A REDEFINED CLASS OF RATIO
ESTIMATORS USING AUXILIARY
INFORMATION FOR THE ESTIMATION OF
POPULATION MEAN UNDER SIMPLE
RANDOM SAMPLING SCHEME

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ABSTRACT
The present paper advocates the improved estimation of population mean through a new class
of estimators using the known informations on an auxiliary variable under the Simple Random
Sampling Scheme. The existing estimators of population mean have been shown as the members
of the proposed class. The Bias and Mean Squared Error (MSE) of the suggested class are derived
upto the first order of approximation. The values of Bias and the minimum MSE are obtained by
optimizing the characterizing scalar. The MSE of the suggested estimator has also been compared
with the MSEs of the existing estimators for the comparison of these estimators. Some of the new
members of the suggested family are also presented in the paper which are more efficient than the
existing ones empirically which has been shown using a natural population.

KEYWORDS: Study Variable, Auxiliary Variables, Simple Random Sampling, Constants, MSE,
PRE
MSC: 62D05

RESUMEN
El presente artículo aboga por una mejor estimación de la media de la población a través de una
nueva clase de estimadores que utilizan la información conocida sobre una variable auxiliar bajo el
esquema de muestreo aleatorio simple. Los estimadores existentes de la media de la población se
muestran como los miembros de la clase propuesta. El sesgo y el error cuadrático medio (MSE) de
la clase sugerida se derivan hasta el primer orden de aproximación. Los valores de Bias y el MSE
mínimo se obtienen optimizando el escalar caracterizador. El MSE del estimador sugerido también se
ha comparado con los MSE de los estimadores existentes para la comparación de estos estimadores.
Algunos de los nuevos miembros de la familia sugerida también se presentan en el documento, que
son más eficientes que los existentes, que se ha demostrado con una población natural.

PALABRAS CLAVE: Variable de estudio, Variables Auxiliares, Muestreo Aleatorio Simple, Con-
stantes, MSE, PRE

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1. INTRODUCTION

The study variable combined with the auxiliary information provides considerably efficient estimation of the population parameters. Cochran (1940) was the first who used the auxiliary information in development of ratio estimator. Ratio estimator is effective when there is positive correlation between the study and auxiliary variable otherwise in case of negative correlation product estimator would be the wiser choice. The parameters of the auxiliary variable such as the mean, median, variance and the coefficient of variation or the combinations of the parameters of auxiliary variable is used as the components of the concerned estimator. So many researchers have studied on the auxiliary variable and the ratio estimators and tried to obtain the minimum as well as the unbiased estimators. As a result we end up having a lot of estimators. If some are useful for one situation others are useful for other situations. We don’t have any universal estimator. Different estimators are useful in different situations. Considering all of them is a tuff job for us so we have considered some of them in our literature which are similar to the situation we are studying on.

Upadhaya and Singh (1991), Kadilar and Cingi (2003), Al-Omari et al. (2009) suggested new improved ratio estimators. Bahl and Tuteja (1991) proposed exponential type of estimators. Jeelani et al. (2013) used the linear combination of coefficient of skewness and quartile deviation for enhanced estimation. Jerajuddin and Kishun (2016) suggested the use of sample size selected from the population to modify the ratio estimator. His estimator did not require the auxiliary information. Singh, Tailor and Kakran (2004) used the power transformation for the improvement. Singh and Tailor (2003), Sisodia and Dwivedi (1981), Yan and Tian (2010), Subramani and Kumarpandian (2012), Yadav et al. (2019) took the use of various characteristics of auxiliary information and their combinations to obtain the efficient estimators. Encouraged by these researchers we proposed a generalized class of estimators which has more precision than the existing competing estimators.

