A GENERALIZED CLASS OF ESTIMATORS FOR MEAN IN PRESENCE OF MEASUREMENT ERRORS

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ABSTRACT
This paper presents a generalized class of mean estimators under simple random sampling using auxiliary variable. The observations on both the study variable and the auxiliary variable are supposed to be recorded with measurement error. The mean square error of the proposed class of estimators is derived and studied under measurement errors. Several commonly known estimators are shown as special cases of the proposed class of estimators. Simulation and numerical studies are carried out to evaluate the performance of the estimators.

KEYWORDS: Measurement error, percentage relative efficiency, mean square error, simulation.

MSC: 62D05.

RESUMEN
Este artículo presenta una clase generalizada de estimadores de medias bajo muestreo aleatorio simple usando variable auxiliar. Se supone que las observaciones tanto en la variable de estudio como en la variable auxiliar deben registrarse con un error de medición. El error cuadrado medio de la clase propuesta de estimadores se deriva y estudia bajo errores de medición. Varios estimadores conocidos comúnmente se muestran como casos especiales de la clase propuesta de estimadores. Se realizan simulaciones y estudios numéricos para evaluar el desempeño de los estimadores.

PALABRAS CLAVE: Error de medición, porcentaje de eficiencia relativa, error cuadrático medio, simulación.

1. INTRODUCTION

In survey sampling literature, auxiliary variable is used at the estimation stage of a parameter when some characteristics of the study variable are closely related to auxiliary variables. Ratio, product and regression methods are pioneered methods that use auxiliary variable. Thompson (2012), Sharma et al. (2016), Bouza (2016), Singh et al. (2019), etc. have made their good contribution in the estimation of ratio and product methods of estimation of population mean. In the estimation of the population mean, by using the auxiliary variable, the ratio estimator is best suited when the study variable and auxiliary variable are strongly positively correlated. The product estimator provides a better estimate of mean when study variable and auxiliary variable are negatively correlated. Regression estimator is the most efficient estimator except when regression line passes through the origin. Srivastva (1971) proposed an efficient method for the estimation of population mean in simple random sampling.
The literature available on survey sampling, it is assumed that the data collected during the survey is the actual recorded values of the observation. But the observation under study may be recorded with some error called as measurement error. The measurement error is defined as the discrepancy between the actual value of the parameter and the observed values of the parameter. Cochran (1968) and Murthy (1967) studied the effect of measurement error in the context of survey sampling. The impacts of measurement error commingle with data on the statistical properties of estimators of parameters is discussed in the textbooks by Fuller (1987), Cheng and Vanness (1994) and Carroll et al. (2006). Measurement error can result in serious misleading inference; see Biemer et al. (1991). Estimation of parameters with the use of auxiliary variable in the literature is vast and substantial. Ratio, Product and regression estimators are widely used in the estimation of parameters. Shalabh (1997), Maneesha and Singh (2001), Allen and Singh (2003), Sahoo et al. (2006), Kumar et al. (2011) have studied the effect of measurement error on ratio and regression estimators in the estimation of population mean. Gregoire and Salas (2009) introduced ratio estimation with measurement error in the auxiliary variate, Shalabh and Tsai (2017) have proposed ratio and product method of estimation in the

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presence of correlated measurement error. Singh and Vishwakarma (2019), Vishwakarma et al. (2019) present the method for the estimation of mean in the presence of non-response and measurement error. Singh et al. (2019) studied simultaneously effect of measurement error.

In this paper, we propose a generalized class of estimators of the population mean of study variable under measurement error. Since the proposed class of estimators is in functional form, many preexisting estimators are a member of this class of estimators. The effect of measurement error on the mean square error of the proposed estimators, ratio estimator, product estimator and unbiased mean are shown. Many authors have studied the effects of measurement error on ratio, product and regression estimator. Our aim is to show the effect of measurement error simultaneously on the proposed, ratio, product and unbiased estimator at the different levels of measurement error for the different correlation coefficient.

