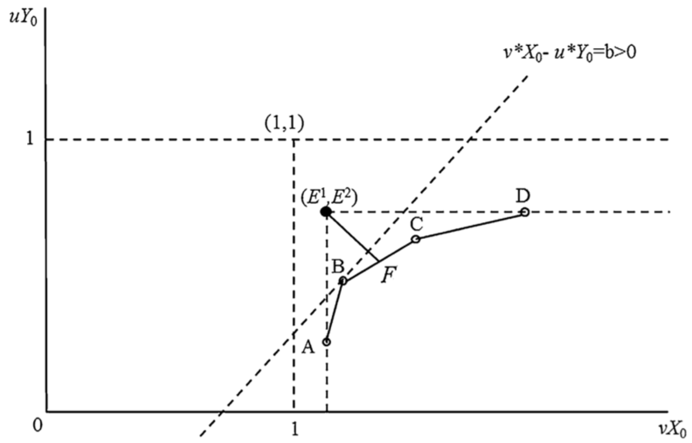

Figure 1 – MOLP Pareto frontier and its optimal solution
(Despotis et al., 2016)

We can find the solution for model 4 where the deviations from the ideal point are equal and minimized. As shown in Figure 1, the solution to the problem is point F, the intersection between the Pareto Frontier (ABCD), and a radius starting from the ideal point. The ideal point for this model is the point represented by the highest value of virtual output obtained from the non-dominated set of solutions and the lowest value of virtual input from that same set.

The main advantage of model 4 over the additive and the relational models is that it provides a single point, not necessarily extreme (vertex), on the Pareto Frontier, that is, we can obtain unique efficiency scores for the two internal stages [7].

Since we can find the optimal solution (v^*, w^*, u^*) in model 4, the efficiency values for each stage for DMU_0 are found by equations 5 for the first stage, and 6 for the second phase. We can note that the efficiency of the first stage is equal to the inverse of the exogenous virtual input, while the efficiency of the second stage is equal to the value of the exogenous virtual output since the C-C Transformation was used to solve the problem and the virtual intermediate variable is always 1 for the DMU under analysis.

$$E_0^1 = \frac{\sum_{d=1}^{D} w_d z_{d0}}{\sum_{i=1}^{m} v_i x_{i0}} = \frac{1}{\sum_{i=1}^{m} v_i x_{i0}} \quad (5)$$

$$E_0^2 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{d=1}^{D} w_d z_{d0}} = \sum_{r=1}^{s} u_r y_{r0} \quad (6)$$

Finally, we need to aggregate the efficiencies of the two internal stages to obtain the efficiency of the overall system, which can be defined as the arithmetic mean of the efficiencies of the internal stages, as presented in 7 or can be aggregated multiplicatively, as shown in 8. In this study, we will use the multiplicative aggregation.

$$E_0 = \frac{1}{2}\left(E_0^1 + E_0^2\right) \quad (7)$$

$$E_0 = E_0^1 * E_0^2 = \frac{1}{\sum_{i=1}^{m} v_i x_{i0}} * \sum_{r=1}^{s} u_r y_{r0} = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \quad (8)$$

To facilitate the interpretation of the results, a two-dimensional representation of the efficient frontier was elaborated for the model, following the logic proposed by Bana e Costa et al.[32], for the classic DEA models. Initially, the graphical representation of the efficiency frontier for the classic DEA was known for the model of constant returns of scale (CCR) when up to one input and two outputs or two inputs and one output are used. In the variable returns of scale model (BCC), the graphical representation of the efficient frontier was known only for the model where there is only one input and one output. Other more complex models were developed, however, they had limitations as to the number of inputs, outputs and DMUs to be evaluated or they were not two-dimensional, which also makes interpretation difficult[1].

To propose a more comprehensive representation, Bana e Costa et al.[1] proposed a model to develop the two-dimensional representation of the efficiency frontier for the classic DEA models (CCR and BCC), without restrictions on the number of DMUs evaluated or the number of inputs and outputs used. The model uses the modified virtual input and the modified virtual output as axes of the graphical representation. They needed to make modifications because the C-C Transformation linearizes the models and, if a graph were plotted with the virtual input and the virtual output as axes, all DMUs would be represented on the same line in the graph.