We have considered the mean estimation and tried to obtain the efficient estimator under the simple random sampling scheme. Let from the finite population \((X, Y)\) of size \(N\), a bivariate sample \((x_i, y_i); i = 1, 2, \ldots, n\) of size \(n\) is taken using SRSWOR scheme. The sample means \(\bar{x}\) and \(\bar{y}\) are unbiased estimators of population means \(\bar{X}\) and \(\bar{Y}\) respectively. If we ignore the property of unbiasedness, we would obtain the Mean Square Error much lower than the usual variance. Though we have obtained the biasedness of our proposed estimator yet we focused on minimizing our Mean Square Error (MSE).

1.1. Notations

- \(N\) : Size of the population
- \(n\) : Size of the sample
- \(N_C_n\) : Number of possible samples of size \(n\) from the population of size \(N\)
- \(Y\) : Study Variable
- \(X\) : Auxiliary Variable
- \(M_y, M_x\) : Medians
- \(\bar{Y}, \bar{X}\) : Population means
- \(\bar{y}, \bar{x}\) : Sample means
- \(\rho\) : Correlation Coefficient between \(X\) and \(Y\)
- \(\beta\) : Regression Coefficient of \(Y\) on \(X\)
• $S^2_y, S^2_x$: Population Mean Squares
• $S_{yx}$: Covariance between X and Y
• $C_y, C_x$: Coefficients of Variation
• $Bias(\cdot)$: Bias of the estimator
• $V(\cdot)$: Variance of the estimator
• $\beta_1(x)$: Coefficient of Skewness
• $\beta_2(x)$: Coefficient of Kurtosis
• $Q_1(x)$: First Quartile
• $Q_3(x)$: Third Quartile
• $QD$: Quartile Deviation
• $Q_{a(x)}$: Quartile Average
• $Q_r(x)$: Quartile Range
• $TM$: Tri Mean
• $MSE(\cdot)$: Mean Squared Error of the Estimator
• $PRE(\bar{y}, t)$: Percentage Relative Efficiency of the proposed estimator with respect to the SRS mean

1.2. Formulae

$$\lambda = \frac{1-f}{n}$$

$$f = \frac{n}{N}$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$V(\bar{y}) = \lambda \bar{y}^2 C_y^2$$

$$V(\bar{x}) = \lambda \bar{x}^2 C_x^2$$

$$\rho = \frac{S_{yx}}{S_x S_y}$$

$$\rho = \frac{C_y}{C_x}$$

$$d = \rho C_y C_x$$

$$QD = \frac{Q_3 - Q_1}{2}$$

$$Q_{a(x)} = \frac{Q_1 + M_x + Q_3}{3}$$

$$Q_r(x) = Q_3 - Q_1$$

$$TM = \frac{Q_1 + 2M_x + Q_3}{4}$$

2. LITERATURE REVIEW OF EXISTING ESTIMATORS

There are a number of estimators which have been developed till date for the elevated estimation of population mean under simple random sampling without replacement (SRSWOR) scheme. Various estimators from the literature are considered for the review and are presented in the Table-1 along with their Mean Squared Errors and corresponding constants up the approximation of order one.

3. PROPOSED CLASS OF ESTIMATORS

Inspired by the literature presented by Yadav et al. (2019), we suggest an improved class of ratio type estimators for the enhanced estimation of population mean of primary variable using information on secondary variable under SRSWOR as follows:

$$t = k \bar{y} \left( \frac{ab \bar{x} + cd}{ab \bar{y} + cd} \right)$$  (3.1)
Table 1: Literature Review

