# EFFECTS OF FUTURE PRICE INCREASE AND TRADE CREDIT ON OPTIMAL ORDERING POLICIES FOR PERISHABLE ITEMS UNDER QUADRATIC DEMAND

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**ABSTRACT**

When a supplier declares a price increase at an assured time in the future, for each retailer it is essential to decide whether to buy a supplementary stock at the current lower price to get benefit or buy at a new price. In economic business, the offer of selling an account at a later date is common practice to boost the market demand by attracting more players. So, a Permissible delay in payment is one of the best policies to gain business. This paper focuses on the possible effects of price increase on a retailer's reordered scheme for constant deterioration of items where the supplier gives a mutually agreed credit limit to the retailer. Here, quadratic demand is debated; which is appropriate for seasonal products. Two scenarios are discussed in this study: (I) when the special-order time coincides with the retailer's replenishment time and (II) when the special-order time falls during the retailer's sales period. An optimal ordering policy for each case is determined by maximizing total cost savings between special and regular orders during the depletion time of the special-order quantity. Scenarios are established and illustrated with numerical examples. Through, sensitivity analysis important inventory parameters are classified. Graphical results, in two dimensions, are exhibited with a supervisory decision.

**KEYWORDS:** Inventory; price increase; quadratic demand; deteriorating items; Trade credit

**MSC:** 90B05

**RESUMEN**

Cuando un proveedor declara un aumento de precio en un momento asegurado en el futuro, para cada minorista es esencial decidir si comprar una acción complementaria al precio actual más bajo para obtener beneficios o comprar a un precio nuevo. En los negocios económicos, la oferta de vender una cuenta en una fecha posterior es una práctica común para impulsar la demanda del mercado al atraer a más jugadores. Por lo tanto, una demora permitida en el pago es una de las mejores políticas para obtener negocios. Este documento se centra en los posibles efectos del aumento de precios en el esquema reordenado de un minorista para el deterioro constante de los artículos donde el proveedor otorga un límite de crédito mutuamente acordado al minorista. Aquí, se debate la demanda cuadrática; que es apropiado para productos de temporada. En este estudio se analizan dos escenarios: (I) cuando el tiempo de pedido especial coincide con el tiempo de reabastecimiento del minorista y (II) cuando el tiempo de pedido especial cae durante el período de ventas del minorista. Una política de pedidos óptima para cada caso se determina maximizando el ahorro total de costos entre pedidos especiales y regulares durante el tiempo de agotamiento de la cantidad de pedido especial. Los escenarios se establecen e ilustran con ejemplos numéricos. Mediante el análisis de sensibilidad se clasifican parámetros importantes del inventario. Los resultados gráficos, en dos dimensiones, se exhiben con una decisión de supervisión.

**PALABRAS CLAVE:** inventario; artículos deteriorados; aumento de precio; demanda cuadrática; Crédito comercial

## 1. INTRODUCTION

Due to the increase in oil prices globally, the prices of merchandise have gone along to increase at the universal level. It has become a severe issue for originalities. Especially once administrators make decisions about their inventory strategy, they wish to consider increments in trade good prices. When the supplier declares a price increase of the product which will affect at a certain time in the future, each retailer needs to conclude whether to buy enough stock earlier to the price increase to take benefit of the current lower price. Researchers have bought a declaration of a price increase problem into account and planned numerous logical representations to earn extra perceptions into the conclusions associating with inventory insurance. When the supplier declares a price increment

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Naddor (1966) was the first researcher who designed “an infinite horizon economic order quantity (EOQ) model” Lev and Soyster (1979) recognized “a finite horizon inventory model” and established an optimum ordering strategy and made acknowledged evidence about a forthcoming price increase. After that Goyal (1979) studied Lev and Soyster's (1979) EOQ model and offered the alternate technique for specifying the optimal strategy. Lev and Weiss (1990) subsequently established the construction of optimum strategies and dealings for calculating the best strategy. Goyal *et al.* (1991) analyzed and given a written report of inventory strategies, under one-time incentives. Erel (1992) and Khouja and Park (2003) represented new EOQ models considered the continuous nature of increasing or decreasing costs. Sharma (2009) has established the inventory model on price increases and also considered temporary price reductions with partial backordering. Ouyang *et al*. (2014) detected optimal order policy under-declared price increases with limited special-order quantity. Shah *et al*. (2016) extended the Ouyang *et al*. (2014) model and obtained an optimal ordering policy under the impact of the future price increase.

Demand plays a vital role in business policies and all situations the customer’s demand for the products is very crucial for the seller. Therefore, the researchers consider the demand concept into account, and Harris (1913) was the first researcher who developed a basic EOQ model, in which demand was constant. In reality, the demand for some products like fashionable items, electronic items, and food items can be increase or decrease with time. Considering this fact, Hariga (1996) established an EOQ model with time-dependent demand. Jalan and Chaudhuri (1999) developed an order level inventory model for a perishable item under a time-dependent demand. Also, price owns a huge effect on demand. So, a general decrement in the selling price of some products will automatically increase customer demand, results in higher sales of such products. Thus, written reports of some inventory models through price-dependent demand have received too much consideration into account. Datta and Paul (2001) analyzed an inventory system with price dependent demand rate. Shinn and Hwang (2003) determined the optimal ordering policy by considering price and order size simultaneously, in which the demand was the function of the selling price. Many of the researchers observed that there is some relation between demand rate and inventory level, they both are correlated with each other positively. Therefore, stock-dependent demand inventory models are also taken into consideration. Teng and Chang (2005) established an economic production quantity (EPQ) model in which demand was dependent on display stock level and selling price per unit. Soni (2013) determined an optimal replenishment policy for non-instantaneous deteriorating items with price-stock-sensitive demand. Hossen *et al.* (2016) developed a fuzzy inventory model with a price and time-dependent model. Shah *et al* (2017) developed an EOQ model on optimum strategies and considered a price credit-dependent trapezoidal demand. Mashud *et al.* (2018) developed an inventory model with stock and price-dependent demand under partially backlogged shortages. Recently, Agi and Soni (2020) presented a deterministic model jointly pricing and inventory control policies with price-dependent demand.

