OPTIMAL INVENTORY DECISIONS WITH ADVANCED SALES FOR DETERIORATING ITEMS

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ABSTRACT
This paper studies retailer's optimal policies which include optimal replenishment, pricing and promotional cost for a two-phase inventory system with a constant rate of deterioration. The first phase is advance booking period in which the product is offered at a discounted price and delivery of the product is given as soon as the replenishment arrives. The second phase is spot sales period in which the product is sold at a normal price. Products are considered to be deteriorated at a constant rate. Demand of the product is influenced by promotion of the product as well as by the selling price of the product. The objective is to maximize the total profit of the retailer with respect to cycle time, selling price and promotional cost through advertisement.

KEYWORDS: Inventory model; Marketing; Advance booking; Deterioration; Selling price

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RESUMEN
Este documento estudia las políticas óptimas del minorista que incluyen reposición óptima, precios y costos promocionales para un sistema de inventario de dos fases con una tasa constante de deterioro. La primera fase es el período de reserva anticipada en el que el producto se ofrece a un precio con descuento y la entrega del producto se otorga tan pronto como llega la reposición. La segunda fase es el período de ventas al contado en el que el producto se vende a un precio normal. Los productos se consideran deteriorados a una tasa constante. La demanda del producto está influenciada por la promoción del producto, así como por el precio de venta del producto. El objetivo es maximizar el beneficio total del minorista con respecto al tiempo de ciclo, el precio de venta y el costo promocional a través de la publicidad.

PALABRAS CLAVE: Modelos de Inventario; Marketing; Registro avanzdo ; Deterioro ; precio de venta

1. INTRODUCTION

In present time, advertisements and promotional efforts play a major role to introduce the product to a large volume of customers very effectively and attract them to buy it. Further by booking the product at a discounted rate for some specific period of time, gives a boost to sales. Also the selling price of product is a key factor in generating revenue and increasing the demand.

Table 1: Literature Survey

Authors	Advertisement	Price dependant demand	Deterioration	Advance booking
Abad et al.,2003	×	✓	×	×
Adekoya et al.,2011	✓	×	×	×
Chen & D.Simchi Levi,2004	×	✓	×	×
Chen & D.Simchi Levi,2004	×	✓	×	×
Dye & Hsieh,2013	×	✓	✓	✓
Li , Wang & Yan,2013	✓	✓	×	×
Seyedesfahani et al.,2011	✓	✓	×	×
Shah & Naik,2018	×	✓	✓	×
Shah, Patel & Shah,2014	×	×	✓	×
Shah, Patel & Shah,2014	×	✓	✓	×
Shah & Naik,2018	×	✓	✓	×
Szmerekovsky & Zhang,2009	✓	✓	×	×
Tsao et al.,2009	×	×	×	✓
Xie & Wei,2009	✓	✓	×	×
You et al.,2006	×	✓	×	✓
You & Wu.,2007	×	✓	×	✓

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Yue , Austin, Wang & Huang,2006	✓	✓	×	×
Shah, Shah & Rabari	✓	✓	✓	✓

In this paper, optimal ordering, pricing and discount on advance booking is worked out for retailer's inventory system. When units are stored in warehouse they are considered to be deteriorated at a constant rate. A discount on price is available at the time of advanced booking. Demand is considered to be additive function of selling price and investments in advertisement for promoting the product in the market. Total profit is maximized with respect to cycle time, selling price and advertisement cost.

The paper is organized as follows. Section 1 is introduction. Section 2 deals with assumption and notations followed by various inventory costs as a part of the objective function. Mathematical model is developed in section 3.The results are validated in Section 4 through numeric hypothetical inventory parametric values and the managerial issues are discussed through sensitivity analysis, where one inventory parameter is varied by $-20\%, -10\%, 10\% \, and \, 20\%.$ Section 5 concludes the study.

2. ASSUMPTIONS AND NOTATIONS

We will use following assumptions and notations in development of mathematical model.

2.1 Assumptions

(1) Replenishment rate is infinite and there is no lead time.

(2) Inventory model is for a single cycle $[0,T]$, which includes two phases: advance sales phase $[0,T_a]$ and spot sales phase $[T_a,T]$.

(3) Product is considered to be deteriorated at a constant rate during the spot sales phase.

(4) Product is offered at a discounted price P_a during advance sales phase $[0,T_a]$ and at price p during spot sales $[T_a,T]$.

