AN INVENTORY MODEL FOR NON-INSTANTANEOUS DETERIORATING ITEM UNDER PROGRESSIVE TRADE CREDIT POLICY

Chandra K. Jaggi*, Monalisha Tripathy**, Anuj Kumar Sharma***, Geetanjali Sharma**
*Department of Operational Research, Faculty of Mathematical Sciences, New Academic Block, University of Delhi, Delhi 110007, India
**Department of Mathematics and Statistics, Banasthali Vidyapith, Newai, Rajasthan 304022, India
***Department of Mathematics, Shyam Lal College, University of Delhi, Delhi 110032, India

ABSTRACT
In this paper, authors consider optimal replenishing strategies for constant demand in various financial scenarios by considering non-instantaneous deteriorating item under the progressive trade credit policy. The aim of this work is to develop a cost function for various situations depending on the trade credit period in economic environment. An algorithm is established to obtain the average cost, the replenishment time and the optimal order quantity. A thorough sensitivity analysis was carried out to assess the importance of the model.

KEYWORDS: Inventory Theory, Non-Instantaneous Deterioration, Progressive Trade Credit, Different Financial Scenario, EOQ.

MSC: 90B05

RESUMEN
En este paper los autores consideran optimas estrategias de reaprovisionamiento para demandas constantes en varios escenarios financieros considerando ítems no-instantáneamente deteriorables bajo la política progresiva de crédito comercial. El interés de este trabajo es desarrollar una función de costo para varias situaciones dependientes del periodo del crédito comercial en un ambiente económico. Un algoritmo es establecido para obtener el costo promedio , de reaprovisionamiento y la óptima cantidad a ordenar. Un análisis de sensibilidad fue desarrollado para soportar la importancia del modelo.

PALABRAS CLAVE: Teoría de Inventario, Deterioro No-instantáneo, Crédito Comercial Progresivo, Diferente Escenario Financiero, EOQ.

1. INTRODUCTION

In deterministic as well as probabilistic inventory models of classical type, deterioration plays an essential role. Deterioration can be defined as the decay, obsolescence, damage, disappearance, harm of utility or loss of an initial product's marginal values. Earlier, researchers have already recognised a lot of research work in Inventory management and Inventory control system considering deterioration as an important factor. Many researchers for example Philip (1974), Ghare and Schrader (1963), Wee (1995), Aggrawal and Jaggi (1995), Geetha and Udayakumar (2015) accept that the items in the inventory, deteriorates once they arrive. Raafat (1991), Bakker et al. (2012), Goyal and Giri, Li et al. (2010) and Janssen et al. (2016) presented the analyses of progress of deteriorating inventory literature. M. Maragatham et al. (2017) established an inventory model by considering time relative deterioration rate, demand rate is based on selling cost and ordering cost, holding cost and deterioration rate are all time-based. Dr. Jayjayanti Ray (2017) created a model to study different fuzzy EOQ models for deteriorating items.
However, over time, most products, including medicinal products, unstable liquids and blood banks, are deteriorated or damaged. This highlights that there is no deterioration of products for an initial period. This phenomenon was described by Wu et al. (2006) as the non-instantaneous or slow deterioration and called these products as non-instantaneous products. They have created a mathematical model and are looking at the issue of defining the optimum strategy of replenishing non-instantaneous and stock-dependent products. Chang et al. (2010) define the ideal refilling strategy for a non-instant inventory design with deterioration based on stock dependent demand. Maihami, R. et al, (2012) developed a non-instant deteriorating joint

ckjaggi@yahoo.com , monalisha.tripathy1980@gmail.com

pricing and inventory model. M. A fuzzy inventory model was established by Maragatham, P.K. Lakshmi Devi (2016) in order to determine relevant stock cost per unit time for non-instant deteriorating products over a definite time period with significantly declining demands for n-cycles. A deterministic stock model was constructed by Geetha, K.V. et al (2017) with two stages of storage for non-instant deteriorating products.

Commercial credit is another important factor that governs the world of company today. Commercial credit or trade credit is an important instrument for many enterprises to finance development. A firm providing the credit shall determine the amount of days for which a loan is provided and agreed on by both the corporation and the business receiving the loan. It is a typical conception that when the purchaser buys anything from the supplier, she/he pays for the bought items as soon as the items once received from the supplier. It was trusted in traditional corporate trade that the retailers must pay for the things she/he had requested once she/he got them. Although, in today's highly competitive industry, such an assumption seems no longer practical. Instead a delayed payment is presently a well-known fact in business exchanges and an invaluable advertising instrument for suppliers. In addition, this instrument is gainful for both the merchant as well as the purchaser. Trade credit framework assists the seller, with increasing benefit through stimulating more deals, but is an opportunity to reduce demand uncertainty and related risks for distributors. When we examine the structure for trade credit carefully, we can say that when the provider sends the units to the dealer without payment, the provider exchanges storage accountability and expenses. In addition, he accepts the danger of insecurity in demand.

