THE CUBIC TRANSMUTED TYPE I GENERALIZED HALF LOGISTIC DISTRIBUTION

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ABSTRACT
In this paper, we proposed a new distribution called the Cubic Transmuted Type I Generalized Half Logistic Distribution using the Cubic Transmuted-G distribution which is more flexible than its submodels. The statistical properties of the distribution such as survival function, hazard function, mode, median, order statistics were studied. Parametric estimation of the distribution was done using the maximum likelihood method. The distribution was applied to two real life data in which the results revealed that the cubic transmuted type I generalized half logistic distribution performs better than its submodels.

KEYWORDS: Cubic Transmuted- G Distribution; Type I Generalized Half logistic distribution;statistical properties; Submodels

MSC: 62E05

RESUMEN
En este paper, propusimos una nueva distribución llamada Medio Logística Transmutada Cúbica de tipo I Generalizada. Usando la G distribución transmutada que es más flexible que sus submodelos. Las propiedades estadísticas de la distribución como las funciones de sobrevivencia, hazard, moda, mediana, estadísticos de orden, fueron estudiadas. La estimaci'pn paramétrica de la distribución fue realizada usando el método de máxima verosimilitud. La distribución fue aplicada a dos datos de la vida real en los que los resultados revelaron que la Medio Logística Transmutada Cúbica de tipo I Generalizada se desempeña mejor que los submodelos.

PALABRAS CLAVE: Medio logística transmutada Cúbica, Generalizada de tipo I Submodelos.

1. INTRODUCTION

The Half logistic has been generalized to form new distributions that are more flexible and fit data better. A generalized form of the half logistic distribution is the type I generalized half logistic distribution as obtained by Olapade(2014). The type I generalized half logistic distribution has been further generalized as obtained by Awodutire *et al*(2020a), Awodutire *et al*(2020b), Bello *et al*(2018),

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Olapade *et al*(2011). The generalized distribution has been applied to life situations, for example Awodutire *et al*(2016) obtained a survival model and Awodutire *et al*(2017) applied the derived model to survival time of Breast Cancer patients in Nigeria. The type I generalized half logistic distribution has cumulative density function G(x) as

$$G(x) = 1 - \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{b} x > 0, \psi > 0 \tag{1.1}$$

with corresponding probability density function g(x) as

$$g(x) = \frac{b2^{b}e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{b+1}} \tag{1.2}$$

The generalization of the half logistic(a submodel of the type I generalized half logistic) called Transmuted Half Logistic was studied by Adeyinka and Olapade(2019) using the transmutation derived by Shaw and Beckley(2007). The quadratic transmuted family of distribution is given as

$$F(x) = (1+\rho)G(x) - \rho G^{2}(x) \quad |\rho| < 1 \tag{1.3}$$

where ρ is the transmuted parameter. The quadratic transmuted distributions are, generally, not capable to handle the bi-modality of the data. Recently, Rahman *et al*(2018) proposed an extension of equation 1.3 by the addition of a new parameter named the derived family the cubic transmuted family of distributions. The cdf of cubic transmuted family of distributions is given as

$$F(x) = G(x)[(1+\rho_1) + (\rho_2 - \rho_1)G(x) - \rho_2 G^{2}(x)] \tag{1.4}$$

where $\rho_1 \in$ [-1,1], $\rho_2 \in$[-1,1] are the transmutation parameters of the distribution. The resulting pdf is given as

$$f(x) = (1+\rho_1)G'(x) + 2(\rho_2 - \rho_1)G(x)G'(x) - 3\rho_2 G^{2}(x)G'(x) \tag{1.5}$$

which is further simplified as

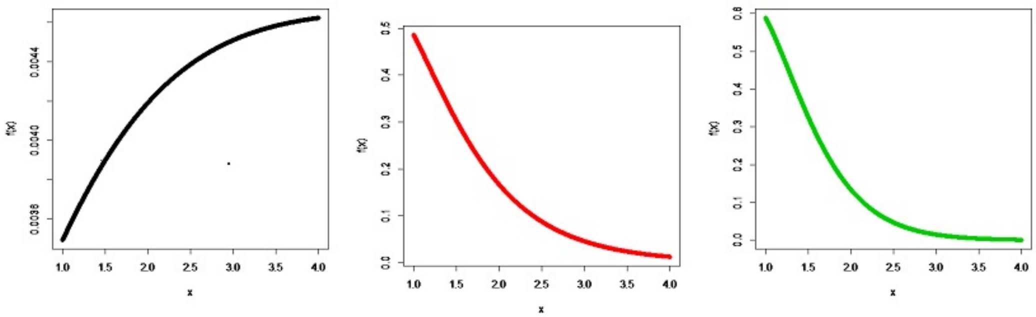
$$f(x) = G'(x)[(1+\rho_1) + 2(\rho_2 - \rho_1)G(x) - 3\rho_2 G^{2}(x)] \tag{1.6}$$