As in the composition NDEA model, the constraint used to guarantee the linearization of the model is related to the intermediate variable, it is possible to use the original values of the virtual exogenous input and the virtual exogenous output (without modifications) for the development of the two-dimensional representation of the overall efficient frontier, because, unlike the classic DEA model, in a graph that relates the virtual exogenous input to the virtual exogenous output, none of the axes would have values equal to 1 for all DMUs.

The closer to the efficient frontier the better the overall performance of a DMU. Furthermore, since the efficiency of the first stage is equal to the inverse of the virtual exogenous input (as seen in 5), the efficiency of this stage is equal to 1 when the exogenous virtual input is equal to 1. Therefore, it is possible to represent the efficient DMUs in the first stage on the virtual line where the exogenous virtual input is equal to 1. Similarly, since the efficiency of the second stage is equal to the exogenous virtual output, a DMU is only efficient in the second stage if the exogenous virtual output is equal to 1 (as seen in 6). Therefore, it is possible to represent efficient DMUs in the second stage on a horizontal line where the exogenous virtual output is equal to 1.

Another characteristic of this representation is the separation of DMUs into two groups. The first group is made up of DMUs that are more efficient in the first stage compared to the second stage, while the second group is the opposite. To determine these groups, we must find the curve where the efficiency of the first stage is equal to the efficiency of the second stage. This hyperbola can be obtained through equation 9. We can note that the hyperbola is not an efficient frontier, but a function that will separate the DMUs according to their performance in the internal stages.

$$E_0^1 = E_0^2 \rightarrow \frac{1}{\sum_{i=1}^{m} v_i x_{i0}} = \sum_{r=1}^{s} u_r y_{r0} \rightarrow \frac{1}{I} = O \quad (9)$$

We represent DMUs that are more efficient in the first stage in relation to the second below the hyperbola, while we represent DMUs that are more efficient in the second stage in relation to the first above the hyperbola. DMUs represented on the hyperbola have the same efficiency value for both stages, as is the case for overall efficient DMUs. This representation was used in the study proposed by Mariano et al.[20].

3. METHODOLOGY

This study proposes a ranking to evaluate the performance of countries in the Rio 2016 Summer Olympics, using the NDEA model of overall system efficiency composition. We will use population and GDP as inputs. GDP per capita was not used because it is an index, which is not recommended in DEA. We will use the number of athletes from each country as an intermediate variable, and the number of gold, silver, and bronze medals obtained as outputs. Figure 2 illustrates the model used.

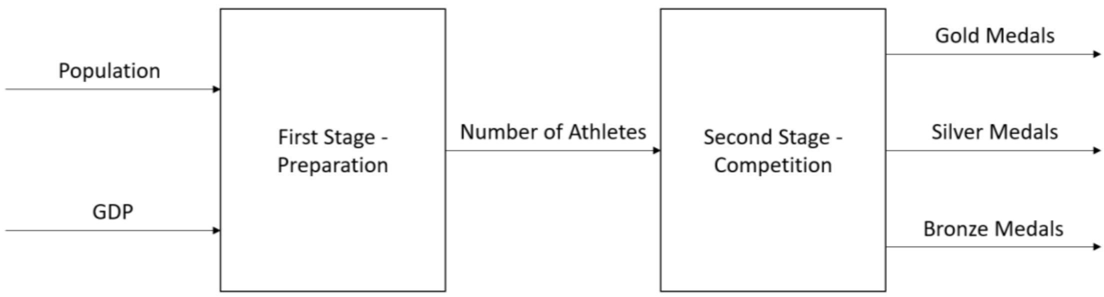

Figure 2 – The proposed model for country assessment

The NDEA model of overall system efficiency composition was chosen because it is a model that is oriented to outputs in the first stage and oriented to inputs in the second stage. In the first stage, we analyze the preparation of the athletes, comparing the number of athletes with the population and the GDP of the countries. The larger the population of a country, the greater the chances of obtaining Olympic athletes. GDP, on the other hand, has a direct relationship with a nation's economy, and the better an economy, the more it can be invested in sports, which also generates a greater number of athletes. At this stage, it is intended to increase the number of athletes in relation to the inputs, since these inputs are difficult to change in the analyzed context, considering that the decision-maker would be some Olympic committee, in addition, it is not recommended to reduce the population nor any country's GDP.