<table>
<thead>
<tr>
<th>SNo</th>
<th>Estimators</th>
<th>MSE/Variances</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$t_0 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ Sample Mean</td>
<td>$\lambda \bar{Y}^2 C_y^2$</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>$t_1 = \bar{y} \left( \frac{\bar{x}}{\bar{Y}} \right)$ Cochran(1940)</td>
<td>$\lambda \bar{Y}^2 \left( C_y^2 + C_x^2 - 2C_{yx} \right)$</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>$t_2 = \bar{y} \exp \left( \frac{\bar{x} - \bar{X}}{\bar{Y} + \bar{x}} \right)$ Bahl and Tuteja(1991)</td>
<td>$\lambda \bar{Y}^2 \left( C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x \right)$</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>$t_3 = \bar{y} \left( \frac{\bar{x} + C_x}{\bar{x} + \bar{C}_x} \right)$ Sisodia and Dwivedi (1981)</td>
<td>$\lambda \bar{Y}^2 \left( C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_{yx} \right)$</td>
<td>$\theta_3 = \frac{\bar{X}}{\bar{X} + C_x}$</td>
</tr>
<tr>
<td>5</td>
<td>$t_4 = \bar{y} \left( \frac{\bar{X} C_x + \beta_2(\bar{x})}{\bar{X} C_x + \beta_2(\bar{x}) + C_x} \right)$ Upadhyaya and Singh(1999)</td>
<td>$\lambda \bar{Y}^2 \left( C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_{yx} \right)$</td>
<td>$\theta_4 = \frac{XC_x}{XC_x + \beta_2(\bar{x})}$</td>
</tr>
<tr>
<td>6</td>
<td>$t_5 = \bar{y} \left( \frac{\beta_2(\bar{x}) + C_x}{\beta_2(\bar{x}) + C_x} \right)$ Upadhyaya and Singh(1999)</td>
<td>$\lambda \bar{Y}^2 \left( C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_{yx} \right)$</td>
<td>$\theta_5 = \frac{\beta_2(\bar{x})}{\beta_2(\bar{x}) + C_x}$</td>
</tr>
</tbody>
</table>

where constant $k$ is suitably chosen such that the MSE of the suggested estimator is minimum and the constants $a, b, c, d$ are either the constants or the known parameters of the auxiliary variable. Also $(a, b, c, d)$ are free to take those real and parametric values which makes the MSE minimum.

3.1. Bias and MSE

To derive the Bias and MSE for the suggested class of estimators, we define the following standard approximations as:

$$e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \quad \text{and} \quad e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$$

So,

$$\bar{y} = \bar{Y}(1 + e_0) \quad \text{and} \quad \bar{x} = \bar{X}(1 + e_1)$$

$$E(e_0) = E(e_1) = 0 \quad \text{and} \quad E(e_0 e_1) = \lambda C_{yx}$$

$$E(e_0^2) = \lambda C_y^2 \quad \text{and} \quad E(e_1^2) = \lambda C_x^2.$$
Putting the value of \( k \) in equation (3.4), we get the Bias as,

\[
E(t - Y) = \bar{Y}[k(1 - \theta \lambda C_{yx} + \theta^2 \lambda C_x^2) - 1]
\]

(3.3)

Taking expectation on both sides of equation (3.3), we have,

\[
E(t - Y) = \bar{Y}[k(1 - \theta \lambda C_{yx} + \theta^2 \lambda C_x^2) - 1]
\]

(3.4)

Squaring the equation (3.3), we get,

\[
(t - Y)^2 = \bar{Y}^2[k^2(1 + e_0 - \theta e_1 - \theta e_0 e_1 + \theta^2 e_1^2) + 1 - 2k(1 + e_0 - \theta e_1 - \theta e_0 e_1 + \theta^2 + e_1^2)]
\]

(3.5)

Applying the method of maxima and minima for obtaining the optimum value of the constant kappa. The optimum value of \( k \) which makes the MSE in equation (3.5) minimum, is given by,

\[
k = \frac{1 - \theta \lambda C_{yx} + \theta^2 \lambda C_x^2}{1 + \lambda C_y^2 + 3\theta^2 \lambda C_x^2 - 4\theta \lambda C_{yx}} = \frac{A}{B}
\]

(3.6)

Thus the minimum value of MSE of \( t \) is given as,

\[
M(t)_{\text{min}} = \bar{Y}^2 \left( 1 - \frac{A^2}{B} \right)
\]

(3.7)

Putting the value of \( k \) in equation (3.4), we get the Bias as,

\[
B(t) = \bar{Y} \left( \frac{A^2}{B} - 1 \right)
\]

(3.8)

