IMPROVED SHEWHART CONTROL CHART USING MINIMAX RANKED SET SAMPLING

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ABSTRACT
The sampling technique plays a vital role in the performance of control charts. This study leads to the development of a new Shewhart-type mean control chart to monitor the process by employing cost-effective MiniMax ranked set sampling (MMRSS) technique. This competes with known mean control charts based on simple random sampling (SRS) and some of the existing ranked set sampling techniques. The average run length (ARL) is utilized as performance measures to assess the efficiency of MMRSS mean control chart and other considered SRS, ranked set sampling (RSS) and extreme ranked set sampling (ERSS) charts by using Monte Carlo simulations. The simulation results of the MMRSS control chart are in some cases better than the results obtained using SRS, RSS and ERSS control charts. The procedure is demonstrated with a case study using a real dataset that supported the findings of the simulation study.

KEYWORDS: Average run length; Extreme ranked set sampling; Median ranked set sampling; Minimax ranked set sampling; Monte Carlo simulation Shewhart control chart.

MSC: 62D05

RESUMEN
La técnica del muestreo juega un rol vital en comportamiento de la cartas de control. Este estudio lleva a desarrollar un nueva carta de control de media del tipo Shewhart para monitorear el proceso empleando la técnica efectiva respecto al costo la tecnica MiniMax ranked set sampling (MMRSS). Esta compite con la conocida carta de control basada en muestreo simple aleatorio (SRS) y algunas otras existentes técnicas de ranked set sampling. El average del largo de la corrida (ARL) es utilizada como medidas de desempeño para medir la eficiencia de la carta de control MMRSS y otras cartas consideradas SRS, ranked set sampling (RSS) y extremal ranked set sampling (ERSS) usando simulación de Monte Carlo. Los resultados de la simulación de la carta de control MMRSS son en algunos casos mejores que los obtenidos por SRS, RSS y ERSS. SE demuestra que el procedimiento con un caso de estudio usando datos reales soporta los hallazgos del estudio de simulación .

KEYWORDS: Average run length; Extreme ranked set sampling; Median ranked set sampling; Minimax ranked set sampling; Monte Carlo simulation Shewhart control chart.

1. INTRODUCTION

There are many various ways to enhance the quality, one of which is quality control charts. Quality control charts are important statistical tools for quality monitoring with their long history of successful implementations and many practical applications such as in scientific research and industry. Quality improvement techniques have been used in the last decade to fulfill the needs of consumers.
Control charts are very useful tools to detect the undesirable shift in the process, and to determine when to take corrective actions. Indeed, one of their application may serve as an “early warning” index regarding potential “out-of-control” processes. *The basic idea of control charts* consists of upper and lower control limits and the natural variations are expected to lie within these limits. *Many control charts have been* proposed, *among which the* most widely used control chart *is the Shewhart* X-*bar* which was introduced by Shewhart (1924).
ollowing the pioneering work of Shewhart, several improved quality control charts have been suggested with new techniques being proposed. Most of the techniques reported in the literature are based on simple random sampling (SRS), which to a certain extent is considerably less effective in estimating the population mean compared with a new sampling technique, such as ranked set sampling (RSS) and its modifications.

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In recent decades, ranked set sampling (RSS) has attracted a considerable amount of interest and research. The concept of RSS was first proposed by McIntyre (1952) in the context of estimating mean pasture and forage yields. McIntyre noted that RSS is much superior to the simple random sampling (SRS) when the observations are easier ranked than measured. Therefore, RSS is used as an alternate data collection technique to SRS in the situations where measuring the sample observations is not easy, costly or time-consuming but ranking them is much easier and relatively reliable. Later on, Takahasi and Wakimoto (1968) development the theory and properties of RSS.

Shewhart X-*bar* control chart has been considered to monitor the mean of a quality characteristic for a given process. Control charts have been extensively discussed and extended in numerous textbooks and papers. Interested *readers are referred to* (e.g. Brown, 1991; Claro et al., 2008; Haridy et al., 2016; Yaqub et al., 2016; Huang et al., 2017; Al-Nasser and Gogah, 2017; Bouza and Al-Omari, 2018; Gogah and Al-Nasser, 2018; Al-Nasser and Aslam, 2019; Al-Nasser et al, 2020; and Montgomery, 2020).

McIntyre's concept of RSS depends on drawing a SRS of size m^2 from the population of interest, and those are partitioned randomly into m sets each of equal size. Then, we rank each unit of size m according to a character of interest without measuring them. After that, we measure the lowest unit from the first set, the second ranked unit from the second set and so on, until we reach the maximum unit from the last set. In other words, the units in the first set $X_{11}, X_{12}, \dots, X_{1m}$ are ranked by judgment and the smallest is measured. Then the units in the second set $X_{21}, X_{22}, \dots, X_{2m}$ are ranked by judgment and the second smallest is measured. The procedure is continued until in the last set $X_{m1}, X_{m2}, \dots, X_{mm}$, the maximum unit is measured. This entire procedure completes a one sampling cycle and the set $X_{(1)1}, X_{(1)2}, \dots, X_{(1)m}$ is called a ranked set sample in the first cycle and yields □ units out of the m^2 selected ones. Since m is typically taken to be small in order to facilitate the ranking procedure, there may not be enough measurements for reasonable analysis and the cycle can be repeated r times to obtain a RSS of size rm measurements.

