ECONOMIC ORDER QUANTITY MODEL FOR PRICE-SENSITIVE QUADRATIC DEMAND WITH IMPERFECT QUALITY UNDER INSPECTION

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ABSTRACT
In conventional EOQ models, it is anticipated that all the products arrived are unspoiled in the quality. But it is not the case in practice. In this article, an EOQ model with two different situations is presented. In the first one, the buyer receives imperfect items which need to be inspected. This incurs inspection cost to the buyer. While in second situation buyer gets no imperfect items. But for this he has to pay some more amount. This concept is elaborated here. The price and time sensitive demand is incorporated in analysis to study the effect of imperfect quality. The model is confirmed through numerical example and sensitivity analysis is figured out to deduce the managerial insights.

KEYWORDS: EOQ Model, Imperfect Quality, Inspection, Price-sensitive Quadratic Demand

MSC: 90B05

RESUMEN
En los modelos EOQ convencionales, se anticipa que todos los productos que se hayan llegado no tengan la calidad intacta. Pero no es el caso en la práctica. En este artículo, se presenta un modelo EOQ con dos situaciones diferentes. En el primero, el comprador recibe artículos imperfectos que deben ser inspeccionados. Esto incurre en costos de inspección para el comprador. Mientras que en la segunda situación, el comprador no obtiene artículos imperfectos. Pero para esto tiene que pagar algo más. Este concepto se elabora aquí. La demanda sensible al precio y al tiempo se incorpora en el análisis para estudiar el efecto de la calidad imperfecta. El modelo se confirma mediante un ejemplo numérico y se resuelve el análisis de sensibilidad para deducir las ideas gerenciales.

PALABRAS CLAVE: Modelo EOQ, calidad imperfecta, inspección, demanda cuadrática sensible al precio

1. INTRODUCTION

Traditional EOQ models have an incredible supposition that all items created are of required feature. However this is not the situation consistently. Some products are having imperfect quality also. Rosenblatt & Lee (1986) studied the results of a faulty production method on the best production cycle time. Salameh & Jaber (2000) developed further the EPQ / EOQ model by considering for faulty items. Goyal & Cardenas - Barron (2002) offered a simple methodology for deciding the economic production quantity for an item with faulty quality. Rezaei (2005) extended traditional EOQ/EPQ model with backorder for faulty items. Papachristos & Konstantaras (2006) studied EOQ models with shortages for items with faulty quality. Wee *et al.*(2007)and Chang and Ho (2010) established an best inventory model for items with faulty quality.
Chung & Huang (2006) contributed for a inventory model of the retailer to allow items with faulty quality under allowable delay for the payments. Hsu & Yu (2009) explored an inventory model for faulty items under single time discount. Chan *et al.* (2003) delivered an outline to incorporate low price, re-work and reject conditions for imperfect items. In place of whole inspection, Wang (2005) studied an inventory model in which inspection of the goods are executed at last. Haji and Haji (2010) considered a production system which produces defective as well as perfect items from which defective items can be altered. Maddah & Jaber (2008) corrected weakness in an economic order quantity (EOQ) model by Salameh & Jaber (2000) with variable supply. Goyal *et al.*(2003) and Huang (2002, 2004) derived that combined decision can decrease the total cost of supply chain for EOQ model with faulty items. Chen & Kang (2007, 2010) considered delay in payment for the ordered items. Rezaei & Davoodi (2008) derived deterministic inventory model with more than one item with selection by supplier and faulty standards. Lin (2009) gave optimal policy for a

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simple supply chain system with faulty items and reverted cost with selection errors. Rezaei and Davodi (2011) studied models with multi objective for lot size with supplier choice. Khan *et al.*(2011) gave a analysis of the extension of a improved EOQ models for faulty quality items. Rezaeia and Salimi (2012) derived an inventory model in which inspection for faulty items is done by buyer and supplier both. Ouyang *et al.* (2002) inspected the order quantity size, re-order time investigate model including flexible lead time with fractional back orders, where the production process is defective. Sebatjane and Adetunji (2019) incorporated of unsatisfactory quality for the EOQ model for developing products.

In above referred articles, generally every one of the scientists considered demand rate to be steady. In any case, the market review says that the demand barely stays consistent. In this paper, we believed demand to be value touchy time quadratic. Quadratic demand at first increments with time for quite a while and after that diminishes.

