OPTIMAL REPLENISHMENT POLICY OF RAMP TYPE INVENTORY MODEL UNDER DISCOUNTED PRICE AND IMPRECISION[1]

P.K. Tripathy[2] and Sujata Sukla
P.G. Department of Statistics, Utkal University,
Bhubaneswar-751004 , India.

ABSTRACT
Demand plays a crucial role in supply chain system. Proper knowledge on behaviour of demand improves the effectiveness of the decision making process in supply chain system. Demand of some products may not always be linear or quadratic or exponential but ramp type. For newly launched fashionable goods, garments, automobiles, etc. demand rises initially but it becomes stagnant after a certain period of time. Ramp type function rigorously depicts such type of demand pattern. Moreover, price discount which is a way of alluring the customers in the market has become a strategy for promoting the business. Further, it becomes difficult to assess the parameters involved in supply chain due to its increasing complexity. That is why it is essential to effectively deal with such type of uncertainty which occurs in business process. The present study endeavours to develop a ramp type inventory model under imprecision and price discount where deterioration follows weibull distribution. The model is exemplified through numerical illustration. Sensitivity analysis is conducted to discern the effect of various system parameters on optimality. The outcomes of the paper provide inspiring and instrumental insights about the uncertainty vis-à-vis price discount.

KEYWORDS: Weibull distribution, price discount, fuzzy, defuzzification

MSC 90B05

RESUMEN
La demanda juega un papel crucial en la cadena del sistema de suministro. Un apropiado conocimiento del comportamiento de la demanda mejora la efectividad del proceso de toma de decisión en la cadena del sistema de suministro. La demanda de algunos productos puede no ser lineal , cuadrática o exponencial sino ser del tipo “ramp”. Para nuevos productos de moda, adornos, automóviles, etc. la demanda inicialmente crece, pero se estanca después de un periodo de tiempo. Las funciones del tipo “ramp” describen rigurosamente tal tipo de patrón de demanda. Más aun, el descuento en los precios, busca atraer los clientes en el mercado y se ha convertido en una estrategia para promover los negocios; más aun es difícil asesorarse sobre los parámetros envueltos en la cadena del sistema de suministro, debido a que se incrementa la complejidad. Esto es por lo que es esencial tratar efectivamente con tal tipo de incertidumbre, que aparece en el proceso de negociación. Este estudio lleva a desarrollar un modelo de inventario del tipo “ ramp” ante la imprecisión y el precio descontado, donde el deterioro sigue una distribución de weibull. El modelo es ejemplificado a través de una lustración numérica. Un análisis de sensibilidad se lleva a cabo para discernir sobre el efecto de varios parámetros del sistema en la optimalidad . Los resultados de este paper provee una inspiración y una visión instrumental sobre a incertidumbre cara-a-cara del precio de descuento

PALABRAS CLAVE: distribución de Weibull distribution, precio de descuento, fuzzy, defuzzification

1. INTRODUCTION

Demand plays a vital role in supply chain system. It entirely controls the whole supply chain system. The success of business completely depends on knowing the proper behaviour of demand. Researchers have focused on various kinds of demand patterns in their research work. Manna and Chaudhuri [6] proposed an EOQ model for deteriorating items with linear demand where finite production rate is proportional to time dependent demand rate and deterioration rate is time proportional. Shah and Raykundaliya [12] attempted to develop an optimal ordering policy for deteriorating items with linearly declining demand under delay in payment. Yadav and Vats [19] explored a deterministic deteriorating inventory model with quadratic demand under inflation. Mishra et al. [7] focused on an inventory model for weibull deteriorating items with quadratic demand where shortages are partially backlogged and salvage value is associated to deteriorated units. Chatterji and Gothi [1] analysed an inventory model with time dependent demand constant holding cost where the deterioration follows

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2 msccompsc@gmail.com, suklasujata086@gmail.com

two and three parameter weibull distribution. Singh et al. [13] investigated a supply chain system considering a trapezoidal type demand dependent production rate. Debata et al. [2] discussed an inventory model for perishable items with quadratic trapezoidal type demand and constant deterioration. Tripathy and Pradhan [16] formulated an inventory model with weibull demand and variable deterioration rate where unsatisfied demand is partially backlogged and delay in payment is allowed. Tripathy and Pradhan [17] endeavoured to develop an inventory model with ramp type demand under permissible delay. Panda et al. [10] analysed a single cycle perishable inventory model with quadratic ramp type demand and partial backlogging.
Price discount is a way of promotional aid for the seller in the modern market. It also serves as a medium for attraction of customers' willing to purchase habit. It helps in accumulating the seller's profit and aids in growing business paradigm. Price discount acts as an essential business supplement for short life span products and for the products which gradually decay with time. This has highly tremendous effect on business for occasional selling products. Many researchers have adorned their research process by embarking upon price discount. Panda et al. [9] explored an inventory model with stock dependent demand to find out the actual amount of discount that to be provided to increase the profit. Thangam [15] attempted to develop a market friendly inventory model where the retailer offers a price discount and a credit period to promote his sales. Sarkar et al. [11] discussed an EOQ model for various type of time dependent demand when delay in payment and price discount are permitted. Pal and Chandra[8] proposed a periodic review inventory model under permissible deal under stock dependent demand and backorder price discount.
Moreover, uncertainty is an inherent issue which can arise at any stage of business process. Every business organisation struggles to withstand in uncertainty. Uncertainty cannot be weeded out from the supply chain system. The vast growing marketing system is gaining complexity day by day therefore it becomes very strenuous to appraise the exact values of the parameters involved in the inventory system. This fact led the researchers to bring the concept of fuzziness into the field of research. Tripathy and Sukla [18] explored a fuzzy inventory model under trade credit system involving default risk. Jaggi et al. [4] suggested a fuzzy inventory model for deteriorating items with linear demand and shortages. Sujatha and Parvathi [14] developed a fuzzy inventory model for deteriorating items with two parameter weibull demand in partially backlogged situation allowing permissible delay. Mahata and Mahata [5] analysed an EOQ model to reflect the supply chain management situation under two level trade credit in fuzzy sense. Jaggi et al. [3] evoked a fuzzy inventory model with constant demand under inflationary conditions.

Table-1: Contribution of authors

Reference	Demand	Deterioration	Price Discount	Both Pre & Post Deterioration Discount	Model	Fuzzification	Defuzzification
Manna & Chaudhuri [6]	Linear	Time dependent	No	--	Crisp	--	--
Yadav & Vats[19]	Quadratic	Constant	No	--	Crisp	--	--
Mishra, Singh & Pattnayak [7]	Quadratic	Weibull	No	--	Crisp	--	--
Singh, Vaish & Singh [13]	Trapezoidal	Constant	No	--	Crisp	--	--
Debata, Acharya & Samanta [2]	Trapezoidal	Constant	No	--	Crisp	--	--
Shah & Raykundaliya [12]	Time dependent	Constant	No	--	Crisp	--	--
Chatterji & Gothi [1]	Time dependent	Weibull	No	--	Crisp	--	--
Tripathy & Pradhan [16]	Weibull	Time dependent	No	--	Crisp	--	--
Tripathy & Pradhan [17]	Ramp	Weibull	No	--	Crisp	--	--
Panda, Senapati & Basu [10]	Ramp	Heaviside's function	No	--	Crisp	--	--
Sarkar, Sana & Chaudhuri[11]	Constant & Time	No	Yes	No	Crisp	--	--

	dependent						
Thangam[15]	Constant	No	Yes	No	Crisp	--	--
Panda, Shah & Basu[9]	Stock dependent	Heaviside's function	Yes	Yes	Crisp	--	--
Pal and Chandra[8]	Stock dependent	No	Yes	No	Crisp	--	--
Sujatha & Parvathi [14]	Weibull	Time dependent	No	--	Fuzzy	Trapezoidal	Signed distance
Mahata & Mahata [5]	Constant	Constant	No	--	Fuzzy	Triangular	Graded mean
Jaggi, Pareek, Khanna & Nidhi [3]	Constant	Constant	Yes	No	Fuzzy	Triangular	Signed distance
Jaggi, Pareek, Sharma & Nidhi [4]	Linear	Constant	No	--	Fuzzy	Triangular	Centoid, Signed distance, Graded mean
Tripathy & Sukla[18]	Linear	No	No	--	Crisp & Fuzzy	Triangular & Trapezoidal	Signed distance & Graded mean
Present paper	Ramp	Weibull	Yes	Yes	Crisp & Fuzzy	Triangular & Trapezoidal	Signed distance & Graded mean

The present study develops an inventory model under discounted selling price and imprecision. Here demand is considered as a ramp type quadratic function and deterioration as a three parameter weibull distribution. Both pre and post deterioration discounts are considered where the former helps in maintaining constancy in the demand rate and the latter boosts the demand of decreased quality items. The effect of both types of discounts in optimising the profit is examined. Fuzziness has been introduced to deal with imprecision. The cost parameters governing the inventory model like holding cost, purchase cost, disposal cost, ordering cost and selling price are treated as triangular and trapezoidal fuzzy numbers. Both signed distance and graded mean integration methods are employed to defuzzify the total profit. The model is assessed through numerical illustration. Behaviour of the parameters associated with the model in optimising the profit is studied through sensitivity analysis. The model helps in attaining optimality in uncertainty and furnishes a clear and concrete idea about the offer of discounts when impreciseness is present.