Let N be the size of a finite population and n be size of sample drawn from it. In order to obtain the MSE under measurement error we consider that each data value is observed with error. It is considered that (x_i, y_i) be the observed value and (X_i, Y_i) be the true values for every i^{th} $(i = 1, 2, ..., n)$ unit. In such a way, these values are expressible in additive form as $y_i = Y_i + U_i$, and $x_i = X_i + V_i$. The errors (U, V) are normally distributed with mean zero and variance (σ_U^2, σ_V^2). Also, the error variables U and V are uncorrelated to each other and uncorrelated with X and Y. That implies $Cov(X, Y) \neq 0$ and $Cov(X, U) = Cov(X, V) = Cov(Y, V) = Cov(Y, U) = Cov(U, V) = 0$. Let μ_Y, μ_X be the population mean and σ_Y^2, σ_X^2 be the population variance of the study and the auxiliary variables respectively. Further, $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$, $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$ be the sample mean of the observed data and are unbiased estimators of the population mean μ_X and μ_Y respectively. We found that $s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ and $s_y^2 = (n-1)^{-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ are not an unbiased estimator of the population variance σ_X^2 and σ_Y^2 respectively under measurement error. The expected value of s_x^2 and s_y^2 in the presence of measurement error is given by $E(s_x^2) = \sigma_X^2 + \sigma_V^2$ and $E(s_y^2) = \sigma_Y^2 + \sigma_U^2$ where σ_U^2, σ_V^2 are variance of U and V respectively.

Let define $v = \dfrac{\bar{x}}{\mu_X}$, $\dfrac{\bar{y} - \mu_Y}{\mu_Y} = e_0$ and $(v-1) = \left(\dfrac{\bar{x}}{\mu_X} - 1\right) = e_1$. Also $E(e_0) = 0$, $E(e_1) = 0$. Ignoring finite population correction (fpc) term we have the following results. $E(e_0^2) = \dfrac{C_Y^2}{n\eta_Y}$, $E(e_1 e_0) = \dfrac{\rho C_X C_Y}{n}$ and $E(e_1^2) = \dfrac{C_X^2}{n\eta_X}$ where $C_Y = \sigma_Y / \mu_Y$ and $C_X = \sigma_X / \mu_X$ are coefficient of variation of study and auxiliary variable respectively. $\eta_Y = \dfrac{\sigma_Y^2}{\sigma_Y^2 + \sigma_U^2}$, $\eta_X = \dfrac{\sigma_X^2}{\sigma_X^2 + \sigma_V^2}$ are the reliability ratio of study and auxiliary variable respectively and always lies between 0 to 1. ρ is the correlation coefficient between X and Y and $R = \mu_Y / \mu_X$ is the ratio of the population mean of study variable to the auxiliary variable.

The expression for ratio estimator, product estimator and unbiased estimator in the presence of measurement error was studied by Shalabh (1997). The expression for ratio, product, and usual unbiased estimator in the presence of measurement error is given as

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}} \mu_X \tag{1}$$

The Bias and *MSE* of (1) are as follows

$$B(\hat{\bar{Y}}_R) = \frac{\mu_Y}{n}\left[C_X(C_X - \rho C_Y) + \frac{\sigma_V^2}{\mu_X^2}\right] \tag{2}$$

$$M(\hat{\bar{Y}}_R) = \mu_Y^2\left[\frac{C_Y^2}{n} + \frac{C_X^2}{n} - \frac{2\rho C_X C_Y}{n}\right] + \mu_Y^2\left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2} + \frac{C_X^2}{n}\frac{\sigma_V^2}{\sigma_X^2}\right] \tag{3}$$

where $\mu_Y^2\left[\frac{C_Y^2}{n}+\frac{C_X^2}{n}-\frac{2\rho C_X C_Y}{n}\right]$ is the *MSE* of $\hat{\hat{Y}}_R$ under no measurement error case and $\mu_Y^2\left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2}+\frac{C_X^2}{n}\frac{\sigma_V^2}{\sigma_X^2}\right]$ is the additional term due to measurement error in *MSE* of the estimator.