### 3.2. Existing Members of the Proposed Class of Estimators

Many of the existing estimators of population mean \( Y \) are identified as the members of the proposed class for different values of \((a, b, c, d)\) in (3.1), which are given in Table 2 as,
3.3. New Members of Proposed Class of Estimators

We have suggested some new members of the proposed class of estimators which come out to be more efficient than the existing estimators of population mean. These are given as follows,

- \( t_1 = k_1\bar{y} \left[ \frac{\beta_2(x)M_xX + \rho C_x}{\beta_2(x)M_x + \rho} \right] \)
- \( t_2 = k_2\bar{y} \left[ \frac{\beta_2(x)M_xX + pC_x}{\beta_2(x)M_x + p} \right] \)
- \( t_3 = k_3\bar{y} \left[ \frac{N_{C_M}X + M_x}{N_{C_M} + M_x} \right] \)
- \( t_4 = k_4\bar{y} \left[ \frac{M_xX + pf}{M_x + p} \right] \)
- \( t_5 = k_5\bar{y} \left[ \frac{TMQ_3(x)X + Q_a(x)}{TMQ_3(x) + Q_a} \right] \)
- \( t_6 = k_6\bar{y} \left[ \frac{QDQ_a(x)X + \beta_3(x)\beta_1(x)}{QDQ_a(x) + \beta_3(x)\beta_1(x)} \right] \)
- \( t_7 = k_7\bar{y} \left[ \frac{M_xS_xX + \rho C_x}{M_xS_x + \rho} \right] \)

4. THEORETICAL EFFICIENCY COMPARISON

Following are the conditions under which proposed class of estimators is more efficient than the existing estimators, where,

- \( D = \theta(1 - \lambda \rho^2 C_x^2 - \theta^2 \lambda C_x^3 + 2\theta \lambda C_{yx}) - 6d \)
- \( D' = \theta D - B\theta_i^2 + 2B\theta_id \quad ; \quad i = 3, \ldots, 20 \)

5. COMPUTATIONAL STUDY

To prove the theoretical results numerically we have considered a Natural Population with sample size

Data Source : Daroga Singh and F.S. Chaudhary (1986, Page-177)

Data Details : Study Variable :

- Area under wheat in a region during year 1974

Auxiliary Variable :

- Cultivated Area under wheat in a region during year 1973

To compute the Percent Relative Efficiency (PRE) for different estimators with respect to Simple Random Sample Mean we use the following :

\[
PRE = \frac{V(t_0)}{MSE(t)}
\]

6. RESULTS AND CONCLUSION

1. Table 1 Reviews the Existing literature. Table 2 shows the values for which the Existing estimators become special case of the proposed class of estimators. Table 3 shows the conditions for which our proposed class estimators is better than the existing estimators. Table 4 consists of parametric values of the data with which we verified our results empirically. Table 5 shows the MSE and PRE of existing and proposed class of estimators.
2. We study the Bias and MSE of the proposed class up to first order of approximation. Since Efficiency is stronger property than the unbiasedness. Hence here we prefer the biased estimator with minimum MSE instead of unbiased estimator with higher MSE.

3. From table 5 we can easily notice that the proposed class of estimators have lesser MSEs thus greater PREs which proves that our class of proposed estimators is efficient enough for the practical purposes.

REFERENCES


A  APPENDIX

R CALCULATION

# Secondry data (From Daroga and Singh)
Y=[50,149,284,381,278,111,634,278,112,355,99,498,111,6,339,80,105,27,515,249,85,221,133,144,103,175,335,219,62,79,60,100,141,263]
X=[70,163,320,440,250,125,558,254,101,359,109,481,125,5,427,78,75,45,564,238,92,247,134,131,129,190,363,235,73,62,71,137,196,255]

# mean
y=mean(Y)
x=mean(X)

# standard deviation
sy=sqrt(var(Y))
sx=sqrt(var(X))

# coefficient of variation
cy=sy/y
cx=sx/x

# median
mx=median(X)
my=median(Y)

# correlation coefficient
r=cor(X,Y)
cyx=r*cy*cx

library(moments)