(5) $g\,\%$ discount is given to customers during advance booking phase, which gives $P_a = (1-g)p$ .where g is discount such that $0 < g < 1$.

(6) Demand is additive function of selling price and investments in advertisement cost of the product. $D(p,M) = d(p).d(M)$, where $d(p)$ is demand component due to influence of selling price p on demand and $d(M)$ is demand component due to influence of marketing of the product through advertisement cost per unit item.

(7) Price sensitive demand component $d(p) = a - bp; a,b > 0$ and Marketing influenced demand component $d(M) = \alpha + \beta M; \alpha, \beta > 0$.

2.2 Notations

A	Ordering cost in \$ / order
c	Cost price in \$ / unit
g	Discounting factor
h	Holding cost in \$ per unit per unit time
$I_a(t)$	Inventory level during advance sales phase at time t
$I_s(t)$	Inventory level during spot sales phase at time t
M	Advertisement expenditure in \$ / unit (decision variable)
p	Selling price in \$ / unit in spot sales phase (decision variable)
P_a	Selling price in \$ / unit in advance sales phase
$R(p,M)$	Demand rate in spot sales phase
$R(P_a,M)$	Demand rate in advance sales phase
T	Cycle length of inventory

T_a Advance sales period length

θ Rate of deterioration

3. MATHEMATICAL MODEL

During the advance sales period demand rate is $R(P_a, M)$ for the interval $[0, T_a]$, so corresponding inventory level at any of time t in the interval is governed by the differential equation,

$$\frac{d}{dt} I_a(t) = -R(P_a, M), 0 \le t \le T_a \tag{1}$$

Using the boundary condition $I_a(0) = 0$ and Solving (1) we get,

$$I_a(t) = -R(P_a, M)t; \quad 0 \le t \le T_a \tag{2}$$

During the spot sales period demand rate is $R(p, M)$ for the interval $[T_a, T]$, hence corresponding inventory level at any time t in the interval considering constant rate of deterioration is governed by the differential equation,

$$\frac{d}{dt} I_s(t) = -R(p, M) - \theta I_s(t),\ T_a \le t \le T \tag{3}$$

Using the boundary condition $I_s(T) = 0$ and solving (3) we get,

$$I_s(t) = \frac{R(p,M)}{\theta}[e^{(T-t)\theta} - 1];\ T_a \le t \le T \tag{4}$$

Now the ordering quantity is given as,

$$Q = R(P_a, M)T_a + \left(\frac{1}{1-\theta}\right)R(p,M)(T-T_a) = (a - gbp)(\alpha + \beta M)T_a + \left(\frac{1}{1-\theta}\right)(a-bp)(\alpha+\beta M)(T-T_a) \tag{5}$$

Number of items sold in advance sales phase and number of items sold in spot sales phase is denoted as N_a and N_s respectively and given as,

$$N_a = R(P_a, M)T_a = (a - gbp)(\alpha + \beta M)T_a \tag{6}$$

$$N_s = R(p, M)(T - T_a) = (a - bp)(\alpha + \beta M)(T - T_a) \tag{7}$$

This leads to total revenue TR, given as

$$TR = P_a N_a + pN_s \tag{8}$$

The holding cost is;

$$HC = \int_{T_a}^{T} hI_s(t)dt = \frac{h(a-bp)(\alpha+\beta M)}{\theta^2}\left[e^{(T-T_a)\theta} - 1 - (T-T_a)\theta\right] \tag{9}$$

Marketing cost through advertisement is given as,

$$MC = MT[R(P_a, M) + R(p, M)] \tag{10}$$

The total profit of complete inventory cycle $[0, T]$ is given as

$$\Pr(M, p, T) = TR - \frac{1}{T}(MC + A + cQ + HC) \tag{11}$$

4. NUMERIC EXAMPLE

The objective is to maximize total profit which can be obtained by differentiating equation (11) with respect to decision variables and setting them zero in order to get solution. This is shown in following procedure.
Step 1: Allocate values to all inventory parameters other than decision variables.

Step 2: Work out $\frac{\partial \Pr}{\partial M}=0, \frac{\partial \Pr}{\partial p}=0$ *and* $\frac{\partial \Pr}{\partial T}=0$ to get optimum values of decision variables M, p and T respectively.