In this work, the type I generalized half logistic is futher generalized to obtain a new distribution using the transmutation family of distributions of Rahman *et al*(2018) The derived distribution is named Cubic Transmuted Type I Generalized Half Logistic Distribution(CTTIGHLD). The statistical properties of the derived distribution will be studied, estimation of parameters will be done and real life applications will be considered.

2. CUBIC TRANSMUTED TYPE I GENERALIZED HALF LOGISTIC DISTRIBUTION(CTTIGHLD)

To derive the cumulative distribution function of CTTIGHLD, we put equation in 1.1 in 1.4. Therefore, the cdf is obtained as

$$F(x) = 1 - \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{b}\left[(1+\rho) + (\rho_2 - \rho_1)\left(1 - \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{b}\right) - \rho_2\left(1 - \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{b}\right)^{2}\right] \tag{2.1}$$



ψ=1,ρ_1=0.6,ρ_2=-0.2, b= 0.003 | ψ=1,ρ_1=-0.3,ρ_2=0.6,b= 1.3 | ψ=1,ρ_1=0.6,ρ_2=-0.2, b= 0.003

Figure 1: Graph of the p.d.f of CTTIGHLD with various parameter values

which further gives

$$F(x) = 1 - \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b \left[1 + (\rho_2 + \rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b - \rho_2 \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\right] \quad x > 0, \psi, b > 0 \qquad (2.2)$$

The corresponding probability distribution function of CTTIGHLD can be obtained by putting 1.2 in 1.5. Doing this, we derived the pdf as

$$f(x) = \frac{b2^b e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{b+1}} \left[1 - \rho_1 - \rho_2 + 2(2\rho_2 + \rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b - 3\rho_2\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\right] \qquad (2.3)$$

where the ρ_1 and ρ_2 are the transmuted parameters, further satisfying the conditions $-2 \leq \rho_1 + \rho_2 \leq \infty$. Figure 1 shows different shapes of the p.d.f of the distribution with different parameters assigned. The submodels of the CTTIGHLD are as follows:

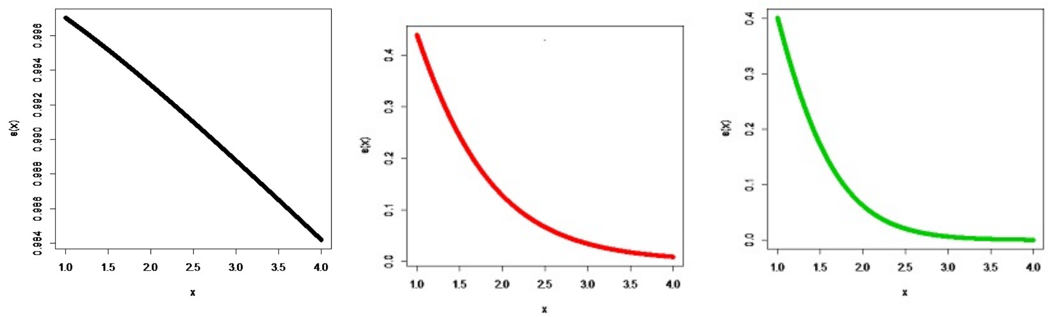
1. When $\rho_1 = 0$ and $\rho_2 = 0$, the resulting distribution is 1.2 as obtained by Olapade(2014)
2. When $\rho_2 = 0$ and b=1, we have the transmuted half logistic distributin of Adeyinka and Olapade(2019)
3. When $\rho_1 = 0$, $\rho_2 = 0$ and b= 1, we have the half logistic distribution
4. When $\rho_2 = 0$, the resulting distribution is

$$f(x) = \frac{b2^b e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{b+1}} \left[1 - \rho_1 + 2\rho_1\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b\right]$$

which is the transmuted type I generalized half logistic distribution

3. STATISTICAL PROPERTIES OF CTTIGHLD

In this section, the statistical properties of the CTTIGHLD were studied. The properties are discussed in the subsections below



