A CLASS OF RATIO ESTIMATORS USING AUXILIARY INFORMATION FOR THE ESTIMATION OF POPULATION MEAN UNDER STRATIFIED SAMPLING

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ABSTRACT
In this paper we proposed a class of ratio estimators using auxiliary information for the estimation of population mean under stratified sampling. The proposed estimator in this work is based on the Yadav and Baghel (2021) estimator. The expression of the mean square error for this class of estimators is derived and the performance of the proposed estimator is compared with competing estimators. Numerical illustration of efficiency comparison is also worked out in the paper.

KEYWORDS: Auxiliary Variable, Study Variable, Stratified Sampling, Constants, Bias, MSE, PRE.

MSC: 62D05

RESUMEN
En este paper proponemos una clase de estimadores de razón usando información auxiliar de la media de la población en el muestreo estratificado. El propuesto estimador en este trabajo se basa en Yadav and Baghel (2021). La expresión del error cuadrático medio para esta clase de estimadores es derivada y su comportamiento es comparado con competidores. Numéricas comparaciones de la eficiencia También son trabajados en este paper.

PALABRAS CLAVE: Variable Auxiliar, Variable de Estudio, Muestreo Estratificado, Constantes, Sesgo, MSE, PRE.

1. INTRODUCTION

A ratio estimators commonly used when the study variable Y is highly correlated with the auxiliary variable X. Cochran (1940) was the first who used auxiliary information in development of ratio estimator. Several authors who used information on auxiliary variable for the estimation of population parameters are among Srivastava and Jhajj (1981, 83), Sahai and Rai (1980), Bahl and Tuteja (1991), Singh and Vishwakarma (2007).

There are numerous authors who have suggested different estimators by utilizing different population parameter of an auxiliary variable such as the mean, variance, coefficient of variation, coefficient of kurtosis, coefficient of skewness or the combination of the parameters as the component of concerned estimator. Different estimators are useful in different situation. Sisodia Dwivedi (1981) presented a proportion estimator by utilizing coefficient of variation of an auxiliary variable. Upadhaya and Singh (1999) examined a ratio type estimator by using the linear combination of coefficient of variation and kurtosis of an auxiliary variable. Kadilar and Cingi (2003) utilized the stratified forms of these specified estimators keeping in mind the end goal to improve the efficiency of the estimator. Yan and Tian (2010) suggested a ratio type estimator in simple random sampling using coefficient of skewness. Jerajuddin and Kishun (2016) modified ratio estimator for population mean using size of the sample selected from the population.

In this paper we proposed a class of ratio estimators using coefficient of variation, coefficient of kurtosis, and coefficient of skewness in stratified sampling and compared them, with estimators that are existing in literature in terms of their efficiency.

2. NOTATIONS

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Consider a finite population $U = \{U_1, U_2 \dots \dots \dots U_N\}$having N distinct and identifiable unit partioned into L Strata. Let y and x be the study and auxiliary variables taking values y_{hi} and x_{hi}, respectively for i^{th}unit ($i = 1,2, \dots.., N$) in h^{th} stratumConsisting of N_h units($h = 1,2 \dots L$) such that $\sum_{h=1}^{L} N_{h=} \ N$. Let n_h be the size of the sample for h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$.

Let, $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_{h,}$ Where $\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi}$; $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$, Where $\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{n_h} y_{hi}$ and $W_h = \frac{N_h}{N}$Is the stratum weight, we can define similar expressions for $\bar{x}_{st}$and $\bar{X}$ also.

$S^2{}_{yh} = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} (y_{hi} - \bar{y}_h)^2$and $S^2{}_{xh} = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} (x_{hi} - \bar{x}_h)^2$ Be the sample variance of y and x respectively in h^{th} stratum corresponding to the population variances.

$C^2{}_{xh} = \frac{S^2{}_{xh}}{\bar{X}^2}$, $C^2{}_{yh} = \frac{S^2{}_{yh}}{\bar{Y}^2}$ be thecoefficient of variation ofh^{th} stratum.

$C_{x(st)} = \sum_{h=i}^{L} W_h C_{xh}$, is the coefficient of variation of x.

$\beta_{1(x)st} = \sum_{h=1}^{L} W_h \, \beta_{1h}(x)$ And $\beta_{2(x)st} = \sum_{h=1}^{L} W_h \, \beta_{2h}(x)$ is the coefficient of skewness and coefficientkurtosis of x.

$C_{y(st)} = \sum_{h=i}^{L} W_h C_{yh}$, is the coefficient of variation of x.