The second stage makes a relationship between the medals won and the number of athletes that a country sent to the Olympics. A larger delegation is expected to achieve better results than a smaller delegation. For this stage, it is intended to reduce the number of athletes in relation to the number of medals to obtain efficiency. This must happen because an Olympic committee obtains expenses for each athlete that sends to the Olympic Games, so, a reduction of the delegation, with the maintenance of the medals obtained, is interesting for the country. In addition, the decision to reduce the number of athletes is a responsibility of an Olympic committee while the increase of the number of medals is not, since to obtain more medals, you must remove achievements from other countries.

The overall system analyzes the relationship between population and GDP with a nation's medal achievements. It is expected that the larger the population and the economy of a country, the more Olympic achievements will be obtained since countries invest in athletes to demonstrate the power to other countries.

As gold medals are more valuable than silver medals, which in turn are more valuable than bronze medals, weight restrictions have been added to the proposed NDEA model. The restrictions were the same as those proposed by Soares de Mello et al. [25] and guarantee that the weights obtained for the gold medals are greater than the weights of the silver medals, that the weights of the silver medals are greater than the weights of the bronze medals and that the difference between the weights of the gold and silver medals must be greater than the difference between silver and bronze medals. The global efficiency composition model with weight restrictions is described in 10, considering ε equal to 0.000001.

4. RESULTS

We evaluate the 85 countries that won at least one medal at the Rio 2016 Summer Olympics. In 2016, a delegation of 9 independent Olympic athletes participated in the Olympics and won a gold medal and a bronze medal. This delegation was made up of Kuwaiti athletes who were banned from competing for their country, however, as these athletes do not officially represent a nation, they were not included in the analysis. The number of medals distributed and the number of athletes from each country were obtained from the website of the International Olympic Committee [5]. Population and GDP numbers (in dollars) were obtained from the World Bank [29].

$$
\begin{aligned}
& min\ \delta \\
& s.t.\ \ v_1x_{10} + v_2x_{20} - \delta \le E_{jo}^1 \\
& u_1y_{10} + u_2y_{20} + u_3y_{30} + \delta \ge E_{jo}^2 \\
& w_1z_{10} = 1 \\
& w_1z_{1j} - (v_1x_{1j} + v_2x_{2j}) \le 0,\ j = 1, 2, \dots, n \\
& u_1y_{1j} + u_2y_{2j} + u_3y_{3j} - w_1z_{1j} \le 0,\ j = 1, 2, \dots, n \\
& \mathrm{u}_1 - \mathrm{u}_2 \ge \varepsilon \\
& \mathrm{u}_2 - \mathrm{u}_3 \ge \varepsilon
\end{aligned}
\qquad (10)
$$

$u_1 - 2u_2 + u_3 \geq \varepsilon$
$u_1, u_2, u_3, v_1, v_2, w_1, \delta \geq 0$

The calculated efficiencies are shown in Table 1. E_1 corresponds to the value of efficiency obtained for the first stage, E_2 refers to the value of efficiency for the second stage while E_0 is the value calculated for the overall system. The countries are identified by the acronyms used by the IOC and the indexes presented are related to the position of each country in the lexicographic method and, in case of ties, through alphabetical order.