Nowadays, many modifications and improvements of RSS have been suggested: Samawi et al. (1996) investigated the extreme ranked set sampling (ERSS), Muttlak (1997) suggested the median ranked set sampling (MRSS), Jemain and Al-Omari (2006) proposed double quartile ranked set samples, Al-Nasser (2007) proposed a generalized robust sampling technique called L ranked set sampling (LRSS). Moreover, Al-Nasser and Mustafa (2009) used robust extreme ranked set sampling (RERSS) as an alternative sampling technique. On the other hand, Mahdizadeh and Zamanzade (2017) and Al-Omari and Haq (2019) estimated the parameters of some distributions using RSS, For more details, an extensive review on the RSS design, its extensions, theory and applications are presented by Al-Omari and Bouza (2014) and the references therein.

The use of RSS to develop quality control charts for monitoring the process mean was first suggested by Salazar and Sinha (1997). They found that control charts based on RSS perform better than the classical one based one SRS. Later on, Muttlak and Al-Sabah (2003) and Al-Nasser and Al-Rawwash (2007) employed several RSS techniques to improve the achievement of Shewhart-type mean control charts for detecting the large shifts in the process mean.

This article deals with the idea of developing an efficient Shewhart control chart based on mini-max ranked set sampling (MMRSS) and investigate the average run length (ARL) performance for this chart compared with the control chart for the mean based on SRS, RSS and ERSS with the same sample sizes. The newly developed control charts are considered as alternatives, and more efficient than the usual control charts based on the SRS technique.

The remainder of the article is outlined as follows: in Section 2, we describe the MMRSS sampling technique. The proposed control chart using MMRSS is explained in Section 3. The run length evaluation and performance comparisons are given in Sections 4. An illustrative application is presented in Section 5 to support the proposed control chart by analyzing a real dataset. Finally, Section 7 ends the article with some concluding remarks.

2. MINI-MAX RANKED SET SAMPLING

MMRSS is an efficient and cost-effective technique for estimating the population parameters. In this technique, only the extreme unit is measured from each sample. So, it is cost-effective in such situations when the sampling is very expensive.

In order to apply the MMRSS design (Al-Nasser and Al-Omari, 2018), we need to draw m simple random samples each of size $1, 2, 3, \dots, m$, sampling units respectively. Then, we rank each sampling units by judgment without measuring them. After that, if the sample size is odd, we measure the lowest unit otherwise we measure the largest unit. The MMRSS design can be executed as by applying the following steps:

Step I: Draw m simple random samples from the target population each of size $i = 1, 2, 3, \dots, m$, respectively.

$$\begin{array}{llllll}
SRS\ \mathbf{1} & \Rightarrow & x_1 & & & \\
SRS\ \mathbf{2} & \Rightarrow & x_1 & x_2 & & \\
SRS\ \mathbf{3} & \Rightarrow & x_1 & x_2 & x_3 & \\
\vdots & & \vdots & \vdots & \vdots & \vdots \\
SRS\ \mathbf{m} & \Rightarrow & x_1 & x_2 & \cdots & x_m
\end{array}$$

Step II: Rank the units within each sample increasingly by using visual inspection, expert knowledge or any other costless way.

$$\begin{array}{llllll}
SRS\ \mathbf{1} & \Rightarrow & x_{(1:1)} & & & \\
SRS\ \mathbf{2} & \Rightarrow & x_{(1:2)} & x_{(2:2)} & & \\
SRS\ \mathbf{3} & \Rightarrow & x_{(1:3)} & x_{(2:3)} & x_{(3:3)} & \\
\vdots & & \vdots & \vdots & \vdots & \vdots \\
SRS\ \mathbf{m} & \Rightarrow & x_{(1:m)} & x_{(2:m)} & \cdots & x_{(m:m)}
\end{array}$$

Step III: From the first ranked sample of size $i = 1$, measure the unit with rank 1; $x_{(1:1)}$; the minimum.
Step IV: From the second ranked samples of size $i = 2$, measure the unit with rank 2; $x_{(2:2)}$; the maximum.
Step V: From the third ranked samples of size $i = 3$, measure the unit with rank 1; $x_{(1:3)}$; the minimum.
Step VI: The cycle is completed by continuing the above procedure till in the last sample the minimum is selected if m is odd, otherwise the maximum is selected.

$$\begin{array}{llllllll}
 & & \textit{Ranked samples} & & & & & \textit{MMRSS} \\
SRS\ \mathbf{1} & \Rightarrow & x_{(1:1)} & & & & \Rightarrow & x_{[1:1]} \\
SRS\ \mathbf{2} & \Rightarrow & x_{(1:2)} & x_{(2:2)} & & & \Rightarrow & x_{[2:2]} \\
SRS\ \mathbf{3} & \Rightarrow & x_{(1:3)} & x_{(2:3)} & x_{(3:3)} & & \Rightarrow & x_{[1:3]} \\
\vdots & & \vdots & \vdots & \vdots & \vdots & & \\
SRS\ \mathbf{m} & \Rightarrow & x_{(1:m)} & x_{(2:m)} & \cdots & x_{(m:m)} & \Rightarrow & \begin{cases} x_{[1:m];if\ m\ is\ odd} \\ x_{[m:m];if\ m\ is\ even} \end{cases}
\end{array}$$

