STOCHASTIC MODELING OF SINGLE UNIT SYSTEMS WITH DELIBERATED FAILURES SUBJECT TO WARRANTY

Ashish Kumar*, Ravi Chaudhary*, Kapil Kumar** and Monika Saini[1]*
*Department of Mathematics & Statistics, Manipal University Jaipur, Jaipur, India
[2]Department of Statistics, Central University of Haryana, Mahendragarh, India

ABSTRACT
The prominent objective of present study is to analyse the impact of warranty and deliberated failures on single unit systems. A novel stochastic model is developed using regenerative point technique (RPT) and semi-Markovian approach (SMA) for a single unit system. The system suffers due to two types of failures with warranty period namely normal failure and deliberated failure. Warranty period for the system is prespecified and after that no provision of extended warranty is made. All kinds of failure rates, repair rates are Lindley distributed while warranty period follows exponential distribution. Various measures of system effectiveness derived. The numerical values of these measures are derived for a particular case with respect to failure and repair rates.

KEYWORDS: Single-Unit System, Warranty, Deliberated Failure, Lindley distribution.

MSC: 90B05, 60J10

RESUMEN
El prominente objetivo del presente estudio es analizar el impacto de la garantía y las deliberadas fallas en sistema de unidades sencillas. Un novedoso modelo estocástico es desarrollado mediante la técnica de regeneración puntual (RPT) y el enfoque semi-Markoviano (SMA), en sistemas de unidades sencillas. El sistema sufre debido a dos tipos de falla con periodo de garantía llamado de fallas normales y deliberadas. El periodo de garantía para el sistema es pre-especificado y después no se provee de extensión de esta. Todo tipo de tasas de fallas es hecho. Todos ellos , y de reparación se distribuyen Lindley mientras que la garantía sigue una es una exponencial . Varias medidas de la efectividad del son derivadas. Los valores numéricos de ellas se derivan para casos particulares respecto a las tasas de fallas y de reparación.

PALABRAS CLAVE: Sistema de Unidades Sencillas, Garantía. Falla , Deliberada, Distribución de Lindley.

1. INTRODUCTION

Reliability is the prime characteristic to measure the performance of systems like industrial, communication, medical and transport etc. Several techniques have been used to improve the reliability of the system. Redundancy is one of such technique that is utilized for reliability improvement. Several studies performed on reliability evaluation of redundant systems like Pundir et al. [13-14] and Niwas and Kadyan [10]. But it doesn't feasible for all the systems to provide redundant unit. Sometime the cost of the system is so high that management of plant cannot provide second unit. In such situations use of single unit is recommended.
Therefore, the study of single unit systems get in focus and reliability models of single-unit systems with different failure modes have also been probed by the researchers like Ram Niwas et al. [11], Kumar and Saini [6], Gupta [4], Kumar et al. [7] and Saini and Kumar [15] with the assumptions that the unit has a constant failure rate, work perfectly as a new one after repair and maintenance, immediate repair whenever failure is occurred. The system suffers due to several types of failure like normal failure, common cause failure, human failure, and deliberated failures. But the impact of deliberated failures is not investigated yet on the single unit systems.
Though the concept of warranty is discussed in reliability investigation and used by several researchers like Dai et al. [2], Kumari [9], and Afsahi and Shafiee [1] under certain assumptions but it needs more investigation. Kumar [8] suggested a stochastic model for cost analysis of a repairable system under abnormal environmental conditions. Most of the studies conducted so far in reliability investigation by considering either exponential distribution or Weibull distribution. But some systems' behaviour does not exhibit by these distributions and requirement of other distributions is felt. Generally, biological, medical and engineering system's lifetime does not exhibit accurately by using constant failure rates. Many systems failure rate show non-monotone behaviour. And in such situations Lindley distribution is a best fitted distribution. Ghitany et al. [3] developed Lindley distribution and highlighted its application in reliability modelling and estimation of systems. Krishna and Kumar [5] estimated the reliability of Lindley distribution using progressive type II censored data. Recently, Nandal and Malik [12] developed a stochastic model for