ψ=1,ρ_1=0.6,ρ_2=-0.2, b= 0.003 | ψ=1,ρ_1=-0.3,ρ_2=0.6,b= 1.3. | ψ=1,ρ_1=0.6,ρ_2=-0.2, b= 0.003

Figure 2: Graph of the survival function of CTTIGHLD with various parameter values

3.1. SURVIVAL FUNCTION

To derive the survival function s(x),

$$s(x) = 1 - F(x)$$

Therefore, we have the survival function as

$$s(x) = 1 - \left(1 - \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b \left[1 + (\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b - \rho_2\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\right]\right) \tag{3.1}$$

In Figure 2, different plots of the survival function on the distribution with assinged values of the parameters

3.2. HAZARD FUNCTION

To derive the hazard function h(x),

$$h(x) = \frac{f(x)}{s(x)}$$

Therefore, we have the hazard function as

$$h(x) = \frac{\frac{b2^b e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{b+1}}\left[1 - \rho_1 - \rho_2 + 2(2\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b - 3\rho_2\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\right]}{1 - \left(1 - \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b \left[1 + (\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b - \rho_2\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\right]\right)} \tag{3.2}$$

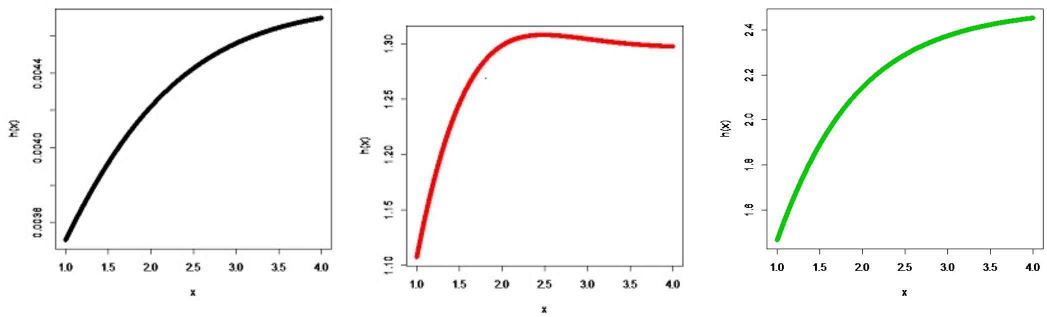
Figure 3 reveals different curves of the hazard function which shows it flexibility.

3.3. CUMMULATIVE HAZARD FUNCTION

$$H(x) = -logs(x) \tag{3.3}$$

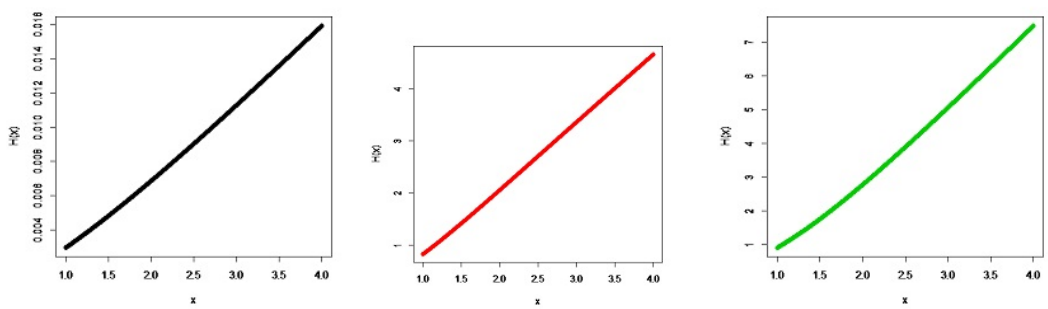
$$H(x) = -log\left(1 - \left(1 - \left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b \left[1 + (\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b - \rho_2\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\right]\right)\right) \tag{3.4}$$

Figure 4 shows the different plots of the cummulative hazard function.