Step 3: In order to check concavity of profit function work out the Hessian matrix,

$$H=\begin{bmatrix} \frac{\partial^2 \Pr}{\partial M^2} & \frac{\partial^2 \Pr}{\partial M \partial p} & \frac{\partial^2 \Pr}{\partial M \partial T} \\ \frac{\partial^2 \Pr}{\partial p \partial M} & \frac{\partial^2 \Pr}{\partial p^2} & \frac{\partial^2 \Pr}{\partial p \partial T} \\ \frac{\partial^2 \Pr}{\partial T \partial M} & \frac{\partial^2 \Pr}{\partial T \partial p} & \frac{\partial^2 \Pr}{\partial T^2} \end{bmatrix}$$

Calculate principal minor determinants,

$$D_1=\frac{\partial^2 \Pr}{\partial M^2}, \quad D_2=\begin{vmatrix} \frac{\partial^2 \Pr}{\partial M^2} & \frac{\partial^2 \Pr}{\partial M \partial p} \\ \frac{\partial^2 \Pr}{\partial p \partial M} & \frac{\partial^2 \Pr}{\partial p^2} \end{vmatrix} \text{ and } D_3=\begin{vmatrix} \frac{\partial^2 \Pr}{\partial M^2} & \frac{\partial^2 \Pr}{\partial M \partial p} & \frac{\partial^2 \Pr}{\partial M \partial T} \\ \frac{\partial^2 \Pr}{\partial p \partial M} & \frac{\partial^2 \Pr}{\partial p^2} & \frac{\partial^2 \Pr}{\partial p \partial T} \\ \frac{\partial^2 \Pr}{\partial T \partial M} & \frac{\partial^2 \Pr}{\partial T \partial p} & \frac{\partial^2 \Pr}{\partial T^2} \end{vmatrix}$$

H is Negative definite if $D_1<0, D_2>0 \, and \, D_3<0$. This ensures concavity of the profit function.

Step 4: Use values obtained above in equation (5) and equation (11) to get optimum values of ordering quantity and profit respectively.

We consider following example to validate the mathematical formulation.

Example: Consider $A=\$400$ per order, $a=680$, $b=15$, $c=\$12$ per unit, $h=\$0.5$/unit/month, $g=0.04$, $T_a=0.4$ month, $\alpha=1.4$, $\beta=0.25$.

We follow the procedure mentioned in steps shown above. We get optimum values of decision variables as, marketing cost $M=\$1.54/unit$, selling price $p=\$30.85/unit$ and cycle time $T=1.7\,months$. For concavity of the profit function we work out $D_1, D_2 \, and \, D_3$ as per step 3. We get $D_1=-228.71<0, \; D_2=26691.87>0 \; and \; D_3=-1.07<0$ that ensures concavity. Further we can have the concavity from following graphs.

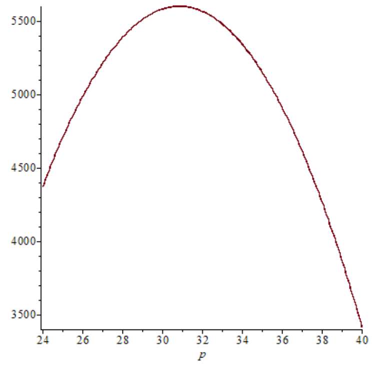

Figure 1 Concavity of Profit function with respect to Selling price p

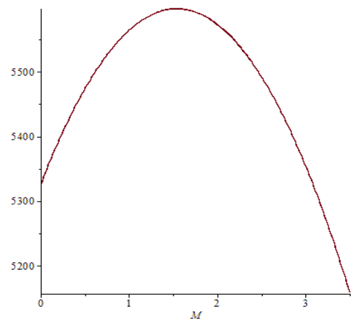

Figure 2 Concavity of profit function with respect to Marketing cost M

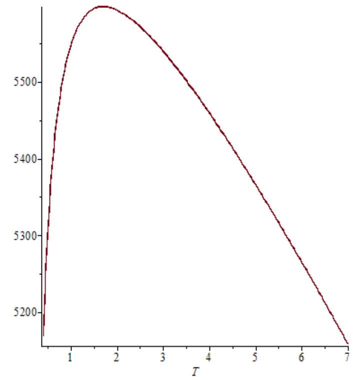

Figure 3 Concavity of profit function with respect to cycle time T

Using these optimum values of M, p, T and following step 4 we get optimum order quantity $Q = 699$ units and profit $\Pr = \$5597$.
Next, we determine the variations in cycle time, marketing cost, Selling price, Order quantity and Total profit with respect to change in inventory parameters by $-20\%, -10\%, 10\%$ *and* 20%.