2. NOTATIONS AND ASSUMPTIONS

Notations

i. C_0 set up cost
ii. $\widetilde{C_0}$ fuzzy set up cost
iii. S constant selling price of the product per unit
iv. $\tilde{S}$ fuzzy selling price of the product per unit
v. h holding cost per unit per unit time
vi. $\tilde{h}$ fuzzy holding cost per unit per unit time
vii. d disposal cost per unit
viii. $\tilde{d}$ fuzzy disposal cost per unit
ix. P purchase cost of the product per unit
x. $\tilde{P}$ fuzzy purchase cost of the product per unit
xi. r_1 pre deterioration discount per unit
xii. r_2 post deterioration discount per unit
xiii. T_1 the total cycle time
xiv. π the total average profit
xv. $\tilde{\pi}$ fuzzy total average profit
xvi. $\tilde{\pi}_{SD}$ defuzzified profit using Signed distance method
xvii. $\tilde{\pi}_{GM}$ defuzzified profit using Graded mean integration method

Assumptions

i. Replenishment rate is infinite.

ii. The deterioration rate is assumed to follow three parameter weibull distribution function.

$$\theta = \alpha\,\beta\,(t-\tau)^{\beta-1}$$

where α is the shape parameter,

β is the scale parameter

and τ is the location parameter

i. Demand rate is a ramp type quadratic function defined as

$$D(t) = a + b\{t-(t-\mu)H(t-\mu)\} + c\{t-(t-\gamma)H(t-\gamma)\}^2, \quad a>0,\ b>0$$

, where

$$H(t-\mu) = \begin{cases} 1, & t \ge \mu \\ 0, & t < \mu \end{cases} \quad \text{and} \quad H(t-\gamma) = \begin{cases} 1, & t \ge \gamma \\ 0, & t < \gamma \end{cases}$$

a is the initial demand rate, b is the rate with which the demand rate increases. The rate of change in demand itself increases at a rate c.

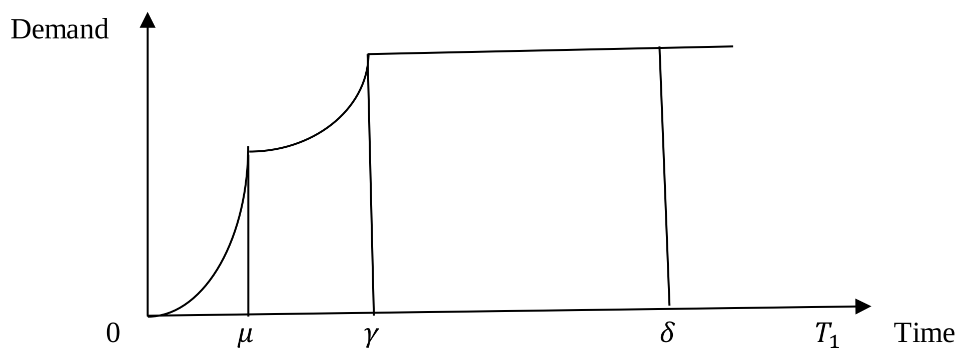

Figure.1: Behaviour of demand with respect to time

iii. r_1 $(0 \le r_1 \le 1)$ is the percentage pre deterioration discount offer on unit selling price. $\propto_1 = (1-r_1)^{-n_1}$, $n_1 \in R$ is the effect of pre deterioration discount on demand. r_2 $(0 \le r_2 \le 1)$ is the percentage post deterioration discount offer on unit selling price. $\propto_2 = (1-r_2)^{-n_2}$, $n_2 \in R$ is the effect of post deterioration discount on demand.

2.1. Model Formulation

Case-I

Here deterioration starts at the time period δ. So pre deterioration discount is provided during the time period $\gamma \le t \le \delta$ and the post deterioration discount is provided during the time period $\delta \le t \le T_1$.

$$\frac{dI(t)}{dt} = -(a + bt + ct^2), \qquad 0 \le t \le \mu \qquad (1)$$

$$\frac{dI(t)}{dt} = -(a + b\mu + ct^2), \qquad \mu \le t < \gamma \qquad (2)$$

$$\frac{dI(t)}{dt} = -\alpha_1(a + b\mu + c\gamma^2), \quad \gamma \le t < \delta \qquad (3)$$

$$\frac{dI(t)}{dt} = -\alpha_2(a + b\mu + c\gamma^2) - \theta I(t), \quad \delta \le t < T_1 \qquad (4)$$

With boundary conditions $I(0) = Q_1$ and $I(T_1) = 0$.

Solving these equations, we can have

$$I_1(t) = -\left(at + b\frac{t^2}{2} + c\frac{t^3}{3}\right) + Q_1 \tag{5}$$

$$I_2(t) = -\left(at + b\mu t + c\frac{t^3}{3}\right) + b\frac{\mu^2}{2} + Q_1 \tag{6}$$

$$I_3(t) = \begin{bmatrix} -\alpha_1 t\left(at + b\mu + c\gamma^2\right) + \alpha_1\gamma\left(at + b\mu + c\gamma^2\right) \\ -(a + b\mu) - c\frac{\gamma^3}{3} + b\frac{\mu^2}{2} + Q_1 \end{bmatrix} \tag{7}$$

$$I_4(t) = -\alpha_2\left(at + b\mu + c\gamma^2\right)\left((T_1 - t)(1 - \alpha t^\beta) + \frac{\alpha}{\beta + 1}(T_1^{\beta+1} - \delta^{\beta+1})\right) \tag{8}$$

and the order quantity of the system is

$$Q_1 = \begin{bmatrix} -\alpha_2\left(at + b\mu + c\gamma^2\right)\left((T_1 - t)(1 - \alpha t^\beta) + \frac{\alpha}{\beta + 1}(T_1^{\beta+1} - \delta^{\beta+1})\right) \\ -\alpha_1 t\left(at + b\mu + c\gamma^2\right) + \alpha_1\gamma\left(at + b\mu + c\gamma^2\right) - (a + b\mu) + c\frac{\gamma^3}{3} - b\frac{\mu^2}{2} \end{bmatrix} \tag{9}$$

The sales revenue is

$$SR = S\left[\int_0^\mu D(t)\;dt + \int_\mu^\gamma D(t)\,dt + \alpha_1(1 - r_1)\int_\gamma^\delta D(t)\,dt + \alpha_2(1 - r_2)\int_\delta^{T_1} D(t)\,dt\right]$$

The holding cost and disposal cost of the system in this case is

$$HC + DC = \left[h\int_0^\mu I_1(t)\;dt + h\int_\mu^\gamma I_2(t)\;dt + h\int_\gamma^\delta I_3(t)\;dt + (h + \theta d)\int_\delta^{T_1} I_4(t)\;dt\right]$$

Purchase cost in the cycle is given by $PC = PQ_1$

Thus the total profit per unit time of the system is

$$\pi = \frac{1}{T_1}\left[SR - PC - HC - DC - C_0\right]$$

$$= \frac{1}{T_1}\begin{bmatrix} S\left[\int_0^\mu D(t)\;dt + \int_\mu^\gamma D(t)\,dt + \alpha_1(1 - r_1)\int_\gamma^\delta D(t)\,dt + \alpha_2(1 - r_2)\int_\delta^{T_1} D(t)\,dt\right] - PQ_1 \\ -\left[h\int_0^\mu I_1(t)\;dt + h\int_\mu^\gamma I_2(t)\;dt + h\int_\gamma^\delta I_3(t)\;dt + (h + \theta d)\int_\delta^{T_1} I_4(t)\;dt\right] - C_0 \end{bmatrix} \tag{10}$$