The above inventory model’s interpretation of the results of price variations and conclusions of the optimum special-order quantity for the vendor. All the above researchers did not consider the perishable items (deteriorating inventory), which is a usual phenomenon. By keeping the deteriorating products like vegetables, juice, medicines, and fruits in storage for a long period, the products will damage with time. Thus, such types of products that are damaging because of deterioration cannot be ignored while determining the optimum order strategy. In the previous research, inventory-related problems of deteriorating items have been studied extensively. Ghare and Schrader (1963) were the first researchers who recognized a time-dependent inventory EOQ model in which an exponential distribution rate was considered for deteriorating items. After that Philip (1974) established an EOQ model and he considers the Weibull distribution deterioration rate with three parameters. Wee (1993) formulated an economic production policy for deteriorating items with partial backordering. Skouri *et al.* (2009) developed an inventory model and considered a Weibull deterioration rate with ramp type demand rate and partial backlogging. Sarkar and Sarkar (2014) developed an EOQ model and considered stock-dependent demand with a time-varying deterioration rate. Tripathi and Tomar (2015) studied the optimal setting up a strategy for deteriorating items with time-varying demand in response to an impermanent price discount related to order quantity, they considered a constant rate of deterioration. Tiwari (2016) developed a two-warehouse inventory model for non-instantaneous deteriorating items. Chen *et al.* (2019) examined optimal pricing and replenishment policies for deteriorating inventories under different nature of demands. Recently, Jani *et al*. (2020) discussed an EOQ model by considering the maximum lifetime of the products and variable demand.

Expected commercial practices are created by cash-on-delivery. Nowadays, the commercial world is motivated by giving credit periods to fix the financial statement in contradiction of purchase. It is noticed that giving credit period will boost the demand and results in higher sales of the products and a decrease in stock. Goyal (1985) recognized an EOQ model with fixed upstream trade credits. Huang (2003) encouraged retailers to permit some portion of trade credit which is acknowledged by the dealer to his customers. Liao (2008) established a model for finite production rate under a two-level credit period. Teng and Lou (2012) established an economic order quantity model with up-stream and down-stream trade credits. Shah *et al*. (2015) derived an inventory model, in which the supplier gives a mutually agreed fixed credit period to the retailer under a price-sensitive quadratic demand. Shah (2017) formulated the three-layered integrated production inventory model under a two-level trade credit policy. Aljazzar *et al.* (2018) developed a strategy to reduce carbon emissions from the supply chain under the permissible delay in payments. Shi *et al.* (2019) developed an optimal order policy for a single deteriorating item with ramp-type demand under permissible delay in payments. Recently, Cardenas-Barron *et al.* (2020) derived an EOQ model with non-linear holding costs and also considered a single trade credit policy.

**Table 1.**
**Summary of works related to a Price increase, Demand, Deterioration, and Trade credit.**

| Source | Price increase | Demand | Deterioration | Trade credit |
|---|---|---|---|---|
| Philip (1974) | NO | Constant | Weibull distribution | NO |
| Erel (1992) | YES | Constant | NO | NO |
| Sarkar and Sarkar (2014) | NO | Stock dependent | Maximum fixed life-time | NO |
| Ouyang *et al.* (2014) | YES | Constant | Constant | NO |
| Shah *et al.* (2016) | YES | Quadratic | Constant | NO |
| Shah *et al* (2017) | NO | Price-trade credit-dependent quadratic | Maximum fixed life-time | Two-layered |
| Mashud *et al.* (2018) | NO | Stock-price dependent | Non-instantaneous | NO |
| Shi *et al.* (2019) | NO | Ramp-type | Constant | Up-stream |
| Cardenas-Barron *et al.* (2020) | NO | Non-linear stock dependent | NO | Up-stream |
| Jani *et al.* (2020) | NO | Price-time dependent | Maximum fixed life-time | Two-layered |
| In this paper | YES | Quadratic | Constant | YES |

Some researchers revealed that sellers are motivated to take a special order after the special-order amount is largely limitless when the supplier announces a cost increase. In reality, to protect against the vendor's advertisement goods for future sale at a higher selling price, the dealer is eager to provide a limited amount of goods at the present lower cost before the cost increase. So, the amount of goods that vendors can demand is also limited. Thus, to deliberate the above commercial subjects, this survey notes the probable effects of a cost increase in a vendor's replacement strategies A role of this article related to earlier studies, this study determines the inventory conclusions and the three events of the traditional economical order quantity model at the same time. These include the following: (1) the vendor will imagine the cost increase at a certain time in the future as declared by the dealer and determines whether to localizes a special-order; (2) the goods are deteriorating at a constant rate; and (3) retailer's best policies are analyzed under upstream trade credit and the demand is quadratic; Additionally, the time for hiring the special-order may or may not concur with the replenishment time, this article consists two-phases: (I) when the special-order time matches with the retailer's replenishment time and (II) when the special-order time occurs during the retailer's sales period. In Case I, the retailer's optimal order policy is to decide whether to place a larger order which is always larger than the regular EOQ. In Case II, the retailer's optimal order policy is to decide whether to place an additional order which is not necessarily larger than the regular EOQ. This survey determines to see the retailer's optimal order policies in response to a price increase by maximizing the total cost saving between special and regular orders during the depletion time of the special-order quantity. Moreover, two numerical examples are furnished to illustrate the theoretical outcomes, and sensitivity analysis of the optimal result concerning major parameters is also held out.

The resulting flow of the article is governed as follows. Portion 2 demonstrates the notations and the assumptions that are applied. Portion 3 is about the building of a mathematical model of the inventory problem. Portion 4 authorizes the resultant inventory model with numerical illustrations and an optimum solution. The sensitivity analysis of the main parameters is held out in Portion 5. Finally, Portion 6 contains the conclusion of this clause and its future possibilities.

## 2. NOTATION AND ASSUMPTIONS

The following notation and assumptions are used to build up the mathematical model of the problem under consideration.