Table 1 – Country efficiencies at the 2016 Olympics

Index	Country	E_1	E_2	E_0	Index	Country	E_1	E_2	E_0
1	USA	0,023	1,000	0,023	42	ARM	0,284	0,613	0,174
2	GBR	0,075	0,878	0,066	43	CZE	0,148	0,314	0,046
3	CHN	0,005	0,748	0,004	44	ETH	0,042	0,799	0,033
4	RUS	0,030	0,922	0,027	45	SLO	0,429	0,289	0,124
5	GER	0,070	0,450	0,032	46	INA	0,003	0,565	0,002
6	JPN	0,036	0,503	0,018	47	ROU	0,081	0,161	0,013
7	FRA	0,083	0,442	0,037	48	BRN	0,342	0,326	0,111
8	KOR	0,056	0,476	0,026	49	VIE	0,010	0,496	0,005
9	ITA	0,069	0,399	0,028	50	TPE	0,037	0,213	0,008
10	AUS	0,233	0,291	0,068	51	BAH	1,000	0,353	0,353
11	NED	0,194	0,389	0,075	52	CIV	0,032	0,824	0,026
12	HUN	0,260	0,478	0,124	53	FIJ	1,000	0,137	0,137
13	BRA	0,037	0,186	0,007	54	JOR	0,020	0,875	0,017
14	ESP	0,092	0,254	0,023	55	KOS	0,110	0,875	0,096
15	KEN	0,111	0,789	0,088	56	PUR	0,173	0,167	0,029
16	JAM	0,457	0,865	0,395	57	SIN	0,060	0,280	0,017
17	CRO	0,333	0,596	0,198	58	TJK	0,097	1,000	0,097
18	CUB	0,177	0,436	0,077	59	MAS	0,017	0,608	0,010
19	NZL	0,577	0,386	0,223	60	MEX	0,017	0,135	0,002
20	CAN	0,117	0,254	0,030	61	VEN	0,047	0,122	0,006
21	UZB	0,140	0,771	0,108	62	ALG	0,040	0,132	0,005
22	KAZ	0,096	0,618	0,059	63	IRL	0,221	0,114	0,025
23	COL	0,051	0,241	0,012	64	LTU	0,365	0,186	0,068
24	SUI	0,168	0,322	0,054	65	BUL	0,121	0,183	0,022
25	IRI	0,014	0,553	0,008	66	IND	0,005	0,050	0,000
26	GRE	0,136	0,312	0,043	67	MGL	0,373	0,145	0,054
27	ARG	0,077	0,119	0,009	68	BDI	0,282	0,490	0,138

28	DEN	0,290	0,436	0,126	69	GRN	0,892	0,735	0,655
29	SWE	0,209	0,302	0,063	70	NIG	0,073	0,735	0,054
30	RSA	0,041	0,322	0,013	71	PHI	0,004	0,339	0,001
31	UKR	0,180	0,213	0,038	72	QAT	0,190	0,116	0,022
32	SRB	0,249	0,339	0,084	73	NOR	0,160	0,201	0,032
33	POL	0,102	0,164	0,017	74	EGY	0,051	0,078	0,004
34	PRK	0,200	0,881	0,176	75	TUN	0,152	0,153	0,023
35	BEL	0,129	0,245	0,032	76	ISR	0,079	0,130	0,010
36	THA	0,011	0,575	0,007	77	AUT	0,110	0,044	0,005
37	SVK	0,144	0,447	0,064	78	DOM	0,047	0,107	0,005
38	GEO	0,257	0,729	0,187	79	EST	0,513	0,069	0,035
39	AZE	0,136	1,000	0,136	80	FIN	0,139	0,056	0,008
40	BLR	0,221	0,264	0,058	81	MAR	0,046	0,061	0,003
41	TUR	0,021	0,267	0,006	82	NGR	0,020	0,041	0,001
42	ARM	0,284	0,613	0,174	83	POR	0,133	0,034	0,005
43	CZE	0,148	0,314	0,046	84	TTO	0,361	0,097	0,035
44	ETH	0,042	0,799	0,033	85	UAE	0,019	0,239	0,004

For a better interpretation of the results, we developed a two-dimensional representation of the efficient frontier. This representation is illustrated in Figure 3.

