EXPONENTIAL TYPE ESTIMATOR FOR MISSING DATA UNDER IMPUTATION TECHNIQUE

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ABSTRACT
In this paper, we suggest an exponential type estimator for estimation of population mean for missing data under suggested imputation techniques. Family of proposed estimator is obtained for missing data. Expression for Bias and MSE's are acquired in the form of population parameters up to the terms of first order of approximation. Theoretical results depict the superiority of proposed estimator and its family over other estimators. The empirical study in support of theoretical results is also included to verify the results numerically.

KEYWORDS: Bias, Mean square error, Missing data, SRSWOR.

MSC: 62D05

RESUMEN
En este artículo, sugerimos un estimador de tipo exponencial para estimar la media poblacional de los datos faltantes bajo las técnicas de imputación sugeridas. Se obtiene la familia de estimadores propuestos para los datos faltantes. La expresión de Bias y MSE se adquiere en forma de parámetros de población hasta los términos de primer orden de aproximación. Los resultados teóricos muestran la superioridad del estimador propuesto y su familia sobre otros estimadores. También se incluye el estudio empírico en apoyo de los resultados teóricos para verificar los resultados numéricamente.

PALABRAS CLAVE: Sesgo, Error cuadrática medio, Data Faltante, SRSWOR

1. INTRODUCTION

It is experienced that surveys involving human population (E.g.: medical and social science surveys) often face the problem of non-response. These missing values in turn create complications in handling and analysis of data. In course of time many methods have been developed to deal with the problem of estimation of unknown parameters in the presence of missing values. Imputation is a common technique employed to deal with situations where missing data is present. Imputation methods make use of auxiliary information in improving precision of the estimates of the population parameters.Sande (1979) gave a view of hot deck imputation procedure. Rubin and Schenker (1986) gave multiple imputation procedure for Interval Estimation from Simple Random Samples with Ignorable Nonresponse.Singh and Horn (2000) introduced compromised type imputation technique and Singh and Deo (2003) suggested imputation by power transformation. Singh et al. (2014) defined exponential-type compromised method of imputation.For estimating unknown parameters when missing values are present Kadilar and Cingi (2008) suggested estimation of population mean for missing data values. Omari et. al (2013) suggested estimation of population mean using known correlation coefficient for missing values. Pandeyet. al (2015) proposed exponential ratio type imputation method for estimation of population mean. Other authors including Gira (2015),Prasad (2016) and Mishra et al. (2017) have also defined estimators under missing values. Several authors including Singh et al. (2018) andSingh et al. (2019) have also proposed estimators in the presence of measurement and non response error and a known non-sampling error i.e, measurement error. In the present context we propose an estimator under exponential type imputation method for the case of missingness.
Consider a population P= (P_1, P_2,…………, P_N) of size N and let us draw a sample of size n from it using SRSWOR technique in such a way that n units respond and (n-r) units do not respond.For every responding unit belonging to set S, y_i is observed value. Correspondingly, for every non-responding unit belonging to set S', y_i is the non- responding (missing) observation to be imputed using appropriate technique. Further, let (Y_i, X_i) be the value of study and auxiliary variable with population mean $\left(\overline{Y}, \overline{X}\right)$ respectively (for

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i=1,2,…,n). Let S_x^2 and S_y^2 be the population mean square and $C_Y = \frac{S_Y}{\bar{Y}}$ and $C_X = \frac{S_X}{\bar{X}}$ be corresponding Coefficient of variation for study and auxiliary variable respectively and ρ be the correlation coefficient between Y and X. Finite population correction is given as follows:

$$\theta_{n,N} = \left(\frac{1}{n} - \frac{1}{N}\right), \theta_{r,N} = \left(\frac{1}{r} - \frac{1}{N}\right) \text{ and } \theta_{r,n} = \left(\frac{1}{r} - \frac{1}{n}\right)$$

2. EXISTING IMPUTATION TECHNIQUES

In this section we study different existing estimators in the literature. The expression for Bias and MSE's is also shown.