### 2.1 Notation

| | |
|---|---|
| $A$ | Ordering cost per order (\$/order) |
| $C$ | Purchasing cost per unit (\$/unit) |
| $h$ | Inventory holding cost (\$/unit/unit time) |
| $a$ | Total market potential demand; $a \geq 0$ |
| $b$ | The linear rate of change of demand; $0 \leq b < 1$ |
| $c$ | Quadratic rate of change of demand; $0 \leq c < 1$ |
| $R(t)$ | $= a\left(1+bt-ct^2\right)$; quadratic demand rate at the time $t$ |
| $\theta$ | Constant deterioration rate; $0 \leq \theta \leq 1$ |
| $T$ | The length of the replenishment cycle time before the price increase (in years) |
| $I(t)$ | Inventory level at any instant of time $t$ before the price increase, $0 \leq t \leq T$ (units) |
| $Q$ | Economic Order Quantity before the price increase per cycle (units) |
| $M$ | The up-stream trade credit period offered by the supplier (in years) |
| $p$ | Selling price (\$/unit) |
| $I_e$ | The retailer's interest earned per \$ per year |
| $I_c$ | The retailer's interest charged per \$ per year, $I_c > I_e$ |
| $TCT_1(T)$ | Retailer's total cost before the price increase during the replenishment period $T$ for scenario $M < T$ (in \$) |
| $TCT_2(T)$ | Retailer's total cost before the price increase during the replenishment period $T$ for scenario $M \geq T$ (in \$) |
| $k$ | Price increase of the product (\$/unit) |
| $T_1$ | The length of the replenishment cycle time after the price increase (in years) |
| $I_1(t)$ | Inventory level at any instant of time $t$ after the price increase, $0 \leq t \leq T_1$ (units) |
| $Q_1$ | Economic Order Quantity after the price increase per cycle (units) |
| $TC(T_1)$ | Retailer's total cost after price increase during the replenishment period $T_1$ (in \$) |
| $Q_s$ | Economic Order Quantity of the special-order (units) (decision variable) |
| $T_s$ | Depletion time for the special-order quantity $Q_s$ (in years) (decision variable) |
| $I_s(t)$ | Inventory level at any instant of time t when the special-order is adopted, $0 \leq t \leq T_s$ |
| $q$ | Residual inventory level when the special-order is placed (units) |
| $t_q$ | The length of time until the special order is placed during the retailer's regular replenishment period (in years) |
| $T_q$ | Depletion time for the inventory quantity $Q_s + q$ (in years) |
| $I_q(t)$ | Inventory level at time t during the time interval $\left[0, T_q\right]$ (units) |
| $TCS_i(T_s)$ | The total cost of a special-order during the time interval $\left[0, T_s\right]$ for case i, i=1,2 (in \$) |
| $TCN_i(T_s)$ | The total cost of a regular order during the time interval $\left[0, T_s\right]$ for $i = 1,2,3,4$ (in \$) |
| $g_i(T_s)$ | Total cost saving between the special-order and regular order during the special cycle time for $i = 1,2,3,4$ (in \$) |

$*$ The superscript represents optimal value

### 2.2 Assumptions

1. The scheme under review contracts with a single item.
2. The demand rate is $R(t) = a\left(1 + bt - ct^2\right)$, where $a > 0$ denotes the scaling demand, $b$ and $c$ denotes linear and quadratic demand rates respectively. The functional form of demand rate suggests that demand increases linearly and decreases quadratically. This demand is especially justified for sessional products.
3. To reflect the increased price of raw materials, the supplier announces that the unit price of an item will increase by a given amount $k$ , at a certain future date.
4. Under the upstream trade credit policy, the supplier offers a credit limit $M$ to the retailer.
5. Due to permissible delays in payments, the retailer can earn interest on the customer's payment with an interest rate $I_e$ per unit per year. The retailer will have to settle the account at $M$ , a credit period offered by the supplier. Retailers will pay interest charges on the unsold stock with rate, $I_c$ to the supplier.
6. The retailer has only one chance to refill its stock at the current amount earlier to the price increase.
7. The replenishment rate is infinite and the lead-time is zero.
8. Shortages are not allowed.
9. There is no replacement or repair of deteriorated units during the period under consideration.

### 3. MATHEMATICAL MODEL

In this model, depletion of the inventory occurs due to the combined effects of demand and physical deterioration. Thus, the change in inventory level before the price increase is described by the following differential equation:

$$\frac{dI(t)}{dt} = -R(t) - \theta(t) I(t), \qquad 0 < t < T \tag{1}$$

given the boundary condition $I(T) = 0$, the solution of equation (1) is represented by

$$I(t) = \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 Tb + 2\theta Tc - cT^2\theta^2\right)e^{-\theta t + \theta T}}{\theta^3} - \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 tb + 2\theta tc - ct^2\theta^2\right)}{\theta^3}, \qquad 0 \le t \le T \tag{2}$$

Hence, the order quantity is given by

$$Q = I(0) = \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 Tb + 2\theta Tc - cT^2\theta^2\right)e^{\theta T}}{\theta^3} - \frac{a\left(\theta^2 - \theta b - 2c\right)}{\theta^3} \tag{3}$$

Earlier to the price increase, the purchasing cost $C$ follows the fixed economic order strategy with a unit purchasing cost, $C$ .

The total cost per unit time of retailer per cycle time $T$ is comprised by

- Ordering Cost: $OC = A$
- Holding Cost: $HC = hC\int_0^T I(t)\,dt$
- Purchasing Cost: $PC = CQ$

Now, the supplier offers a fixed credit period $M$ to the retailer, and the following two possible scenarios are given by

1. $M < T$
2. $M \ge T$

**Scenario 1:** $M < T$

Here the retailer needs to pay on time $M$ .

In this case, the allowable credit period is less than cycle time. During $[0, M]$ the buyer earns $I_e$ interest at the rate on the generated income. The interest earned per unit time is

$$IE_1 = pI_e \int_0^M t\,R(t)\,dt \tag{4}$$

In time $[M,T]$ the supplier will charge at the interest rate $I_c$ for unsold stock. Hence, the interest charged per unit time is

$$IC_1 = CI_c \int_M^T I(t)\,dt \tag{5}$$

Therefore, the total cost during the replenishment period $T$ per unit time is

$$\begin{aligned} TCT_1(T) &= \frac{1}{T}\left(OC + PC + HC + IC_1 - IE_1\right) \\ &= \frac{1}{T}\left(A + CQ + hC\int_0^T I(t)\,dt + CI_c\int_M^T I(t)\,dt - pI_e\int_0^M t\,R(t)\,dt\right) \end{aligned} \tag{6}$$

**Scenario 2:** $M \geq T$

In this case, the retailer has sold all the purchased items on or before the offered payment time. Therefore, Interest charge $IC_2 = 0$;

Also, the retailer can earn interest in generated revenue at the rate $I_e$ in $[0, M]$ which is given by

$$IE_2 = pI_e \int_0^T t\,R(t)\,dt \tag{7}$$

Therefore, the total cost during the replenishment period $T$ per unit time is

$$\begin{aligned} TCT_2(T) &= \frac{1}{T}\left(OC + PC + HC + IC_2 - IE_2\right) \\ &= \frac{1}{T}\left(A + CQ + hC\int_0^T I(t)\,dt - pI_e\int_0^T t\,R(t)\,dt\right) \end{aligned} \tag{8}$$