$\beta_{1(y)st} = \sum_{h=1}^{L} W_h \, \beta_{1h}(y)$ And $\beta_{2(y)st} = \sum_{h=1}^{L} W_h \, \beta_{2h}(y)$ is the coefficient of skewness and coefficientkurtosis of y.

$\delta_h = \frac{1}{n_h} - \frac{1}{N_h}$ is f p c within h^{th}stratum.

For deriving the expression of Bias and MSE of various estimators we define the following terms by the motivation of Singh, S. (2003)

$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h = \bar{Y}(1 + e_{0st})$

$\bar{y}_{st} = \bar{Y} + \bar{Y} e_{0st} = \sum_{h=1}^{L} W_h \bar{y}_h$

$$e_{0st} = \frac{\sum_{h=1}^{L} W_h \bar{y}_h - \bar{Y}}{\bar{Y}}$$

$$E(e_{0st}) = 0 \qquad (2.1)$$

And

$\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$, $\bar{x}_{st} = \bar{X}(1 + e_{1st})$, $\sum_{h=1}^{L} W_h \bar{x}_h = \bar{X}(1 + e_{1st})$, $e_{1st} = \frac{\sum_{h=1}^{L} W_h \bar{x}_h - \bar{X}}{\bar{X}}$

$$E(e_{1st}) = 0 \qquad (2.2)$$

Where $\bar{y}_{st}$ and $\bar{x}_{st}$ are unbiased estimators of population mean $\bar{Y}$ $and \bar{X}$ respectively

Then

$$E(e_{0st}{}^2) = \frac{1}{\bar{Y}^2} \sum_{h=1}^{L} W_h{}^2 \, \delta_h S^2{}_{hy} \qquad (2.3)$$

$$E(e_{1st}{}^2) = \sum_{h=1}^{L} W_h{}^2 \, \delta_h \frac{S_{xh}{}^2}{\bar{X}^2} \qquad (2.4)$$

$$E(e_{0st} e_{1st}) = \sum_{h=1}^{L} W_h{}^2 \, \delta_h \frac{S_{yxh}}{\bar{Y}\bar{X}} \qquad (2.5)$$

3. SOME OF THE EXISTING ESTIMATORS IN STRATIFIED SAMPLING

Some estimators from the literature are considered for the review and are presented in Table1 along with their Mean square Error and corresponding constant up to the approximation of order one, Following Koyuncu, N. and Kadilar, C. (2009) we adapted these estimators in stratified sampling.

Table (1) the stratified estimator

S.No	Estimator	MSE	Constant

1	$\boldsymbol{t_{st(0)}}=\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_{h,}$ Sample mean	$\bar{Y}^2 \sum_{h=1}^{L} {W_h}^2 \delta_h {C^2}_{hy}$	-
2	$t_{st1} = \bar{y}_{st} \frac{\bar{X}}{\bar{x}_{st}}$	$\bar{Y}^2 \sum_{h=1}^{L} W_h^2 \delta_h \left(C_{hy}^2 + C_{hx}^2 - 2C_{hxy}\right)$	-
3	$t_{st2} = \bar{y}_{st} exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right)$ Stratified version of Bahl, Tuteja(1991)	$\bar{Y}^2 \sum_{h=}^{L} W_h^2 \delta_h \left(C_{hy}^2 + \frac{1}{4} C_{xh}^2 - C_{hxy}\right)$	-
4	$t_{st3}=\bar{y}_{st}\left(\frac{\bar{X}C_{xst}+\beta_{2(x)st}}{\bar{x}_{st}C_{xst}+\beta_{2(x)st}}\right)$ Adapted from Upadhaya and Singh (1999)	$\bar{Y}^2 \sum_{h=1}^{L} W_h^2 \delta_h \left(C_{hy}^2 + \theta^2 C_{hx}^2 - 2\theta C_{hxy}\right)$	$\boldsymbol{\theta} = \left(\frac{\bar{X}C_{xst}}{\bar{X}C_{xst} + \beta_{2(x)st}}\right)$
5	$t_{st4}=\bar{y}_{st}\left(\frac{\bar{X}+\beta_{(x)st}}{\bar{x}_{st}+\beta_{(x)st}}\right)$ Adapted from Yan and Tian (2010)	$\bar{Y}^2 \sum_{h=1}^{L} W_h^2 \delta_h \left(C_{hy}^2 + \theta^2 C_{hx}^2 - 2\theta C_{hxy}\right)$	$\boldsymbol{\theta} = \left(\frac{\bar{X}}{\bar{X} + \beta_{1(x)st}}\right)$
6	$t_{st5}=\bar{y}_{st}\left(\frac{\bar{X}+n}{\bar{x}_{st}+n}\right)$ Adapted from Jerajuddin and Kishun (2016)	$\bar{Y}^2 \sum_{h=1}^{L} W_h^2 \delta_h \left(C_{hy}^2 + \theta^2 C_{hx}^2 - 2\theta C_{hxy}\right)$	$\boldsymbol{\theta} = \left(\frac{\bar{X}}{\bar{X} + n}\right)$