[1] Email: ashish.kumar@jaipur.manipal.edu; ravifunspace@gmail.com; kapilstats@gmail.com; drmnksaini4@gmail.com

three-unit system using Lindley distribution. But for single unit systems its applicability is not evaluated so far. So, in present study an effort has been made to analyse the impact of warranty and deliberated failures on single unit systems under Lindley distribution. A novel stochastic model is developed using regenerative point technique (RPT) and semi-Markovian approach (SMA) for a single unit system. The system suffers due to two types of failures withing warranty period namely normal failure and deliberated failure. Warranty period for the system is prespecified and after that no provision of extended warranty is made. All kinds of failure rates, repair rates are Lindley distributed while warranty period follows exponential distribution. Various measures of system effectiveness derived. The numerical values of these measures derived for a particular case with respect to failure and repair rates.
The whole manuscript is organized into six sections including the introductory first section. Notations and system descriptions given in section 2 and 3 respectively. Reliability measures derived in section 4 while numerical results appended in section 5. Section 6 presents the concluding remarks.

2. NOTATIONS

Warrantable failure	**WF**	Deliberated failure	**DF**
Normal failure after warranty	**NF**	Unit in operation under warranty	N_o
Unit fails deliberately under warranty period	DN_F	Unit fails in normal operation under warranty	WN_F
Unit in operation after warranty	N_{WO}	Unit fails deliberately after warranty	DN_{WF}
Unit fails in normal operation after warranty	N_{WF}	$\alpha_0 = \alpha_0 e^{-\alpha_0 t}, \alpha_0 > 0, t > 0$: Maximum warranty time	
$f_i(t) = \frac{\theta_i^2}{\theta_i + 1}(1+t)e^{-\theta_i t}, \theta_i > 0, t > 0, i = 1,2,3$		Probability density function of deliberated failures, failure within warranty time and failure beyond warranty time respectively.	
$g_j(t) = \frac{\beta_j^2}{\beta_j + 1}(1+t)e^{-\beta_j t}, \beta_j > 0, t > 0, j = 1,2,3$		Probability density function of repair rates of deliberated failures, failure within warranty time and failure beyond warranty time respectively.	

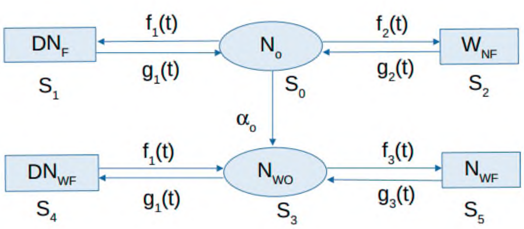
3. SYSTEM DESCRIPTION

The system consists of a single unit which may suffer due to various kind of failures. The system has a predefined maximum warranty period. The system may fail during operation in normal conditions during warranty period and it may also suffer due to deliberate failures. The customer may deliberately hamper the working of the system and resulted in its failure. There is no provision of extended warranty is made. Therefore, after completion of warranty period unit may suffer two kinds of failures. All the failure and repair rates follow Lindley distribution while maximum warranty time is exponentially distributed. The provision of single repairman is made to perform repair activities. All the repairs and switches Figure 1: State Transition Diagram of Single unit
system under warranty.
are perfect. The diagrammatic representation of the proposed model is shown in Figure 1. The applications of present study can be observed in the equipment's from automobile industry, mechanical machinery, aviation, medical sector etc.