The pre deterioration discount on selling price is to be given in such a way that the discounted selling price is not less than the unit cost of the product i.e., $S(1 - r_1) - c > 0$. Similarly, $S(1 - r_2) - c > 0$. Applying these constraints on the unit total profit function we have the following maximisation problem

$$\begin{gathered} maximize\ \pi(\delta, T_1) \\ \text{Subject to } \{r_1, r_2\} < 1 - \frac{c}{S} \\ r_1, r_2, \delta, T_1 \geq 0 \end{gathered} \tag{11}$$

The optimum values of μ and γ which maximize the unit profit, can be obtained by solving the equations

$$\frac{\partial \pi}{\partial \delta} = 0 \text{ and } \frac{\partial \pi}{\partial T_1} = 0 \tag{12}$$

Provided that these values should satisfy the sufficient conditions

$$\frac{\partial^2 \pi}{\partial \delta^2} < 0,\ \frac{\partial^2 \pi}{\partial T_1^2} < 0 \tag{13}$$

$$\text{and} \quad \frac{\partial^2 \pi}{\partial \delta^2}\frac{\partial^2 \pi}{\partial T_1^2} - \frac{\partial^2 \pi}{\partial \delta \, \partial T_1} < 0$$

Case-II

Here deterioration starts at the time period γ. So pre deterioration discount is provided during the time period $\mu \le t \le \gamma$ and the post deterioration discount is provided during the time period $\gamma \le t \le T_1$. The differential equations governing the model are

$$\frac{dI(t)}{dt} = -(a + bt + ct^2), \ 0 \le t \le \mu \tag{14}$$

$$\frac{dI(t)}{dt} = -\alpha_1 (a + b\mu + ct^2), \ \mu \le t < \gamma \tag{15}$$

$$\frac{dI(t)}{dt} = -\alpha_2 (a + b\mu + c\gamma^2) - \theta I(t), \ \gamma \le t < T_1 \tag{16}$$

with boundary conditions $I(0) = Q_1$ and $I(T_1) = 0$.

Thus the total profit per unit time of the system is

$$\pi = \frac{1}{T_1}\left[SR - PC - HC - DC - C_0\right]$$

$$= \frac{1}{T_1}\left[\begin{array}{l} S\left[\int_0^{\mu} D(t)\ dt + \alpha_1 (1-r_1)\int_{\mu}^{\gamma} D(t)\, dt + \alpha_2 (1-r_2)\int_{\gamma}^{T_1} D(t)\, dt\right] - P\, Q_1 \\ -\left[h\int_0^{\mu} I_1(t)\ dt + h\int_{\mu}^{\gamma} I_2(t)\ dt + (h+\theta d)\int_{\gamma}^{T_1} I_4(t)\ dt\right] - C_0 \end{array}\right] \tag{17}$$

The maximisation problem in this case is

$$\begin{gathered} maximize\ \pi(\gamma, T_1) \\ \text{Subject to } \{r_1, r_2\} < 1 - \frac{c}{s} \\ r_1, r_2, \gamma, T_1 \ge 0 \end{gathered} \tag{18}$$

The optimum values of μ and γ which maximize the unit profit, can be obtained by solving the equations

$$\frac{\partial \pi}{\partial \gamma} = 0 \ \text{ and } \ \frac{\partial \pi}{\partial T_1} = 0 \tag{19}$$

Provided that these values should satisfy the sufficient conditions

$$\begin{gathered} \frac{\partial^2 \pi}{\partial \gamma^2} < 0, \ \frac{\partial^2 \pi}{\partial T_1^2} < 0, \\ \text{and} \quad \frac{\partial^2 \pi}{\partial \gamma^2}\frac{\partial^2 \pi}{\partial T_1^2} - \frac{\partial^2 \pi}{\partial \gamma \, \partial T_1} < 0 \end{gathered} \tag{20}$$

Case-III

Here deterioration starts at the time period μ. So there is no pre deterioration discount. Only the post deterioration discount is provided during the time period $\mu \le t \le T_1$.

$$\frac{dI(t)}{dt} = -(a + bt + ct^2), \ 0 \le t \le \mu \tag{21}$$

$$\frac{dI(t)}{dt} = -\alpha_2 (a + b\mu + ct^2) - \theta I(t), \ \mu \le t < \gamma \tag{22}$$

$$\frac{dI(t)}{dt} = -\alpha_2 (a + b\mu + c\gamma^2) - \theta I(t), \ \gamma \le t < T_1 \tag{23}$$

with boundary conditions $I(0) = Q_1$ and $I(T_1) = 0$.

$$\pi = \frac{1}{T_1}\left[SR - PC - HC - DC - C_0\right]$$

$$= \frac{1}{T_1}\left[\begin{array}{l} S\left[\int_0^{\mu} D(t)\ dt + \alpha_2(1-r_2)\int_{\mu}^{\gamma} D(t)\,dt + \alpha_2(1-r_2)\int_{\gamma}^{T_1} D(t)\,dt\right] - P\,Q_1 \\ \left[h\int_0^{\mu} I_1(t)\ dt + (h+\theta d)\int_{\mu}^{\gamma} I_2(t)\ dt + (h+\theta d)\int_{\gamma}^{T_1} I_3(t)\ dt\right] - C_0 \end{array}\right] \qquad (24)$$

The maximisation problem in this case is

$$\begin{array}{c} maximize\ \pi(\mu, T_1) \\ \text{Subject to } \{r_1, r_2\} < 1 - \frac{c}{s} \\ r_1, r_2, \mu, T_1 \geq 0 \end{array} \qquad (25)$$

The optimum values of μ and γ which maximize the unit profit, can be obtained by solving the equations

$$\frac{\partial \pi}{\partial \mu} = 0 \ \text{ and } \ \frac{\partial \pi}{\partial T_1} = 0 \qquad (26)$$

Provided that these values should satisfy the sufficient conditions

$$\begin{array}{c} \frac{\partial^2 \pi}{\partial \mu^2} < 0,\ \frac{\partial^2 \pi}{\partial T_1^2} < 0, \\ \text{and} \quad \frac{\partial^2 \pi}{\partial \mu^2}\frac{\partial^2 \pi}{\partial T_1^2} - \frac{\partial^2 \pi}{\partial \mu \partial T_1} < 0 \end{array} \qquad (27)$$

3. FUZZY MODEL

Due to uncertainty the cost parameters involved in the model are treated as fuzzy in nature.

Cost parameters are Triangular fuzzy numbers

Treating Ordering cost $\widetilde{C_0} = (C_{0_1}, C_{0_2}, C_{0_3})$, selling price $\tilde{S} = (S_1, S_2, S_3)$, purchase cost $\tilde{P} = (P_1, P_2, P_3)$, holding cost $\tilde{h} = (h_1, h_2, h_3)$, disposal cost $\tilde{d} = (d_1, d_2, d_3)$ as triangular fuzzy numbers and applying signed distance method for defuzzification, the defuzzified profit in Case-I is obtained as

Case-I

$$\tilde{\pi}_{SD} = \frac{1}{4T_1}\left[\begin{array}{l} (S_1+2S_2+S_3)\left[\int_0^{\mu} D(t)\ dt + \int_{\mu}^{\gamma} D(t)\,dt + \alpha_1(1-r_1)\int_{\gamma}^{\delta} D(t)\,dt + \alpha_2(1-r_2)\int_{\delta}^{T_1} D(t)\,dt\right] \\ -(h_1+2h_2+h_3)\int_0^{\mu} I_1(t)\ dt - (h_1+2h_2+h_3)\int_{\mu}^{\gamma} I_2(t)\ dt - (h_1+2h_2+h_3)\int_{\gamma}^{\delta} I_3(t)\ dt \\ -\left((h_1+2h_2+h_3)+\theta(d_1+2d_2+d_3)\right)\int_{\delta}^{T_1} I_4(t)\ dt - (P_1+2P_2+P_3)Q_1 - (C_{01}+2C_{02}+C_{03}) \end{array}\right] \qquad (28)$$

Equation (28) satisfies the conditions (11), (12) and (13).

Similarly, the total defuzzified profit can also be obtained in other cases.