It is easily checked that $TCT_i(T)$ ; $i = 1,2$ are the convex functions of $T$ . Therefore, the optimum cycle time $T_i^*$; $i = 1,2$ that minimizes $TCT_i(T)$ ; $i = 1,2$ is obtained by solving the equation

$$\frac{dTCT_i(T)}{dT} = 0 \quad ; i = 1,2 .$$

Next, when the unit purchasing cost increases from $C$ to $C + k$ , the inventory level at a time $t$ is

$$\begin{aligned} I_1(t) = &\frac{a\left(\theta^2 - \theta b - 2c + \theta^2 T_1 b + 2\theta T_1 c - cT_1^2\theta^2\right)e^{-\theta t + \theta T_1}}{\theta^3} \\ &- \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 tb + 2\theta tc - ct^2\theta^2\right)}{\theta^3}, \quad 0 \leq t \leq T_1 \end{aligned} \tag{9}$$

and the order quantity is given by

$$Q_1 = I_1(0) = \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 T_1 b + 2\theta T_1 c - cT_1^2\theta^2\right)e^{\theta T_1}}{\theta^3} - \frac{a\left(\theta^2 - \theta b - 2c\right)}{\theta^3} \tag{10}$$

Now, the total cost per unit time of retailer per cycle time $T_1$ is comprised by

- Holding Cost: $$HC_1 = h(C + k)\int_0^{T_1} I_1(t)\,dt$$
- Purchasing Cost: $$PC_1 = (C + k)Q_1$$

Therefore, the total cost during the replenishment period $T_1$ per unit time becomes

$$TC(T_1) = \frac{1}{T_1}\left(OC + PC_1 + HC_1\right) \tag{11}$$
$$= \frac{1}{T_1}\left(A + (C+k)Q_1 + (C+k)h\int_0^{T_1} I_1(t)\,dt\right)$$

Similarly, there is a unique value for $T_1$ (say $T_1^*$) that minimizes $TC(T_1)$ and the value $T_1^*$ can be obtained by solving the equation $\frac{dTC(T_1)}{dT_1} = 0$.

Afterward, when the supplier declares a cost increment that is effective for a particular forthcoming time, the retailer may place a special order to take benefit of the present lower price, $C$, earlier to the cost increment. Alternatively, the retailer can ignore this announcement and place its regular order. To ward off a reduction in profit, the dealer is only willing to offer the retailer a limited quantity, before the price increase. This study determines to determine the optimal special-order quantity by maximizing the total cost saving between the special and regular order during the depletion time of the special-order quantity. As specified earlier, two explicit circumstances arise, which deliberates in this study: when the special-order time (1) coincides with the retailer's replenishment time or (2) occurs during the retailer's sales period. Following, this study will frame the reliable total applicable maximizing the total cost saving function for the above two cases.

### 3.1. Case I: the special-order time matches with the retailer's replenishment time

In this case, if the retailer decides to adopt a special-order and orders $Q_s$ units, then the inventory level at a time $t$ is

$$I_s(t) = \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 T_s b + 2\theta T_s c - cT_s^2\theta^2\right)e^{-\theta t + \theta T_s}}{\theta^3} - \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 tb + 2\theta tc - ct^2\theta^2\right)}{\theta^3}, \quad 0 \le t \le T_s \tag{12}$$

The special-order quantity at the original unit purchasing price, $C$, is

$$Q_s = I_s(0) = \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 T_s b + 2\theta T_s c - cT_s^2\theta^2\right)e^{\theta T_s}}{\theta^3} - \frac{a\left(\theta^2 - \theta b - 2c\right)}{\theta^3} \tag{13}$$

Next, the total cost per unit time $T_s$ of the retailer is comprised of

- Holding Cost: $HC_2 = hC\int_0^{T_s} I_s(t)\,dt$
- Purchasing Cost: $PC_2 = CQ_s$

The total cost of the special order during the time interval $[0, T_s]$ (denoted by $TCS_1(T_s)$ ) and is represented by

$$TCS_1(T_s) = \left(OC + PC_2 + HC_2\right) \tag{14}$$
$$= \left(A + CQ_s + Ch\int_0^{T_s} I_s(t)\,dt\right)$$

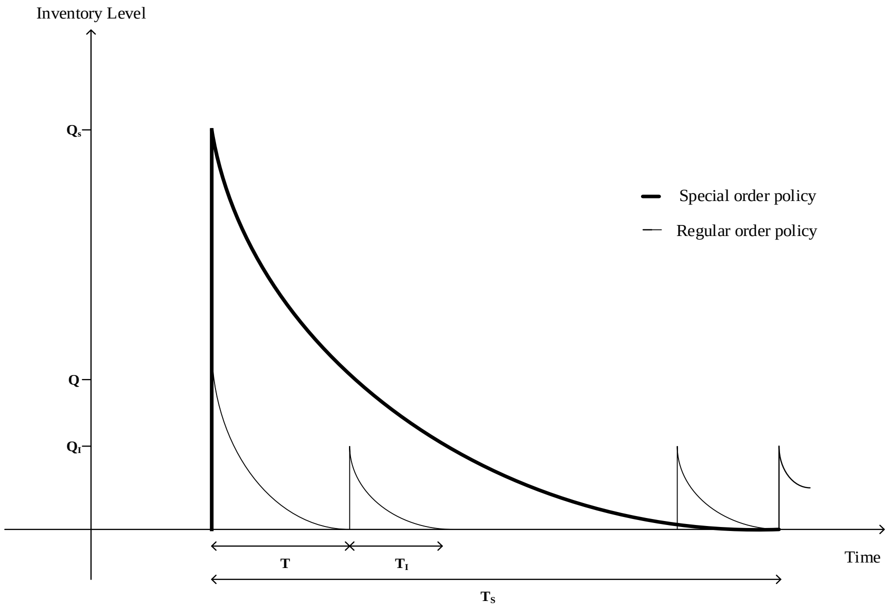
replenishment time (Shah *et al.* (2016))

If the retailer places its regular order, then the total cost of a regular order during the time interval $[0, T_s]$ will be divided into two periods (see fig. (1)). In the first period, the retailer orders $Q^*$, units at the unit purchasing price $C$. The total cost of the first period corresponding to scenario 1 and scenario 2 is similar to equations (6) and (8) respectively and given by

$$\frac{1}{T^*}\left(A + CQ^* + hC\int_0^{T^*} I(t)\,dt + CI_c\int_M^{T^*} I(t)\,dt - pI_e\int_0^M t\,R(t)\,dt\right) \tag{15}$$

$$\frac{1}{T^*}\left(A+CQ^*+hC\int_0^{T^*} I(t)\,dt-pI_e\int_0^{T^*} t\,R(t)\,dt\right) \tag{16}$$

**Figure 1.** Special vs. regular order policies when the special-order time coincides with the retailer's




As to the rest period, the retailer follows a regular EOQ policy for the unit purchasing price $(C+k)$. Hence, the total cost during the rest period is given by

$$\frac{T_s-T^*}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*} I_1(t)\,dt\right) \tag{17}$$