4. PROPOSED CLASS OF ESTIMATORS

Following Yadav and Baghel (2021), we suggest a class of ratio type estimator for estimating the population mean in stratified sampling

$$t_{stp} = k\bar{y}_{st}\left(\frac{a_{st}\bar{X}+b_{st}}{a_{st}\bar{x}_{st}+b_{st}}\right) \tag{4.1}$$

Where constant k is suitably chosen such that the MSE of the suggested estimator is minimum and a_{st}, b_{st} are either real constant or the function of the known parameters for hth stratum of the auxiliary variable x, such as coefficient of variation $C_{x(st)} = \sum_{h=i}^{L} W_h C_{xh}$, coefficient of skewness,$\beta_{1(x)st} = \sum_{h=1}^{L} W_h\, \beta_{1h}(x)$,coefficient of kurtosis$\beta_{2(x)st} = \sum_{h=1}^{L} W_h\, \beta_{2h}(x)$.

4.1. Bias and MSE of the proposed estimator

Expressing t_{st} in terms of $e_{ist}(i = 0,1)$ we can write (4.1) as

$$t_{stp} = k\bar{Y}\,(1 + e_{0st})\left(\frac{a_{st}\bar{X}+b_{st}}{a_{st}\bar{X}+b_{st}+e_{1st}a_{st}\bar{X}}\right)$$

$$t_{stp} = k\bar{Y}\,(1 + e_{0st})\left(\frac{a_{st}\bar{X}+b_{st}}{(a_{st}\bar{X}+b_{st})\left(1+\frac{e_{1st}a_{st}\bar{X}}{(a_{st}\bar{X}+b_{st})}\right)}\right)$$

$$t_{stp} = k\bar{Y}\,(1 + e_{0st})\left(\frac{1}{1+\theta e_{1st}}\right)$$

Where $\theta = \frac{a_{st}\bar{X}}{(a_{st}\bar{X}+b_{st})}$

$$t_{stp} = k\bar{Y}\,(1 + e_{0st})(1 + \theta e_{1st})^{-1} \tag{4.2}$$

$t_{stp} = k\bar{Y}\,(1 + e_{0st})\,(1 - \theta e_{1st} + \theta^2 e_{1st}^2 - \cdots \ldots \ldots)$
$t_{stp} = k\bar{Y}$ - $k\bar{Y}\,\theta e_{1st} + k\bar{Y}\,\theta^2 e_{1st}^2$ $+k\bar{Y}\,e_{0st}$-$k\bar{Y}\,\theta e_{0st}e_{1st}$ $+k\bar{Y}\,\theta^2 e_{0st}e_{1st}^2+\ldots\ldots.$
Using Taylor series expression in equation (4.2) then multiplying its term and relating the term up to the first order of approximation

$$t_{stp} = k\bar{Y}\,[1 + e_{0st} - \theta e_{1st} - \theta e_{0st}e_{1st} + \theta^2 e_{1st}^2 + ..]$$

$$t_{stp} - \bar{Y} = \bar{Y}\,[k(1 + e_{0st} - \theta e_{1st} - \theta e_{0st}e_{1st} + \theta^2 e_{1st}^2 + \cdots.) - 1] \tag{4.3}$$

$$E\left(t_{stp} - \bar{Y}\right) = \bar{Y}\,\left[k\left\{1 - \theta\sum_{h=1}^{L} W_h^2\delta_h\frac{S_{yxh}}{\bar{Y}\bar{X}} + \theta^2\sum_{h=1}^{L} W_h^2\delta\frac{S_{xh}^2}{\bar{X}^2}\right\} - 1\right]$$

$$B\left(t_{stp}\right) = \bar{Y}\left[k\left\{1 - \theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy} + \theta^2\sum_{h=1}^{L} W_h^2\delta_h C_{xh}^2\right\} - 1\right] \tag{4.4}$$