The product estimator

$$\hat{\hat{Y}}_P=\frac{\bar{y}}{\mu_X}\bar{x} \tag{4}$$

$$B\left(\hat{\hat{Y}}\right)=\mu_Y\rho C_X C_Y \tag{5}$$

$$M\left(\hat{\hat{Y}}_P\right)=\mu_Y^2\left[\frac{C_Y^2}{n}+\frac{C_X^2}{n}+\frac{2\rho C_X C_Y}{n}\right]+\mu_Y^2\left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2}+\frac{C_X^2}{n}\frac{\sigma_V^2}{\sigma_X^2}\right] \tag{6}$$

where $\mu_Y^2\left[\frac{C_Y^2}{n}+\frac{C_X^2}{n}+\frac{2\rho C_X C_Y}{n}\right]$ is the *MSE* of $\hat{\hat{Y}}_P$ under no measurement error and $\mu_Y^2\left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2}+\frac{C_X^2}{n}\frac{\sigma_V^2}{\sigma_X^2}\right]$ is the additional term due to measurement error in *MSE* of the estimator.

The usual unbiased estimator in the presence of measurement errors is

$$\hat{\hat{Y}}_M=\bar{y} \tag{7}$$

The Variance of $\bar{y}$ in the presence of measurement error

$$Var(\bar{y})=\left(\frac{\sigma_Y^2}{n}+\frac{\sigma_U^2}{n}\right) \tag{8}$$

where $\frac{\sigma_Y^2}{n}$ is the variance of unbiased estimator and $\frac{\sigma_U^2}{n}$ is the additional variance of measurement errors in the variance of the unbiased estimator.

2. THE PROPOSED ESTIMATOR

Following Srivastava (1971), we propose a generalized class of estimators for the estimation of population mean, when both study, as well as auxiliary variables, are commingled with measurement error as

$$\hat{\hat{Y}}_M=\bar{y}\,M(v) \tag{9}$$

where $v=\frac{\bar{x}}{\mu_X}$ is such that $M(v)$ is continuous and bounded in R also its first and second-order derivative exist and are continuous and bounded in R. Ratio, Product, unbiased estimator and some other pre-existing estimators can be the members of this family.

To obtain bias and *MSE* of we can write $\hat{\hat{Y}}_M$ in terms of error we have

$$M(v)=M(1)+(v-1)M_1(1)+\frac{1}{2}\left\{(v-1)^2M_2\left(v^*\right)\right\}, \tag{10}$$

$$M(v)=1+e_1M_1(1)+\frac{1}{2}\left\{e_1^2M_2\left(v^*\right)\right\}, \tag{11}$$

Using the value of (11) in (9), we can get

$$\hat{\hat{Y}}_M-\mu_Y=\mu_Y\left[e_0+e_1M_1(1)+e_0e_1M_1(1)+\frac{1}{2}\left\{e_1^2M_2\left(\mathrm{v}^*\right)\right\}\right]. \tag{12}$$

where $v*=1+\delta e_1,\ 0<\delta<1;$. M_1, denotes first order derivatives of $M(v)$ at the point $v=e$. M_2 denotes second-order derivatives of $M(v)$ at the point $v=v*$.