In the recent paper, EOQ model with two different scenarios is discussed. In the first scenario, checkup of the produced items is completed by the buyer. While in second scenario, buyer receives all the perfect items at higher cost. We derived the maximum purchase price for the buyer which is acceptable for him to pay for getting no imperfect items.

The next part of this paper is structured as here. 2nd Section describes notations & assumptions made for this paper. 3rd Section develops mathematical model of the case under consideration with two different scenarios. Section 4 deals with solution procedure. 5th section includes example and sensitivity study. Section 6 concludes paper.

2. NOTATIONS & ASSUMPTIONS

2.1. Notations

A	Order Cost per Order
C	Regular Procurement Cost per Unit ($p>0$)
M_c	Maximum Procurement Cost per Unit ($p=0$)
s	Selling Price per Perfect Unit (Decision Variable)
s_i	Selling Price / Faulty Unit
h	Cost of Holding items Per single Unit Per Unit Time
x	Rate of inspection
p	Rate of faulty items
C_i	Screening Cost / Inspection cost per unit
$I(t)$	Level of inventory at t, $0 \le t \le T$
T	Time for single cycle (Decision Variable)
Q	Order size
$\pi(s,T)$	Buyer's Gross Profit / Unit Time

2.2. Assumptions

1. A supply chain of only one buyer is used.
2. An inventory structure has with only one item.
3. The demand rate is $R(s,t) = a\left(1+bt-ct^2\right)s^{-\eta}$; s is selling price per unit ,

$a>0$ is scale demand and $0 \le b < 1$ is the rate of change of demand, $0 \le c < 1$ is the quadratic rate of change of demand and η is price elasticity.

4. Planning horizon is unbounded.
5. From each batch of produced items, $p\%$ items are imperfect; where $p>0$.
6. Inspection rate for the imperfect items is x per time unit.
7. We have not considered any shortages.
8. Rate of replenishment of the product is instant and lead time is zero.

3. MATHEMATICAL MODEL

The buyer's opening stock of Q units lessens to zero at $t = T$ because of demand. Thus, the pace of progress of stock level at any moment of time t is represented by the differential equation

$$\frac{dI(t)}{dt} = -a\left(1+bt-ct^2\right)s^{-\eta}, \quad 0 \le t \le T \tag{3. 1}$$

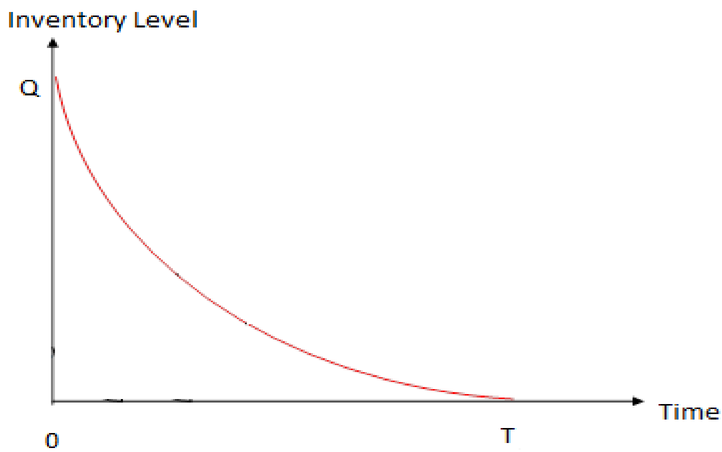

Figure 1: Inventory Level with respect to time

Figure 1 exhibits the inventory stock at any instantaneous t.

With $I(T)=0$, differential equation (1) is satisfied when

$$I(t) = -\frac{1}{6}as^{-\eta}\left(2c\left(T^3-t^3\right)-3b\left(T^2-t^2\right)-6\left(T-t\right)\right) \tag{3.2}$$

Buyer's ordered size is

$$Q = I(0) = -\frac{1}{6}as^{-\eta}\left(2cT^3-3bT^2-6T\right) \tag{3.3}$$

Revenue generated by sales is

$$SR = s(1-p)\int_0^T R(t)\,dt + s_i p\int_0^T R(t)\,dt$$

Cost of order; $OC = A$

Cost of purchase of Q unit is; $PC = CQ$

Cost of holding items ; $HC = h(1-p)\int_0^T I(t)\,dt + \frac{hpQ^2}{x}$

Inspection Cost; $IC = C_i Q$

Now we consider two scenarios as follows.