Hence, from equations (15) and (17) the total cost of the regular order(denoted by $TCN_1(T_s)$) for scenario 1 during the time interval $[0,T_s]$ is given by

$$\begin{aligned}TCN_1(T_s)=&\frac{1}{T^*}\left(A+CQ^*+hC\int_0^{T^*} I(t)\,dt+CI_c\int_M^{T^*} I(t)\,dt-pI_e\int_0^{M} t\,R(t)\,dt\right)\\&+\frac{T_s\text{-}T^*}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*} I_1(t)\,dt\right)\end{aligned}$$

(18)

Similarly, from equations (16) and (17) the total cost of the regular order (denoted by $TCN_2(T_s)$) for scenario 2 during the time interval $[0,T_s]$ is given by

$$\begin{aligned}TCN_2(T_s)=&\frac{1}{T^*}\left(A+CQ^*+hC\int_0^{T^*} I(t)\,dt-pI_e\int_0^{T^*} t\,R(t)\,dt\right)\\&+\frac{T_s\text{-}T^*}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*} I_1(t)\,dt\right)\end{aligned}$$

(19)

Compering Equation (14) with Equation (18), the total cost saving when the special-order coincides with the retailer's replenishment time (i.e., Case I) for scenario 1 can be expressed as follows:

$$g_1(T_s)=TCN_1(T_s)-TCS_1(T_s)$$
$$=\frac{1}{T^*}\left(A+CQ^*+hC\int_0^{T^*}I(t)\,dt+CI_c\int_M^{T^*}I(t)\,dt-pI_e\int_0^{M}t\,R(t)\,dt\right)$$
$$+\frac{T_s-T^*}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*}I_1(t)\,dt\right)-\left(A+CQ_s+Ch\int_0^{T_s}I_s(t)\,dt\right) \quad (20)$$

Simply, it can be shown that $g_1(T_s)$ is a concave function of $T_s$. Hence, there is a unique value for $T_s$ (say $T_{s1}^*$) that maximizes $g_1(T_s)$. The value of $T_{s1}^*$ can be obtained by solving the equations $\frac{dg_1(T_s)}{dT_s}=0$ and the corresponding optimal order quantity $Q_{s1}^*$ is given by

$$Q_{s1}^*=\frac{a\left(\theta^2-\theta b-2c+\theta^2T_{s1}^*b+2\theta T_{s1}^*c-c\left(T_{s1}^*\right)^2\theta^2\right)e^{\theta T_{s1}^*}}{\theta^3}-\frac{a\left(\theta^2-\theta b-2c\right)}{\theta^3} \quad (21)$$

Similarly, comparing Equation (14) with Equation (19), the total cost saving when the special-order coincides with the retailer's replenishment time (i.e., Case I) for scenario 2 can be expressed as follows:

$$g_2(T_s)=TCN_2(T_s)-TCS_1(T_s)$$
$$=\frac{1}{T^*}\left(A+CQ^*+hC\int_0^{T^*}I(t)\,dt-pI_e\int_0^{T^*}t\,R(t)\,dt\right)$$
$$+\frac{T_s-T^*}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*}I_1(t)\,dt\right)-\left(A+CQ_s+Ch\int_0^{T_s}I_s(t)\,dt\right) \quad (22)$$

Again, it can be shown that $g_2(T_s)$ is a concave function of $T_s$. Hence, there is a unique value for $T_s$ (say $T_{s2}^*$) that maximizes $g_2(T_s)$. The value of $T_{s2}^*$ can be obtained by solving the equations $\frac{dg_2(T_s)}{dT_s}=0$ and the corresponding optimal order quantity $Q_{s2}^*$ is given by

$$Q_{s2}^*=\frac{a\left(\theta^2-\theta b-2c+\theta^2T_{s2}^*b+2\theta T_{s2}^*c-c\left(T_{s2}^*\right)^2\theta^2\right)e^{\theta T_{s2}^*}}{\theta^3}-\frac{a\left(\theta^2-\theta b-2c\right)}{\theta^3} \quad (23)$$

### 3.2. Case II: the special-order time takes place during the retailer's sales period

Occasionally, the time of the price increase occurs during the retailer's sales period. In this condition, if the retailer prefers to adopt a special-order of quantity $Q_s$ at the current price $C$ and the special-order quantity has arrived, then at that time the stock level will increase immediately from $q$ to $Q_s+q$ (see Figure 2). Conversely, if the retailer ignores the notification about the price increase, that retailer will not place any order until the next replenishment. Now, this study will frame the total cost functions for the special and regular order policies and then compare the two. As the special-order quantity attains, the maximum inventory is given by

$$Q_s+q=\frac{a(\theta^2-\theta b-2c+\theta^2T_sb+2\theta T_sc-cT_s^2\theta^2)e^{\theta T_s}}{\theta^3}-\frac{a(\theta^2-\theta b-2c)}{\theta^3}$$
$$+\frac{a(\theta^2-\theta b-2c+\theta^2\left(T^*-t_q\right)b+2\theta\left(T^*-t_q\right)c-c\left(T^*-t_q\right)^2\theta^2)e^{\theta\left(T^*-t_q\right)}}{\theta^3}$$
$$-\frac{a(\theta^2-\theta b-2c)}{\theta^3} \quad (24)$$

Additionally, the inventory level at a time $t$ during the time interval $\left[0,T_q\right]$ can be obtained by

$$I_q(t)=\frac{a(\theta^2-\theta b-2c+\theta^2 T_q b+2\theta T_q c-cT_q^{\,2}\theta^2)e^{(-\theta t+\theta T_q)}}{\theta^3}-\frac{a(\theta^2-\theta b-2c+\theta^2 tb+2\theta tc-ct^2\theta^2)}{\theta^3},\qquad 0\le t\le T_q \tag{25}$$

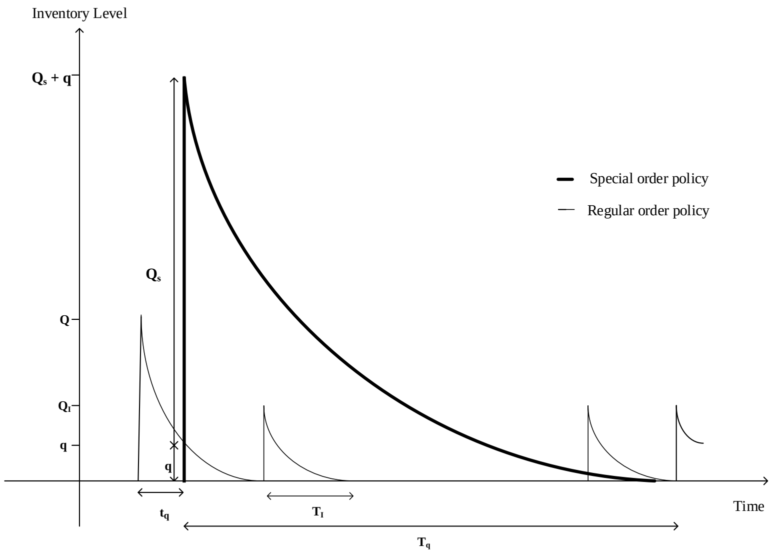



**Figure 2.** Special vs. regular order policies when the special-order time occurs during the retailer's sales period (Shah *et al.* (2016))