Expanding right-hand side of the equation and neglecting term higher than second degree of e and by taking expectation we have

$$B(\hat{\bar{Y}}_M) = \mu_Y \left[e_0 e_1 M_1(1) + \frac{1}{2}\left\{ e_1^2 M_2(v*) \right\} \right], \tag{13}$$

Substituting the value of $E(e_0^2)$, $E(e_0 e_1)$ and

$$B\left(\hat{\bar{Y}}_M\right) = \frac{\mu_Y}{2n}\left[2\rho C_X C_Y M_1(1) - \frac{C_X^2}{n\eta_X} M_2(v*) \right] \tag{14}$$

and for the mean square error of $\hat{\bar{Y}}_M$ we have

$$M(\hat{\bar{Y}}_M) = \mu_Y \left(e_0 + e_1 M_1(1) \right)^2, \tag{15}$$

$$M(\hat{\bar{Y}}_M) = \mu_Y^2 \left(\frac{C_Y^2}{n\eta_Y} + \frac{C_X^2}{n\eta_X} M_1(1)^2 + 2\rho C_X C_Y M_1(1) \right). \tag{16}$$

Differentiating partially (16) with respect to M_1 and equate to zero, we have $M_1 = -\rho\left(C_Y / C_X\right)\eta_X$,
Thus the resulting optimum mean square error of the proposed estimator is

$$M(\hat{\bar{Y}}_M)_{Opt} = \frac{\sigma_Y^2}{n}\left[1 - \rho^2 \eta_X + \frac{\sigma_U^2}{\sigma_Y^2} \right]. \tag{17}$$

3. THEORETICAL EFFICIENCY COMPARISON

Suppose that the observation for both the variable X and Y are recorded without error. In that case, mean square error of the proposed class of estimators can be obtained from (17) by substituting σ_U^2 and σ_V^2 to zero. The *MSE* of the estimator is similar as the obtained by Sahai (1979)

$$M_1(\hat{\bar{Y}}_M)_{\min} = \frac{\sigma_Y^2}{n}(1 - \rho^2) \tag{18}$$

From (17) and (18) we can infer $M(\hat{\bar{Y}}_M)$ has larger *MSE* then $M_1(\hat{\bar{Y}}_M)$ which follow that the estimator has larger *MSE* when the variables are recorded with measurement error.

Theorem 3.1: To the first degree of approximation,

$$\min . M(\hat{\bar{Y}}_M) \geq \frac{\sigma_Y^2}{n}\left[1 - \rho^2 \eta_X + \frac{\sigma_U^2}{\sigma_Y^2} \right]$$

with the equality holding if $M_1 = -\rho\left(C_Y / C_X\right)\eta_X$.

The ratio, product and mean estimator are the member of the proposed estimator $\hat{\bar{Y}}_M$ under measurement error. The ratio estimator

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}} \mu_X, \tag{19}$$

$$M(\hat{\bar{Y}}_R) = \mu_Y^2 \left[\frac{C_Y^2}{n} + \frac{C_X^2}{n} - \frac{2\rho C_X C_Y}{n} \right] + \mu_Y^2 \left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2} + \frac{C_X^2}{n}\frac{\sigma_V^2}{\sigma_X^2} \right] \tag{20}$$

Further, from (17) and (20), the *MSE* of the estimator $M(\hat{\bar{Y}}_M)_{\min} \leq M(\hat{\bar{Y}}_R)_{\min}$ provides

$$\frac{\sigma_Y^2}{n}\left[-\rho^2 \eta_X \right] < \left[\frac{R^2}{n} - \frac{2\rho R \sigma_X \sigma_Y}{n} + \frac{R^2}{n}\sigma_v^2 \right]. \tag{21}$$

$$\hat{\bar{Y}}_P = \frac{\bar{y}}{\mu_X}\bar{x}, \tag{22}$$

$$M(\hat{\bar{Y}}_P) = \mu_Y^2 \left[\frac{C_Y^2}{n} + \frac{C_X^2}{n} + \frac{2\rho C_X C_Y}{n} \right] + \mu_Y^2 \left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2} + \frac{C_X^2}{n}\frac{\sigma_V^2}{\sigma_X^2} \right] \tag{23}$$