4. RELIABILITY MEASURES

4.1 Transition Probabilities:

It is noted that the time to restart by system in any regenerative state known as regenerative point. Suppose T_i: 0,1,2,3….. be the entry time points in any regenerative state and X_i denote the visited state then $\{X_i, T_i\}$ becomes a Markov renewal process on space of regenerative states and $Q_{ij}(t)$represent semi-Markov kernel over set of regenerative states. The transition probabilities are derived using simple probabilistic arguments and equation (1).



$$p_{ij} = \lim_{t \to \infty} Q_{ij}(t) = \int_0^\infty q_{ij}(t)dt \quad (1)$$

All the non-zero transition probabilities are defined below:

$$p_{01} = \lim_{s\to 0} Q_{01}^{**}(s) = \frac{\theta_1^2}{(\theta_1+1)(\theta_2+1)}[\frac{1+\theta_2}{\theta_1+\theta_2+\alpha_0} + \frac{1+2\theta_2}{(\theta_1+\theta_2+\alpha_0)^2} + \frac{2\theta_2}{(\theta_1+\theta_2+\alpha_0)^3}]$$
$$p_{02} = \lim_{s\to 0} Q_{02}^{**}(s) = \frac{\theta_2^2}{(\theta_1+1)(\theta_2+1)}[\frac{1+\theta_1}{\theta_1+\theta_2+\alpha_0} + \frac{1+2\theta_1}{(\theta_1+\theta_2+\alpha_0)^2} + \frac{2\theta_1}{(\theta_1+\theta_2+\alpha_0)^3}]$$
$$p_{03} = \lim_{s\to 0} Q_{03}^{**}(s) = \frac{\alpha_0}{(\theta_1+1)(\theta_2+1)}[\frac{1+\theta_1+\theta_2+\theta_1\theta_2}{\theta_1+\theta_2+\alpha_0} + \frac{\theta_1+\theta_2+2\theta_1\theta_2}{(\theta_1+\theta_2+\alpha_0)^2} + \frac{2\theta_1\theta_2}{(\theta_1+\theta_2+\alpha_0)^3}]$$
$$p_{20} = \lim_{s\to 0} Q_{20}^{**}(s) = \frac{\beta_2^2}{\beta_2+1}[\frac{1}{\beta_2} + \frac{1}{(\beta_2)^2}] = 1$$
$$p_{34} = \lim_{s\to 0} Q_{34}^{**}(s) = \frac{\theta_1^2}{(\theta_1+1)(\theta_3+1)}[\frac{1+\theta_3}{\theta_1+\theta_3} + \frac{1+2\theta_3}{(\theta_1+\theta_3)^2} + \frac{2\theta_3}{(\theta_1+\theta_3)^3}]$$
$$p_{35} = \lim_{s\to 0} Q_{35}^{**}(s) = \frac{\theta_3^2}{(\theta_1+1)(\theta_3+1)}[\frac{1+\theta_1}{\theta_1+\theta_3} + \frac{1+2\theta_1}{(\theta_1+\theta_3)^2} + \frac{2\theta_1}{(\theta_1+\theta_3)^3}]$$
$$p_{43} = \lim_{s\to 0} Q_{43}^{**}(s) = \frac{\beta_1^2}{\beta_1+1}[\frac{1}{\beta_1} + \frac{1}{(\beta_1)^2}] = 1; p_{53} = \lim_{s\to 0} Q_{53}^{**}(s) = \frac{\beta_3^2}{\beta_3+1}[\frac{1}{\beta_3} + \frac{1}{(\beta_3)^2}] = 1$$

4.2 Mean Sojourn Times

The average time spent by system in any regenerative state before moving to any other state is termed as mean sojourn times at that state. If T_i represent the sojourn times, then mean sojourn time is give by equation (2).

$$\mu_i = \int_0^\infty P_r(T_i > t) = \sum_j m_{ij}\ ; \qquad \text{where} \qquad m_{ij} = -\frac{d}{ds}[Q_{ij}^{**}(s)]_{s=0} \tag{2}$$