Applying graded mean integration method for defuzzification, the defuzzified profit in Case-I is obtained as

Case-I

$$\tilde{\pi}_{GM} = \frac{1}{6T_1}\left[\begin{array}{l} (S_1+4S_2+S_3)\left[\int_0^{\mu} D(t)\ dt + \int_{\mu}^{\gamma} D(t)\,dt + \alpha_1(1-r_1)\int_{\gamma}^{\delta} D(t)\,dt + \alpha_2(1-r_2)\int_{\delta}^{T_1} D(t)\,dt\right] \\ -(h_1+4h_2+h_3)\int_0^{\mu} I_1(t)\ dt - (h_1+4h_2+h_3)\int_{\mu}^{\gamma} I_2(t)\ dt - (h_1+4h_2+h_3)\int_{\gamma}^{\delta} I_3(t)\ dt \\ -\left((h_1+4h_2+h_3)+\theta(d_1+4d_2+d_3)\right)\int_{\delta}^{T_1} I_4(t)\ dt - (P_1+4P_2+P_3)Q_1 - (C_{01}+4C_{02}+C_{03}) \end{array}\right] \qquad (29)$$

Equation (29) satisfies the conditions (11), (12) and (13).

In similar manner the total defuzzified profit can also be obtained in other cases.

Cost parameters are Trapezoidal fuzzy numbers

Treating Ordering cost $\widetilde{C_0} = (C_{01}, C_{02}, C_{03}, C_{04})$, selling price $\tilde{S} = (S_1, S_2, S_3, S_4)$, purchase cost $\tilde{P} = (P_1, P_2, P_3, P_4)$, holding cost $\tilde{h} = (h_1, h_2, h_3, h_4)$ and disposal cost $\tilde{d} = (d_1, d_2, d_3, d_4)$ as trapezoidal fuzzy numbers and applying signed distance method for defuzzification, the defuzzified profit in Case-I is obtained as

Case-I

$$\tilde{\pi}_{SD} = \frac{1}{4T_1}\begin{bmatrix} (S_1+S_2+S_3+S_4)\left[\int_0^{\mu} D(t)\ dt + \int_{\mu}^{\gamma} D(t)dt + \alpha_1(1-r_1)\int_{\gamma}^{\delta} D(t)dt + \alpha_2(1-r_2)\int_{\delta}^{T_1} D(t)dt\right] \\ -(h_1+h_2+h_3+h_4)\int_0^{\mu} I_1(t)\ dt - (h_1+h_2+h_3+h_4)\int_{\mu}^{\gamma} I_2(t)\ dt\ - (h_1+h_2+h_3+h_4)\int_{\gamma}^{\delta} I_3(t)\ dt \\ -\left((h_1+h_2+h_3+h_4)+\theta(d_1+d_2+d_3+d_4)\right)\int_{\delta}^{T_1} I_4(t)\ dt - (P_1+P_2+P_3+P_4)Q_1 - (C_{01}+C_{02}+C_{03}+C_{04}) \end{bmatrix} \quad (30)$$

Equation (30) satisfies the conditions (11), (12) and (13).

In similar manner the total defuzzified profit can also be obtained in other cases.

Applying Graded mean integration distance method for defuzzification, the defuzzified profit in Case-I is obtained as

Case-II

$$\tilde{\pi}_{GM} = \frac{1}{6T_1}\begin{bmatrix} (S_1+2S_2+2S_3+S_4)\left[\int_0^{\mu} D(t)\ dt + \int_{\mu}^{\gamma} D(t)dt + \alpha_1(1-r_1)\int_{\gamma}^{\delta} D(t)dt + \alpha_2(1-r_2)\int_{\delta}^{T_1} D(t)dt\right] \\ -(h_1+2h_2+2h_3+h_4)\int_0^{\mu} I_1(t)\ dt - (h_1+2h_2+2h_3+h_4)\int_{\mu}^{\gamma} I_2(t)\ dt \\ -(h_1+2h_2+2h_3+h_4)\int_{\gamma}^{\delta} I_3(t)\ dt - \left((h_1+2h_2+2h_3+h_4)+\theta(d_1+2d_2+2d_3+d_4)\right)\int_{\delta}^{T_1} I_4(t)\ dt \\ -(P_1+2P_2+2P_3+P_4)Q_1 - (C_{01}+2C_{02}+2C_{03}+C_{04}) \end{bmatrix} \quad (45)$$

Equation (45) satisfies the conditions (11), (12) and (13).

The defuzzified total profit in other cases can also be obtained in the same way.

3.1. Empirical Investigation

The values of the system parameters are

$a = 400, b = 10, c = 2, h = 0.3, d = 10, S = 30, C_0 = 200, n_1 = 1, n_2 = 2, \alpha = 0.0002, \beta = 2, \tau = 1.8$

$r_1 = 0.15, r_2 = 0.25, \alpha_1 = 1.17647, \alpha_2 = 1.77778$

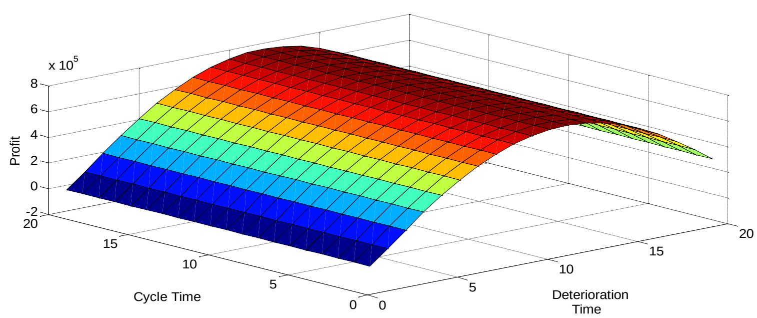

Figure.2: Concavity of the profit in Case-I

Case-I: Considering $\mu = 3$ and $\gamma = 12$

$\delta = 13.8379, T_1 = 14.0828, \pi = 9453.62, Q_1 = 8132.52$

Case-II: Considering $\mu = 3$

$\gamma = 7.44345, T_1 = 7.76801, \pi = 13011.2, Q_1 = 4125.30$

Case-III: Considering $\gamma = 12$

$\mu = 11.6857, T_1 = 13.3268, \pi = 10104.3, Q_1 = 8647.43$

3.2. Sensitivity Analysis

Table-2: Sensitivity Analysis in Case-I

Parameters	% change	δ	T_1	π	Q
a	-60%	13.7967	16.6433	5335.37	6834.19
	-40%	13.7955	15.6031	6685.61	7327.0
	-20%	13.8120	14.7693	8060.03	7755.42
	+20%	13.8687	13.5058	110862.8	8466.99
	+40%	-	-	-	-
	+60%	-	-	-	-
b	-60%	13.8345	14.1575	9176.08	7991.04
	-40%	13.8356	14.1322	9268.57	8038.16
	-20%	13.8368	14.1072	9361.08	8085.08
	+20%	13.8390	14.0586	9546.19	8179.19
	+40%	13.8401	14.0348	9638.78	8226.19
	+60%	13.8413	14.0113	9731.38	8272.88
c	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	-	-	-	-
	+20%	13.8151	14.6671	9860.05	9330.86
	+40%	13.8024	15.1505	10278.1	10498.1
	+60%	13.7960	15.5577	10704.7	11642.7
α	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	-	-	-	-
	+20%	13.1451	13.6991	9447.53	7942.51
	+40%	12.5640	13.3602	9437.78	7762.22
	+60%	12.0672	13.0574	9425.46	7591.95
τ	-60%	13.4297	13.8678	8034.52	9450.77
	-40%	13.5655	13.9392	9451.91	8066.84
	-20%	13.7015	14.0109	9452.86	8099.50
	+20%	13.9746	14.1549	9454.21	8165.20
	+40%	14.1117	14.2272	9454.61	8198.23
	+60%	14.2491	14.2997	9454.83	8231.42
h	-60%	17.6663	19.5483	10560.5	13470.5
	-40%	16.2977	17.4175	10124.9	11331.4
	-20%	15.0194	15.6117	9759.57	9574.74
	+20%	12.7572	12.7830	9198.10	6939.55
	+40%	-	-	-	-
	+60%	-	-	-	-
S	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	8.2787	9.0626	6302.46	4125.26
	+20%	-	-	-	-
	+40%	-	-	-	-
	+60%	-	-	-	-
P	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	-	-	-	-
	+20%	10.6167	11.3442	8340.67	6028.36
	+40%	-	-	-	-
	+60%	-	-	-	-
C_0	-60%	13.8386	14.0675	9462.15	8112.42
	-40%	13.8384	14.0726	9459.31	8119.02
	-20%	13.8381	14.0777	9456.47	8125.67
	+20%	13.8311	14.0829	9450.78	8132.49
	+40%	13.8374	14.0930	9447.95	8145.52
	+60%	13.8372	14.0981	9445.11	8152.13
d	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	-	-	-	-
	+20%	13.0881	13.6634	9446.56	7921.43
	+40%	12.4966	13.3110	9435.48	7727.88
	+60%	12.0104	13.0061	9421.88	7549.41
n_1	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	-	-	-	-