On Squaring both side Equation (4.3) and taking expectations

$$E\left(t_{stp} - \bar{Y}^2\right)^2 = \bar{Y}^2 E[k^2(1 + e_{0st} - \theta e_{1st} - \theta e_{0st}e_{1st} + \theta^2 e_{1st}^2)^2 + 1 - 2k(1 + e_{0st} - \theta e_{1st} - \theta e_{0st}e_{1st} + \theta^2 e_{1st}^2)]$$

$$E\left(t_{stp} - \bar{Y}^2\right)^2 = \bar{Y}^2 E[k^2(1 + e_{0st}^2 + \theta^2 e_{1st}^2 + 2e_{0st} - 2\theta e_{1st} - 2\theta e_{0st}e_{1st} + 2\theta^2 e_{1st}^2 - 2\theta e_{0st}e_{1st}) + 1 - 2k(1 + e_{0st} - \theta e_{1st} - \theta e_{0st}e_{1st} + \theta^2 e_{1st}^2)]$$

$$MSE\left(t_{stp}\right) = \bar{Y}^2[k^2(1 + E(e_{0st}^2) + \theta^2 E(e_{1st}^2) + 2E(e_{0st}) - 2\theta E(e_{1st}) - 2\theta E(e_{0st}e_{1st}) + 2\theta^2(e_{1st}^2) - 2\theta E(e_{0st}e_{1st}) + 1 - 2k(1 + E(e_{0st}) - \theta E(e_{1st}) - \theta E(e_{0st}e_{1st}) + \theta^2(e_{1st}^2)]$$

From equation (2.1), (2.2), (2.3), (2.4), (2.5) we can write the following expression

$$MSE(t_{stp}) = \bar{Y}^2\left[k^2\left\{1 + \sum_{h=1}^{L} W_h^2\,\delta_h\frac{S_{yh}^2}{\bar{Y}^2} + \theta^2\sum_{h=1}^{L} W_h^2\,\delta_h\frac{S_{xh}^2}{\bar{X}^2} - 2\theta\sum_{h=1}^{L} W_h^2\delta_h\frac{S_{yxh}}{\bar{Y}\bar{X}} + 2\theta^2\sum_{h=1}^{L} W_h^2\,\delta_h\frac{S_{xh}^2}{\bar{X}^2} - 2\theta\sum_{h=1}^{L} W_h^2\delta_h\frac{S_{yxh}}{\bar{Y}\bar{X}}\right\} + 1 - 2k\left\{1 - \theta\sum_{h=1}^{L} W_h^2\delta_h\frac{S_{yxh}}{\bar{Y}\bar{X}} + \theta^2\sum_{h=1}^{L} W_h^2\,\delta_h\frac{S_{xh}^2}{\bar{X}^2}\right\}\right]$$

$$MSE(t_{stp}) = \bar{Y}^2\left[k^2\left\{1 + \sum_{h=1}^{L} W_h^2\,\delta_h C_{yh}^2 + 3\theta^2\sum_{h=1}^{L} W_h^2\,\delta_h C_{xh}^2 - 4\theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy}\right\} + 1 - 2k\left\{1 - \theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy} + \theta^2\sum_{h=1}^{L} W_h^2\,\delta_h C_{xh}^2\right\}\right] \tag{4.5}$$

Differentiate equation (4.5) w. r.to k and equating to zero for obtaining the optimum value of k, The optimum value of k which makes the MSE in equation (4.5) minimum is given by,

$$\frac{\partial MSE(t_{stp})}{\partial k} = \bar{Y}^2\left[2k\left\{1 + \sum_{h=1}^{L} W_h^2\,\delta_h C_{yh}^2 + 3\theta^2\sum_{h=1}^{L} W_h^2\,\delta_h C_{xh}^2 - 4\theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy}\right\} - 2\left\{1 - \theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy} + \theta^2\sum_{h=1}^{L} W_h^2\,\delta_h C_{xh}^2\right\}\right]$$

$$0 = \left[k\left\{1 + \sum_{h=1}^{L} W_h^2\,\delta_h C_{yh}^2 + 3\theta^2\sum_{h=1}^{L} W_h^2\,\delta_h C_{xh}^2 - 4\theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy}\right\} - \left\{1 - \theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy} + \theta^2\sum_{h=1}^{L} W_h^2\,\delta_h C_{xh}^2\right\}\right]$$

$$k\left\{1 + \sum_{h=1}^{L} W_h^2\,\delta_h C_{yh}^2 + 3\theta^2\sum_{h=1}^{L} W_h^2\,\delta_h C_{xh}^2 - 4\theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy}\right\} = \left\{1 - \theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy} + \theta^2\sum_{h=1}^{L} W_h^2\,\delta_h C_{xh}^2\right\}$$

$$k = \frac{1 - \theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy} + \theta^2\sum_{h=1}^{L} W_h^2\delta_h C_{xh}^2}{1 + \sum_{h=1}^{L} W_h^2\delta_h C_{yh}^2 + 3\theta^2\sum_{h=1}^{L} W_h^2\delta_h C_{xh}^2 - 4\theta\sum_{h=1}^{L} W_h^2\delta_h C_{hxy}} = \frac{A}{B} \tag{4.6}$$