ψ=1,ρ_1=0.6,ρ_2=-0.2, b= 0.003 ψ=1,ρ_1=-0.3,ρ_2=0.6,b= 1.3. ψ=1,ρ_1=0.6,ρ_2=-0.2, b= 0.003

Figure 3: Graph of the hazard function of CTTIGHLD with various parameter values



ψ=1,ρ_1=0.6,ρ_2=-0.2, b= 0.003 ψ=1,ρ_1=-0.3,ρ_2=0.6,b= 1.3. ψ=1,ρ_1=0.6,ρ_2=-0.2, b= 0.003

Figure 4: Graph of the cummulative hazard function of CTTIGHLD with various parameter values

3.4. MEDIAN

$$Median = \int_0^{x_m} f(x)dx = 1 - \int_{x_m}^{\infty} f(x)dx = \frac{1}{2} \tag{3.5}$$

$$= \int_0^{x_m} G'(x_m)[(1+\rho_1) + 2(\rho_2 - \rho_1)G(x_m) - 3\rho_2 G^2(x_m)] \tag{3.6}$$

$$= F(x_m/b) - \rho_1 F(x_m/b) - \rho_2 F(x_m/b) + (2\rho_2 + \rho_1)F(x_m/2b) - 3\rho_2 F(x_m/3b) \tag{3.7}$$

$$= 1 - \left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b - \rho_1\left(1 - \left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right) - \rho_2\left(1 - \left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right)$$

$$+(2\rho_2 + \rho_1)\left(1 - \left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right) - 3\rho_2\left(1 - \left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^{2b}\right) = \frac{1}{2} \tag{3.8}$$

The values of the median(x_m) can be obtained using Newton Raphson Iteration method by assigning values to the parameters

3.5. MODE

The mode (M_x) of the cubic transmuted type I generalized half logistic distribution is

$$M_x = b2^b[\frac{(1+e^x)e^x - e^{2x}(b+1)}{(1+e^{b+2})}][1-\rho_1-\rho_2+2(2\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b - 3\rho_2\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^{2b}]$$

$$+\frac{b2^b e^{\frac{x}{\psi}}}{\psi(1+e^{\frac{x}{\psi}})^{b+1}}[-(2\rho_2+\rho_1)\frac{b2^b e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{b+1}} + 3\rho_2\frac{2b2^{2b}e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{2b+1}}] \tag{3.9}$$

3.6. ORDER STATISTICS

The importance of Order statistics in non-parametric statistics and inference cannot be overemphasized. Given a random sample $x_1, x_2, ..., x_n$ from the CTTIGHLD, the p.d.f of the ith order statistics is given by

$$f_{(i:n)}(x) = n\binom{n-1}{n-j} f(x)[1-F(x)]^{i-1}F(x)^{n-i} \tag{3.10}$$

Therefore, putting 2.1 and 2.3 in 3.10, we have the result as

$$n\binom{n-1}{n-j}\frac{b2^b e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{b+1}}\left[1-\rho_1-\rho_2+2(2\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b - 3\rho_2\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right]$$

$$1-\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\left[1+(\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b - \rho_2\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right]^{i-1}$$

$$\left(1-\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right)\left[1+(\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b - \rho_2\left(\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right)\right]^{n-i} \tag{3.11}$$

in which the first Order Statistics $X_{(1)}$ has the marginal p.d.f as

$$n\frac{b2^b e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{b+1}}\left[1-\rho_1-\rho_2+2(2\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b - 3\rho_2\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\right]$$

$$1-\left(\left(1-\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b\left[1+(\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b - \rho_2\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\right]\right)\right)^{n-1} \tag{3.12}$$

and the n^{th} order statistics (X_n) having the marginal pdf as

$$n\binom{n-1}{n-j}\frac{b2^b e^{\frac{x}{\psi}}}{\psi\left(1+e^{\frac{x}{\psi}}\right)^{b+1}}\left[1-\rho_1-\rho_2+2(2\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b - 3\rho_2\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right]$$

$$\left(1-\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right)\left[1+(\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b - \rho_2\left(\left(\frac{2}{1+e^{\frac{x_m}{\psi}}}\right)^b\right)\right]^{n-i} \tag{3.13}$$

3.7. QUANTILE FUNCTION

We obtained the quantile function x_h of the CTTIGHLD by solving for x in equation 8 and is given as

$$x_h = \psi \ln(2e^{\frac{1}{b}\ln(\Gamma)} - 1) \tag{3.14}$$

where

$$\Gamma = \frac{-b}{3a} - \frac{2^{\frac{1}{3}}\Psi}{3a(\Theta + \sqrt{4\Psi^3 + \Theta^2})^{\frac{1}{3}}} + \frac{(\Theta + \sqrt{4\Psi^3 + \Theta^2})^{\frac{1}{3}}}{3(2^{\frac{1}{3}})a} \tag{3.15}$$

and

$$\Psi = -f^2 + 3ac, \Theta = -2f^3 + 9afc - 27a^2 d \tag{3.16}$$

given that

$$a = \rho_2, f = -\rho_1 - 2\rho_2, c = \rho_1 + \rho_2 - 1 \;\; and \; d = 1 - h \tag{3.17}$$

h = 0.25, 0.50 and 0.75 for Q_1,Q_2 and Q_3 respectively. Other partitions can also be derived by setting h to the value of the partition.