# coefficient of skewness b1(beta 1)
be1=skewness(X)

# coefficient of kurtosis b2( beta 2)
be2=kurtosis(X)+3

# Function of MSE calculation
mse=function(c){mse=l*y^2+(cy^2+(c^2*cx^2)-(2*c*cyx))
print(mse)
}

# Cochran (1940)
m1=l*y^2*(cy^2+cx^2-2*(cyx))

# Bahl and Tuteja (1991)
m2=l*y^2*(cy^2+(cx^2/4)-cyx)

# Sisodia and Dwivedi (1981)
c3=x/(x+cx)

# Upadhyaya and Singh (1999)........
c4=(x*cx)/((x*cx)+be2)

c5=(x*be2)/((x*be2)+cx)

# Singh and Tailor (2003)......
c6=x/(x+r)

# singh et al.(2004)........
c7=x/(x+be2)

# Al-Omari et al. (2009).
c8=x/(x+q1)

qd=(q3-q1)/2
qr=q3-q1
qr=q3-q1
qa=(q1+mx+q3)/3

# quartile range
# quartile average

tm=(q1+2*mx+q3)/4

# Review of literature
# Sample mean

n0=l*y^2*cy^2
# Cochran (1940)
m1=l*y^2*(cy^2+cx^2-2*(cyx))

# Bahl and Tuteja(1991)
m2=l*y^2*(cy^2+(cx^2/4)-cyx)

# Sisodia and Dwivedi(1981)
c3=x/(x+cx)

# Upadhyaya and Singh (1999)........
c4=(x*cx)/((x*cx)+be2)

c5=(x*be2)/((x*be2)+cx)

# Singh and Tailor (2003)......
c6=x/(x+r)

# singh et al.(2004)........
c7=x/(x+be2)

# Al-Omari et al. (2009).
c8=x/(x+q1)

# quartile deviation

mse=l*y^2+(cy^2+(c^2*cx^2)-(2*c*cyx))

print(mse)
# # Al-Omari et al. (2009)
c9\_x/(x+q3)
m(c9)
# Yan and Tian(2010)
c10\_x/(x+be1)
c11\_x/(x* be1)+be2)
c12\_x/(x* cx)+be1)
m(c10)
m(c11)
m(c12)
# Subramani and Kumarpandiyan(2012a)
c13\_x/(x+mx)
c14\_x/(x* cx)+mx)
m(c13)
m(c14)
# Subramani and Kumarpandiyan(2012b)
c15\_x/(x+qr)
c16\_x/(x+qd)
c17\_x/(x+qa)
m(c15)
m(c16)
m(c17)
# Jeelani et al.(2013)............
c18\_x/(x* be1)/(x* be1)+qd)
m(c18)
# Jerajuddin and Kishun(2016)....
c19\_x/(x+n)
m(c19)
# Yadav et al. (2019)......
c20\_1\_x/(be2* mx* x)/((be2* mx* x)+(r))
c20\_2\_x/(be2* mx* x)/((be2* mx* x)+(r* cx))
c20\_3\_x/(be1* mx* x)/((be1* mx* x)+(r))
c20\_4\_x/(be1* mx* x)/((be1* mx* x)+(r* cx))
c20\_5\_x/(n* x)/((n* x)+(r))
c20\_6\_x/(n* x)/((n* x)+(cx))
c20\_7\_x/(n* x)/((n* x)+(cx* r))
c20\_8\_x/(n* r* x)/((n* r* x)+(cx))
c20\_9\_x/(n* cx* x)/((n* cx* x)+(r))
m(c20\_1)
m(c20\_2)
m(c20\_3)
m(c20\_4)
m(c20\_5)
m(c20\_6)
m(c20\_7)
m(c20\_8)
m(c20\_9)