Scenario 1

The buyer may get some imperfect items for which he has to perform inspection.

Here total profit of the buyer is

$$\pi_1(s,T) = \frac{(SR-PC-OC-HC-IC)}{T}$$

$$= \left[\begin{array}{l} s(1-p)\left(-\frac{1}{3}acs^{-\eta}T^3+\frac{1}{2}abs^{-\eta}T^2+as^{-\eta}T\right)+s_i p\left(-\frac{1}{3}acs^{-\eta}T^3+\frac{1}{2}abs^{-\eta}T^2+as^{-\eta}T\right) \\ +\frac{1}{6}Cas^{-\eta}\left(2T^3c-3T^2b-6T\right)-A-h(1-p)\left(-\frac{1}{4}\frac{acT^4}{s^{\eta}}+\frac{1}{3}\frac{abT^3}{s^{\eta}}+\frac{1}{2}\frac{aT^2}{s^{\eta}}\right) \\ -\frac{1}{36}\frac{hpa^2\left(s^{-\eta}\right)^2\left(2T^3c-3T^2b-6T\right)^2}{x}+\frac{1}{6}C_i as^{-\eta}\left(2T^3c-3T^2b-6T\right) \end{array}\right] / T$$

Here there are $p\%$ imperfect items where $p > 0$ and unit purchase price is C per unit.

Scenario 2

The buyer gets no imperfect items. So he doesn't incur inspection cost in this case.

Here total profit of the buyer is

$$\pi_2(s, T) = \frac{1}{T}(SR - PC - OC - HC)$$

$$= \frac{1}{T}\left[\begin{array}{l} s\left(-\frac{1}{3}acs^{-\eta}T^3 + \frac{1}{2}abs^{-\eta}T^2 + as^{-\eta}T\right) + \frac{1}{6}Cas^{-\eta}\left(2T^3c - 3T^2b - 6T\right) \\ -A - h\left(-\frac{1}{4}\frac{acT^4}{s^\eta} + \frac{1}{3}\frac{abT^3}{s^\eta} + \frac{1}{2}\frac{aT^2}{s^\eta}\right) + \frac{1}{6}C_i as^{-\eta}\left(2T^3c - 3T^2b - 6T\right) \end{array}\right]$$

Here there is noimperfect items i.e. $p = 0$ and unit purchase price is C' per unit.

4. OPTIMAL SOLUTION

In scenario 1,the buyer receives imperfect itemstherefore he requires to inspect the itemsand in scenario 2,there is no imperfect itemswhich results no inspection cost to buyer.Hence it is clear that in scenario 2, buyer agrees to pay more purchase price C' than regular purchase price C.

Now, we determine the maximum purchase cost M_c for lot without imperfect items. M_c is the maximum value of purchase price C'.

The buyer will be ready to pay more if and only if

$$\pi_2\left(s^{**}, T^{**}, C'\right) - \pi_1\left(s^*, T^*\right) \geq 0 \qquad (4.1)$$

Where s^*, T^* are optimum values for π_1 and s^{**}, T^{**} are optimum values for π_2. We consider $p = 0$ while finding s^{**}, T^{**} for π_2.

We derive the value of C' using (4.1).

For the buyer, the necessary condition to have maximum profit for unit time w.r.t. cycle time and retail price is

$$\frac{\partial \pi_1(s,T)}{\partial s} = 0 \text{ and } \frac{\partial \pi_1(s,T)}{\partial T} = 0 \qquad (4.2)$$

However, the non-linearity of $\pi_1(s, T)$ and partial differentiations of it will not give us closed form of $\pi_1(s, T)$. Hence, proposed process to find out solution is here:

1: Input the hypothetical data to different parameters.
2: Solve (4.2) by some software. We here have taken help of Maple 14.
3: Test 2nd order conditions analytically or graphically.
4: Determine profit $\pi_1(s, T)$.

5. NUMERICAL EXAMPLE

We consider following example for validating the mathematical formulation.