Thus the minimum value of MSE of t_{st} is given as

$$M\left(t_{stp}\right)_{min} = \bar{Y}^2\left(1 - \frac{A^2}{B}\right) \tag{4.7}$$

Putting the value of k in equation (4.4) we get the bias as

$$B\left(t_{stp}\right) = \bar{Y}\left(\frac{A^2}{B} - 1\right) \tag{4.8}$$

5. SOME MEMBERS OF PROPOSED CLASS OF ESTIMATORS

We have suggested some members of proposed class of estimators [by the motivation of Yadav and Baghel (2021)]which come out to be more efficient than the existing estimators of population mean. These are given as follows

$t_{st(p1)}=k_1\bar{y}_{st}\left(\frac{\bar{X}+C_{xst}}{\bar{x}_{st}+C_{xst}}\right)$

$t_{st(p2)}=k_2\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)$

$t_{st(p3)}=k_3\bar{y}_{st}\left(\frac{\bar{X}+\beta_{2(x)st}}{\bar{x}_{st}+\beta_{2(x)st}}\right)$

$t_{st(p4)}=k_4\bar{y}_{st}\left(\frac{\bar{X}+1}{\bar{x}_{st}+1}\right)$

$t_{st(p5)}=k_5\bar{y}_{st}\left(\frac{\bar{X}\beta_{2(x)st}+1}{\bar{x}_{st}\beta_{2(x)st}+1}\right)$

$t_{st(p6)}=k_6\bar{y}_{st}\left(\frac{\bar{X}\beta_{2(x)st}+C_{xst}}{\bar{x}_{st}\beta_{2(x)st}+C_{xst}}\right)$

$t_{st(p7)}=k_7\bar{y}_{st}\left(\frac{\bar{X}\beta_{1(x)st}+C_{xst}}{\bar{x}_{st}\beta_{1(x)st}+C_{xst}}\right)$

$t_{st(p8)}=k_8\bar{y}_{st}\left(\frac{\bar{X}\beta_{2(x)st}+\beta_{1(x)st}}{\bar{x}_{st}\beta_{2(x)st}+\beta_{1(x)st}}\right)$

$t_{st(p9)}=k_9\bar{y}_{st}\left(\frac{\bar{X}+\beta_{1(x)st}}{\bar{x}_{st}+\beta_{1(x)st}}\right)$

$t_{st(p10)}=k_{10}\bar{y}_{st}\left(\frac{\bar{X}C_{xst}+1}{\bar{x}_{st}C_{xst}+1}\right)$

$t_{st(p11)}=k_{11}\bar{y}_{st}\left(\frac{\bar{X}\beta_{1(x)st}+\beta_{2(x)st}}{\bar{x}_{st}\beta_{1(x)st}+\beta_{2(x)st}}\right)$

$t_{st(p12)}=k_{12}\bar{y}_{st}\left(\frac{\bar{X}C_{xst}+\beta_{1(x)st}}{\bar{x}_{st}C_{xst}+\beta_{1(x)st}}\right)$

Table (2) some member of proposed class of estimators

S.No.	**Estimators**	a_{st}	b_{st}	**Constant**
1	$t_{st(p1)}$	1	C_{xst}	$\theta_1 = \frac{\bar{X}}{\bar{X}+C_{xst}}$
2	$t_{st(p2)}$	1	0	$\theta_2 = 1$
3	$t_{st(p3)})$	1	$\beta_{2(x)st}$	$\theta_3 = \frac{\bar{X}}{\bar{X}+\beta_{2(x)st}}$
4	$t_{st(p4)}$	1	1	$\theta_4 = \frac{\bar{X}}{\bar{X}+1}$
5	$t_{st(p5)}$	$\beta_{2(x)st}$	1	$\theta_5 = \frac{\bar{X}\beta_{2(x)st}}{\bar{X}\beta_{2(x)st}+1}$