Step VII: The above Steps I through VI can be repeated r times (cycles) if needed to obtain a MiniMax RSS of size $n = rm$.
Consequently, the form of the MMRSS samples will be in the following form:

$$\begin{cases} \left\{x_{[1:2i-1]k};\ x_{[2j:2j]k}; i = 1,2,\dots,\dfrac{m+1}{2}; j = 1,2,\dots,\dfrac{m-1}{2}; k = 1,2,\dots,r\right\}; & \text{if } m \text{ is odd} \\ \left\{x_{[1:2i-1]k};\ x_{[2i:2i]k}; i = 1,2,\dots,\dfrac{m}{2}; k = 1,2,\dots,r\right\} & ;\text{if } m \text{ is even} \end{cases}$$

It follows that the sample mean of MMRSS can be identified as:

$$\bar{X}_{MMRSS} = \begin{cases} \dfrac{1}{mr}\left\{\sum_{k=1}^{r}\left(\sum_{i=1}^{(m+1)/2} X_{[1:2i-1]k} + \sum_{i=1}^{(m-1)/2} X_{[2i:2i]k}\right)\right\}; & m \text{ is odd} \\ \dfrac{1}{mr}\left\{\sum_{k=1}^{r}\left(\sum_{i=1}^{m/2} X_{[1:2i-1]k} + \sum_{i=1}^{m/2} X_{[2i:2i]k}\right)\right\}; & m \text{ is even} \end{cases}$$

Without loss of generality, let r =1; then expected value of the sample mean from MMRSS is given by:

$$E(\bar{X}_{MMRSS}) = \begin{cases} \dfrac{1}{m}\left\{\sum_{i=1}^{(m+1)/2} \int x_{[1:2i-1]} dF(x_{[1:2i-1]}) + \sum_{i=1}^{(m-1)/2} \int x_{[2i:2i]} dF(x_{[2i:2i]})\right\}; & m \text{ is odd} \\ \dfrac{1}{m}\left\{\sum_{i=1}^{m/2} \int x_{[1:2i]} dF(x_{[1:2i]}) + \sum_{i=1}^{m/2} \int x_{[2i:2i]} dF(x_{[2i:2i]})\right\}; & m \text{ is even} \end{cases}$$

where $dF(x_{[i:m]})$ is the probability density functions of $x_{[i:m]}$ and is defined as:

$$dF(x_{[i:m]} = f(x_{[i:m]}) = \frac{m!}{(i-1)!\,(m-i)!}\, x^{i-1}(1-x)^{m-i};$$

Moreover, the associated variance of this estimator is σ^2_{MMRSS} and is given in the following equation:

$$\sigma^2_{MMRSS} = \begin{cases} \dfrac{1}{m^2}\left\{ \displaystyle\sum_{i=1}^{(m+1)/2} \sigma^2{}_{[1:2i-1]} + \sum_{i=1}^{(m-1)/2} \sigma^2{}_{[2i:2i]} \right\}; & m \text{ is odd} \\ \dfrac{1}{m^2}\left\{ \displaystyle\sum_{i=1}^{m/2} \sigma^2{}_{[1:2i-1]} + \sum_{i=1}^{m/2} \sigma^2{}_{[2i:2i]} \right\}; & m \text{ is even} \end{cases}$$

3. ESTIMATING *X*-BAR CHART USING MMRSS

As mentioned earlier, the Shewhart control charts are determined via the lower and upper control limits as well as the central limit term. The estimates of the three part are necessary when the population mean and variance are unknown. This leads us to present new set of estimates of (μ,σ^2) using MMRSS so that we may construct the quality control charts as:

$$LCL = \mu - 3\sigma_{\bar{X}_{MMRSS}}$$
$$CL = \mu$$
$$UCL = \mu + 3\sigma_{\bar{X}_{MMRSS}}$$

where $\sigma_{\bar{X}_{MMRSS}}$ is the standard deviation obtained based on MMRSS technique. The UCL, C and LCL represent the upper, central and lower control limits of Mean-MMRSS chart respectively.

It is remarkable to mention that one or both population parameters are expected to be unknown in practical situations, hence the control limits can be estimated using the sample mean and the sample standard deviation based on the MMRSS scheme to be:

$$LCL = \bar{X}_{MMRSS} - 3\,\hat{\sigma}_{\bar{X}_{MMRSS}}$$
$$CL = \bar{X}_{MMRSS}$$
$$UCL = \bar{X}_{MMRSS} + 3\,\hat{\sigma}_{\bar{X}_{MMRSS}}$$

4. COMPARISONS BETWEEN MMRSS AND SEVERAL RSS TECHNIQUES

In this section, a comprehensive simulation study is conducted to compare the performance of MMRSS control chart with the SRS, RSS, and ERSS control charts based on the average run length (ARL) for different values of shift (δ). In fact, if we define W to be the number of observations plotted on the chart until the first observation gets out-of-control limits, then *W* has a geometric distribution and the mean of *W* is called the ARL.