The mean sojourn time at all states of present model is derived as follows:

$$\mu_0 = \int_0^\infty \overline{F_1(t)F_2(t)\overline{\alpha_0}}dt$$
$$\mu_0^{**}(s=0) = \frac{1}{(\theta_1+1)(\theta_2+1)}[\frac{1+\theta_1+\theta_2+\theta_1\theta_2}{\theta_1+\theta_2+\alpha_0} + \frac{\theta_1+\theta_2+2\theta_1\theta_2}{(\theta_1+\theta_2+\alpha_0)^2} + \frac{2\theta_1\theta_2}{(\theta_1+\theta_2+\alpha_0)^3}]$$
$$\mu_1^{**}(s=0) = \frac{1}{\beta_1+1}; \qquad \mu_2^{**}(s=0) = \frac{1}{\beta_2+1}$$
$$\mu_3^{**}(s=0) = \frac{1}{(\theta_1+1)(\theta_2+1)}[\frac{1+\theta_1+\theta_2+\theta_1\theta_2}{\theta_1+\theta_2} + \frac{\theta_1+\theta_2+2\theta_1\theta_2}{(\theta_1+\theta_2)^2} + \frac{2\theta_1\theta_2}{(\theta_1+\theta_2)^3}]$$
$$\mu_4^{**}(s=0) = \frac{1}{\beta_2+1};\ \ \mu_5^{**}(s=0) = \frac{1}{\beta_3+1}$$

4.3 Mean Time to System Failure (MTSF)

Let $\emptyset_i(t)$ be the c.d.f. of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\emptyset_i(t)$:

$$\emptyset_0(t) = Q_{03}(t)®\emptyset_3(t) + Z_0(t)$$
$$\emptyset_3(t) = Z_3(t) \tag{3}$$

Employing L.S.T. on equation (3), and simplifying for $\emptyset_0^{**}(s)$, we get

$$MTSF = \lim_{s\to 0}\frac{1-\emptyset_0^{**}(s)}{s} = \lim_{s\to 0}\frac{1-Z_0^{**}(s)-Q_{03}^{**}(s)Z_3^{**}(s)}{s} \tag{4}$$

After simplification equation (5) can be expressed as equation (6).

$$MTSF = \frac{1}{(\theta_1+1)(\theta_2+1)}[\frac{1+\theta_1+\theta_2+\theta_1\theta_2}{\theta_1+\theta_2+\alpha_0} + \frac{\theta_1+\theta_2+2\theta_1\theta_2}{(\theta_1+\theta_2+\alpha_0)^2} + \frac{2\theta_1\theta_2}{(\theta_1+\theta_2+\alpha_0)^3}].$$
$$[1 + (\frac{\alpha_0}{(\theta_1+1)(\theta_2+1)})(\frac{1+\theta_1+\theta_2+\theta_1\theta_2}{\theta_1+\theta_2} + \frac{\theta_1+\theta_2+2\theta_1\theta_2}{(\theta_1+\theta_2)^2} + \frac{2\theta_1\theta_2}{(\theta_1+\theta_2)^3})] \tag{6}$$

4.4 Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are given as:

$$A_0(t) = Z_0(t) + q_{01}(t)®A_1(t) + q_{02}(t)®A_2(t) + q_{03}(t)®A_3(t)$$
$$A_1(t) = q_{10}(t)®A_0(t)$$
$$A_2(t) = q_{20}(t)®A_0(t)$$
$$A_3(t) = Z_3(t) + q_{34}(t)®A_4(t) + q_{35}(t)®A_5(t)$$
$$A_4(t) = q_{43}(t)®A_3(t)$$
$$A_5(t) = q_{53}(t)®A_3(t) \tag{7}$$

Employing L.T. on equation (7), and simplifying for $A_0^{**}(s)$, we get

$$A(\infty) = \lim_{s\to 0} A_0^{**}(s) = \lim_{s\to 0} \frac{sN_1}{D_1}$$
$$N_1 = \mu_0(1 - P_{34}P_{43} - P_{35}P_{53}) + P_{03}\mu_3$$

$$D_1 = [1 - P_{34}P_{43} - P_{35}P_{53}][\mu_0 + P_{01}m_{10} + P_{02}m_{20}] + P_{03}[\mu_3 + P_{34}m_{43} + P_{35}m_{53}]$$