The mean method of imputation is given by:

t_1: $y_{.i} = \begin{cases} y_{.i} & \text{if } i \in N \\ \bar{y}_r & \text{if } i \in N' \end{cases}$

and its Bias and MSE is given by Equation (2.0) and Equation (2.1) respectively.

$$Bias(\bar{y}_m) = 0 \qquad (2.0)$$

$$and\ MSE(\bar{y}_m) = \theta_{r,N}\bar{Y}^2 C_Y^2 \qquad (2.1)$$

Ratio method of imputation is given by t_2,

where t_2 is $y_{.i} = \begin{cases} y_i & i \in N \\ \hat{b}x_i & i \in N' \end{cases}$,

$$\hat{b} = \frac{\sum_{i=1}^{r} x_i}{\sum_{i=1}^{r} y_i}$$

Bias and MSE of Ratio method of imputation are given in Equation (2.2) and Equation (2.3),

$$Bias(\bar{y}_{rt}) = \theta_{r,n}\bar{Y}\left(C_X^2 - \rho C_Y C_X\right) \qquad (2.2)$$

$$MSE(\bar{y}_{rt})\theta_{r,N}\bar{Y}^2 C_Y^2 + \theta_{r,n}\bar{Y}^2\left(C_X^2 - 2\rho C_X C_Y\right) \qquad (2.3)$$

Compromised method of imputation (Singh and Horn(2000)) is given by t_3

$$y_{.i} = \begin{cases} \delta\frac{n}{r} y_i + (1-\delta)\hat{b}x_i & i \in N \\ (1-\delta)\hat{b}x_i & i \in N' \end{cases}$$

Its bias and MSE is given in Equation (2.4) and Equation (2.5),

$$Bias(\bar{y}_{cmp}) = (1-\delta)\theta_{r,n}\bar{Y}\left(C_X^2 - \rho C_Y C_X\right) \qquad (2.4)$$

$$MSE(\bar{y}_{cmp}) = \theta_{r,N}\bar{Y}^2 C_Y^2 + \theta_{r,n}\bar{Y}^2\left[(1-\delta)C_X^2 - 2(1-\delta)\rho C_X C_Y\right] \qquad (2.5)$$

Exponential type compromised Method (Singh et al (2014)) is given by t_4,

$$y_{.i} = \begin{cases} p\frac{n}{r} y_i + (1-p)\bar{y}_r \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) & \text{if } i \in N \\ (1-p)\bar{y}_r \exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right) & \text{if } i \in N' \end{cases}$$

Bias and MSE are given by Equation (2.6) and Equation (2.7),

$$Bias(\bar{y}_{\exp}) = (1-p)\left(\frac{1}{r} - \frac{1}{N}\right)\bar{Y}\left(\frac{3}{8}C_X^2 - \frac{1}{2}\rho_{YX}C_X C_Y\right) \qquad (2.6)$$

$$Min.\,MSE(\bar{y}_{et}) = \left(\frac{1}{r} - \frac{1}{N}\right)\bar{Y}^2\left[C_Y^2\left(1-\rho_{XY}^2\right)\right] \qquad (2.7)$$

3. PROPOSED ESTIMATOR UNDER IMPUTATION TECHNIQUE

We propose an exponential type imputation method given in Equation (3.0),

$$\bar{y}_{Rp} = \begin{cases} y_{.i} & \text{if } i \in A \\ \bar{y}_r\left(\kappa_1 + \kappa_2 \dfrac{\bar{X}}{\bar{x}_r}\right)\exp\left(\dfrac{\eta(\bar{X}-\bar{x}_r)}{\eta(\bar{X}+\bar{x}_r)+2\beta}\right) & \text{if } i \in A^c \end{cases} \tag{3.0}$$

The point estimator of the population mean under this method of imputation is given by:

$$\frac{r}{n}\bar{y}_r + \left(1-\frac{r}{n}\right)\bar{y}_r\left(\kappa_1 + \kappa_2\frac{\bar{X}}{\bar{x}_r}\right)\exp\left(\frac{\eta(\bar{X}-\bar{x}_r)}{\eta(\bar{X}+\bar{x}_r)+2\beta}\right) \tag{3.1}$$

To obtain bias and mean square error (MSE) of the proposed estimator up to the first order of approximation, we use following transformations:

$$\bar{y}_r = \bar{Y}(1+\varepsilon_0) \Rightarrow \varepsilon_o = \left(\frac{\bar{y}_r - \bar{Y}}{\bar{Y}}\right),\ \bar{x}_r = \bar{X}(1+\varepsilon_1) \Rightarrow \varepsilon_1 = \left(\frac{\bar{x}_r - \bar{X}}{\bar{X}}\right) \text{ and } \bar{x}_n = \bar{X}(1+\varepsilon_2) \Rightarrow \varepsilon_2 = \left(\frac{\bar{x}_n - \bar{X}}{\bar{X}}\right)$$

such that $|\varepsilon_i| < 1, \quad \forall\ i = (0,\ 1,\ 2)$.