Since, $I_q(0)=Q_s+q$ , from Equation (24) and (25),

$$\begin{aligned}&\frac{a(\theta^2-\theta b-2c+\theta^2 T_s b+2\theta T_s c-cT_s^{\,2}\theta^2)e^{\theta T_s}}{\theta^3}-\frac{a(\theta^2-\theta b-2c)}{\theta^3}\\&+\frac{a(\theta^2-\theta b-2c+\theta^2\left(T^*-t_q\right)b+2\theta\left(T^*-t_q\right)c-c\left(T^*-t_q\right)^2\theta^2)e^{\theta\left(T^*-t_q\right)}}{\theta^3}-\frac{a(\theta^2-\theta b-2c)}{\theta^3}\\&=\frac{a(\theta^2-\theta b-2c+\theta^2 T_q b+2\theta T_q c-cT_q^{\,2}\theta^2)e^{(-\theta t+\theta T_q)}}{\theta^3}-\frac{a(\theta^2-\theta b-2c)}{\theta^3}\end{aligned} \tag{26}$$

Now, avoiding higher-order terms of $\theta$, one can find $T_q$ as follows

$$T_q=-\frac{1}{\theta^3}\left(\begin{aligned}&-e^{\theta T_s}\theta^2+e^{\theta T_s}\theta b+2e^{\theta T_s}c-e^{\theta T_s}\theta^2 T_s b\\&-2e^{\theta T_s}\theta T_s c+e^{\theta T_s}\theta^2 T_s^{\,2}c+2\theta^2-2\theta b\\&-4c-e^{\theta\left(T-t_q\right)}\theta^2+e^{\theta\left(T-t_q\right)}\theta b+2e^{\theta\left(T-t_q\right)}c\\&-e^{\theta\left(T-t_q\right)}\theta^2 Tb+e^{\theta\left(T-t_q\right)}\theta^2 bt_q-2e^{\theta\left(T-t_q\right)}\theta Tc\\&+2e^{\theta\left(T-t_q\right)}\theta ct_q+e^{\theta\left(T-t_q\right)}cT^2\theta^2-2e^{\theta\left(T-t_q\right)}\theta^2 cTt_q+e^{\theta\left(T-t_q\right)}\theta^2 ct_q^{\,2}\end{aligned}\right) \tag{27}$$

Here, the retailer's average holding cost per unit time $T_s$ is comprised by $HC_3=hC\int_0^{T_q} I_q(t)\,dt$

Subsequently, the total cost of the special-order (denoted by $TCS_2(T_s)$) during the time interval $\left[0,T_q\right]$ can be formulated as follows:

$$\begin{aligned}TCS_2(T_s)&=\left(OC+PC_2+HC_3\right)\\&=\left(A+CQ_s+hC\int_0^{T_q} I_q(t)\,dt\right)\end{aligned} \tag{28}$$

Conversely, if the retailer doesn't agree to buy the special-order quantity and just placed its regular order then the total profit between the time interval $\left[0,T_q\right]$ will be divided into two phases. In the first phase, the retailer only has the cost during the depletion time of residual $q$, (i.e, $T^* - t_q$ ) and by the use of average cost analysis approach on scenario 1 and scenario 2 which gives us the following:

$$\frac{T^*-t_q}{T^*}\left(A+CQ^*+hC\int_0^{T^*} I(t)\,dt+CI_c\int_M^{T^*} I(t)\,dt-pI_e\int_0^M t\,R(t)\,dt\right) \tag{29}$$

$$\frac{T^*-t_q}{T^*}\left(A+CQ^*+hC\int_0^{T^*} I(t)\,dt-pI_e\int_0^{T^*} t\,R(t)\,dt\right) \tag{30}$$

Next, the retailer places the regular order with the unit purchasing cost $C+k$ during the second phase. Here, by using the average cost analysis method to obtain the total cost of the second phase and it is given by

$$\frac{T_q-\left(T^*-t_q\right)}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*} I_1(t)\,dt\right) \tag{31}$$

Hence, from equations (29) and (31) the total cost of the regular order (denoted by $TCN_3\left(T_s\right)$) for scenario 1 during the time interval $\left[0,T_q\right]$ is given by

$$TCN_3(T_s)=\frac{T^*-t_q}{T^*}\left(A+CQ^*+hC\int_0^{T^*} I(t)\,dt+CI_c\int_M^{T^*} I(t)\,dt-pI_e\int_0^M t\,R(t)\,dt\right)+\frac{T_q-\left(T^*-t_q\right)}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*} I_1(t)\,dt\right) \tag{32}$$

Similarly, from equations (30) and (31) the total cost of the regular order (denoted by $TCN_4\left(T_s\right)$) for scenario 2 during the time interval $\left[0,T_q\right]$ is given by

$$TCN_4(T_s)=\frac{T^*-t_q}{T^*}\left(A+CQ^*+hC\int_0^{T^*} I(t)\,dt-pI_e\int_0^{T^*} t\,R(t)\,dt\right)+\frac{T_q-\left(T^*-t_q\right)}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*} I_1(t)\,dt\right) \tag{33}$$

Compering Equation (28) with Equation (32), the total cost saving when the special-order takes place during the retailer's sales period (i.e., Case II) for scenario 1 can be expressed as follows:

$$g_3(T_s)=TCN_3(T_s)-TCS_2(T_s)=\frac{T^*-t_q}{T^*}\left(A+CQ^*+hC\int_0^{T^*} I(t)\,dt+CI_c\int_M^{T^*} I(t)\,dt-pI_e\int_0^M t\,R(t)\,dt\right)$$
$$+\frac{T_q-\left(T^*-t_q\right)}{T_1^*}\left(A+(C+k)Q_1^*+(C+k)h\int_0^{T_1^*} I_1(t)\,dt\right) \tag{34}$$
$$-\left(A+CQ_s+hC\int_0^{T_q} I_q(t)\,dt\right)$$