	+20%	12.6484	14.0126	9482.68	8579.50
	+40%	11.3325	14.0168	9482.58	9110.83
	+60%	9.87766	14.1191	9446.98	9748.18
r_1	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	12.4061	14.0071	9485.06	8649.22
	+20%	-	-	-	-
	+40%	-	-	-	-
	+60%	-	-	-	-

Table-3: Sensitivity Analysis in Case-II

Parameters	% change	δ	T_1	π	Q
a	-60%	7.19275	8.46441	14566.1	2415.33
	-40%	7.33343	8.60997	19760.3	3305.78
	-20%	7.42037	8.69908	24870.1	4190.03
	+20%	7.52290	8.80330	34968.6	5950.24
	+40%	7.55589	8.83652	39985.1	6828.22
	+60%	7.58183	8.86261	44988.8	7705.39
b	-60%	7.46798	8.74648	28743.6	4898.95
	-40%	7.47199	8.75093	29140.4	4956.33
	-20%	7.47591	8.75528	29537.1	5013.71
	+20%	7.48346	8.76366	30330.0	5128.43
	+40%	7.48711	8.76770	30726.1	5185.77
	+60%	7.49068	8.77165	31122.1	5243.11
c	-60%	7.65844	8.94094	28136.5	4830.41
	-40%	7.59464	8.87658	28775.4	4913.11
	-20%	7.53527	8.81628	29372.7	4993.26
	+20%	7.42753	8.70588	30462.3	5146.77
	+40%	7.37826	8.65502	30962.7	5220.50
	+60%	7.33161	8.60667	31438.0	5292.44
α	-60%	7.48805	8.77814	30243.7	5085.74
	-40%	7.48527	8.77192	30139.8	5080.84
	-20%	7.48249	8.76571	30036.5	5075.96
	+20%	7.47698	8.75333	29830.9	5066.20
	+40%	7.47424	8.74716	29728.8	5061.33
	+60%	7.47150	8.74100	29627.1	5056.46
β	-60%	12.7276	13.3271	11629.0	8583.46
	-40%	10.2602	11.7800	20387.7	7492.93
	-20%	8.63319	10.1589	26707.5	6168.13
	+20%	6.65766	7.69443	31309.4	4181.96
	+40%	6.06058	6.90407	31762.9	3722.39
	+60%	5.61474	6.31041	31741.5	3316.13
τ	-60%	6.49507	7.77999	32569.6	4401.67
	-40%	6.8252	8.11317	31724.0	4626.10
	-20%	7.15339	8.43964	30845.0	4849.26
	+20%	7.8043	9.07288	28990.5	5291.44
	+40%	8.12719	9.37981	28017.6	5510.23
	+60%	8.44853	9.68036	27016.1	5727.24
h	-60%	9.36954	11.2610	32569.6	7151.81
	-40%	8.4744	10.0819	31724.0	6131.66
	-20%	7.89643	9.31554	30845.0	5507.07
	+20%	7.15906	8.32896	28990.5	4742.60
	+40%	6.90137	7.98086	28017.6	4482.47
	+60%	6.6878	7.69062	27016.1	4269.07
S	-60%	5.99203	6.53792	3820.42	3411.4
	-40%	6.57278	7.44495	11707.9	4073.34
	-20%	7.05776	8.15778	20458.4	4607.32
	+20%	7.85674	9.28743	40055.9	5489.46
	+40%	8.19979	9.76198	50776.5	5875.78
	+60%	8.51616	10.1958	62061.7	6238.08
P	-60%	7.58354	8.97302	36680.1	5251.26
	-40%	7.54771	8.90132	34396.0	5190.80
	-20%	7.51311	8.83017	32147.7	5130.76
	+20%	7.44757	8.68930	27751.6	5011.68
	+40%	7.41661	8.61948	25600.9	4952.53
	+60%	7.38684	8.54998	23479.4	4893.55
C_0	-60%	7.47969	8.75897	29936.8	5070.56
	-40%	7.47971	8.75915	29935.6	5070.73
	-20%	7.47972	8.75933	29934.5	5070.90
	+20%	7.47974	8.75970	29932.5	5071.25

	+40%	7.47976	8.75988	29931.3	5071.42
	+60%	7.47977	8.76006	29930.2	5071.59
d	-60%	7.48897	8.7788	30246.2	5087.31
	-40%	7.48588	8.77236	30141.6	5081.89
	-20%	7.48280	8.76593	30037.3	5076.47
	+20%	7.47668	8.75312	29830.2	5065.69
	+40%	7.47364	8.74674	29727.4	5060.32
	+60%	7.47061	8.74037	29625.0	5054.95
n_1	-60%	7.44166	8.74352	29652.6	4828.7
	-40%	7.45402	8.74879	29744.7	4906.46
	-20%	7.46671	8.75413	29838.4	4987.24
	+20%	7.49309	8.76496	30030.2	5158.16
	+40%	7.50680	8.77045	30128.3	5248.58
	+60%	7.52086	8.77597	30227.5	5342.80
n_2	-60%	7.63380	8.72172	21500.4	4684.73
	-40%	7.57792	8.73847	24029.2	4793.32
	-20%	7.52666	8.75073	26828.0	4921.57
	+20%	7.43687	8.76564	33386.8	5244.09
	+40%	7.39781	8.76974	37233.5	5443.10
	+60%	7.36227	8.77229	41524.2	5670.09
r_1	-60%	7.52292	8.82363	31071.4	4872.87
	-40%	7.5095	8.80367	30714.4	4935.12
	-20%	7.49513	8.78235	30335.9	5001.08
	+20%	7.46317	8.73500	29516.0	5145.70
	+40%	7.44531	8.70861	29048.1	5224.66
	+60%	7.42601	8.68013	28559.6	5309.16
r_2	-60%	7.85222	9.12808	25749.6	4949.19
	-40%	7.73058	9.01274	27037.2	4974.07
	-20%	7.60643	8.88999	28426.1	5013.52
	+20%	7.35041	8.62096	31582.0	5151.37
	+40%	7.21842	8.47387	33398.9	5260.56
	+60%	7.08368	8.31766	35419.4	5406.94

Table-4: Sensitivity Analysis in Case-III

parameters	% change	μ	T_1	π	Q
a	-60%	11.7334	15.4717	5862.98	7356.27
	-40%	11.7157	14.6235	7257.66	7833.02
	-20%	11.7000	13.9209	8653.59	8260.67
	+20%	11.6726	12.8162	11549.7	8999.58
	+40%	11.6603	12.3715	13006.7	9322.22
	+60%	11.6488	11.9797	14473.8	9618.88
b	-60%	11.7020	12.8042	9436.05	7344.19
	-40%	11.6973	12.9932	9657.46	7783.31
	-20%	11.6918	13.1669	9880.26	8217.7
	+20%	11.6790	13.4746	10329.3	9073.3
	+40%	11.6717	13.6115	10555.2	9495.57
	+60%	11.6640	13.7385	10782.0	9914.32
c	-60%	-	-	-	-
	-40%	11.6275	12.3740	9399.53	6753.10
	-20%	11.6621	12.8980	9747.45	7711.99
	+20%	11.7033	13.6855	10467.6	9566.22
	+40%	11.7170	13.9905	10836.0	10472.6
	+60%	11.7281	14.2535	11208.2	11369.8
α	-60%	11.7439	14.3970	10109.2	10220.7
	-40%	11.7231	14.0011	10102.7	9641.54
	-20%	11.7038	13.6470	10101.3	9120.69
	+20%	11.6687	13.0348	10111.2	8213.96
	+40%	11.6527	12.7664	10121.6	7813.90
	+60%	11.6376	12.5182	10135.2	7445.87
β	-60%	10.9226	15.5516	10144.1	12341.6
	-40%	11.3797	15.2327	10140.2	11643.3
	-20%	11.5737	14.6319	10122.0	10655.2
	+20%	-	-	-	-
	+40%	-	-	-	-
	+60%	-	-	-	-
τ	-60%	11.5981	13.2298	10066.6	8535.64
	-40%	11.6301	13.2562	10083.2	8562.81
	-20%	11.6593	13.2889	10095.4	8600.61
	+20%	11.7096	13.3690	10110.4	8701.80
	+40%	11.7310	13.4146	10114.4	8762.39