Hence we have

$$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0 \quad \begin{array}{l} E(\varepsilon_0^2) = \theta_{r,N}C_Y^2,\ E(\varepsilon_1^2) = \theta_{r,N}C_X^2,\ E(\varepsilon_2^2) = \theta_{n,N}C_X^2,\ E(\varepsilon_0\varepsilon_1) = \theta_{r,N}\rho C_Y C_X, \\ E(\varepsilon_0\varepsilon_2) = \theta_{n,N}\rho C_Y C_X,\ E(\varepsilon_1\varepsilon_2) = \theta_{n,N}C_X^2 \end{array}$$

Where $\rho = \dfrac{S_{XY}}{S_X S_Y}$

Under the above transformations Equation (3.1) is defined as

$$\bar{y}_{Rp} = \frac{r}{n}\bar{Y}(1+\varepsilon_0) + \left(1-\frac{r}{n}\right)\bar{Y}\kappa_1\left(1+\varepsilon_0 - \alpha\varepsilon_1 + \frac{3}{2}\alpha^2\varepsilon_1^2 - \alpha\varepsilon_0\varepsilon_1\right) + \left(1-\frac{r}{n}\right)\bar{Y}\kappa_2$$
$$\left(1+\varepsilon_0 - (\alpha+1)\varepsilon_1 + \left(\frac{3}{2}\alpha^2 + \alpha + 1\right)\varepsilon_1^2 - (\alpha+1)\varepsilon_0\varepsilon_1\right)$$

Where $\alpha = \dfrac{\eta\bar{X}}{2(\eta\bar{X}+\beta)}$

$$\bar{y}_{Rp} - \bar{Y} = \bar{Y}\left[\begin{array}{l} \left\{\dfrac{r}{n}(1+\varepsilon_0) - 1\right\} + \left(1-\dfrac{r}{n}\right)\kappa_1\left(1+\varepsilon_0 - \alpha\varepsilon_1 + \dfrac{3}{2}\alpha^2\varepsilon_1^2 - \alpha\varepsilon_0\varepsilon_1\right) + \\ \left(1-\dfrac{r}{n}\right)\kappa_2\left(1+\varepsilon_0 - (\alpha+1)\varepsilon_1 + \left(\dfrac{3}{2}\alpha^2 + \alpha + 1\right)\varepsilon_1^2 - (\alpha+1)\varepsilon_0\varepsilon_1\right) \end{array}\right] \tag{3.2}$$

Now, taking expectation both the sides of Equation (3.2), we get the bias of proposed estimator, given by

$$Bias(\bar{y}_{Rp}) = \bar{Y}\left[\begin{array}{l} \left(\dfrac{r}{n} - 1\right) + \kappa_1\left(1-\dfrac{r}{n}\right)\left(1 + \dfrac{3}{2}\alpha^2\theta_{r,N}C_X^2 - \theta_{r,N}\rho C_Y C_X\right) + \\ \kappa_2\left(1-\dfrac{r}{n}\right)\left(1 + \left(\dfrac{3}{2}\alpha^2 + \alpha + 1\right)\theta_{r,N}C_X^2 - (\alpha+1)\theta_{r,N}\rho C_Y C_X\right) \end{array}\right] \tag{3.3}$$

Again squaring and taking expectation on both the sides of Equation (3.2), we get the mean square error (MSE) of proposed estimator, given by

$$MSE(\bar{y}_{Rp}) = \bar{Y}^2\left[\left\{1 - \frac{2r}{n} + \frac{r^2}{n^2}\left(1+\theta_{r,N}C_Y^2\right)\right\} + \kappa_1^2 A + \kappa_2^2 B + 2\kappa_1 C + 2\kappa_2 D + 2\kappa_1\kappa_2 E\right] \tag{3.4}$$

where,

$$A = \left(1-\frac{r}{n}\right)^2\left[1+\theta_{r,N}\left(C_Y^2 + 4\alpha^2 C_X^2 - 4\alpha\rho C_Y C_X\right)\right]$$

$$B = \left(1-\frac{r}{n}\right)^2\left[1+\theta_{r,N}\left(C_Y^2 + (3+4\alpha^2+4\alpha)C_X^2 + 4(\alpha+1)\rho C_Y C_X\right)\right]$$