Now, to define the ARL, we use type I error (α) when the process is under control such that:

$$ARL_0 = \frac{1}{\alpha}$$

On the other hand, if the process gets out of control, then the ARL is written in terms of type II error (β) as following:

$$ARL_1 = \frac{1}{1-\beta}$$

Based on the ARL criterion, the process remains in control with mean μ_0 and standard deviation σ_0 and sometimes it may get out of control in terms of a mean shift of the amount $\delta \frac{\sigma_0}{\sqrt{m}}$, where δ is nonnegative and selected to dominate the shift in the mean μ.

To carry on our task, we use a simulation study to illustrate the quality control mechanism via SRS, RSS, ERSS as well as MMRSS techniques. The simulation study is conducted under the normality assumption with zero mean and unity variance assuming the ranking is perfect.

The program codes were prepared by the authors using the R software. Note that under the SRS technique, the ARL of the *X* chart will be 370. This represents the reciprocal of the probability that a single point falls outside the control limits when the process is in fact under control. In other words, the out-of-control signal will flash once every 370 observed samples even though the process is already under control.

We followed the same procedure of Muttlak and Al-Sabah (2003) to conduct a simulation study using one million iterations for each value of δ and for all sampling methodologies. At each iteration, we simulate a sample of size m = 3, 4, 5, 6 which represent the most recommended sample size in the RSS literature. As another simulation option, we set the shift-in-mean δ to vary between 0 and 3.4 to cover the under and out-of-control process.

Monte Carlo simulation experiments were used to study the performance of the extreme ranked control charts under the following assumptions:

Step I: Mean and variance of the samples

1. Generate 1000000 MMRSS samples of size $m = 3, 4, 5,$ and 6 for an in-control process i.e. from $N(0,1)$.
   - Remark: the exact value of variance under normal distribution using MMRSS can be calculated:

Table 1: The exact value of variance under normal distribution using MMRSS.

Sample size	Variance based on MMRSS
3	0.258
4	0.207
5	0.134
6	0.119

2. Calculate the mean of the sample.

Step II: Setting up the control limits

1. Select an initial value of k for a fixed ARL_0 (here $k = 3$).
2. Evaluate the control chart limits (LCL, UCL).

Step III: Evaluating the out-of-control ARL

1. Check the mean for out-of-control process. If the process is declared as in-control, go to sub-step III. If the process is declared to be out-of-control, record the number of samples so far as the in-control run-length.
2. Repeat **steps I** and **II** 1000000 times to compute in-control ARL.
3. Assume that the number the in-control run length is R. Then the ARL= R/1000000
4. Compute ARL for $\delta = 0.1, 0.2, \ldots, 3.4$.

The comparisons between the three sampling techniques are given in Tables 2 - 5 and Figures 1 - 5:

Table 2: ARL using several ranked data techniques when m=3.

δ	SRS	RSS	ERSS	MMRSS
0	369.6858	340.5995	340.4835	326.7974
0.1	361.2717	321.8539	319.1829	284.0909
0.2	305.2503	254.7771	247.5247	224.2152
0.3	254.7122	185.1852	184.2978	152.9052
0.4	202.4701	128.5017	133.7614	120.9190
0.5	153.2332	93.7910	91.2909	87.9508
0.6	120.7146	65.0280	65.6901	63.9386
1.0	44.0393	18.8929	18.8743	22.5026
1.4	18.2282	6.9544	6.9678	9.4464
1.8	8.6675	3.2767	3.2947	4.6292
2.2	4.7417	1.9340	1.9308	2.6602
2.6	2.9033	1.3782	1.3774	1.7877
3.0	1.9985	1.1425	1.1426	1.3734
3.4	1.5246	1.0461	1.0463	1.1649

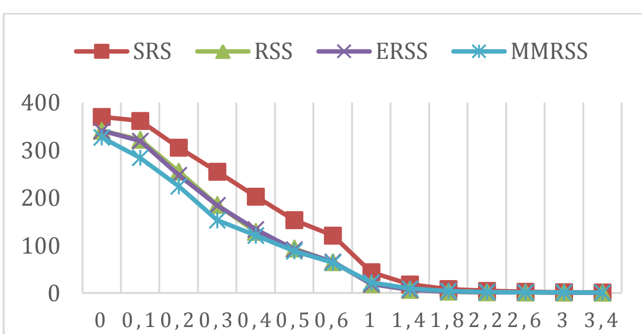

Figure 1: Comparison between ARL using several ranked data techniques when *m*=3.

Table 3: ARL using several ranked data techniques when *m*=4.