Example: Consider $A = \$100/$ order, $C = \$\ 25$ / unit, $h = \$5$ / unit / year, $a = 500000$ units, $b = 10\%$, $c = 20\%$, $\eta = 1.2$, $p = 4\%$, $x = 1$ unit/min, $C_i = \$0.5$/unit and $S_i = \$20$/unit.

Hence optimum T is 4.175 yrs and related profit is \$9768.05 and selling price is \$145.51. The buyer's purchases 247 units and the maximum purchase cost is \$49.02.

Figure 2 shows, concavity of π_1 w. r. t. 's' and 'T'.

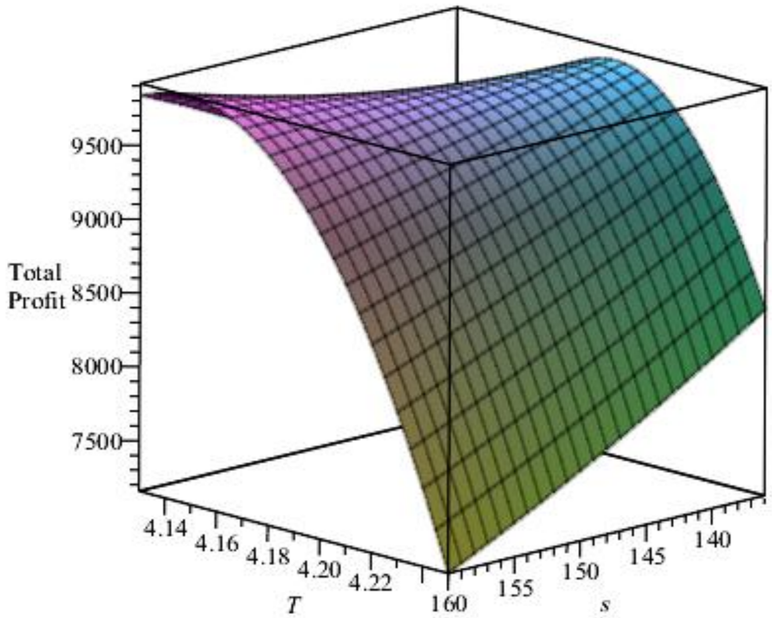

Figure 2: concavity of π_1 w. r. t. 's' and 'T'

Now, changes in T (fig. 3), selling price (fig. 4), maximum purchase cost (fig. 5) and profit realization (fig. 6) by altering inventory parameters as -10%, -5%, 5% and 10% with the help of numerical data given in table 1.

Table 1: Sensitivity study w. r. t. the key variables

	Value	T (Years)	s ($/unit)	Profit ($)	Maximum Cost ($/unit)
A	90	4.17530833	145.51078460	9770.44706900	49.02103033
	95	4.17530819	145.51111790	9769.24945500	49.02102047
	100	4.17530806	145.51145110	9768.05200400	49.02101072
	105	4.17530792	145.51178440	9766.85449300	49.02100109
	110	4.17530778	145.51211760	9765.65700800	49.02099147
H	4.5	4.179111585	136.7140095	9453.388016	47.88935774
	4.75	4.177175941	141.1536062	9614.548046	48.46126598
	5	4.175308055	145.5114511	9768.052004	49.02101072
	5.25	4.173440169	149.869296	9921.555962	49.58075546
	5.5	4.171572283	154.2271409	10075.05992	50.1405002
A	450000	4.172600564	141.3028342	9068.378207	48.67186937
	475000	4.173998362	143.4446733	9421.266556	48.8520178
	500000	4.175308055	145.5114511	9768.052004	49.02101072
	525000	4.176539109	147.5093271	10109.1243	49.18002444
	550000	4.177699587	149.4436921	10444.83185	49.33006358
B	0.09	4.135120783	144.379454	9690.135054	48.88476736
	0.095	4.155169898	144.9439342	9728.963342	48.95275855