# Proposed Estimators............
mse\_function(th){
A=1-(th* l* r* cy* cx)+(th^2* l* cx^2)
B=1+(l* cy^2)+(th^2* l* cx^2)-4*(th* l* r* cy* cx)+(2* th^2* l* cx^2)
M=y^2*(1-(A^2/B))
print(M)
}

th1=((be2* mx* x)/((be2* mx* x)+(r)))
mse(th1)
th2=((be2* mx* x)/((be2* mx* x)+(r* cx)))
mse(th2)

th3=(Ncn* x)/((Ncn* x)+(mx))
mse(th3)

th4=(mx* x)/((mx* x)+(f* r))
mse(th4)

th5=(tm* q3* x)/((tm* q3* x)+(qa/qd))
mse(th5)

th6=(qd* qa* x)/((qd* qa* x)+(be2* be1))
mse(th6)

th7=(mx* sx* x)/((mx* sx* x)+(r* l))
mse(th7)

PRE\_function(me){
PRE=(m0/me)* 100
print(PRE)
}
Table 2: *

<table>
<thead>
<tr>
<th>SNo</th>
<th>Estimators</th>
<th>MSE/ Variance</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>$t_6 = \bar{y}\left(\frac{\sum_{i}X_i + \bar{x}}{\sum_{i}p_i}\right)$ Singh and Tailor(2003)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2 C_x^2 - 2\theta_6 C_{yx}\right)$</td>
<td>$\theta_6 = \frac{X}{X + p}$</td>
</tr>
<tr>
<td>8</td>
<td>$t_7 = \bar{y}\left(\frac{\sum_{i}X_i + \beta_{2(x_i)}}{\sum_{i}p_i}\right)$ Singh et al. (2004)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2 C_x^2 - 2\theta_7 C_{yx}\right)$</td>
<td>$\theta_7 = \frac{X}{X + \beta_{2(x)}}$</td>
</tr>
<tr>
<td>9</td>
<td>$t_8 = \bar{y}\left(\frac{\sum_{i}X_i + Q_{1(x_i)}}{\sum_{i}p_i}\right)$ Al-Omari et al. (2009)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2 C_x^2 - 2\theta_8 C_{yx}\right)$</td>
<td>$\theta_8 = \frac{X}{X + Q_{1(x)}}$</td>
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<tr>
<td></td>
<td>$t_9 = \bar{y}\left(\frac{\sum_{i}X_i + Q_{3(x_i)}}{\sum_{i}p_i}\right)$ Al-Omari et al. (2009)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2 C_x^2 - 2\theta_9 C_{yx}\right)$</td>
<td>$\theta_9 = \frac{X}{X + Q_{3(x)}}$</td>
</tr>
<tr>
<td>10</td>
<td>$t_{10} = \bar{y}\left(\frac{\sum_{i}X_i + \beta_{1(x_i)}}{\sum_{i}p_i}\right)$ Yan and Tian(2010)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2_{10} C_x^2 - 2\theta_{10} C_{yx}\right)$</td>
<td>$\theta_{10} = \frac{X}{X + \beta_{1(x)}}$</td>
</tr>
<tr>
<td>11</td>
<td>$t_{11} = \bar{y}\left(\frac{\sum_{i}X_i + \beta_{1(x_i)} + \beta_{2(x_i)}}{\sum_{i}p_i}\right)$ Yan and Tian(2010)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2_{11} C_x^2 - 2\theta_{11} C_{yx}\right)$</td>
<td>$\theta_{11} = \frac{X}{X + \beta_{1(x)} + \beta_{2(x)}}$</td>
</tr>
<tr>
<td>12</td>
<td>$t_{12} = \bar{y}\left(\frac{\sum_{i}X_i + \beta_{1(x)}}{\sum_{i}p_i}\right)$ Yan and Tian(2010)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2_{12} C_x^2 - 2\theta_{12} C_{yx}\right)$</td>
<td>$\theta_{12} = \frac{X}{X + \beta_{1(x)} + \beta_{2(x)}}$</td>
</tr>
<tr>
<td>13</td>
<td>$t_{13} = \bar{y}\left(\frac{\sum_{i}X_i + M_{i(x)}}{\sum_{i}p_i}\right)$ Subramani and Kumarpandiyan(2012a)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2_{13} C_x^2 - 2\theta_{13} C_{yx}\right)$</td>
<td>$\theta_{13} = \frac{X}{X + M_{i(x)}}$</td>
</tr>
<tr>
<td>14</td>
<td>$t_{14} = \bar{y}\left(\frac{\sum_{i}X_i + M_{i(x)}}{\sum_{i}p_i}\right)$ Subramani and Kumarpandiyan(2012a)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2_{14} C_x^2 - 2\theta_{14} C_{yx}\right)$</td>
<td>$\theta_{14} = \frac{X}{X + M_{i(x)}}$</td>
</tr>
<tr>
<td>15</td>
<td>$t_{15} = \bar{y}\left(\frac{\sum_{i}X_i + Q_{1(x)}}{\sum_{i}p_i}\right)$ Subramani and Kumarpandiyan(2012b)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2_{15} C_x^2 - 2\theta_{15} C_{yx}\right)$</td>
<td>$\theta_{15} = \frac{X}{X + Q_{1(x)}}$</td>
</tr>
<tr>
<td>16</td>
<td>$t_{16} = \bar{y}\left(\frac{\sum_{i}X_i + Q_{D(x)}}{\sum_{i}p_i}\right)$ Subramani and Kumarpandiyan(2012b)</td>
<td>$\lambda X^2 \left(C_y^2 + \theta^2_{16} C_x^2 - 2\theta_{16} C_{yx}\right)$</td>
<td>$\theta_{16} = \frac{X}{X + Q_{D}}$</td>
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### Literature Review