δ	SRS	RSS	ERSS	MMRSS
0	369.4126	349.0401	331.785	329.8277
0.1	337.7238	312.3048	304.5995	263.1914
0.2	312.9890	229.4104	243.2498	178.5714
0.3	266.0990	166.7500	179.6945	140.8451
0.4	200.7226	115.9420	126.3584	95.2371
0.5	158.1778	76.7048	88.1213	74.7269
0.6	119.2890	52.7816	60.2882	56.1698
1.0	43.7101	14.1495	17.4028	18.2148
1.4	18.3006	5.1341	6.3553	7.4139
1.8	8.6781	2.4803	3.0136	3.8332
2.2	4.7293	1.5504	1.8029	2.2627
2.6	2.9022	1.1932	1.3136	1.5776
3.0	1.9999	1.0584	1.1109	1.2586
3.4	1.5244	1.0138	1.0332	1.1021

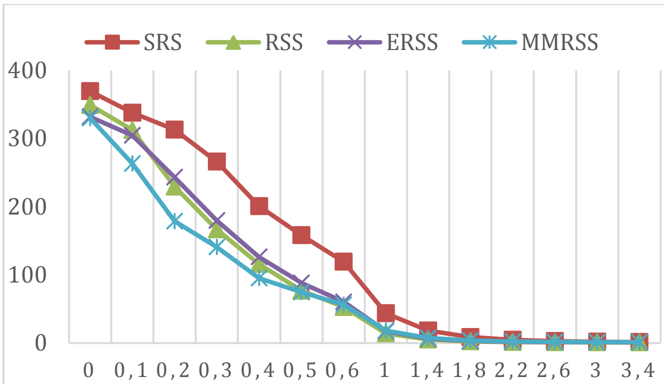

Figure 2: Comparison between ARL using several ranked data techniques when *m*=4.

Table 4: ARL using several ranked data techniques when *m*=5.

δ	SRS	RSS	ERSS	MMRSS
0	372.0238	356.7606	350.7541	340.1361
0.1	346.2604	301.9324	298.8643	279.3296
0.2	313.8732	225.8356	229.8322	207.9002
0.3	249.4388	152.4623	164.4466	143.4720
0.4	205.6767	98.4252	107.1352	106.1571
0.5	157.3812	65.3339	74.0631	71.9425
0.6	120.4094	44.0238	51.2453	52.7148
1.0	43.9638	11.0552	13.5932	18.1258
1.4	18.1831	3.9908	4.9395	7.3714
1.8	8.6989	2.0078	2.3922	3.5957
2.2	4.7118	1.3390	1.5117	2.1614
2.6	2.9120	1.1008	1.1761	1.5121
3.0	2.0007	1.0237	1.0514	1.2188
3.4	1.5254	1.0040	1.0119	1.0847

Table 5: ARL using several ranked data techniques when *m*=6.

δ	SRS	RSS	ERSS	MMRSS
0	375.5163	346.1405	331.1258	359.7122
0.1	349.406	300.8423	309.8853	322.5066
0.2	309.0235	218.7705	232.6664	226.3319
0.3	247.0356	137.1178	158.4033	156.3788
0.4	196.8891	87.0019	110.6072	109.7508
0.5	154.9427	55.9503	75.0356	97.7133
0.6	120.8021	37.0508	51.0882	54.1938
1.0	43.5749	9.0035	13.6605	18.0631
1.4	18.2435	3.2467	4.9405	7.3586
1.8	8.6944	1.7118	2.3992	3.4323

2.2	4.7149	1.2144	1.5167	2.0768
2.6	2.9026	1.0530	1.1770	1.4695
3.0	1.9961	1.0095	1.0521	1.1870
3.4	1.5244	1.0012	1.0118	1.0708

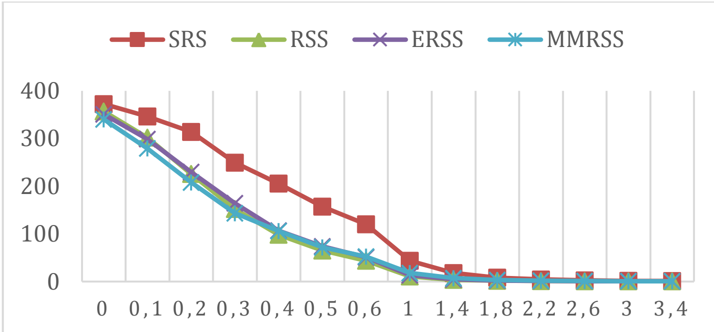

Figure 3: Comparison between ARL using several ranked data techniques when *m*=5.

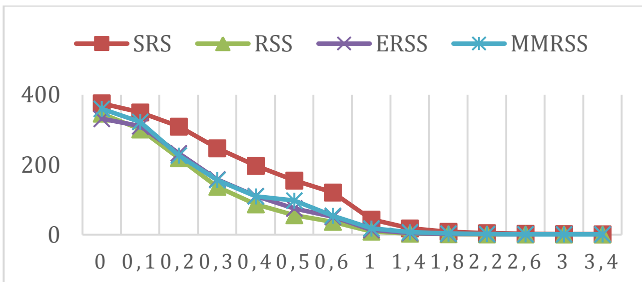

Figure 4: Comparison between ARL using several ranked data techniques when *m*=6.