	+60%	11.7501	13.4630	10116.8	8828.21
h	-60%	11.5278	19.2747	11677.7	17442.3
	-40%	11.6458	16.9993	11025.1	14011.9
	-20%	11.6797	15.0369	10502.6	11125.3
	+20%	-	-	-	-
	+40%	-	-	-	-
	+60%	-	-	-	-
S	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	-	-	-	-
	+20%	11.7547	17.0463	14126.5	14016.6
	+40%	11.8004	19.8716	18572..3	18182.1
	+60%	11.8288	22.1927	23260.5	21672.7
P	-60%	11.8840	19.5361	14892.1	17632.6
	-40%	11.8229	17.6928	13142.8	14928.9
	-20%	11.7570	15.6618	11528.5	11987.8
	+20%	-	-	-	-
	+40%	-	-	-	-
	+60%	-	-	-	-
C_0	-60%	11.6857	13.3123	10113.3	8626.54
	-40%	11.6857	13.3171	10110.3	8633.46
	-20%	11.6857	13.3220	10107.3	8640.52
	+20%	11.6857	13.3317	10101.3	8654.49
	+40%	11.6857	13.3365	10098.3	8661.41
	+60%	11.6857	13.3413	10095.3	8668.33
d	-60%	11.7131	14.2660	10190.8	9986.61
	-40%	11.7036	13.9358	10157.9	9514.95
	-20%	11.6944	13.6234	10129.1	9069.60
	+20%	11.6773	13.0443	10083.2	8246.0
	+40%	11.6692	12.7743	10065.7	7862.90
	+60%	11.6614	12.5157	10051.7	7496.48
n_2	-60%	-	-	-	-
	-40%	-	-	-	-
	-20%	-	-	-	-
	+20%	11.7718	14.7619	10313.3	11194.4
	+40%	11.8470	15.9041	10624.5	13800.4
	+60%	11.9126	16.8382	11026.0	16563.4
r_2	-60%	11.6849	15.8749	10367.7	10494.6
	-40%	11.6945	15.1965	10230.7	10233.9
	-20%	11.6950	14.3601	10201.6	9670.15
	+20%	11.6660	12.0335	10036.3	6879.17
	+40%	-	-	-	-
	+60%	-	-	-	-

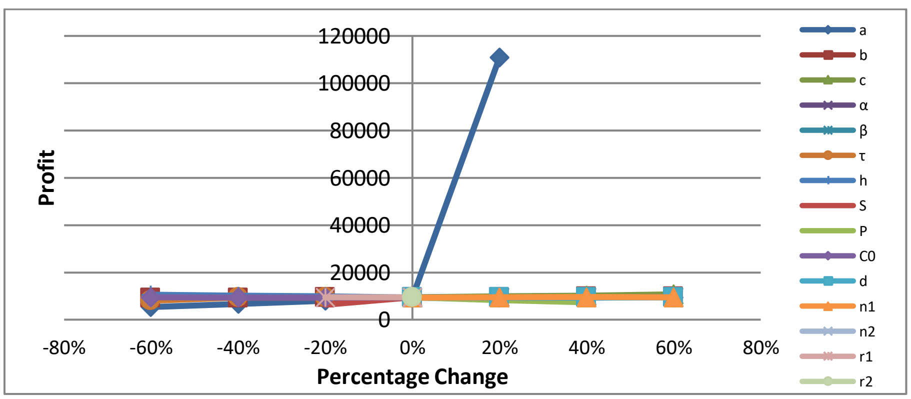

Figure.4: Behaviour of parameters in Case-I

Fuzzy Model

When ordering cost $\widetilde{C_0} = (180, 200,230)$, selling price $\tilde{S} = (25, 28, 33)$, purchase cost $\tilde{P} = (5, 10, 12)$, holding cost $\tilde{h} = (0.1, 0.3,\ 0.4)$, disposal cost $\tilde{d} = (5,10,12)$ are treated as triangular fuzzy numbers.

<u>Case-I</u>

Signed distance method: $\delta = 15.5854, T = 28.1424, \pi = 7461.95, Q = 26047.3$
Graded mean integration method: $\delta = 14.9314, T = 27.9114, \pi = 7002.21, Q = 26062.8$
Case-II
Signed distance method: $\gamma = 17.8907, T = 25.6153, \pi = \mathbf{2608790}, Q = 28385.9$
Graded mean integration method: $\gamma = 15.5731, T = 22.7521, \pi = 1595350, Q = 22514.0$
Case-III
Signed distance method: $\mu = 11.7102, T = 25.4720, \pi = 32873.9, Q = 27932$
Graded mean integration method: $\mu = 11.7051, T = 25.2205, \pi = 7543.26, Q = 26403$

When ordering cost $\widetilde{C_0} = (150,170,220,250)$, selling price $\tilde{S} = (25,27,33,35)$, purchase cost $\tilde{P} = (5,8,12,16)$, holding cost $\tilde{h} = (0.1,0.2,0.4,0.7)$, disposal cost $\tilde{d} = (3,8,12,15)$ are treated as trapezoidal fuzzy numbers.

Case-I
Signed distance method: $\delta = 13.4143, T = 24.5196, \pi = 7377.21, Q = 23705.8$
Graded mean integration method: $\delta = 14.0614, T = 26.3741, \pi = 7235.32, Q = 26288.6$
Case-II
Signed distance method: $\gamma = 17.8105, T = 25.5223, \pi = \mathbf{3263550}, Q = 28176.8$
Graded mean integration method: $\gamma = 15.5716, T = 22.7440, \pi = 1872540, Q = 22499.9$
Case-III
Signed distance method: $\mu = 11.7141, T = 22.0433, \pi = 8004.11, Q = 21478.7$
Graded mean integration method: $\mu = 11.7170, T = 23.6529, \pi = 7889.66, Q = 23939.1$

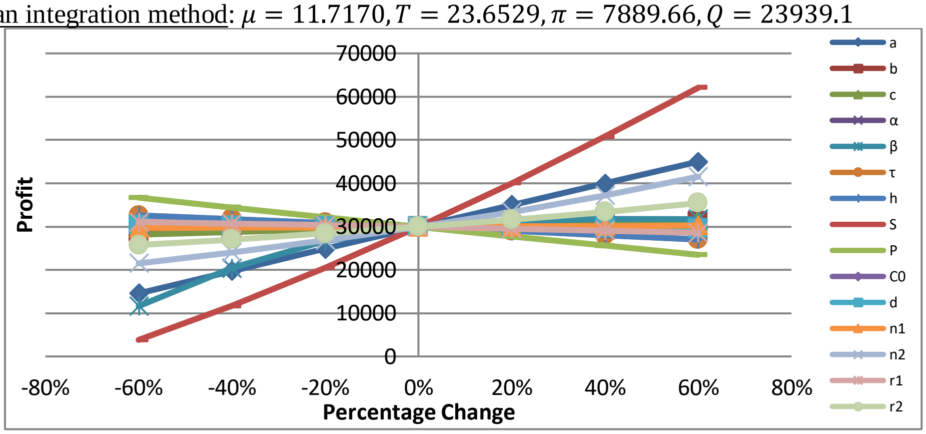

Figure.5:Behaviour of parameters in Case-II

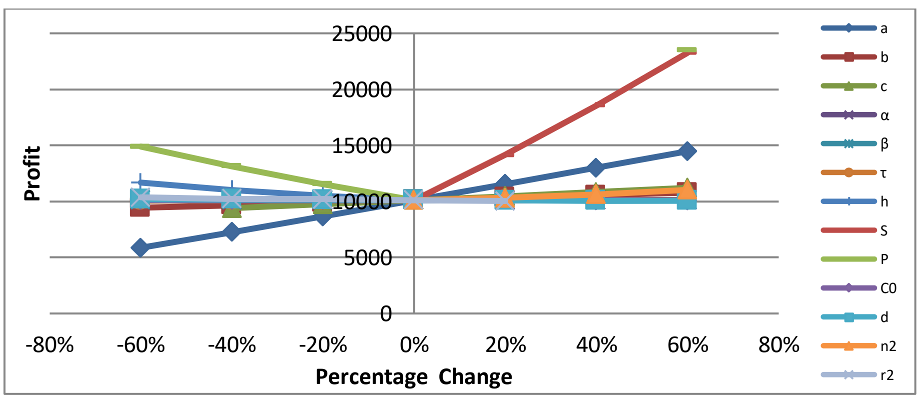

Figure.6:Behaviour of parameters in Case-III

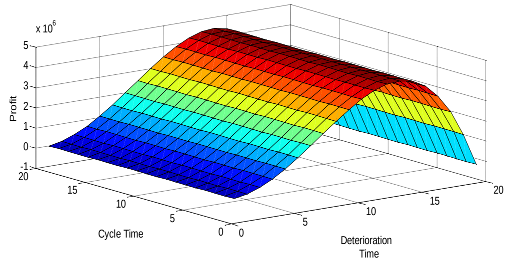

Figure.3: Concavity of the profit in Case-I for Signed Distance method using Triangular fuzzy number.