4.5 Busy Period Due to Repair

Let $B_i(t)$ be the probability that server is busy in repairing the failed unit at epoch "t" given that the system entered state S_i at t = 0. The recursive relations for $B_i(t)$ are given as:

$$B_0(t) = q_{01}(t)®B_1(t) + q_{02}(t)®B_2(t) + q_{03}(t)®B_3(t)$$
$$B_1(t) = Z_1(t) + q_{10}(t)®B_0(t)$$
$$B_2(t) = Z_2(t) + q_{20}(t)®B_0(t)$$
$$B_3(t) = q_{34}(t)®B_4(t) + q_{35}(t)®B_5(t)$$
$$B_4(t) = Z_4(t) + q_{43}(t)®B_3(t)$$
$$B_5(t) = Z_5(t) + q_{53}(t)®B_3(t) \quad (8)$$

Employing L.T. on equation (8), and simplifying for $B_0^{**}(s)$, we get

$$B(\infty) = \lim_{s\to 0} \frac{N_2 + sN'_2}{D'_2}$$
$$N_2 = [\mu_1 P_{01} + \mu_2 P_{02}][1 - P_{34}P_{43} - P_{35}P_{53}] + P_{03}[\mu_4 P_{34} + \mu_5 P_{35}]$$
$$D_2 = [1 - P_{34}P_{43} - P_{35}P_{53}][\mu_0 + P_{01}m_{10} + P_{02}m_{20}] + P_{03}[\mu_3 + P_{34}m_{43} + P_{35}m_{53}]|$$

4.6 Profit Function

Any production unit is essentially a profit-making enterprise, and no business can thrive for long without a reasonable return on its investments. There must be a perfect balance between the reliability of the system and the price of the product. The most important elements that contribute to the overall cost is comprised of the server's availability, peak usage, and estimated revenue. The reliability of things varies depending on the mean time to system failure. We would like to improve the product's reliability by necessitate an equally large research effort. As the demand for more reliability grows, so does the price. The profit function of any system can be evaluated as follows:

Profit = K_0 A_0 – K_1 B_0 (9)

where K_0 and K_1 are the constants.

$$profit = \frac{(1-P_{34}P_{43}-P_{35}P_{53})[K_0\mu_0 - K_1(\mu_1 P_{01} + \mu_2 P_{02})] - P_{03}[K_0\mu_3 + K_1(\mu_4 P_{34} + \mu_5 P_{35})]}{[1-P_{34}P_{43}-P_{35}P_{53}][\mu_0 + P_{01}m_{10} + P_{02}m_{20}] + P_{03}[\mu_3 + P_{34}m_{43} + P_{35}m_{53}]} \quad (10)$$

The profit function of proposed model can be derived using equations (9-10) for particular values of the failure and repair rates.

5. NUMERICAL RESULTS

In this section, for a particular case mean time to system failure, availability and profit function are derived and appended in tables 1-5.

Table 1: Impact of various failure rates on mean time to system failure with respect to deliberated failures

θ1	**θ2**= 0.0046, **θ3** = 0.0056 , **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ2**= 0.0055, **θ3** = 0.0056 , **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ2**= 0.0046, **θ3** = 0.00672 , **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ2**= 0.0046, **θ3** = 0.0056 , **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 6
0.003	0.5064144	0.5264486	0.5064144	0.4220125
0.004	0.4730397	0.4959463	0.4730397	0.3942004
0.005	0.4448453	0.4691540	0.4448453	0.3707052
0.006	0.4211379	0.4459188	0.4211379	0.3509492
0.007	0.4011279	0.4258117	0.4011279	0.3342743
0.008	0.3841213	0.4083686	0.3841213	0.3201023
0.009	0.3695518	0.3931679	0.3695518	0.3079612
0.01	0.3569693	0.3798500	0.3569693	0.2974760

Table 2: Impact of various failure rates on availability with respect to deliberated failures