$$C=\left(1-\frac{r}{n}\right)\left[\frac{r}{n}\left\{1+\theta_{r,N}\left(\frac{3}{2}\alpha^2C_X^2-2\alpha\rho C_YC_X\right)\right\}-\left\{1+\theta_{r,N}\left(\frac{3}{2}\alpha^2C_X^2-\alpha\rho C_YC_X\right)\right\}\right]$$

$$D=\left(1-\frac{r}{n}\right)\left[\begin{array}{l}\frac{r}{n}\left\{1+\theta_{r,N}\left(\left(\frac{3}{2}\alpha^2+\alpha+1\right)C_X^2-2(\alpha+1)\rho C_YC_X\right)\right\}-\\ \left\{1+\theta_{r,N}\left(\left(\frac{3}{2}\alpha^2+\alpha+1\right)C_X^2-(\alpha+1)\rho C_YC_X\right)\right\}\end{array}\right]$$

$$E=\left(1-\frac{r}{n}\right)^2\left[1+\theta_{r,N}\left(C_Y^2+\left(4\alpha^2+2\alpha+1\right)C_X^2-2(2\alpha+1)\rho C_YC_X\right)\right]$$

Partially differentiating Equation (2.4) w. r. to κ_1 and κ_2, we get the optimum value of $\kappa_1$and $\kappa_2$in form of A, B, C, D, E, as given below:

$$\kappa_1^{opt}=\frac{BC-DE}{E^2-AB}\ \text{ and }\ \kappa_2^{opt}=\frac{AD-CE}{E^2-AB}$$

On substituting the optimum value of κ_1 and κ_2 in Equation (2.4), we get the minimum MSE of proposed estimator given by Equation (3.5).

$$\min.MSE\left(\bar{y}_{Rp}\right)=\bar{Y}^2\left[\left\{1-\frac{2r}{n}+\frac{r^2}{n^2}\left(1+\theta_{r,N}C_Y^2\right)\right\}+\frac{BC^2+AD^2-2CDE}{E^2-AB}\right] \qquad (3.5)$$

Now, for distinct choice of η and β, we generate distinct point estimators:

Table 1: Family of Proposed estimator under imputation for distinct η,β.

η	β	**Estimators**
1	-1	$\bar{y}_{Rp1}=\frac{r}{n}\bar{y}_r+\left(1-\frac{r}{n}\right)\bar{y}_r\left(\kappa_1+\kappa_2\frac{\bar{X}}{\bar{x}_r}\right)\exp\left(\frac{\left(\bar{X}-\bar{x}_r\right)}{\left(\bar{X}+\bar{x}_r\right)-2}\right)$
1	0	$\bar{y}_{Rp2}=\frac{r}{n}\bar{y}_r+\left(1-\frac{r}{n}\right)\bar{y}_r\left(\kappa_1+\kappa_2\frac{\bar{X}}{\bar{x}_r}\right)\exp\left(\frac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right)$
1	1	$\bar{y}_{Rp3}=\frac{r}{n}\bar{y}_r+\left(1-\frac{r}{n}\right)\bar{y}_r\left(\kappa_1+\kappa_2\frac{\bar{X}}{\bar{x}_r}\right)\exp\left(\frac{\left(\bar{X}-\bar{x}_r\right)}{\left(\bar{X}+\bar{x}_r\right)+2}\right)$
0	1	$\bar{y}_{Rp4}=\frac{r}{n}\bar{y}_r+\left(1-\frac{r}{n}\right)\bar{y}_r\left(\kappa_1+\kappa_2\frac{\bar{X}}{\bar{x}_r}\right)$

4. EFFICIENCY COMPARISON

1.) From Equation (2.1) and Equation (3.5),

Min. MSE $(\bar{y}_{rp})$ < min. MSE $(\bar{y}_m)$ if

$$\bar{Y}^2\left[\left\{1-\frac{2r}{n}+\frac{r^2}{n^2}\left(1+\theta_{r,N}C_Y^2\right)\right\}+\frac{BC^2+AD^2-2CDE}{E^2-AB}\right]-\theta_{r,N}\bar{Y}^2C_Y^2\ \geq 0$$

2.) From Equation (2.3) and Eq. (3.5)