Simply, it can be shown that $g_3\left(T_s\right)$ is a concave function of $T_s$. Hence, there is a unique value for $T_s$ (say $T_{s3}^*$ ) that maximizes $g_3\left(T_s\right)$. The value of $T_{s3}^*$ can be obtained by solving the equations $\frac{dg_3\left(T_s\right)}{dT_s}=0$ and the corresponding optimal order quantity $Q_{s3}^*$ is given by

$$Q_{s3}^*=\frac{a\left(\theta^2-\theta b-2c+\theta^2 T_{s3}^*b+2\theta T_{s3}^*c-c\left(T_{s3}^*\right)^2\theta^2\right)e^{\theta T_{s3}^*}}{\theta^3}-\frac{a\left(\theta^2-\theta b-2c\right)}{\theta^3} \tag{35}$$

Similarly, comparing Equation (28) with Equation (33), the total cost saving when the special-order takes place during the retailer's sales period (i.e., Case II) for scenario 2 can be expressed as follows:

$$g_4(T_s) = TCN_4(T_s) - TCS_2(T_s) = \frac{T^* - t_q}{T^*}\left(A + CQ^* + hC\int_0^{T^*} I(t)\,dt - pI_e\int_0^{T^*} t\,R(t)\,dt\right)$$
$$+\frac{T_q - (T^* - t_q)}{T_1^*}\left(A + (C+k)Q_1^* + (C+k)h\int_0^{T_1^*} I_1(t)\,dt\right) \quad (36)$$
$$-\left(A + CQ_s + hC\int_0^{T_q} I_q(t)\,dt\right)$$

Again, it can be shown that $g_4(T_s)$ is a concave function of $T_s$. Hence, there is a unique value for $T_s$ (say $T_{s4}^*$) that maximizes $g_4(T_s)$. The value of $T_{s4}^*$ can be obtained by solving the equations $\frac{dg_4(T_s)}{dT_s} = 0$ and the corresponding optimal order quantity $Q_{s4}^*$ is given by

$$Q_{s4}^* = \frac{a\left(\theta^2 - \theta b - 2c + \theta^2 T_{s4}^* b + 2\theta T_{s4}^* c - c\left(T_{s3}^*\right)^2 \theta^2\right)e^{\theta T_{s4}^*}}{\theta^3} - \frac{a\left(\theta^2 - \theta b - 2c\right)}{\theta^3} \quad (37)$$

**Remark:** Note that it is worth placing a special order only when the total cost saving is positive in the above cases. Otherwise, the retailer will ignore the opportunity to place a special order.

## 4. NUMERICAL EXAMPLES AND OPTIMAL SOLUTION

The developed mathematical model is illustrated in the following numerical examples. The retailer's total cost per unit time is optimized using Maple XVI is given in Table 1.

**Table 2** Input parameter for the numerical examples

| **Parameters** | **Examples** | |
|---|---|---|
| | Scenario 1( $M < T$ ) | Scenario 2( $M \geq T$ ) |
| $a$ (units) | 100 | 100 |
| $b$ | 0.05 | 0.05 |
| $c$ | 0.05 | 0.05 |
| $A$ ($/order) | 50 | 50 |
| $\theta$ | 0.2 | 0.2 |
| $h$ ($/unit/unit time) | 3 | 3 |
| $C$ ($/unit) | 20 | 20 |
| $k$ ($/unit) | 6 | 6 |
| $p$ ($/unit) | 35 | 35 |
| $I_c$ | 0.10 | 0.10 |
| $I_e$ | 0.09 | 0.09 |
| $M$ (in years) | $\frac{30}{365}$ | $\frac{120}{365}$ |

**Table 3 Optimal Solution**

| | | |
|---|---|---|
| $T_{si}\ ;\ i = 1,2,3,4$ (in years) | $T_{s1} = 1.190675142$ | $T_{s3} = 1.190675142$ |
| | $T_{s2} = 0.1454937942$ | $T_{s4} = 0.1454937942$ |
| $Q_{si};\ i = 1,2,3,4$ (units) | $Q_{s1} = 135.2337537$ | $Q_{s3} = 135.2337537$ |
| | $Q_{s2} = 14.8118448$ | $Q_{s4} = 14.8118448$ |

| | | |
|---|---|---|
| $g_i(T_s); i = 1,2,3,4$ (in $) | **$g_1$ = 1861.45420** | $g_3 = 1835.58843$ |
| | $g_2 = 17.68289$ | $g_4 = 17.68279$ |

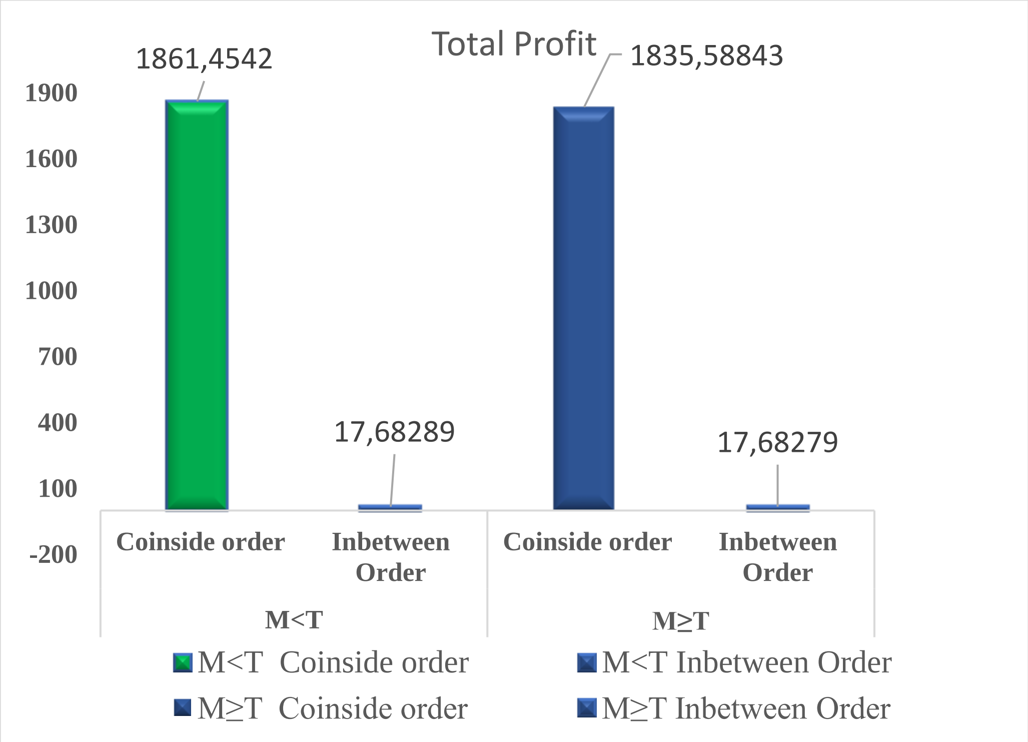



**Figure 3. Optimal solution**

An optimal solution is a feasible solution that has the most favorable value of the objective function. A feasible solution is a solution for which all the constraints are satisfied. Above all optimum solutions, Case I for scenario 1 ( $M < T$ ) has the best optimum solution as the total cost saving is maximum and positive (i.e., $g_1(T_s)$ is our optimal solution, Which is coincided order when $M < T$ ). Moreover, Figure 4 shows the concavity of total cost savings $g_1(T_s)$ between regular and special orders.