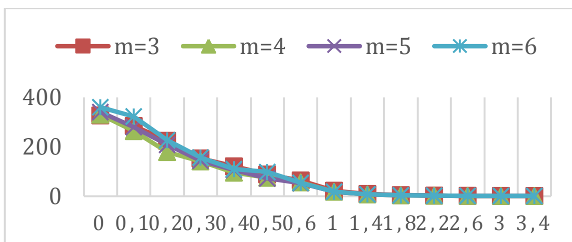

Figure 5: Comparison between ARL using MMRSS technique when *m*=3,4,5 and 6.

5. APPLICATION

MMRSS can be used as a cost-effective sampling method in surveys of natural resources in *agriculture*, *ecology*, forestry, environmental, and others. Total tree height of individual trees is one of the most frequently used variables in forest inventories. This variable is important requisites for developing forest management plans.

In this section, a real dataset is considered to study the performance of suggested Shewhart-type mean control charts. The dataset consists of two variables; the heights of spruce trees measured in meters (*m*), say *Y*, and the diameters of the spruce trees measured at breast height in centimeters (*cm*). We only consider one of them in this study: the height (*Y*). Our objective is to estimate the mean height of a random sample of 1103 spruce trees. We treat this sample as our parent population.

We used this data after removing 31 outlier observations to satisfy normality assumption. The histogram and the Normal probability plot of the heights are shown in Figures 6 and 7, respectively. Based on these figures we can assume that the distribution of height of trees can be approximated by a normal distribution. For a detail description about the dataset, see Prodan (2013). The summary statistics of this dataset is provided in the following Table 6.

Table 6: Summary Statistics of 1072 spruce trees heights (*m*)

N	*Min*	*Q1*	*Median*	*Q3*	*Max*	***μ***	***σ***
1072	13.50	19.7	21.90	23.70	29.50	21.72	2.917259

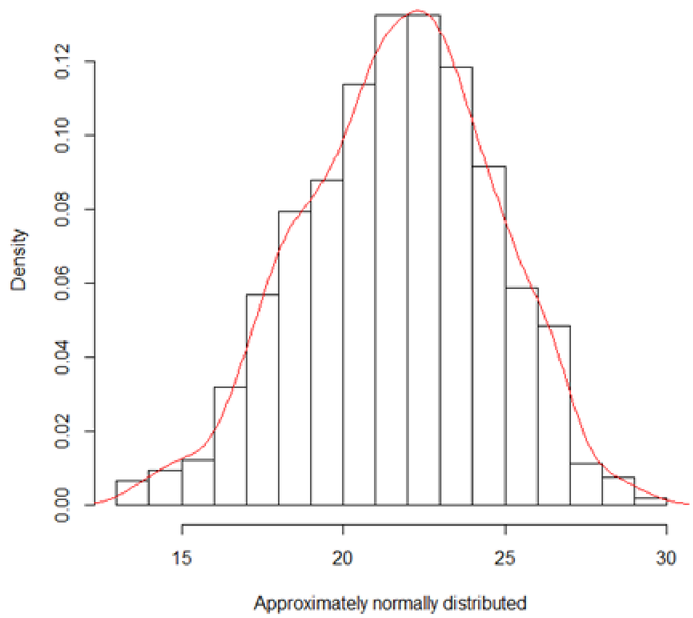

Figure 6: Histogram of 1072 spruce trees heights (*m*)

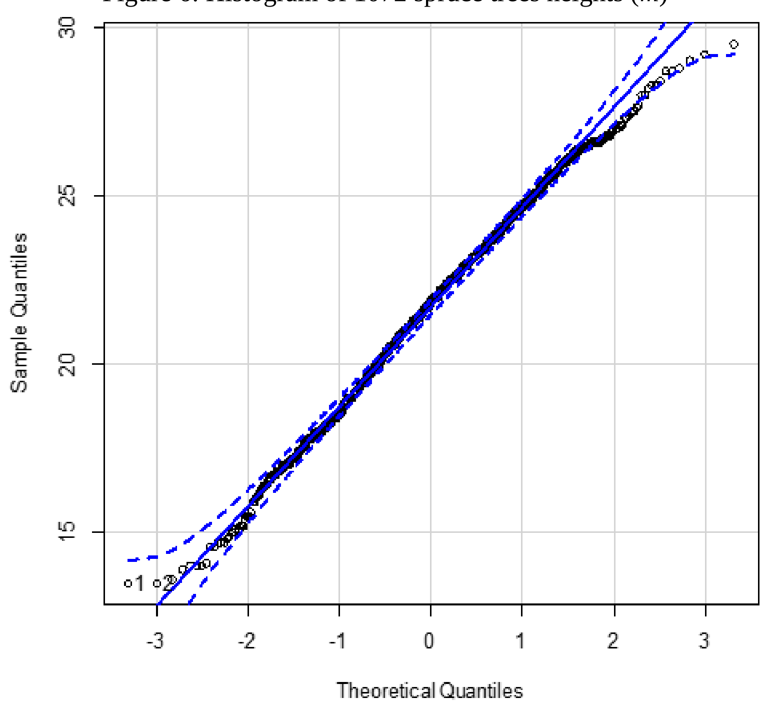

Figure 7: Normal q-q plot of 1072 spruce trees heights (*m*)

In order to draw MMRSS of size n=268, the procedure can be done as follows: randomly divide the tree data into 268 subsets (each of size 4) then take a random sample of size one from the first subset and a random sample of size two from the second subset and the same for the third and the fourth subsets. Repeat the procedure for the rest subsets and the rank each subset according to Y values. Pick the smallest ranked sample from the first ranked subset and the largest from the second ranked subset and continue similarly until you reach the last ranked subset. SRS of size 268 is also selected from same data. A summary of the selected sample units based on ERSS, RRS, MMRSS, and SRS is presented in the following Table 7.