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<th>MSE/Variance</th>
<th>Constants</th>
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<td>$t_{17} = \bar{y} \left( \frac{\bar{X} + Q_{u(x)}}{\bar{x} + Q_{u(x)}} \right)$</td>
<td>$\lambda \tilde{y}^2 \left( C_y^2 + \theta_{17}^2 C_x^2 - 2 \theta_{17} C_{yx} \right)$</td>
<td>$\theta_{17} = \frac{\bar{X}}{\bar{X} + Q_{u(x)}}$</td>
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<td>Subramani and Kumarpandiyan (2012b)</td>
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<td>$t_{18} = \bar{y} \left( \frac{\bar{X} + n}{\bar{x} + n} \right)$</td>
<td>$\lambda \tilde{y}^2 \left( C_y^2 + \theta_{18}^2 C_x^2 - 2 \theta_{18} C_{yx} \right)$</td>
<td>$\theta_{18} = \frac{\bar{X}}{\bar{X} + n}$</td>
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<td>Jerajuddin and Kishum (2016)</td>
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<td>19</td>
<td>$t_{19} = \bar{y} \left( \frac{\bar{X} \beta(x) + QD}{\bar{x} \beta(x) + QD} \right)$</td>
<td>$\lambda \tilde{y}^2 \left( C_y^2 + \theta_{19}^2 C_x^2 - 2 \theta_{19} C_{yx} \right)$</td>
<td>$\theta_{19} = \frac{\bar{X} \beta(x)}{\bar{X} \beta(x) + QD}$</td>
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<td>$t_{20} = \bar{y} \left( \frac{ab \bar{X} + cd}{ab \bar{x} + cd} \right)$</td>
<td>$\lambda \tilde{y}^2 \left( C_y^2 + \theta_{20}^2 C_x^2 - 2 \theta_{20} C_{yx} \right)$</td>
<td>$\theta_{20} = \frac{ab \bar{X}}{ab \bar{x} + p \bar{C}_z}$</td>
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<td>$t_{20(1)} = \bar{y} \left( \frac{\beta_{2(1)} \bar{M}_x \bar{X} + \rho \bar{C}<em>z}{\beta</em>{2(1)} \bar{M}_x \bar{x} + \rho \bar{C}_z} \right)$</td>
<td>$\lambda \tilde{y}^2 \left( C_y^2 + \theta_{20(1)}^2 C_x^2 - 2 \theta_{20(1)} C_{yx} \right)$</td>
<td>$\theta_{20(1)} = \frac{\beta_{2(1)} \bar{M}<em>x \bar{X}}{\beta</em>{2(1)} \bar{M}_x \bar{x} + \rho \bar{C}_z}$</td>
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<td>$t_{20(2)} = \bar{y} \left( \frac{\beta_{2(2)} \bar{M}_x \bar{X} + \rho \bar{C}<em>z}{\beta</em>{2(2)} \bar{M}_x \bar{x} + \rho \bar{C}_z} \right)$</td>
<td>$\lambda \tilde{y}^2 \left( C_y^2 + \theta_{20(2)}^2 C_x^2 - 2 \theta_{20(2)} C_{yx} \right)$</td>
<td>$\theta_{20(2)} = \frac{\beta_{2(2)} \bar{M}<em>x \bar{X}}{\beta</em>{2(2)} \bar{M}_x \bar{x} + \rho \bar{C}_z}$</td>
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<td>$\lambda \tilde{y}^2 \left( C_y^2 + \theta_{20(3)}^2 C_x^2 - 2 \theta_{20(3)} C_{yx} \right)$</td>
<td>$\theta_{20(3)} = \frac{\beta_{2(3)} \bar{M}<em>x \bar{X}}{\beta</em>{2(3)} \bar{M}_x \bar{x} + \rho \bar{C}_z}$</td>
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<td>$\theta_{20(4)} = \frac{\beta_{2(4)} \bar{M}<em>x \bar{X}}{\beta</em>{2(4)} \bar{M}_x \bar{x} + \rho \bar{C}_z}$</td>
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<td>$\theta_{20(6)} = \frac{n \bar{X}}{n \bar{x} + p \bar{C}_z}$</td>
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<td>$t_{20(7)} = \bar{y} \left( \frac{n \bar{X} + p \bar{C}_z}{n \bar{x} + p \bar{C}_z} \right)$</td>
<td>$\lambda \tilde{y}^2 \left( C_y^2 + \theta_{20(7)}^2 C_x^2 - 2 \theta_{20(7)} C_{yx} \right)$</td>
<td>$\theta_{20(7)} = \frac{n \bar{X}}{n \bar{x} + p \bar{C}_z}$</td>
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## Table 4: Some Existing Members of Proposed Class of estimators t