θ1	**θ2**= 0.0046, **θ3** = 0.0056 , **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ2**= 0.0055, **θ3** = 0.0056 , **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ2**= 0.0046, **θ3** = 0.00672 , **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0=** 5	**θ2**= 0.0046, **θ3** = 0.0056 , **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 6
0.003	0.9874788	0.9858829	0.9870721	0.9874788
0.004	0.9867825	0.9853504	0.9862476	0.9867825
0.005	0.9860496	0.9847529	0.9854106	0.9860496
0.006	0.9852863	0.9841022	0.9845648	0.9852863
0.007	0.9844965	0.9834069	0.9837105	0.9844965
0.008	0.9836830	0.9826736	0.9828474	0.9836830
0.009	0.9828481	0.9819076	0.9819751	0.9828481
0.01	0.9819940	0.9811134	0.9810934	0.9819940

Table 3: Impact of various failure rates on profit function with respect to deliberated failures

θ1	**θ2**= 0.0046, **θ3** = 0.0056, **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ2**= 0.0055, **θ3** = 0.0056, **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ2**= 0.0046, **θ3** = 0.00672, **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0=** 5	**θ2**= 0.0046, **θ3** = 0.0056, **β1** = 0.69, **β2**= 0.72, **β3**= 0.356, **α0**= 6
0.003	4936.1409	4928.0016	4934.0928	4936.1409
0.004	4932.5365	4925.2272	4929.8433	4932.5365
0.005	4928.7422	4922.1191	4925.5249	4928.7422
0.006	4924.7911	4918.7386	4921.1580	4924.7911
0.007	4920.7034	4915.1301	4916.7458	4920.7034
0.008	4916.4939	4911.3280	4912.2869	4916.4939
0.009	4912.1752	4907.3594	4907.7798	4912.1752
0.01	4907.7580	4903.2466	4903.2237	4907.7580

Table 4: Impact of various repair rates on availability with respect to repair rate of deliberated failures

β1	**θ1**= 0.0030**,** **θ2**= 0.0046, **θ3** = 0.0056, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ1**= 0.0030, **θ2**= 0.0046, **θ3** = 0.0056, **β2**= 0.864, **β3**= 0.356, **α0**= 5	**θ1**= 0.0030**,** **θ2**= 0.0046, **θ3** = 0.0056, **β2**= 0.72, **β3**= 0.4272, **α0**= 5
0.690	0.98747879	0.98747879	0.98940737
0.700	0.98751038	0.98751038	0.98943908
0.710	0.98754103	0.98754103	0.98946985
0.720	0.98757079	0.98757079	0.98949972
0.730	0.98759968	0.98759968	0.98952874
0.740	0.98762776	0.98762776	0.98955692
0.750	0.98765504	0.98765504	0.98958431
0.760	0.98768157	0.98768157	0.98961094

Table 5: Impact of various repair rates on profit with respect to repair rate of deliberated failures

β1	**θ1**= 0.0030, **θ2**= 0.0046, **θ3** = 0.0056, **β2**= 0.72, **β3**= 0.356, **α0**= 5	**θ1**= 0.0030, **θ2**= 0.0046, **θ3** = 0.0056, **β2**= 0.864, **β3**= 0.356, **α0**= 5	**θ1**= 0.0030, **θ2**= 0.0046, **θ3** = 0.0056, **β2**= 0.72, **β3**= 0.4272, **α0**= 5
0.690	4936.14093	4936.16366	4945.82931
0.700	4936.29883	4936.32156	4945.98783
0.710	4936.45205	4936.47478	4946.14165
0.720	4936.60079	4936.62353	4946.29097
0.730	4936.74525	4936.76798	4946.43599

0.740	4936.88558	4936.90832	4946.57688
0.750	4937.02198	4937.04471	4946.71381
0.760	4937.15458	4937.17732	4946.84693

6. CONCLUSION

In present study, a novel stochastic model for single unit systems is proposed using the concept of deliberated failures and warranty period. For the validation and characterization of results, numerical values of mean time to system failure, availability and profit are derived for a particular case. From table 1, it is observed that mean time to system failure of the system declined sharply with the increase of deliberated failure rates. The mean time to system failure declined with the increase of the maximum warranty period. Table 2 &3 depicted that availability and profit of the system declined sharply with the increase of the various failure rates. From tables 4-5, it is revealed that increment in the repair rate reflect the higher availability. So, it is recommended that single unit system can be made more available, reliable, and profitable by controlling the deliberated failure rates and increasing the warranty period. The proposed model's findings will be implemented in single unit system reliability enhancement. The application of proposed model can be visualized in the area of mobile devices, communication devices, electric equipment's and mechanical systems. The classical and Bayesian estimation of various reliability measures will be performed in the future study.