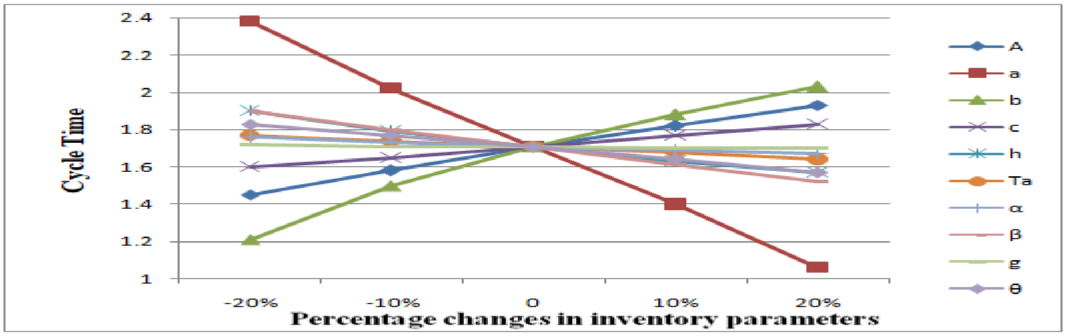

Figure 4 Sensitivity analysis for cycle time (T)

(1) (Fig.4) The graph shows that increase in A, b and c results into increase in cycle time. T is very responsive to a and b. Increase of holding cost results into decrease in the cycle time. Cycle time T is negatively sensitive to β, θ and T_a.

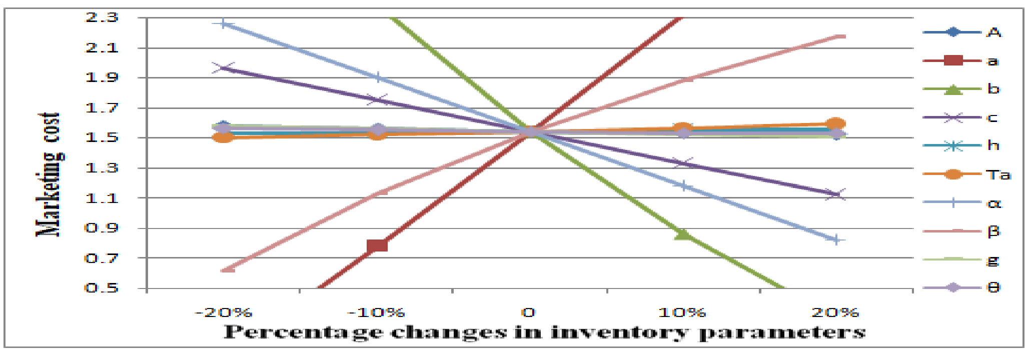

Figure 5 Sensitivity analysis for Marketing cost (M)

(2) (Fig. 5) Marketing cost is very sensitive to the parameters a, b, α, β and c. Other parameter effects negligible to the marketing cost. With increase in a, h, T_a and β marketing price also increase. On the other way, increase in b, g, A and α results into decrease in marketing cost.

(3) (Fig. 6) Increase in a, c, g, β, T_a and θ cause the increase in selling price. Selling price is negatively sensitive to b, A and α, i.e. with the increase in these variables the selling price decrease.

(4) (Fig.7) Ordering quantity is very sensitive with all inventory parameters. It increases with the increase in A and α. On the other hand increase in c, g, h, β, θ and T_a drives to decrease in it.

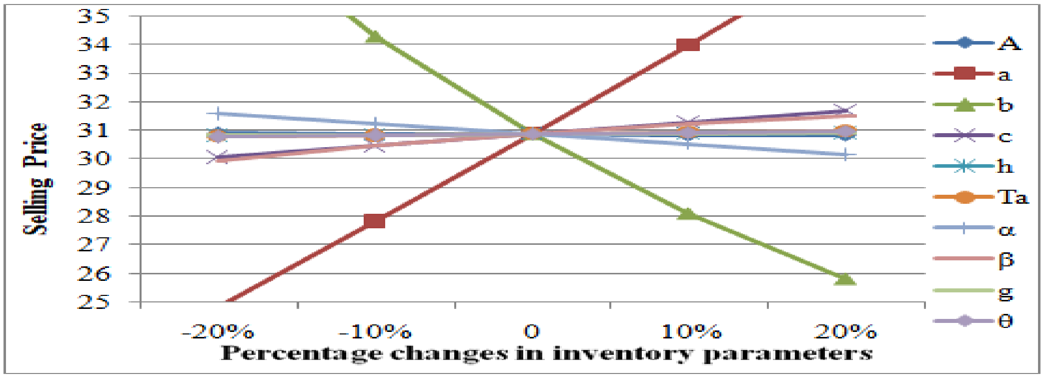

Figure 6 Sensitivity analysis for Selling price (p)