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<td>c</td>
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## Table 5: Efficiency Comparison

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<th>SNo.</th>
<th>MSE($t$) &lt; MSE($\cdot$)</th>
<th>Condition</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>$MSE(t) &lt; V(t_0)$</td>
<td>$C_y^2 &gt; \frac{\theta C^2 D}{B-1}$</td>
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<tr>
<td>2</td>
<td>$MSE(t) &lt; MSE(t_1)$</td>
<td>$C_y^2 &gt; \frac{C_y^2 (\theta D - B_1 + 2BD)}{B-1}$</td>
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<tr>
<td>3</td>
<td>$MSE(t) &lt; MSE(t_2)$</td>
<td>$C_y^2 &gt; \frac{C_y^2 (\theta D - \frac{B-1}{B})}{B-1}$</td>
</tr>
<tr>
<td>4</td>
<td>$MSE(t) &lt; MSE(t_i); i = 3, \ldots 20$</td>
<td>$C_y^2 &gt; \frac{C_y^2 D'}{B-1}$</td>
</tr>
</tbody>
</table>
Table 6: **Parametric Values of the Population**

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<th>Data Set</th>
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<td>2</td>
<td>n</td>
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<td>3</td>
<td>$\bar{Y}$</td>
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<td>4</td>
<td>$\bar{X}$</td>
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<td>$S_y$</td>
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<td>$S_x$</td>
<td>150.506</td>
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Table 7: *Parametric Values of the Population*

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Table 8: MSE and PRE of Estimators

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Table 9: MSE and PRE of Estimators

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