3.8. PARAMETRIC ESTIMATION UNDER COMPLETE OBSERVATION

In this section we have discussed the maximum likelihood estimation (MLE) for parameters of cubic transmuted Pareto distribution. The likelihood function for the cubic transmuted type I generalized half logistic distribution is given as

$$L = \frac{b^n 2^{nb} \prod_{i=1}^{n} e^{\frac{x}{\psi}}}{\psi \prod_{i=1}^{n}(1 + e^{\frac{x}{\psi}})^{b+1}} \prod_{i=1}^{n}(1 - \rho_1 - \rho_2 + 2\frac{2^b}{(1 + e^{\frac{x}{\psi}})^{2b}}(2\rho_2 + \rho_1) - 3\rho_2 \frac{2^{2b}}{(1 + e^{\frac{x}{\psi}})^{2b}}) \tag{3.18}$$

Taking the log of L, we have

$$logL = l = n\ln b + nb\ln 2 + \sum_{i=1}^{n} x_i - (b+1)\sum_{i=1}^{n}\ln(1 + e^{\frac{x}{\psi}}) +$$

$$\sum_{i=1}^{n}\ln(1 - \rho_1 - \rho_2 + 2\frac{2^b}{(1 + e^{\frac{x}{\psi}})^{2b}}(2\rho_2 + \rho_1) - 3\rho_2 \frac{2^{2b}}{(1 + e^{\frac{x}{\psi}})^{2b}}) \tag{3.19}$$

Deriving the maximum likelihood estimates(M.L.E) of the parameters ρ_1,ρ_2, b, ψ by maximizing l and setting the derivatives with the parameters to zero, we have

$$\frac{dl}{d\rho_1} = \sum_{i=1}^{n} \frac{-1 + 2(\frac{2^b}{(1+e^{\frac{x}{\psi}})^b})}{1 - \rho_1 - \rho_2 + 2\frac{2^b}{(1+e^{\frac{x}{\psi}})^{2b}}(2\rho_2 + \rho_1) - 3\rho_2 \frac{2^{2b}}{(1+e^{\frac{x}{\psi}})^{2b}}} \tag{3.20}$$

$$\frac{dl}{d\rho_2} = \sum_{i=1}^{n} \frac{-1 + 4(\frac{2^b}{(1+e^{\frac{x}{\psi}})^b}) - 3(\frac{2^b}{(1+e^{\frac{x}{\psi}})^{2b}})}{1 - \rho_1 - \rho_2 + 2\frac{2^b}{(1+e^{\frac{x}{\psi}})^{2b}}(2\rho_2 + \rho_1) - 3\rho_2 \frac{2^{2b}}{(1+e^{\frac{x}{\psi}})^{2b}}} \tag{3.21}$$

$$\frac{dl}{d\psi} = -\sum_{i=1}^{n}\frac{x}{\psi^2} - (b+1)\sum_{i=1}^{n}\frac{e^{\frac{x}{\psi}}\frac{x}{\psi}}{1 + e^{\frac{x}{\psi}}}$$

$$-2b2^b(2\rho_2+\rho_1)\sum_{i=1}^{n}\frac{\frac{e^{\frac{x}{\psi^2}}x}{(1+e^{\frac{x}{\psi}})^{b+1}}+6b\rho_2 2^{2b}e^{\frac{x}{\psi}}}{1-\rho_1-\rho_2+2(2\rho_2+\rho_1)\frac{2^b}{(1+e^{\frac{x}{\psi}})^b}-3\rho_2\frac{2^{2b}}{(1+e^{\frac{x}{\psi}})^{2b}}} \tag{3.22}$$

$$\frac{dl}{db}=\sum_{i=1}^{n}\left(1+e^{\frac{x}{\psi}}\right)+\frac{2(2\rho_2+\rho_1)\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^b\ln\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)-6\rho_2\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)^{2b}\ln\left(\frac{2}{1+e^{\frac{x}{\psi}}}\right)}{1-\rho_1-\rho_2+2(2\rho_2+\rho_1)\frac{2^b}{\left(1+e^{\frac{x}{\psi}}\right)^{2b}}-3\rho_2\frac{2^{2b}}{\left(1+e^{\frac{x}{\psi}}\right)^{2b}}} \tag{3.23}$$