From (17) and (23), the *MSE* of the estimator $M(\hat{\bar{Y}}_M)_{\min} \leq M(\hat{\bar{Y}}_P)_{\min}$ provide

$$\frac{\sigma_Y^2}{n}\left[-\rho^2\eta_X\right]<\left[\frac{R^2}{n}+\frac{2\rho R\sigma_X\sigma_Y}{n}+\frac{R^2}{n}\sigma_v^2\right] \tag{24}$$

$$\hat{\hat{\bar{Y}}}_M=\bar{y}\,, \tag{25}$$

$$M(\hat{\hat{\bar{Y}}}_M)=\frac{\sigma_Y^2}{n}\left(1+\frac{\sigma_U^2}{\sigma_Y^2}\right)=V(\bar{y})\,. \tag{26}$$

From (17) and (26) for the efficiency comparison $M(\hat{\hat{\bar{Y}}}_M)_{\min}\le V\left(\bar{y}\right)$, which provide

$$\frac{\sigma_Y^2}{n}\left[\rho^2\eta_X\right]>0\,. \tag{27}$$

4. EMPIRICAL STUDY

To analyze the merits of the suggested estimator, we have used the data from Gujrati and Sangeetha (2007). The descriptions of the data and values of the parameters are given below:

Y_i = True consumption expenditure, X_i = True income,

y_i = Measured consumption expenditure, x_i = Measured income.

n	N	μ_X	μ_Y	σ_X^2	σ_Y^2	ρ	σ_V^2	σ_U^2
4	10	170	127	3300	1278	0.964	36	36

The percentage relative efficiency (*PRE*) of the proposed and existing estimators with respect to the usual unbiased estimator $\bar{y}$ are computed using the formula:

$$PRE\,(\phi,\,\bar{y}\,)=\frac{V\left(\bar{y}\right)}{MSE\left(\phi\right)}\times 100 \tag{28}$$

where $\phi=\hat{\hat{\bar{Y}}}_M\,,\,\hat{\hat{\bar{Y}}}_P,\,\hat{\hat{\bar{Y}}}_R,\,\bar{y}$.

The *MSE* with and without measurement error and percent relative efficiency of the estimators is presented in Tables 1.

Table 1: *MSE* and *PRE* of the estimators with and without measurement error for Population 1

Estimators	MSE without Measurement error	PRE	MSE with Measurement error	PRE	Contribution of Measurement error
$\hat{\hat{\bar{Y}}}_M$	22.59	1414.34	34.79	944.24	12.20
$\hat{\hat{\bar{Y}}}_R$	40.45	789.86	54.48	603.00	14.03
$\hat{\hat{\bar{Y}}}_P$	1519.41	21.02	1533.61	21.42	14.02
$\bar{y}$	319.50	100.00	328.50	100.00	9.00

5. SIMULATION STUDY

For the validation of results, a simulation study has been carried out using *R* studio. We have adopted the following steps in proposed algorithm is summarized The steps of the algorithm are given below

Step-1: The generated population of size N =1000 of variables are assumed as Y and X . Such set of four normal variables have been generated using the below mean vector $\left(\mu_Y \quad \mu_X \quad 0 \quad 0\right)$ and covariance matrix

$$\begin{pmatrix} \sigma_Y^2 & \rho\sigma_X\sigma_Y & 0 & 0 \\ \rho\sigma_X\sigma_Y & \sigma_X^2 & 0 & 0 \\ 0 & 0 & \sigma_U^2 & 0 \\ 0 & 0 & 0 & \sigma_V^2 \end{pmatrix}$$

The details of the used parameters are given below:

μ_X = 30, μ_Y = 20, σ_X^2 =(10) , σ_Y^2 = (10), ρ = (-0.9, -0.5, -0.1, 0.9, 0.5, 0.1), σ_U^2 = (0, 2), σ_V^2 = (0, 2).

Step-2: The two random sub-sample of size n =30 and n =100 have been drawn from the population. Then we computed sample mean, sample variance and coefficient of variation for both sub-samples.
Step-3: The *MSE* for the proposed class of estimators, ratio estimator, product estimator and unbiased estimator have been obtained by using derived formula.
Step-4: We calculated *PRE* for the proposed class of estimators, ratio estimator, product estimator and unbiased estimator by using derived formula.
Step-5: The step 3 and step-4 have been repeated for 5000 times using loop.
Step-6: We computed the grand mean for *MSE* and *PRE* for proposed class of estimators, ratio estimator, product estimator and unbiased estimator.