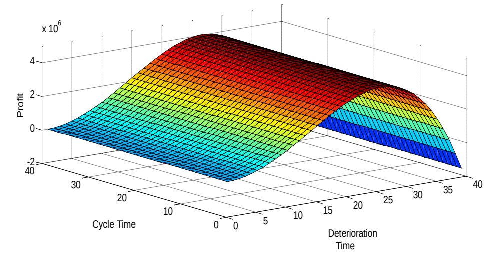

Figure.4: Concavity of the profit in Case-I for Graded Mean Integration method using Triangular fuzzy number

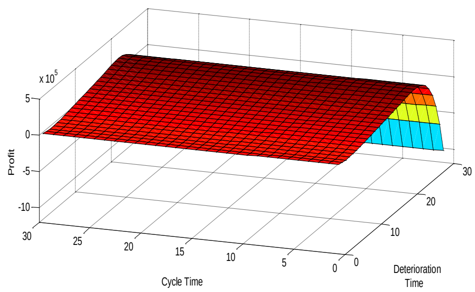

Figure.5: Concavity of the profit in Case-I for Signed Distance method using Trapezoidal fuzzy number.

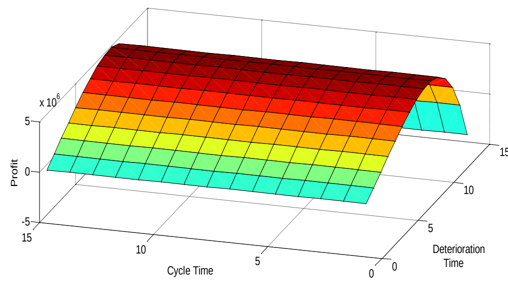

Figure.6: Concavity of the profit in Case-I for Graded Mean Integration method using Trapezoidal fuzzy number

The profit is attaining concavity in other cases also.

Comparative Analysis

Table-5: Comparative Analysis in Case-I

Method	Triangular							
	Signed Distance				Graded Mean Integration			
Fuzzy Parameters	δ	T_1	π	Q_1	δ	T_1	π	Q_1
$\tilde{d}, \tilde{S}, \tilde{P}, \tilde{C}_0$	20.7615	39.6786	8814.01	40122.9	20.7555	41.2046	8599.56	42516.2
$\tilde{S}, \tilde{P}, \tilde{C}_0$	22.2965	59.2738	8757.62	73621.1	20.0197	67.0238	8617.12	92153.3
$\tilde{S}, \tilde{P}$	22.2963	59.2714	8760.46	73616.4	20.0195	67.0222	8619.81	92149.8
$\tilde{C}_0$	13.0985	24.3115	1686.86	21990.3	13.2799	25.3162	1086.49	23284.8
Method	Trapezoidal							
	Signed Distance				Graded Mean Integration			
Fuzzy Parameters	δ	T_1	π	Q_1	δ	T_1	π	Q_1
$\tilde{d}, \tilde{S}, \tilde{P}, \tilde{C}_0$	19.9892	39.1121	**9755.72**	39718	20.7265	41.6795	**9598.31**	43287.8
$\tilde{S}, \tilde{P}, \tilde{C}_0$	20.4080	57.3927	8973.46	71381.3	19.3537	67.0338	9116.54	92751.5
$\tilde{S}, \tilde{P}$	20.4078	57.3905	8976.29	71377.1	19.3536	67.0324	9119.13	92748.4
$\tilde{C}_0$	13.09844	24.31116	1687.08	21989.9	13.2798	25.3158	1086.7	23284.3

Table-6: Comparative Analysis in Case-II

Method	Triangular							
	Signed Distance				Graded Mean Integration			
Fuzzy Parameters	γ	T_1	π	Q_1	γ	T_1	π	Q_1
$\tilde{d}, \tilde{S}, \tilde{P}, \tilde{C}_0$	17.1606	24.6299	607344	22634.2	--			
$\tilde{S}, \tilde{P}, \tilde{C}_0$	16.1674	23.2176	471322	19526.3	--			
$\tilde{S}, \tilde{P}$	16.1675	23.2178	471329	23441	--			
$\tilde{P}$	19.3705	27.7135	1003160	33326.6	17.9796	26.2249	516248	29651
$\tilde{d}, \tilde{P}, \tilde{h}, \tilde{C}_0$	18.6027	26.6094	**3072110**	30654.6	16.46607	24.0236	**2002900**	24957.9
$\tilde{d}, \tilde{h}, \tilde{C}_0$	18.2147	26.0589	2804480	29383.5	16.01239	23.3694	1781300	23677.9
$\tilde{h}, \tilde{C}_0$	18.0520	25.8291	2696800	28864.9	15.8638	23.1565	1712270	23272
$\tilde{C}_0$	17.9857	25.7479	727358	288680.8	15.6124	22.8034	284040	22609.6
Method	Trapezoidal							
	Signed Distance				Graded Mean Integration			

Fuzzy Parameters	γ	T_1	π	Q_1	γ	T_1	π	Q_1
$\tilde{d}, \tilde{S}, \tilde{P}, \tilde{C}_0$	12.9348	18.7540	194020	15995.5	--			
$\tilde{P}$	19.4409	27.8355	1022060	33627.2	18.1402	26.4566	536625	30182
$\tilde{d}, \tilde{P}, \tilde{h}, \tilde{C}_0$	18.5124	26.4856	**3830130**	30364.2	16.3885	23.9076	**2307640**	24727.9
$\tilde{d}, \tilde{h}, \tilde{C}_0$	18.1599	25.9857	3524180	29216.7	15.9661	23.2989	2067720	23542.6
$\tilde{h}, \tilde{C}_0$	18.0279	25.7989	3413590	28796.5	15.8370	23.1133	1997720	23191
$\tilde{C}_0$	17.8380	25.5647	704589	28269.5	15.61244	22.80345	284043	22609.7

Table-7: Comparative Analysis in Case-III

Method	Triangular							
	Signed Distance				Graded Mean Integration			
Fuzzy Parameters	γ	T_1	π	Q_1	γ	T_1	π	Q_1
$\tilde{d}, \tilde{S}, \tilde{P}, \tilde{C}_0$	7.17763	37.7991	10110.8	47801.1	--			
$\tilde{S}, \tilde{P}, \tilde{C}_0$	8.31008	47.7938	10285.5	66028.3	--			
$\tilde{S}, \tilde{P}$	9.70771	47.9557	10259.2	66509.5	--			
$\tilde{P}$	--				17.9796	26.2249	**516248**	29651
$\tilde{C}_0$	11.68516	22.93657	1181.4	22883.8	11.6852	22.9366	1181.4	22883.9
Method	Trapezoidal							
	Signed Distance				Graded Mean Integration			
Fuzzy Parameters	γ	T_1	π	Q_1	γ	T_1	π	Q_1
$\tilde{d}, \tilde{S}, \tilde{P}, \tilde{C}_0$	9.7768	37.6680	**11087.3**	47465.6	Infeasible solution			
$\tilde{C}_0$	11.6814	22.1191	1828.42	21638.5	11.6852	22.9363	1181.63	22883.4