Table 7: A summary of the selected samples using different techniques (m=4)

$\bar{Y}_{(ERSS)}$	21.4944	$s^2{}_{(ERSS)}$	13.2623
$\bar{Y}_{(RSS)}$	21.5489	$s^2{}_{(RSS)}$	8.9053
$\bar{Y}_{(SRS)}$	22.725	$s^2{}_{(SRS)}$	2.6292
$\bar{Y}_{(MMRSS)}$	22.1974	$s^2{}_{(MMRSS)}$	10.0723

Using these sample data, estimated means and variances of the ERSS, RSS, MMRSS, and SRS mean estimators are found then control limits with plotting statistics of each chart are estimated. The control limits with statistics of proposed as well as considered charts for all 268 samples (67 sample means) are shown in the Figures 8 – 11.

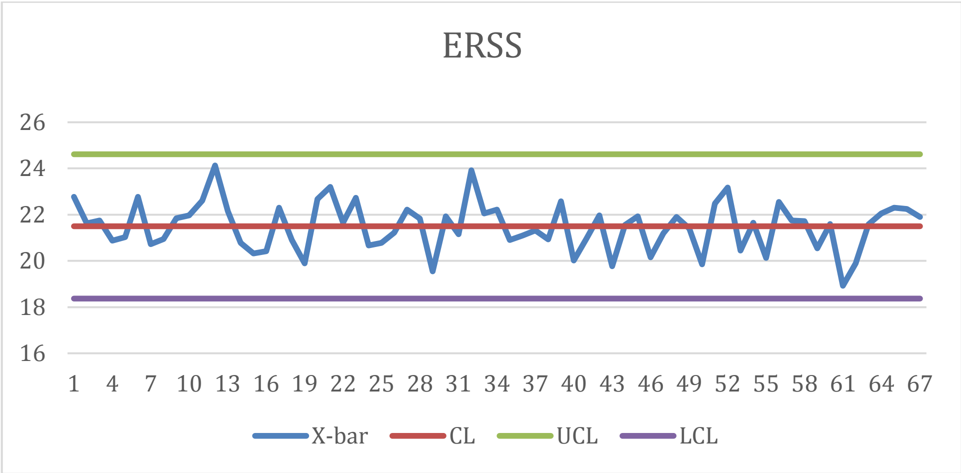

Figure 8: Shewhart-Type Mean Control Chart using ERSS

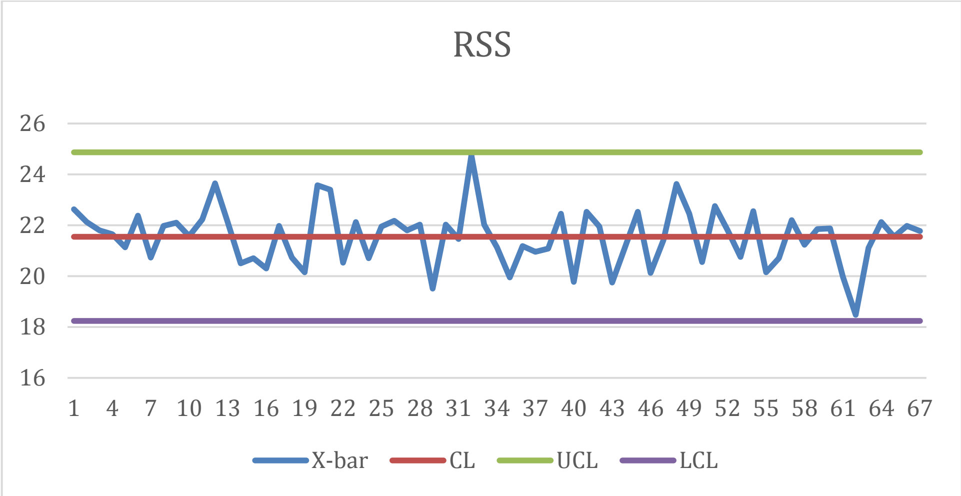

Figure 9: Shewhart-Type Mean Control Chart using RSS

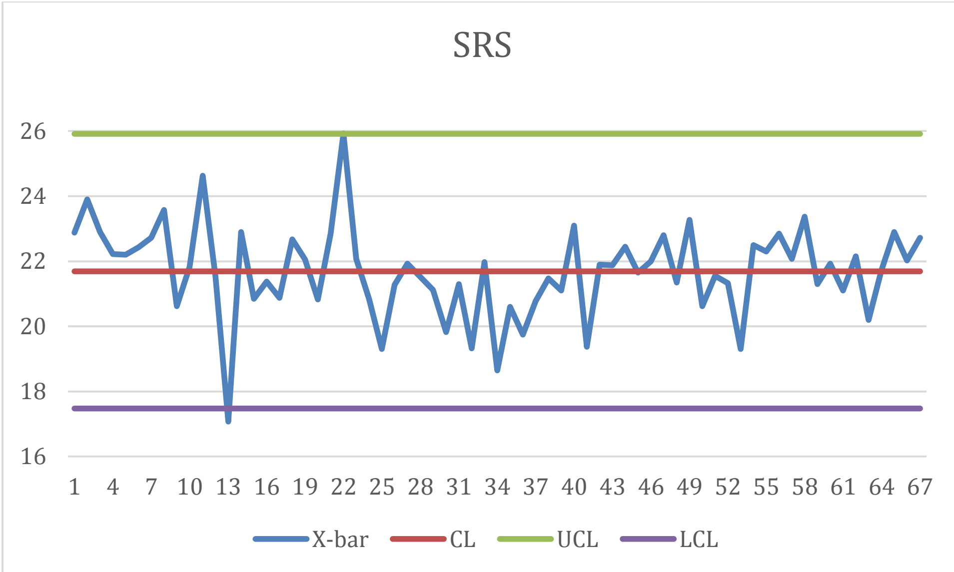

Figure 10: Shewhart-Type Mean Control Chart using SRS

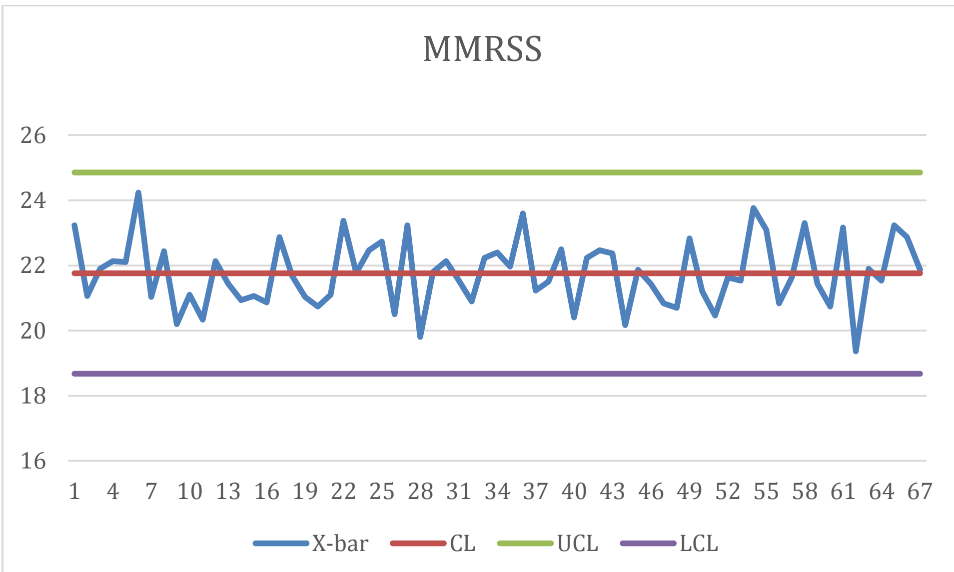

Figure 11: Shewhart-Type Mean Control Chart using MMRSS

Based on Figures 8–11, we can conclude that the MMRSS technique produces an efficient mean control chart which is more accurate than its counterparts based on SRS, RSS and ERSS in detecting a shift in the process mean. This demonstrates that the suggested control chart is less variable as compared with the traditional control charts based on the same sample size.

6. CONCLUDING REMARKS