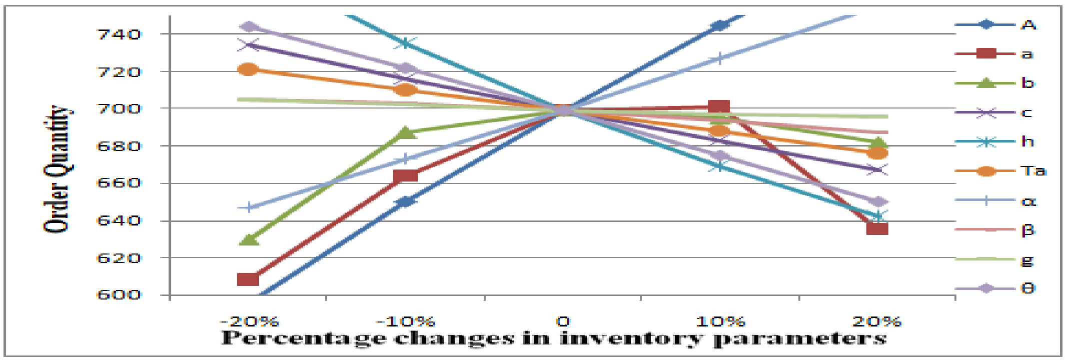

Figure 7 Sensitivity analysis for Ordering quantity (Q)

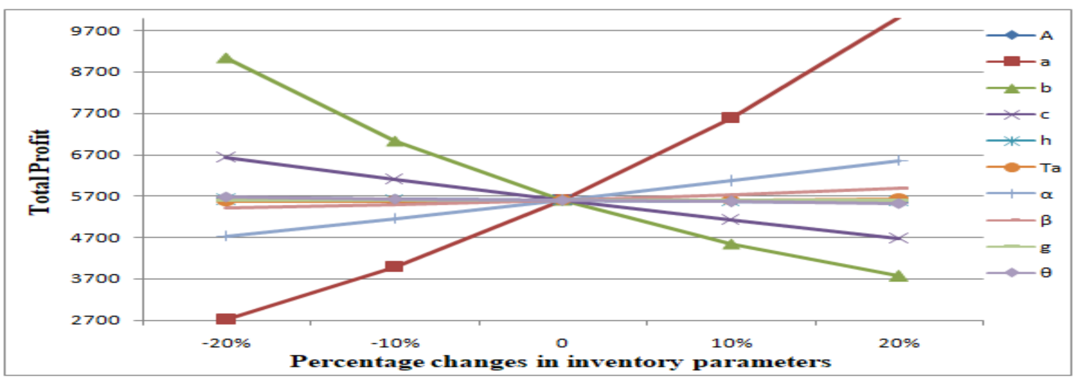

Figure 8 Sensitivity analysis for Total Profit (Pr)

(5) (Fig. 8) If there is increase in advance booking time period, total profit increases. Similarly, increase in parameters a, α and β also results into increase in total profit. Increase in Ordering cost, Cost price per unit, holding cost per unit, discounting factor and rate of deterioration forces the decrement in total profit, which is clearly reflected in the sensitivity graph.

The retailer must take care of balancing between advance sales period, discount factor, advertising expenditure and selling price per unit of the product in order to maximize the total profit. Offering discount in advance booking phase increase the sales but it is necessary to keep it bounded so that it do not cost the decrease in total profit.

5. CONCLUSION

We have studied a deteriorating products inventory model with advance booking and spot sales phases to maximize the total profit of the retailer. We consider demand of the product to be price sensitive as well as effected by advertisement efforts. This study is applicable to products which deteriorate at a constant rate, e.g. Dairy products, Flowers, Fruits, Sweets, Chocolates etc. It is observed from the study that the retailer should wisely choose the advance booking period and the discounting factor to be offered in the period. Retailer should take care of advertising expenditure to promote the product.

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