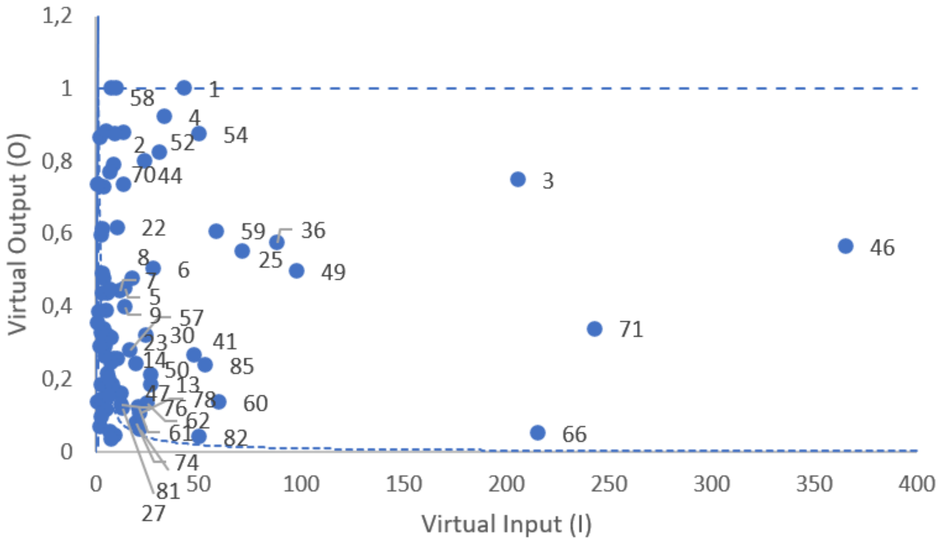

Figure 3 – Two-dimensional representation for country assessment

As in Figure 3, many DMUs are located far from the overall system efficiency frontier, another two-dimensional representation was developed with DMUs that have a virtual input value less than 10. This graph is illustrated in Figure 4. In both representations, the frontier that has a slope of 45 degrees in relation to the x-axis is related to the overall system, the vertical frontier is related to the first stage, while the horizontal frontier is related to the second stage. The hyperbola divides DMUs into those that are more efficient in the first stage than in the second (which are below the hyperbola) and those that are more efficient in the second stage compared to the first (which are above the hyperbola).

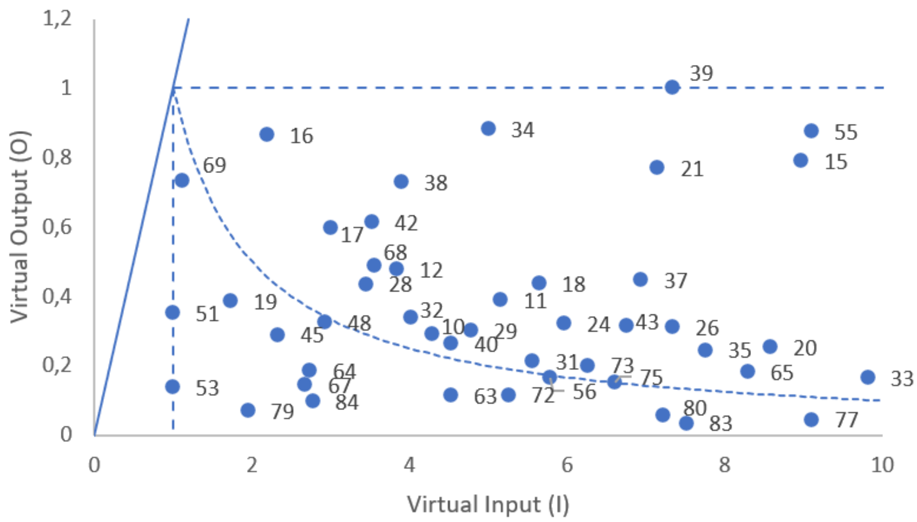

Figura 4 – Two-dimensional representation with zoom for country assessment

5. DISCUSSION

It can be seen from the figures that no DMU is represented at the overall system efficient frontier, however, the closest is DMU 69, which represents the country of Granada, which has an efficiency of 65.5%. DMUs 51 and 53 (Bahamas and Fiji) are efficient countries in the first stage and are represented at the first stage efficient frontier, while DMUs 1, 39, and 58 (United States of America, Azerbaijan, and Tajikistan) are efficient in the second stage and are located at the efficient frontier relative to the efficiency of the second stage.

In the first stage, it is possible to observe that, in general, DMUs with a better performance in the lexicographic method do not have a good result, that is, they are unable to generate many Olympic athletes in the proportion of other countries such as Bahamas and Fiji, which have low GDP and population. Granada, which is the DMU with the best efficiency result globally, has a good result at this stage, because, despite having classified only 6 athletes, it is the country with the lowest population and GDP among those listed.