Setting $\frac{dl}{d\theta}$ =0 , θ=(ρ_1,ρ_2,b,ψ) and solving the resulting equation to obtain $\hat{\theta}$ =($\hat{\rho_1}$,$\hat{\rho_2}$,b,,$\hat{\psi}$) The equations 3.20, 3.21,3.22 and 3.23 are not in closed forms. The maximum likelihood estimates of each parameter is obtained numerically from the non-linear equations using computer programs.
For interval estimation and test of hypothesis on the parameters $(\psi, \rho_1, \rho_2, b)$, we obtain a 4x4 information matrix

$$M=\begin{bmatrix} M_{\rho_1,\rho_1} & M_{\rho_1,\rho_2} & M_{\rho_1,b} & M_{\rho,\psi} \\ M_{\rho_1,\rho_2} & M_{\rho_2,\rho_2} & M_{\rho_2,b} & M_{\rho_2,\psi} \\ M_{\rho_1,b} & M_{,\rho_2,b} & M_{b,b} & M_{b,\psi} \\ M_{\rho_1,\psi} & M_{\rho_2,\psi} & M_{b,\psi} & M_{\psi,\psi} \end{bmatrix}$$

The corresponding elements are derived by the second derivates of l with respect to the parameters Under conditions that are fulfilled for parameters, the asymptotic distribution of $\sqrt{n}(\hat{\theta}-\theta)$ is $N_4(0, M(\hat{\theta})^{-1})$ distribution of θ can be used to construct approximate confidence intervals and confidence regions for the parameters and for the hazard and survival functions. The asymptotic normality is also useful for testing goodness of fit of the Cubic transmuted type I generalized half logistic distribution and for comparing this distribution with some of its special sub-models using one of these two well known asymptotically equivalent test statistics- namely, the likelihood ratio statistic and Wald statistic. An asymptotic confidence interval with significance level α for each parameter θ_i is given by

$$ACI(\theta_i, 100(1-\alpha)) = \hat{\theta} - z_{\frac{\alpha}{2}}\sqrt{M^{\hat{\theta},\hat{\theta}}}, \theta + z_{\frac{\alpha}{2}}\sqrt{M^{\hat{\theta},\hat{\theta}}} \tag{3.24}$$

where $M^{\hat{\theta},\hat{\theta}}$ is the i^th diagonal element of $K_n(\hat{\theta})^{-1}$ for $i = 1, 2, 3, 4$ and $z_{\alpha/2}$ is the quantile of the standard normal distribution.

4. APPLICATION TO REAL DATA

In application to real data, we applied the CTTIGHL distribution and its sumodels to two datasets. They are the survival times of guinea pigs and the . For model comparison, Akaike Information criterion (AIC) , Bayesian Information criterion (BIC) and Corrected Akaike Information criterion (CAIC) were used to compare the performance of the Cubic Transmuted Type I Generalized half logistic distribution (CTTIGHL) and its submodels, which are the Transmuted Half Logistic Distribution(THL),Transmuted Type I generalized half logistic distribution (TTIGHL) and the type I generalized half logistic distribution(TIGHL) and Half Logistic Distribution. The AIC= -2l+2k, BIC= -2l+klog(n), CAIC= AIC+(2k(k+1))/(n-k+1), where l is the maximized value from the loglikelihood function, n is the sample size and k is the number of parameters. The model with the corresponding criterion with the lowest value(i.e lowest AIC,BIC,CAIC) is chosen as the best model that fits the data set.