	0.1	4.175308055	145.5114511	9768.052004	49.02101072
	0.105	4.195446212	146.078968	9807.140666	49.08926289
	0.11	4.215584369	146.6464849	9846.229328	49.15751506
C	0.18	4.422167182	150.77869	9970.57463	49.6907147
	0.19	4.293574667	148.0460564	9866.147372	49.34354121
	0.2	4.175308055	145.5114511	9768.052004	49.02101072
	0.21	4.066069085	143.1513513	9675.59748	48.72017473
	0.22	3.964775859	140.9460081	9588.198168	48.43856119
H	1.08	4.196212153	167.0764023	14395.12822	51.9736613
	1.14	4.185760104	156.2939267	12081.59011	50.49733601
	1.2	4.175308055	145.5114511	9768.052004	49.02101072
	1.26	4.163996	135.7803308	7930.977975	47.64015129
	1.32	4.151804323	126.983078	6464.08839	46.34393651
C_i	0.45	4.175295368	145.4904518	9771.006305	49.00322954
	0.475	4.175301711	145.5009508	9769.52918	49.01212365
	0.5	4.175308055	145.5114511	9768.052004	49.02101072
	0.525	4.175314398	145.5219528	9766.575169	49.02989065
	0.55	4.175320742	145.5324559	9765.098454	49.03876297
P	0.036	4.172486468	141.1292189	10134.63239	48.60895028
	0.038	4.173942046	143.3563398	9945.943564	48.82104892
	0.04	4.175308055	145.5114511	9768.052004	49.02101072
	0.042	4.17659409	147.6005807	9599.835356	49.21009727
	0.044	4.177808328	149.6290027	9440.334557	49.38938664
X	0.9	4.213444485	195.9674507	5429.828212	53.89745916
	0.95	4.19437627	170.7394509	7598.940108	51.45923494
	1	4.175308055	145.5114511	9768.052004	49.02101072
	1.05	4.15623984	120.2834513	11937.1639	46.5827865
	1.1	4.143686353	107.9711864	13336.13911	45.01024649
s_i	18	4.175328354	145.5450614	9763.326844	49.02632211
	19	4.175318205	145.5282545	9765.689115	49.02366675
	20	4.175308055	145.5114511	9768.052004	49.02101072
	21	4.175297905	145.4946512	9770.415474	49.01835402
	22	4.175287756	145.4778549	9772.779463	49.01569674

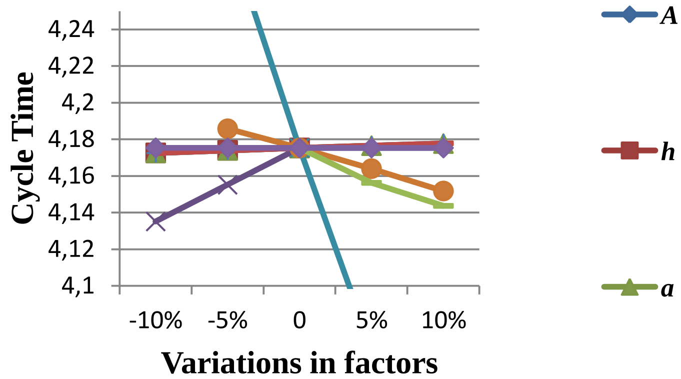

Figure 3: Sensitivity study for *T*

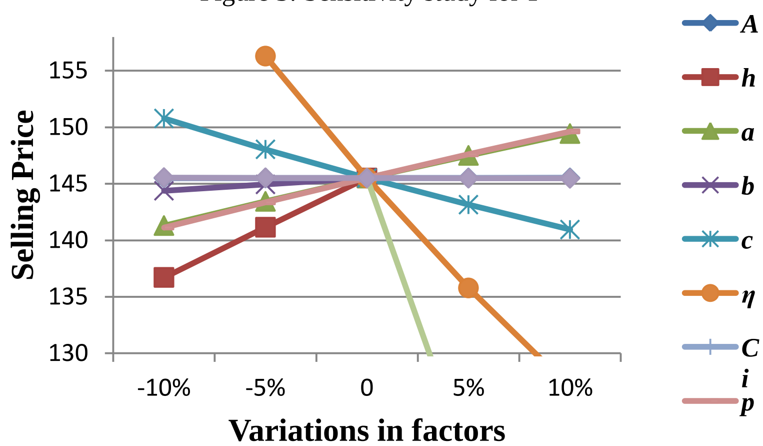

Figure 4: Sensitivity study for '*s*'