Min. MSE $(\bar{y}_{rp})$ < min. MSE $(\bar{y}_{rt})$ if

$$\bar{Y}^2\left[\left\{1-\frac{2r}{n}+\frac{r^2}{n^2}\left(1+\theta_{r,N}C_Y^2\right)\right\}+\frac{BC^2+AD^2-2CDE}{E^2-AB}\right]-\theta_{r,N}\bar{Y}^2C_Y^2+\theta_{r,n}\bar{Y}^2\left(C_X^2-2\rho C_XC_Y\right)\geq 0$$

3.) From Equation (2.5) and Equation (3.5)

Min. MSE $(\bar{y}_{rp})$ <min. MSE $(\bar{y}_{cmp})$ if

$$\bar{Y}^2\left[\left\{1-\frac{2r}{n}+\frac{r^2}{n^2}\left(1+\theta_{r,N}C_Y^2\right)\right\}+\frac{BC^2+AD^2-2CDE}{E^2-AB}\right]-\theta_{r,N}\bar{Y}^2C_Y^2+\theta_{r,n}\bar{Y}^2\left[(1-\delta)C_X^2-2(1-\delta)\rho C_XC_Y\right]\geq 0$$

4.) From Equation (2.7) and Equation (3.5)

Min. MSE $(\bar{y}_{rp})$ <min. MSE $(\bar{y}_{exp})$ if

$$\bar{Y}^2\left[\left\{1-\frac{2r}{n}+\frac{r^2}{n^2}\left(1+\theta_{r,N}C_Y^2\right)\right\}+\frac{BC^2+AD^2-2CDE}{E^2-AB}\right]-\left(\frac{1}{r}-\frac{1}{N}\right)\bar{Y}^2\left[C_Y^2\left(1-\rho_{XY}^2\right)\right]\geq 0$$

5. EMPIRICAL STUDY

For empirical justification of findings, we consider four real data sets. The performance of the proposed estimator is justified by comparing percentage relative efficiency of the proposed estimator with the existing ones.

Population 1: (Source: Singh (2009))

X: the number of laborers (in thousands) and Y: quantity of raw materials required (in lakhs of bales).

$$N=3055, n=611, r=520, \bar{Y}=308582.4, \bar{X}=56.5, S_y=425312.8, S_x=72.3$$
$$S_{yx}=20817828.5, \rho=0.677$$

Population 2: (Source: Diana and Perri (2010))

X: the number of household income earners and Y: the household net disposal income

$$N=8011, n=400, r=360, \bar{Y}=28229.43, \bar{X}=1.69, S_y=22216.56, S_x=0.78$$
$$S_{yx}=7971.302, \rho=0.46$$

Population 3: (Source: Kadilar and Cingi (2008))

$$N=19, n=10, r=8, \bar{Y}=575.00, \bar{X}=13537.68, S_y=858.36, S_x=12945.38$$
$$S_{yx}=9738380.82, \rho=0.88$$

Population 4: (Source: Mukhopadhyay (2000))

$$N=20, n=7, r=5, \bar{Y}=41.50, \bar{X}=441.95, S_y^2=95.937, S_x^2=10215.21$$
$$S_{yx}=644.8782906, \rho=0.6521$$

Table 2: PREs of various estimators under imputation:

Estimators	Population			
	1	2	3	4
t_1	100	100	100	100
t_2	107.633	102.091	132.928	114.122
t_3	108.964	102.266	136.522	119.331
t_4	184.614	126.839	143.262	173.984
$\bar{y}_{Rp1}$	191.252	122.153	207.337	313.383
$\bar{y}_{Rp2}$	189.667	135.639	207.333	312.784
$\bar{y}_{Rp3}$	188.209	131.157	207.328	312.193
$\bar{y}_{Rp4}$	186.192	126.437	163.010	190.584

6. CONCLUSION

From Table 2, it is observed that families of estimators $\bar{y}_{Rpi}$ $(i=1,2,3,4)$ under proposed imputation technique is found to be more efficient than existing mean, ratio, exponential and compromised exponential techniques. Among proposed families $\bar{y}_{Rp1}$ is more efficient than others for high correlation between study and auxiliary variables and correlation is low, then $\bar{y}_{Rp2}$ is more efficient. To enhance the precision of the estimates in case missing values are present as in socio and economic survey data, medical surveys, income surveys etc, it is suggested that proposed estimators be used efficiently for obtaining desirable precision.

ACKNOWLEDGEMENT

The authors are thankful to learned referees for their valuable comments and suggestions regarding the paper.

RECEIVED: OCTOBER, 2020.
REVISED: APRIL, 2021.

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