4.1. SURVIVAL TIMES OF GUINEA PIGS

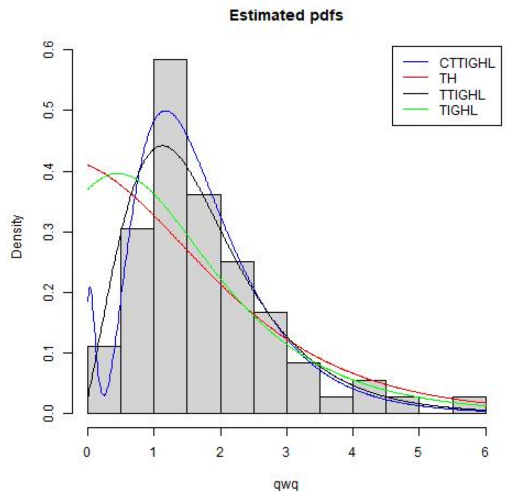
The data represents the survival time of 72 Guinea Pigs infected with virulent tubercle bacilli. The data set is obtained from the work of Bjerkedal, T. (1960). The data are as follows: 0.1, 0.33, 0.44, 0.56, 0.59, 0.72, 0.74, 0.77, 0.92, 0.93, 0.96, 1.00, 1.00, 1.02, 1.05, 1.07, 0.7, 0.08, 1.08, 1.08, 1.09, 1.12, 1.13, 1.15, 1.16, 1.20, 1.21, 1.22, 1.22, 1.24, 1.3, 1.34, 1.36, 1.39, 1.44, 1.46, 1.53, 1.59, 1.60, 1.63, 1.63, 1.68, 1.71, 1.72, 1.76, 1.83, 1.95, 1.96, 1.97, 2.02, 2.13, 2.15, 2.16, 2.22, 2.30, 2.31, 2.40, 2.45, 2.51, 2.53, 2.54, 2.54, 2.78, 2.93, 3.27, 3.42, 3.47, 3.61, 4.02, 4.32, 4.58, 5.55.
Table 1 describes the data. Figure 5 shows the histogram of the data with the fitted distributions. Figure 6 is the TTT plot of the data, revealing the shape of the hazard curve, which is non decreasing. After analysis, the results showing the estimates of the parameters of the distributions and the AIC,BIC and CAIC are presented in table 2 in which it shows that CTTIGHL performs best when compared to its submodels, using the data due to smallest values of AIC,BIC and CAIC.

Table 1: Table displaying Descriptive analysis of survival time of Guinea Pigs

Minimum	First Quartile	Median	Mean	Third Quartile	Maximum
0.080	1.077	1.495	1.749	2.240	5.550

Table 2: Table displaying results of analysis of Survival Times of Guinea Pigs

Model	Parameter	Estimate	L	AIC	BIC	CAIC
CTIGHL	ρ_1	-0.683	94.172	197.24	204.07	197.59
	ρ_2	-1.983				
	b	0.044				
	ψ	0.037				
THL	ρ_1	0.189	107.186	308.64	306.36	309
	ψ	1.451				
TTIGHL	ρ_1	-0.953	96.681	314.682	316.54	315.04
	b	0.690				
	ψ	0.649				
TIGHL	b	0.539	104.711	314.682	316.54	315.04
	ψ	0.731				