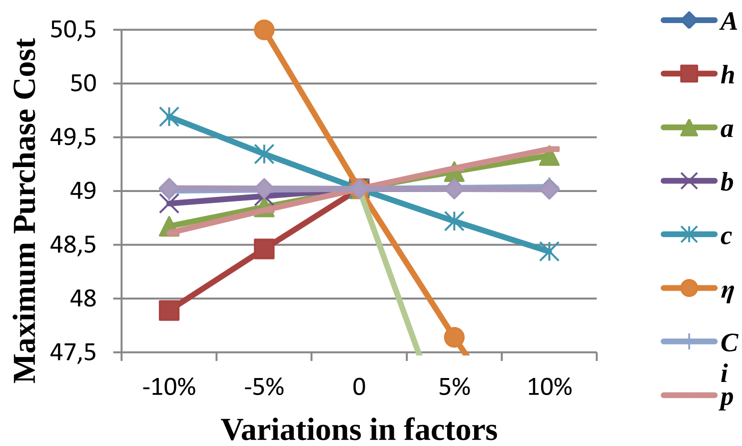

Figure 5: Sensitivity study for '*Mc*'

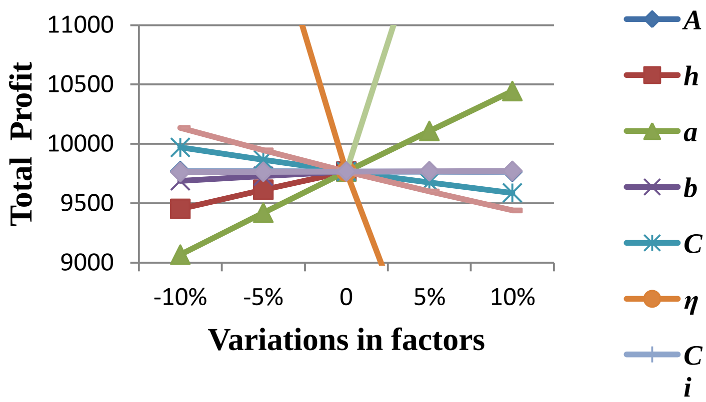

Figure 6: Sensitivity study for π_1

Table 2: Sensitivity study of parameters

Inventory Parameters	**Cycle Time (*T*)**	**Selling Price (*S*)**	**Maximum Purchase Cost**	**Total Profit**
A	--	↑	--	↓
h	↑	↑	↑	↑
a	↑	↑	↑	↑
b	↑	↑	↑	↑
c	↓	↓	↓	↓
η	↑	↓	↓	↓
C_i	↑	↑	↑	↓
p	↑	↑	↑	↓
x	↓	↓	↓	↑
s_i	--	↓	↓	↑

N. B. In table 2, ↑ denotes increasing pattern and ↓ denotes decreasing pattern.

Above table 2 shows the outcome of changes in variables. From that one can derive some managerial implications as follows.

- Total profit decreases with the increase in ordering cost, *c*, η, C_i and p.
- Total profit rises with rise in *a*, *b*, inspection rate, selling price for imperfect items.

So efforts should be made to decrease ordering cost, inspection cost and imperfect rate. Also *a* and *b* can be increased. Inspection rate can be improved. Selling price can be raised upto some extent.

6. CONCLUSION

In real life, output of the production processes does not give all perfect items despite of all efforts. Here, Economic Order Quantity Model is formulated and maximum purchase price buyer can pay to get no imperfect items is decided. To enhance the demand, price of any product plays key role. Hence we have considered quadratic demand to be price sensitive. Quadratic demand rises initially and afterwards falls. Two scenarios are considered in the present article. In first, inspection is done by the buyer for imperfect items and in other buyer need not to inspect the items. We have derived the optimal profit for the buyer in both the cases. Hence we have decided extreme purchase price for buyer to receive no imperfect items. Thus, buyer can make the manufacturer advance production standards by giving higher than regular purchase value. Hence buyer can also save his time and efforts for the inspection.

This paper is applicable where there are more chances of defective items like electronics goods e.g Mobile Phone, Calculator, Trimmers, Health parameter measuring equipment in which inspection requires more time and care.

Limitation and future scope

The present article can be extended by reducing the assumptions made and by incorporating perishable items, more than single product, multi buyers, shortages, different types of demand, trade credit, etc.

ACKNOWLEDGEMENT: We are grateful to the commentators for their fruitful ideas.

RECEIVED: JANUARY 2020.
REVISED: JUNE, 2020.

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