The efficient quality control chart is explored by using the MMRSS technique to improve process monitoring. The ARL is employed to compare the proposed MMRSS mean control chart with the existing mean control charts under ERSS, RSS and SRS for the same sample sizes, shifts and number of iterations. Comparing the results in Tables 2 - 5 allow us to construct the following comments and remarks on the effectiveness of the MMRSS as well as the sampling techniques considered in this study.

- Assuming that the process is under control (i.e. $\delta = 0$), we clearly see that the number of false alarms does not depend on the sample size for all sampling approaches in the sense that the ARL has no monotonic pattern when the sample size varies between 3 and 6.
- The significant role of the ranked set sampling techniques; RSS, ERSS and MMRSS, start to emerge when the process gets out of control gradually (i.e., when δ gets larger than zero). Distinguishable differences between the RSS and the new proposed sampling techniques arise more clearly while comparing results when m = 3 or 4.
- The performance of the ranked set sampling techniques via the ARL values dominate SRS for certain values of k. In fact, ARL reaches a value 1 when the mean shift is 3.4. As a result, we may conclude that the process is already out-of-control and the signal for this purpose flashes every time we choose a sample.
- Despite the convincing results illustrated in Tables 2 - 5, we may still get more supportive remarks. In fact, we clearly notice that the gap between the ARL values using the ranked set sampling techniques compared to the regular SRS reaches its peak when δ is around 1.4 and it reaches more than nine manifolds comparing SRS with RSS, ERSS and MMRSS.

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