pig.jpg

Figure 5: Graph of Guinea Pigs with the density curve

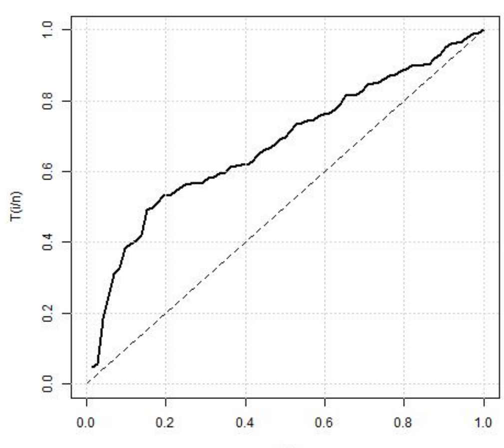


Figure 6: TTT plot of Guinea Pig data

4.2. INTERVALS OF SUCCESSIVE COAL MINING DISASTER

Data set 2 contains intervals in days between 109 successive coal-mining disasters in Great Britain, for the period 1875–1951, published by Mahdavi and Kundu [10]. The data are given as: 1, 4, 4, 7, 11, 13, 15, 15, 17, 18, 19, 19, 20, 20, 22, 23, 28, 29, 31, 32, 36,37, 47, 48, 49, 50, Data set 2: 54, 54, 55, 59, 59, 61, 61, 66, 72, 72, 75, 78, 78, 81, 93, 96, 99, 108, 113, 114, 120, 120, 120,123, 124, 129, 131, 137, 145, 151, 156, 171, 176, 182, 188, 189, 195, 203, 208, 215, 217, 217, 217, 224, 228, 233, 255, 271, 275, 275, 275, 286, 291, 312, 312, 312, 315, 326,326, 329, 330, 336, 338, 345, 348, 354, 361, 364, 369, 378, 390, 457, 467, 498, 517, 566, 644, 745, 871, 1312, 1357, 1613, 1630. To demonstrate the proposed family of distributions, Anderson–Darling (A), Cramer–von Mises (W*), Akaike information criterion (AIC), Bayesian information criterion (BIC), Consistent and Akaike information criterion (CAIC) are calculated for each of the three particular mod els. The general strategy is to choose the model having minimum value of these statistics as the best model. In this study, numerical results (of maximum likelihood estimates and goodness of fit criteria) are calculated by using the **nlm()** command available in **R** language. Analyzing the data, using R, table 3 shows the description of the data.Figure 7, which is the plot of the histogram shows the skewness of the data with the fitted distribution. Figure 8 is the TTT plot of the data.It shows that the hazard curve of the data is non-decreasing.Estimates of the parameters of the distributions and their respective values of the loglikelihood, AIC,BIC and CAIC are in table 4 in which revealed that the CTTIGHL performs best when compared to its submodels.

Table 3: Table displaying Descriptive analysis of Interval of successive coal mining

Minimum	First Quartile	Median	Mean	Third Quartile	Maximum
1.0	54.0	145.0	233.3	312.0	1630.0

Table 4: Table displaying results of analysis of Interval of successive coal mining

Model	Parameter	Estimate	L	AIC	BIC	CAIC
CTIGHL	ρ_1	3.225	699.572	1407.14	1417.905	1407.517
	ρ_2	-0.493				
	b	0.007				
	ψ	3.016				
THL	ρ_1	0.723	704.53	1417.06	1427.825	1417.437
	ψ	241.085				
TTIGHL	ρ_1	0.501	702.16	1412.32	1423.085	1412.697
	b	0.003				
	ψ	1.98				
TIGHL	b	0.022	703.74	1415.48	1426.245	1415.857
	ψ	5.001				

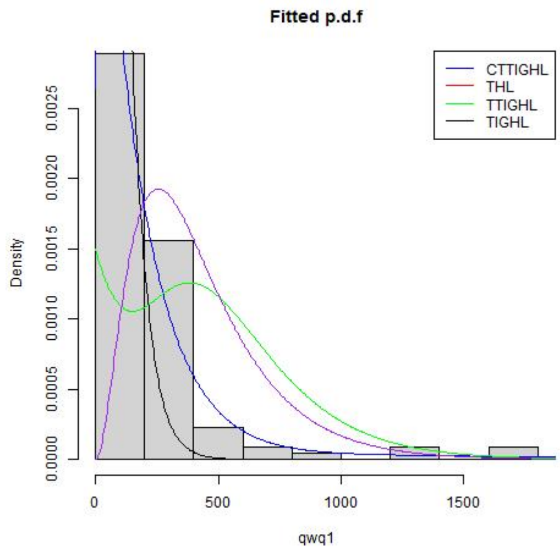


Figure 7: Graph of coal data with the density curve

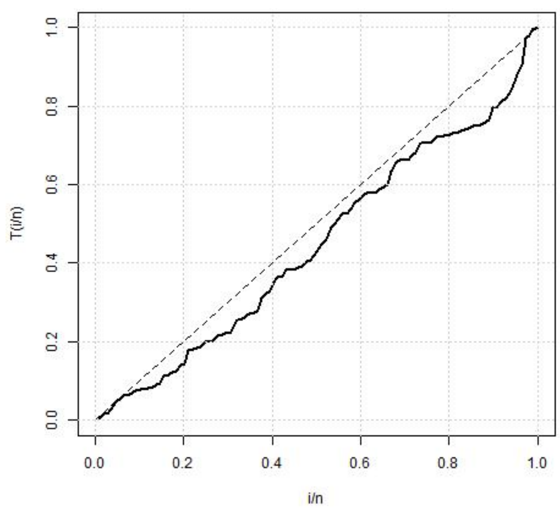


Figure 8: TTT plot of Coal disaster data

5. CONCLUSIONS

In this work, a new probability distribution called the Cubic transmuted type I generalized half logistic distribution was derived and studied. The new distribution generalizes the half logistic distribution by having additional three parameters,the shape parameter and two transmuted parameters. Statistical properties of the new distribution were extensively studied and application to two real data set using the model comparison criterion revealed that the distribution performs better(produce better fit) than its submodels.

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