The results of simulation study are, Table 2 gives the *MSE* and *PRE* of the proposed estimator and other estimators at different levels of correlation coefficients $\left(\rho = -0.9, -0.5, -0.1, 0.1, 0.5, 0.9\right)$ between study and auxiliary variable. The results of simulation study is, Table 2 exhibits that proposed estimator has higher efficiency than other estimators for different levels of correlation coefficient under no measurement error. Table 3 shows the contamination of measurement error in *MSE* and *PRE* in different estimator for (σ_U^2 =2, σ_V^2 =2). Table 3, reveals that *MSE* of proposed estimator and other estimator is higher in the presence of measurement error. From table 2 and table 3, it is also revealed that *MSE* is decreasing and more efficient with increase in sample size.

Table 2: *MSE* and *PRE* of various estimators of μ_Y for ρ = {-0.9, -0.5, -0.1, 0.9, 0.5, 0.1) when $(\sigma_Y^2, \sigma_X^2 = 10, 10)$ **,** $(\sigma_V^2, \sigma_U^2 = 0, 0)$ **and** $(n_1, n_2 = 100, 30)$

σ_U^2	σ_V^2	ρ	Estimator	$n_1 = 100$		$n_2 = 30$	
				MSE	PRE	*MSE*	PRE
0	0	-0.9	$\hat{\bar{Y}}_M$	0.019027	526.32	0.063236	526.32
			$\hat{\bar{Y}}_R$	0.291714	34.33	0.968307	34.36
			$\hat{\bar{Y}}_P$	0.021412	469.73	0.072397	466.17
			$\bar{y}$	0.100144	100.00	0.332823	100.00
0	0	- 0.5	$\hat{\bar{Y}}_M$	0.074947	133.33	0.249849	133.33
			$\hat{\bar{Y}}_R$	0.230857	43.24	0.767704	43.27
			$\hat{\bar{Y}}_P$	0.08149	122.41	0.274166	120.80
			$\bar{y}$	0.09993	100.00	0.333131	100.00
0	0	-0.1	$\hat{\bar{Y}}_M$	0.099249	101.01	0.331217	101.01
			$\hat{\bar{Y}}_R$	0.171552	58.33	0.572124	58.16
			$\hat{\bar{Y}}_P$	0.141639	70.64	0.473437	70.23
			$\bar{y}$	0.100251	100.00	0.334563	100.00
0	0	0 .9	$\hat{\bar{Y}}_M$	0.01896	526.32	0.063513	526.32
			$\hat{\bar{Y}}_R$	0.021301	470.36	0.072512	467.04
			$\hat{\bar{Y}}_P$	0.290883	34.30	0.972574	34.34
			$\bar{y}$	0.099788	100.00	0.334279	100.00

0	0	0.5	$\hat{\hat{Y}}_M$	0.074975	133.33	0.25035	133.33
			$\hat{\hat{Y}}_R$	0.081465	122.49	0.274055	121.03
			$\hat{\hat{Y}}_P$	0.230713	43.28	0.766892	43.34
			$\bar{y}$	0.099966	100.00	0.3338	100.00
0	0	0.1	$\hat{\hat{Y}}_M$	0.098688	101.01	0.327991	101.01
			$\hat{\hat{Y}}_R$	0.141125	70.49	0.471027	69.87
			$\hat{\hat{Y}}_P$	0.170957	58.21	0.569392	57.84
			$\bar{y}$	0.099685	100.00	0.331304	100.00