RECEIVED: AUGUST, 2022.
REVISED: SEPTEMBER, 2022.

REFERENCES

[1] AFSAHI, M., and SHAFIEE, M. (2020): A stochastic simulation-optimization model for base-warranty and extended-warranty decision-making of under-and out-of-warranty products. **Reliability Engineering & System Safety**, 197, 106772.

[2] DAI, A., HE, Z., LIU, Z., YANG, D., & HE, S. (2017): Field reliability modeling based on two-dimensional warranty data with censoring times. **Quality Engineering**, *29*, 468-483.

[3] GHITANY, M.E., ATIEH, B., NADARAJAH, S. (2008): Lindley distribution and its application, Mathematics and Computers in Simulation. 78, 493–506

[4] Gupta, S. (2019): Stochastic modelling and availability analysis of a critical engineering system. International **Journal of Quality & Reliability Management**. 36, 782-796.

[5] KRISHNA, H. and KUMAR, K. (2011): Reliability estimation in Lindley distribution with progressively type II right censored sample. **Mathematics and Computers in Simulation**. 82. 281-294. 10.1016/j.matcom.2011.07.005.

[6] KUMAR, A. and SAINI, M. (2014): Cost -Benefit Analysis of a Single-Unit System with Preventive Maintenance and Weibull Distribution for Failure and Repair Activities, **JAMSI**, 10 , 5-19.

[7] KUMAR, A., CHHILLAR, S. K. and MALIK, S. C. (2016): Analysis of a single-unit system with degradation and maintenance. **Journal of Statistics and Management Systems**, 19, 151–161.

[8] KUMAR, A. (2016): Cost-benefit analysis of a repairable system in abnormal environmental conditions. **Palestine Journal of Mathematics**, *5*, 111-119.

[9] KUMARI, S. (2018): Reliability Modelling of a 2-Out-of-3 Redundant System with Alternate Repair. **International Journal of Statistics and Reliability Engineering**, 5, 101-107.

[10] NIWAS, R., & KADYAN, M. S. (2015): Reliability modeling of a maintained system with warranty and degradation. **Journal of Reliability and Statistical Studies**, 63-75.

[11] NIWAS, R., KADYAN, M. and KUMAR, J. (2013): Stochastic modeling of a single-unit repairable system with preventive maintenance under warranty. **International Journal of Computer Applications,** 75, 36-41.

[12] NANDAL, N. and MALIK, S.C. (2019): Use of Lindley Distribution for Profit Analysis of a Three Unit Cold Standby System Subject to Arrival Time of the Server. **International Journal of Statistics and Reliability Engineering**, 6, 145-151.

[13] PUNDIR, P.S., PATAWA R. & GUPTA P.K. (2018): Stochastic outlook of two non-identical unit parallel system with priority in repair, **Cogent Mathematics & Statistics**, 5:1, 1467208, DOI: 10.1080/25742558.2018.1467208

[14] PUNDIR P.S., PATAWA, R. & GUPTA, P.K. (2020): Analysis of Two Non-Identical Unit Cold Standby System in Presence of Prior Information, **American Journal of Mathematical and Management Sciences**, DOI: 10.1080/01966324.2020.1860840,

[15] SAINI, M., & KUMAR, A. (2020): Stochastic modeling of a single-unit system operating under different environmental conditions subject to inspection and degradation. **Proceedings of the National Academy of Sciences, India Section A: Physical Sciences**, 90, 319-326.