6	$t_{st(p6)}$	$\beta_{2(x)st}$	C_{xst}	$\theta_6 = \frac{\bar{X}\beta_{2(x)st}}{\bar{X}\beta_{2(x)st} + C_{xst}}$
7	$t_{st(p7)}$	$\beta_{1(x)st}$	C_{xst}	$\theta_7 = \frac{\bar{X}\beta_{1(x)st}}{\bar{X}\beta_{1(x)st} + C_{xst}}$
8	$t_{st(p8)}$	$\beta_{2(x)st}$	$\beta_{1(x)st}$	$\theta_8 = \frac{\bar{X}\beta_{2(x)st}}{\bar{X}\beta_{2(x)st} + \beta_{1(x)st}}$
9	$t_{st(p9)}$	1	$\beta_{1(x)st}$	$\theta_9 = \frac{\bar{X}}{\bar{X} + \beta_{1(x)st}}$
10	$t_{st(p10)}$	$\beta_{1(x)st}$	1	$\theta_{10} = \frac{\bar{X}\beta_{1(x)st}}{\bar{X}\beta_{1(x)st} + 1}$
11	$t_{st(p11)}$	$\beta_{1(x)st}$	$\beta_{2(x)st}$	$\theta_{11} = \frac{\bar{X}\beta_{1(x)st}}{\bar{X}\beta_{1(x)st} + \beta_{2(x)st}}$
12	$t_{st(p12)}$	C_{xst}	$\beta_{1(x)st}$	$\theta_{12} = \frac{\bar{X}C_{xst}}{\bar{X}C_{xst} + \beta_{1(x)st}}$

6. THEORETICAL EFFICIENCY COMPARISON

In this section the proposed classes of ratio estimator were compared theoretically with other existing estimator. Described in Table (1) namely $t_{st0}, t_{st1} \ldots\ldots t_{st5}$, we compared the MSE of our proposed estimator for all the 12 special cases described in Table (2)
Efficiency condition over some related existing estimators.

$$MSE(t)_{st} - MSE(t)_{stpi} > 0 \forall i = 1,2, \ldots \ldots 12$$

From the MSE of proposed ratio estimator (t_{stpi}) is better than sample mean per unit estimator (t_{st0}) if
(1)$MSEt_{st0}$ - $MSEt_{stpi} > 0$

$$\bar{Y}^2 \sum_{h=1}^{L} w_h^2 \delta_h C^2_{hy} - \bar{Y}^2 \left(1 - \frac{A^2}{B}\right) > 0$$

$$\left[\sum_{h=1}^{L} w_h^2 \delta_h C^2_{hy} + \frac{A^2}{B}\right] > 1 \quad (6.1)$$

Where, A=$1 - \theta \sum_{h=1}^{l} W_h^2 \delta_h C_{hxy} + \theta^2 \sum_{h=1}^{l} W_h^2 \delta_h C_{xh}^2$,

$$B = 1 + \sum_{h=1}^{l} W_h^2 \delta_h C_{yh}^2 + 3\theta^2 \sum_{h=1}^{l} W_h^2 \delta_h C_{xh}^2 - 4\theta \sum_{h=1}^{l} W_h^2 \delta_h C_{hxy}$$

When equation (6.1) is satisfied than (t_{stpi}) is more efficient than sample mean per unit estimator (t_{st0}).
Similarly we can define the following conditions as follows
(2) $MSE(t)_{st1} - MSE(t)_{stpi} > 0$

$$\left[\sum_{h=1}^{L} w_h^2 \delta_h (C^2_{hy} + C^2_{hx} - 2C_{hxy}) + \frac{A^2}{B}\right] > 1 \quad (6.2)$$

(3) $MSE(t)_{st2} - MSE(t)_{stpi} > 0$

$$\left[\sum_{h=1}^{L} w_h^2 \delta_h (C^2_{hy} + \frac{1}{4} C^2_{hx} - C_{hxy}) + \frac{A^2}{B}\right] > 1 \quad (6.3)$$

(4) $MSE(t)_{st3} - MSE(t)_{stpi} > 0$

$$\left[\sum_{h=1}^{L} W_h^2 \delta_h \left(C_{hy}^2 + \theta^2 C_{hx}^2 - 2\theta C_{hxy}\right) + \frac{A^2}{B}\right] > 1 \tag{6.4}$$

(5) $MSE(t)_{st4}$–$MSE(t)_{stpi}$> 0

$$\left[\sum_{h=1}^{L} W_h^2 \delta_h \left(C_{hy}^2 + \theta^2 C_{hx}^2 - 2\theta C_{hxy}\right) + \frac{A^2}{B}\right] > 1 \tag{6.5}$$

(6) $MSE(t)_{st5}$–$MSE(t)_{stpi}$> 0

$$\left[\sum_{h=1}^{L} W_h^2 \delta_h \left(C_{hy}^2 + \theta^2 C_{hx}^2 - 2\theta C_{hxy}\right) + \frac{A^2}{B}\right] > 1 \tag{6.6}$$

Under the condition derived above proposed estimator proves to be more efficient than the existing traditional estimator.