Table 3: *MSE* and *PRE* of various estimators of μ_Y for ρ = {-0.9, -0.5, -0.1, 0.9, 0.5, 0.1) when $(\sigma_Y^2, \sigma_X^2 = 10,10)$, $(\sigma_V^2, \sigma_U^2 = 2,2)$ and $(n_1, n_2 = 100,30)$

σ_U^2	σ_V^2	ρ	Estimator	$n_1 = 100$		$n_2 = 30$	
				MSE	PRE	*MSE*	PRE
2	2	-0.9	$\hat{\hat{Y}}_M$	0.026396	454.26	0.090628	444.07
			$\hat{\hat{Y}}_R$	0.297862	40.38	1.001888	40.49
			$\hat{\hat{Y}}_P$	0.027878	432.12	0.096471	423.89
			$\bar{y}$	0.12002	100.00	0.402594	100.00
2	2	- 0.5	$\hat{\hat{Y}}_M$	0.080262	149.80	0.266311	149.00
			$\hat{\hat{Y}}_R$	0.237511	50.61	0.787067	50.41
			$\hat{\hat{Y}}_P$	0.088074	136.31	0.29598	133.58
			$\bar{y}$	0.120123	100.00	0.395844	100.00
2	2	-0.1	$\hat{\hat{Y}}_M$	0.10329	116.40	0.344339	116.31
			$\hat{\hat{Y}}_R$	0.177814	67.49	0.593743	67.12
			$\hat{\hat{Y}}_P$	0.147947	81.09	0.495387	80.38
			$\bar{y}$	0.120057	100.00	0.398801	100.00
2	2	0 .9	$\hat{\hat{Y}}_M$	0.02646	454.28	0.090658	442.99
			$\hat{\hat{Y}}_R$	0.027919	432.53	0.096118	424.25
			$\hat{\hat{Y}}_P$	0.298214	40.42	0.998028	40.54
			$\bar{y}$	0.120303	100.00	0.401984	100.00
2	2	0.5	$\hat{\hat{Y}}_M$	0.080197	149.84	0.269109	148.85
			$\hat{\hat{Y}}_R$	0.088057	136.25	0.297602	134.03
			$\hat{\hat{Y}}_P$	0.237552	50.57	0.788558	50.77
			$\bar{y}$	0.120055	100.00	0.399608	100.00
2	2	0.1	$\hat{\hat{Y}}_M$	0.103039	116.45	0.342972	116.35

	$\hat{\bar{Y}}_R$	0.14775	81.07	0.492586	80.51
	$\hat{\bar{Y}}_P$	0.177596	67.46	0.590319	67.24
	$\bar{y}$	0.119825	100.00	0.397366	100.00

6. CONCLUSIONS

By the study obtained from numerical and simulation, we can infer that *MSE* is minimum for the proposed estimator and is more efficient than ratio and product estimator for any correlation coefficient. The *MSE* has been always higher when both the variables are recorded with measurement error. Measurement error highly affects the *MSE* as well as *PRE* of the estimator when its value is high, but the properties of estimators do not change in the presence of measurement error. The effect of measurement error cannot be obtained by the *PRE* of the estimator as PRE is the ratio of two parameters, variance of $\bar{y}$ and *MSE* of estimators. From the Tables, we can also conclude that, with the increase in the size of the sample, the *MSE* will be less and more efficient. The proposed class of estimators more efficient for all possible correlation coefficient than other estimators therefore proposed estimators can be used to estimate the population mean under measurement error aspects. Since the proposed estimator encompasses four well-known estimator viz. ratio, product, mean and regression estimator, thus the proposed class of estimators can be used to obtain the effect of measurement error on the proposed, ratio, product and mean estimators simultaneously.

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REFERENCES

[1] ALLEN, J., SINGH, H.P. and SMARANDACHE, F. (2003): A family of estimators of population mean using multi auxiliary information in presence of measurement error. **International Journal of Social Economics**, 30, 837-848.

[2] BIEMER, P. and STOKES, S.L. (1991): Approaches to the modeling of measurement error, **Measurement error in Surveys**. Wiley, 485-516.