4. RESULT AND DISCUSSION

i. The results obtained clearly exhibit that Case-II earns maximum profit. That is, when the deterioration period starts at the time period γ and pre deterioration discount is provided during the time period $\mu \leq t \leq \gamma$ and the post deterioration discount is provided during the time period $\gamma \leq t \leq T_1$, the situation becomes more beneficial for the decision maker.

ii. The results also depict that signed distance method attains highest profit as compared to crisp and graded mean integration method. More specifically the trapezoidal fuzzy number is found to be more economical in attaining our goal.

iii. Sensitivity analysis for case-I indicates that acceleration in the values of holding cost, disposal cost and ordering cost leads to decline in total profit. Total profit also declines with increase in the values of the shape parameter α and the real number n_1. Increase in the values of the location parameter, initial demand rate, rate of change in demand and the rate at which the demand itself increases lead to decrease in total profit.

iv. Sensitivity analysis for case-II suggests that escalation in the values of the cost parameters like holding cost, disposal cost, ordering cost and purchase cost reduces the profit. Total profit also reduces for enhancement in the values of the shape parameter, location parameter and pre deterioration discount. Increment in the values of the initial demand rate, rate of change in demand and the rate at which the demand itself increases, the scale parameter, selling price, effect of post deterioration discount and the real numbers n_1 and n_2 lead to augmentation in profit.

v. Sensitivity analysis for case-III specifies that acceleration in the values of the initial demand rate, rate of change in demand and the rate at which the demand itself increases, shape parameter, selling price, location parameter and the real number n_2 enhances the profit. Enhancement in the values of holding cost, disposal cost, ordering cost, purchase cost, scale parameter and post deterioration discount leads to reduction in profit.

vi. Careful observation on the sensitivity analysis reveals that the model is highly sensitive towards the change in initial demand, unit selling price and unit purchase cost of the product. It is moderately sensitive towards the change in the values of rate of change in demand, the rate at which the demand itself increases, shape parameter, location parameter, holding cost, ordering cost, disposal cost, the real numbers n_1 and n_2.

vii. It is clear from the comparative analysis of case-I (Table-5) that maximum profit can be attained by treating disposal cost, selling price, purchase cost and ordering cost as fuzzy.

viii. Comparative analysis in case-II (Table-6) suggests that when the disposal cost, purchase cost, ordering cost and holding cost are treated as fuzzy, the situation earns maximum profit.

4.1. Conclusion, applicability, managerial insights, suggestions & future research directions

Price discount is by far the most common strategy of sales promotion implemented by the firms. It is the way of convincing the customers and a drive to improve the footfalls. We believe that the outcomes of the paper will provide inspiring and instrumental insights about profit vis-à-vis pre deterioration discount and post deterioration discount. Moreover, uncertainty cannot be ignored while investigating any part of supply chain system. The current research enables the decision maker to cope with uncertainty through fuzziness and produce competitive bottom-line performances.

The model is very useful to the retail business. It can be used for domestic goods, electronic components and fashionable commodities which are likely to have the above characteristics. The real life implications of this inventory model are constrained because complete inspection of inventory and all its associated cost is very expensive in most of the situations. So the analogue of the model is discussed and the accuracy of the inventory system is monitored. However, we have given an analytic formulation of the problem on the framework described above and have presented an optimal solution procedure to find optimal replenishment policies.

For any business transaction, it is very important to choose the business related costs in more appropriate form. Further, the promotional effort through giving discount is found to be beneficial for the decision maker. Service quality is a major concern in this supply chain system. As major parameters are fuzzy, the decision maker needs to perform the various functions in terms of delivery, responsiveness and reliability taking caution of plausible flexibility. The evaluation of fuzzy system dynamics may provide the decision maker information regarding system behaviour uncertainties.

The results indicate that the effects of selling rate and discount period of items on the system behaviour are significant. Hence, the above situations should be dealt with caution in developing the inventory model. It is required to balance the selling cost vis-à-vis purchase cost for smooth operation of business.

This study might be extended in different directions. Equal lot sizing policy may not be fruitful in some situations particularly in the situation of discounted price and hence equal lot sizing policy may be adopted. For more acceptable results one can extend this work by considering constraints of service level and backordering. Extension of the current work with stochastic demand, internal and external inflation and net present value of the items might be an encouraging future research.

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REFERENCES

[1] CHATTERJI, D. and GOTHI, U.B. (2015): EOQ model for deteriorating items under two and three parameter weibull distribution and constant IHC with partially backlogged shortages. **International Journal of Science, Engineering and Technology Research (IJSETR)**, 4, 3581-3592.

[2] DEBATA S., ACHRYA M. AND SAMANTA G.C., (2015): An inventory model for perishable items with quadratic trapezoidal type demand under partial backlogging. **International Journal of Industrial Engineering Computations**, 6, 185-198.

[3] JAGGI, C.K., PAREEK, S., KHANNA, A. and NIDHI (2016): Optimal replenishment policy for fuzzy inventory model with deteriorating items and allowable shortages under inflationary conditions. **Yugoslav Journal of Operations Research**, 26, 507-526.

[4] JAGGI, C.K., PAREEK, S., SHARMA, A. and NIDHI (2012): Fuzzy inventory model for deteriorating items with time varying demand and shortages. **American Journal of Operational Research**, 2, 81-92.

[5] MAHATA, G.C. and MAHATA, P. (2011): Analysis of a fuzzy economic order quantity model for deteriorating items under retailer partial trade credit financing in a supply chain. **Mathematical and Computer Modelling**, 53, 1621-1636.

[6] MANNA S.K. and CHAUDHURI K.S. (2001): An economic order quantity model for deteriorating items with time-dependent deterioration rate, demand rate, unit production cost and shortages**. International Journal of Systems Science**, 32 , 1003-1009.

[7] MISHRA, P.J., SINGH, T. and PATTNAYAK, H. (2016): An optimal policy with quadratic demand, three parameter weibull distribution deterioration rate, shortages and salvage value. **American Journal of Computational Mathematics**, 6, 200-211.

[8] PAL, M. and CHANDRA S. (2014): A periodic review inventory model with stock dependent demand, permissible delay in payment and price discount on backorders. **Yugoslav Journal of Operations Research**, 24, 99-110.

[9] PANDA, S., SAHA, S., and BASU, M. (2008): An EOQ model for perishable products with discounted selling price and stock dependent demand. **CEJOR**, 17, 31-53.

[10] PANDA, S., SENAPATI, S. and BASU, M. (2009): A single cycle perishable inventory model with time dependent quadratic ramp-type demand and partial backlogging. **International Journal of Operational Research**, 5, 110-129.

[11] SARKAR, B., SANA, S.S., AND CHAUDHURI, K. (2013): An inventory model with finite replenishment rate, trade credit policy and price discount demand. **Journal of Industrial Engineering**, 2013, 1-18.

[12] SHAH, N.H. and RAYKUNDALIYA, N. (2010): Optimal ordering policy for deteriorating items under the delay in payments in demand declining market. **Revista Investigacion Operacional**, 31, 34-44.

[13] SINGH, N., VAISH B. and SINGH S.R. (2016): Manufacturer-supplier cooperative inventory model for deteriorating item with trapezoidal type demand. **Yugoslav Journal of Operations Research**, 26, 103-120.

[14] SUJATHA, J. and PARVATHI, P. (2015): Fuzzy Inventory Model for deteriorating items with weibull demand and time varying holding cost under trade credit. **International journal of innovative research in computer and communication engineering**, 7, 11110-11123.

[15] THANGAM, A. (2015): Retailer's inventory system in a two level trade credit financing with selling price discount and partial order cancelations. **Journal of Industrial engineering International**, 11, 159-170.

[16] TRIPATHY, P.K. AND PRADHAN S. (2011): An integrated partial backlogging inventory model having Weibull demand and variable deterioration rate with the effect of trade credit. **International Journal of Scientific and Engineering Research**. 2, 1-5.

[17] TRIPATHY, P.K. and PRADHAN, S. (2012): Inventory model for ramp type demand with trade credit under extra ordinary purchase. **IORS Journal of Mathematics**, 2, 01-09.

[18] TRIPATHY, P.K. and SUKLA, S. (2017): Decision support in a credit environment with fuzzy behaviour of costs. **International Journal of Statistical Sciences**, 15, 37-54.

[19] YADAV R.K. and VATS A.K. (2014): A deteriorating inventory model for quadratic demand and constant holding cost with partial backlogging and inflation. **IOSR Journal of Mathematics**, 10, 47-52.