7. NUMERICAL ILLUSTRATION

To illustrate numerical meaning of the theoretical results, the following real data sets are considered as:
Population: **1** Here we use the data given in Onyeka (2012) to illustrate the properties of the estimators proposed in the present study. The data statistics consisting mainly of population parameters are shown in table 3, while table 5show PRE and MSE of the estimators.

$$\text{PRE} = \frac{V(t_{0)})}{MSE(.)}*100$$

Table (3) parametric values of the population (1)

Population	Males = Stratum 1	Females = Stratum 2
N=96	N_1 =72	N_2 =24
n=20	n_1 =8	n_2 =12
$\bar{Y}$ =2.44	$\bar{Y}_1$=2.44	$\bar{Y}_2$=2.46
$\bar{X}$ =68.13	$\bar{X}_1$=68.11	$\bar{X}_2$=68.17
S_y^2 =0.33	S_{1y}^2=0.35	S_{2y}^2 =0.25
S_x^2 =49.37	S_{1x}^2=52.97	S_{2x}^2=40.41
S_{xy}=3.26	S_{1xy}=.3.43	S_{2xy} =2.75
C_x =0.10	C_{x1}=0.11	C_{x2}=0.09
C_y=0.23	C_{1y}=0.24	C_{2y} =0.20
$\beta_1(x)$=-1.10	$\beta_{11}(x)$=-1.23	$\beta_{12}(x)$=-0.50
$\beta_2(x)$=3.83	$\beta_{21}(x)$= 4.33	$\beta_{22}(x)$= 1.34
-	W_1=0.75	W_2=0.25
-	δ_1=0.11	δ_2=0.041

θ_1**=0.9985,** θ_2=1, θ_3=0.95004, $\theta_4 = 0.985533, \theta_5 = 0.99592, \theta_6 = 0.99957, \theta_7 = 1.00193, \theta_8 = 1.003278, \theta_9 = 1.01184, \theta_{10} = 1.018751 \theta_{11} = 1.0705, \theta_{12} = 1.125$

Population: 2 The second population, is taken from the Census of India 2011(Uttar Pradesh , Series 10, Part 12B and District census hand book, AGRA).

The considered data relates to total area of 45 villages of Khandauli block at Agra districts (U.P). We consider the numbers of Agricultural laborers in villages as study variable$y$ and the total area of villages as auxiliary variable$x$

We divided the whole population of 45 villages is divided in to 5 strata according to the Area. Accordingly we have:

Strata	Area in Hectare
1	(1-4400) (21 Villages)
2	(4400-8400) (10 Villages)
3	(8400-12400) (6 Villages)
4	(12400-16500) (5Villages)
5	(16500-20900) (3Villages)

Table (4) parametric values of the population (2)

Population	Stratum 1	Stratum 2	Stratum 3	Stratum 4	Stratum 5
$N=45$	N_1 =21	N_2 =10	N_3 =6	N_4 =5	N_5 =3
$n=23$	n_1 =10	n_2 =5	n_2 =3	n_2 =3	n_2=2
$\bar{Y}=173.508$	$\bar{Y}_1$=112.09	$\bar{Y}_2$=175.9	$\bar{Y}_3$=149.83	$\bar{Y}_4$=232.2	$\bar{Y}_5$=545
$\bar{X}=463.37$	$\bar{X}_1$=196.4	$\bar{X}_2$=413.411	$\bar{X}_3$=672.33	$\bar{X}_4$=844.26	$\bar{X}_5$=1445.97
	W_1=0.467	W_2=0.23	W_3=0.14	W_4=0.12	W_5=0.067
	S_{1y}^2=13172.99	S_{2y}^2 =17740.32	S_{3y}^2=9496.96	S_{4y}^2=16815.95	S_{5y}^2 =216549
	S_{1x}^2=6707.41	S_{2x}^2=2483.022	$S_{3x=}^2$6053.912	S_{4x}^2=1287.989	S_{5x}^2=161529
	S_{1xy}=2867.	S_{2xy} =202.84	S_{3xy} =554.05	S_{4xy}=-3291.6	S_{5xy} =123496.7
	C_{x1}=0.1767	C_{x2}=0.1075	C_{x3}=0.1679	C_{x4}=0.077	C_{x5}=0.867
	C_{1y}=0.6614	C_{2y} =0.767	C_{3y}=0.5616	C_{4y}=0.7473	C_{5y}=2.6819
	$\beta_{11}(x)$=-0.055	$\beta_{12}(x)$=0.909	$\beta_{13}(x)$ =-1.8848	$\beta_{14}(x)$=0.723	$\beta_{15}(x)$=-0.9810
	$\beta_{21}(x)$= -1.049	$\beta_{22}(x)$= -0.0050	$\beta_{23}(x)$ =4.02	$\beta_{24}(x)$=-1.663	$\beta_{25}(x)$=-4.024