[3] BOUZA, C.N. (2016): Una clase de estimadores basados en una razón: muestreo simple aleatorio y muestreo por conjuntos ordenados. **Revista Investigacion Operacional**, 37, 58-69.

[4] CARROLL, R.J., RUPPERT, D., STEFANSKI, L.A. and CRAINICEANU, C.M. (2006): **Measurement error in nonlinear models: a modern perspective.** CRC Press, N. York.

[5] CHENG, C.L. and VANNESS, J.W. (1994): On estimating linear relationships when both variables are subject to error. **Journal of the Royal Statistical Society, Series B (Methodological),** 56, 167-183.

[6] COCHRAN, W.G. (1968): Errors of measurement in statistics. **Technometrics**, 10(4), 637-666.

[7] FULLER, W.A. (1987): **Measurement error models**. John Wiley & Sons, Inc., N. York.

[8] GREGOIRE, T.G. and SALAS, C. (2009): Ratio estimation with measurement error in the auxiliary variate. **Biometrics**, 65, 590-598.

[9] GUJARATI, D.N. and SANGEETHA (2007): **Basic econometrics**. Tata McGraw Hill Publishing Company Limited, New Delhi.

[10] KUMAR, M., SINGH, R., SINGH, A.K. and SMARANDACHE, F. (2011): Some ratio type estimators under measurement error. **World Applied Sciences Journal**, 14, 272-276.

[11] MANEESHA and SINGH, R.K. (2001): An estimation of population mean in the presence of measurement error. **Journal of Indian Society of Agricultural Statistics,** 54, 13-18.

[12] MURTHY, M.N. (1967): **Sampling theory and methods**. Statistical Publishing Society, Calcutta.

[13] SAHAI, A. (1979): An efficient variant of the product and ratio estimators. **Statistica Neerlandica,** 33, 27-35.

[14] SAHOO, L.N., SAHOO, R.K. and SENAPATI, S.C. (2006): An empirical study on the accuracy of ratio and regression estimators in the presence of measurement error. **Monte Carlo Methods and Applications**, 12, 495-501.

[15] SHALABH (1997): Ratio method of estimation in the presence of measurement error. **Journal of the Indian Society of Agricultural Statistics**, 50, 150-155.

[16] SHALABH and TSAI, J.R. (2017): Ratio and product methods of estimation of population mean in the presence of correlated measurement error. **Communications in Statistics-Simulation and Computation**, 46, 5566-5593.
[17] SHARMA, P., BOUZA, C.N., VERMA, H. and SINGH R (2016): A generalized class of estimators for the finite population mean when the study variable is qualitative in nature. **Revista Investigacion Operacional,** 37, 163-172.
[18] SINGH N., VISHWAKARMA, G.K. and KIM, J.M. (2019): Computing the effect of measurement error on efficient variant of ratio and product estimator using auxiliary variable. **Communications in Statistics-Simulation and Computation,** DOI:10.1080/03610918.2019.1656742.
[19] SINGH, N. and VISHWAKARMA, G.K. (2019): A generalised class of estimator of population mean with the combined effect of measurement errors and non-response in sample survey. **Revista Investigacion Operacional,** 40, 275-285.
[20] SINGH, P., BOUZA, C.N. and SINGH, R. (2019): Generalized exponential estimator for estimating the population mean using auxiliary variables. **Journal of Scientific Research,** 63, 273-280
[21] SRIVASTAVA, S.K. (1971): A generalized estimator for the mean of a finite population using multi-auxiliary information. **Journal of the American Statistical Association**, 66, 404-407.
[22] THOMPSON, S.K. (2012): **Sampling Third edition**. John Wiley & Sons, Inc., Chichester
[23] VISHWAKARMA, G.K., SINGH, N., and KUMAR, A. (2019): Computing the combined effect of measurement errors and non-response using factor chain-type class of estimator. **The Philippine Statistician**, 68, 27-39.