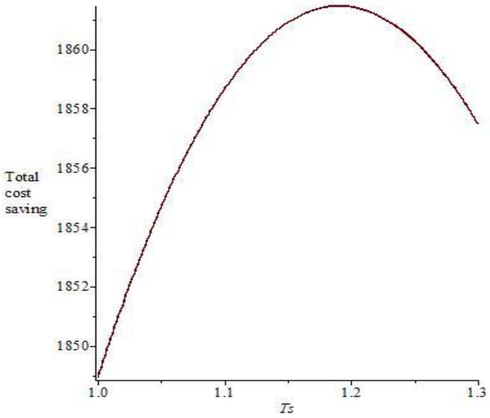



**Figure 4.** Concavity of total cost saving $g_1(T_s)$

## 5. Sensitivity analysis for the inventory parameters

Now, for data of Example 1, this article study the effects of various inventory parameters on total cost savings $g_1(T_s)$, decision variables order quantity $Q_s$ , and period $T_s$ by changing them as -20%, -10%, 10%, and 20%.

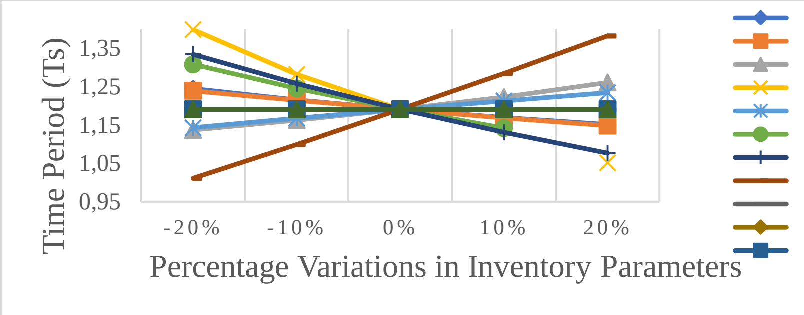



**Figure 5.** Variations in period $T_s$ w. r. t. inventory parameters

From Figure 5, it is noticed that the deterioration rate $(\theta)$ and holding cost $(h)$ have a huge negative impact on the period. Whereas, scaled demand $(a)$, perches cost $(C)$, and linear rate of change of demand $(b)$ drop period slowly. If the rate of quadratic demand $(c)$, ordering cost $(A)$, and price increase $(k)$ then a period increases.

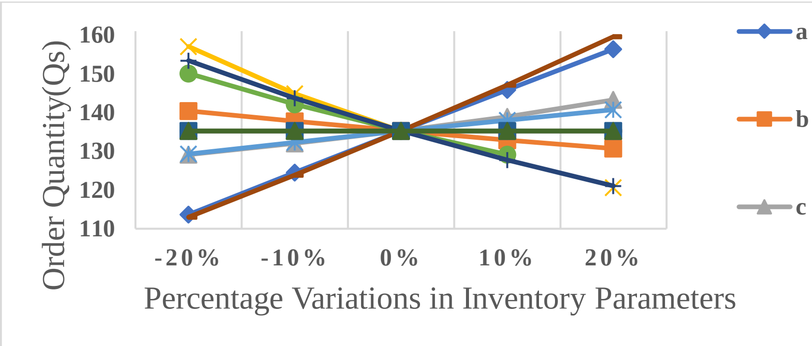



**Figure 6**. Variations in order quantity $Q_s$ w. r. t. inventory parameters

From Figure 6, it is noticed that the deterioration rate $(\theta)$, rate of linear demand $(b)$, and holding cost $(h)$ have a huge negative impact on order quantity. Scaled demand $(a)$, rate of quadratic demand $(c)$, ordering cost $(A)$, and unit price increase $(k)$ have a large positive effect on order quantity.

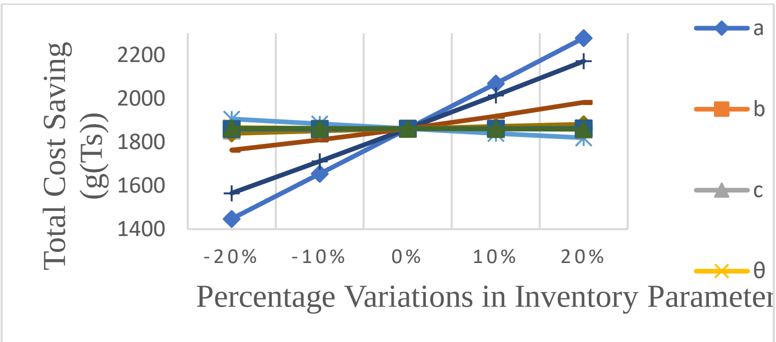



**Figure 7.** Variations in total cost saving $g_1(T_s)$ w. r. t. inventory parameters

From Figure 7, it is observed that scaled demand $(a)$, quadratic rate of change of demand $(c)$, and unit price increase $(k)$ have a huge positive impact on total cost saving. Selling price $(p)$, interest charge $(I_c)$, and interest earn $(I_e)$ an increase in total cost saving slowly. On the other unit, trade-credit $(M)$

decreases total cost savings gradually. Holding cost $(h)$, ordering cost $(A)$, the linear rate of change of demand $(b)$, and deterioration rate $(\theta)$ have an excessively negative effect on total cost saving.

## 6. CONCLUSION

This paper analyzes the impact of the future price increase of a vendor on the buyer's decision policy. Here, time-dependent quadratic demand is discussed which is also called the seasonal demand and it is applicable for seasonal products like ice cream, cold-drink, umbrellas, medicines during epidemics, etc. In this model, the product deteriorates at a constant rate. Moreover, permissible delay in payments is a very realistic approach in today's business world. So, this paper consists of the inventory model in which the vendor gives mutually agreed trade credit to the buyer before the time of price increase to boost the demand of the product. The total cost saving between special and regular order is maximized and results are validated by numerical examples and graphs. The decision-maker should place an order when trade credit offered by a vendor to a buyer is less than the cycle time before the price increase to get the maximum benefit of total cost saving. This paper can be extended in numerous ways. For instance, one can consider variable deterioration rate, price-sensitive demand, and two levels of trade credit.

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