In the second stage, the best-performing countries in the lexicographic method have a better result. The United States of America, Great Britain, China, and Russia achieved a result above 70% at this stage and are close to that frontier. Also noteworthy is Azerbaijan, which with just 56 athletes managed to win 18 medals, half of them in the Olympic fight, and Tajikistan, which won a gold medal in the men's hammer throw, with only 7 athletes in its delegation.

In general, countries performed better in the second stage than in the first stage. This can be seen in Figures 3 and 4, as there are clearly more DMUs located above than below the hyperbola that divides the DMUs. This shows that, in general, there is a greater margin of growth in the first stage than in the second, that is, countries could classify more Olympic athletes in relation to their GDP and population compared to other nations.

In the overall system, DMUs 69, 16, and 51 (Granada, Jamaica, and Bahamas) stand out. Granada, despite having the smallest population and the lowest GDP, still managed to win a silver medal, in the 400 meters of men's athletics. Jamaica, despite also having a relatively low GDP and population, has a long tradition in track and field and has won 11 medals, all in this modality. Usain Bolt and Elaine Thompson, both Jamaican athletes, won 3 medals each, two in individual competitions and one in relays. On the other hand, Bahamas stood out for being efficient in the first stage and by having won 1 gold and 1 bronze medal.

Special attention was paid to the host country. Brazil, despite having won its largest number of medals in an edition of the Summer Olympics, obtained an efficiency of only 0.7%. This was because, in the first stage, Brazil was hampered by the fact of having one of the largest populations and GDPs in the world, which is not reflected in the size of its delegation. Regarding the second stage, as Brazil was the host country, it received qualifications for the participation of athletes who did not obtain Olympic indexes or classifications, thus, it is expected that the delegation will increase in size with athletes who do not have the potential to win a medal, which impairs his performance at this stage. With poor performances in the first and second stages, the overall system, therefore, has a very low value.

Finally, through Figures 3 and 4 we can identify the DMUs that have a very poor performance. For example, we can note that DMUs 3, 66, 71, and 46 (China, India, Philippines, and Indonesia) perform poorly in the first stage. These countries are similarly characterized by the fact that they have a very high population and GDP, so they must analyze how they can increase the size of their delegation in proportion to their indicators. In the second stage, the DMUs that stand out negatively are 83, 82, and 77 (Portugal, Nigeria, and Austria), these DMUs have in common the fact that they have delegations of more than 70 athletes and won only a bronze medal. In this case, if the cited countries want to obtain a better classification in the proposed ranking, it is suggested to invest more in top athletes, to obtain more chances of winning medals, and less in the classification of athletes and teams that have little chance of victory.

6. CONCLUSION

The study evaluated countries that won at least one medal at the Rio 2016 Olympics, in relation to their population, their GDP, and the size of their delegation. For this study, we used the NDEA model of overall system efficiency composition, due to the characteristic of its orientations. The proposed two-dimensional representation helped in the general visualization of the DMUs. Through the proposed representation it was possible to simultaneously analyze the DMUs as to their efficiency in each stage.

Thus, it was possible to analyze the performance of each country, both in the overall system, as well as in the first stage that evaluates the training of athletes and in the second stage that evaluates the effective performance of these athletes.

Also, we have found the DMUs that obtained the best and worst performances, and the DMUs that are more efficient in one internal stage than in another. Bahamas and Fiji were the efficient countries in the first stage and were represented at the first stage efficient frontier, while United States of America, Azerbaijan, and Tajikistan were efficient in the second stage and were represented at the second stage efficient frontier. None of the countries were overall efficient, however, Granada was the most efficient country globally. In general, it was possible to observe a better performance in the second stage than in the first. In addition, the general low efficiency indicates that there is much room for improvement for all countries analyzed, including those that are well ranked in the lexicographic ranking.

As a future study we suggest the adequacy of the proposed modeling to the DEA zero-sum gain methodology, since a country can only obtain a greater number of medals if the others fail to obtain them. In addition, we suggest the application of dynamic DEA models that can analyze countries' performance over time as well.

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