θ_1=**0.999**, θ_2=1, θ_3=1.00086, $\theta_4=0.99784$, $\theta_5=1.0054$, $\theta_6=1.00107$, $\theta_7=1.00723$, $\theta_8=0.9997$, $\theta_9=1.00013$, $\theta_{10}=1.03752$, $\theta_{11}=0.9858$, $\theta_{12}=1.00065$

Table (5) MSE and PRE of Estimators for Population (1)

S No	Estimators	MSE	PRE
1	$t_{st0}=V(t_{0st})$	0.022296875	100
2	t_{st1}	0.10912238	204.33
3	t_{st2}	0.01551878	143.68
4	t_{st3}	0.01308633	170.38
5	t_{st4}	0.011204858	198.99
6	t_{st5}	0.012718053	175.31
7	t_{stp1}	0.010911208	204.35
8	t_{stp2}	0.01090309	204.50
9	t_{stp3}	0.01062527	209.84
10	t_{stp4}	0.01036637	215.09
11	t_{stp5}	0.01029267	216.63
12	t_{stp6}	0.010266984	217.17
13	t_{stp7}	0.010250399	217.52
14	t_{stp8}	0.010241016	217.72
15	t_{stp9}	0.010181509	218.99
16	t_{stp10}	0.01013394	220.02
17	t_{stp11}	0.009790284	227.75
18	t_{stp12}	0.009454697	235.82

Table (6) MSE and PRE of Estimators for Population (1)

S.No	Estimators	MSE	PRE
1	$t_{st0}=V(t_{0st})$	469.66	100
2	t_{st1}	411.15	114.23
3	t_{st2}	432.24	108.66
4	t_{st3}	455.8	103.04
5	t_{st4}	455.9	103.01
6	t_{st5}	456.4	102.90
7	t_{stp1}	405.96	115.71
8	t_{stp2}	405.93	115.70
9	t_{stp3}	405.93	115.70
10	t_{stp4}	406.009	115.67
11	t_{stp5}	405.81	115.73
12	t_{stp6}	405.92	115.70

13	t_{stp7}	405.76	155.74
14	t_{stp8}	405.96	115.69
15	t_{stp9}	405.95	115.69
16	t_{stp10}	405.03	115.96
17	t_{stp11}	406.32	115.58
18	t_{stp12}	405.94	115.69

From the above table 5,6, it can be easily seen that the newly proposed ratio estimators in this study demonstrated high relative efficiency over existing related estimators.

8. CONCLUSION

In this paper we propose a class of ratio estimator for estimating population mean in stratified sampling, utilizing information on different measures based on auxiliary variable. Table I given some of the adapted estimators for stratified sampling. Table 2 listed various estimators as the members of the proposed class. The following interpretations can be made from the numerical illustrations, performed for two populations taken from (1) Onyeka (2012) and (2) District census handbook (2011) India (Agra).
Table 3 and table 4 given the various required parametric values for the above two populations.
Table 5 and table 6 shows the MSE and PRE of the estimators derived from the proposed class of estimator and listed in Table 2 for the above two population respectively
In population (1) the newly proposed ratio estimator $t_{stp12}$is the most efficient estimators with PRE of 235.82, followed by$t_{stp1,,........}t_{stp11}$ in that order, also in population (2) the newly proposed ratio estimator t_{stp10} is the most efficient estimator with PRE of 115.96 followed by $t_{stp11}, t_{stp4,} t_{stp1}, t_{stp2}, t_{stp3}, t_{stp5}, t_{stp6}, t_{stp7}, t_{stp8}, t_{stp9}$ and t_{st12}in that order.
In conclusion the newly proposed class of ratio estimators is stratified version of Yadav and Baghel (2021) estimator of population mean using auxiliary information based on empirically findings. We can easily notice that proposed class of ratio estimator perform better than existing estimators.

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