In recent years, allowable delay in payment or trade credit have been widely researched, but enormous study gaps continue to exist in this region, which has led to future studies. Haley and Higgins (1973) addressed trade credit for the first time. He analysed the effect on optimum inventory and payment time of a two-part trade credit policy for a cash discount. Through various instances such as traditional EOQ (economic order quantity models), payment for sold products in a loan period, and payments after a set time period, Chapman et al. (1983) discovered an ideal refurbishment policy. In anticipation of the provider allowing retailer to pay the account for a period of time, Goyal (1985) developed mathematical models to determine the amount for the financial order. C.K. Jaggi et al. (2008) was a model for assuming that a loan related to demand is associated with the cost of a product and its selling prices are considering different. In the calculation of the interest charged and in a relaxed partnership, Teng and Goyal (2009) complemented the deficit of Huang and Huang (2008).

In a realistic condition, some companies face very large shortages, which require a certain stock level to prevent shortages. On the other side, some circumstances do not have very important shortages at moment of order and their costs are actually minimal. Because shortage is of excellent importance to the amount ordered, especially when the payment model is delayed. Taking this into account, certain research in this field have been carried out. In order to depart from the ideal refuelling and shortage choices, Taleizadeh et al (2013) submitted the economic order quantity model under partial trade credit and partial backlog. Teng et al. (2007) accepted a two-tier trade credit that could exceed the purchase cost in respect of sales prices, and also not necessarily exceed the value earned. They created the business loan funding model of EOQ and supplied an easy, shut-down solution. Teng and Chang worked on Huang (2007) and expanded the job independently by examining the delay period of both the retailer and the customer. Lou and Wang (2013) established a faulty inventory EPQ model for two levels of trade loans and determined an optimum refill time to maximize the manufacturer's overall net profit. The various study documents (Chang et al., 2008; Soni et al., 2010; Seifert et al., 2013; Molamohamadi et al., 2014) summarize further stock works in these areas. D. Yadav et al. (2015) created a stock model for a company that not only continually deteriorates the produced item but has a lifetime. D.J.: D.J. Mohanty et al. (2017) created the stock model in order to explore the joint impact of investment and trade credit policies for conservation technology, where shortages are permitted, and partial backlogs combined with losses of revenues are permitted.

There are a number of researchers who considered two factors simultaneously and formulated different mathematical modelling. One of the biggest factors is progressive trade credit facility. The progressive credit period given by the supplier to settle the loan can be described as: If the retailer covers the due amount by time units M, then the supplier will not charge interest. On the off chance that the retailer settles after M time period yet before N ($N > M$), then supplier will charge an interest on balance amount at I_{c1} rate. If the retailer unable to settles the loan amount at M and goes for the N time period and clears the due after N period, then he has to pay a higher interest I_{c2} ($I_{c2} > Ic1$). Singh C. and Singh S.R. (2015) have developed a progressive trade credit policy for suppliers in the inflationary and fuzzy environment with and without stock-outs for lead time. Taking into account trade credit, Yan Shi et al (2018) created the inventory design for a deteriorating product with ramp-type demand rate. In order to optimize the replenishment strategy for a channel based on stock demand, Longfei H. et al. (2018) created a stock model taking into account deterioration in items and order retrieval in two economical systems of progressive trade credit periods. In the progressive trading phase, we take into account both continuous payments regime (CPR) and discrete payments regime (DPR). Shah N. and Naik M. (2019) established an inventory model for declining products by maximizing the total profit of the retailer. The model included the retailer's cash discount based on the amount of the order and the customer's cash discount as well.

In past years, many researchers worked on the progressive trade credit facility but, none of the authors considering progressive trade credit with non-instantaneous deterioration. To fill this gap, we have set up a non-instantaneous deteriorating inventory model with a steady supply that allows a progressive delay in payment. Various cases depending on the supplier's permissible delay are explored and outcomes are contrasted using numerical examples. The remaining document is structured as follows. Section 2 describes the assumptions and Section 3 describes the notations used across the document presented. The mathematical models created in Section 4 to reduce the total average cost under some constraints. In Section 5, different cases, arising due to permissible delay period are analysed. In Section 6 a solution procedure has been developed in order to get the optimum values. In Section 7 we analyse the model using numerical examples, and Section 8 performs sensitivity analysis. In Section 9 some managerial insights are provided. Concluding remark is given in the Section 10 of the paper and in Section 11 expresses the gratitude towards the anonymous reviewers for their contributions in improving the paper.

2. ASSUMPTION

The mathematical model of the non-instantaneously deteriorated inventory model is based on the following assumptions and notes:

a) The discussions and evaluation in this document shall be limited to a single supplier and retailer of a particular item.
b) Constant demand rate.
c) The replenishment rate is indefinite and the lead period is negligible.
d) The time horizon is unlimited and only one item is included in the inventory system.
e) Unsatisfied demand/shortages are not allowed.
f) The whole product quantity is delivered in one batch.
g) The non-instantaneous period is small and a constant fraction α of the stock is deteriorating after a period of time, per unit time and the inventory deteriorates during the cycle time does not have to be repaired or replenished.
h) The supplier offers a credit period M to the retailer, and after the credit period expires it offers another allowable period N and then the progressive period N1 to settle the amount. During these periods the supplier will earn interest.

3. NOTATIONS

D	the demand rate per unit time
A	the replenishment cost per order ($ / Order)
p	the selling price per unit item $p > c$ ($ / Unit)
c	the purchasing cost per unit item ($ / Unit)
h	holding cost per unit per unit time ($ / Unit)
α	the deterioration rate $\left(0 \le \alpha < 1\right)$
M	credit period offered by the supplier (in Year)
N	next allowable credit period (in Year)
$N1$	progressive credit period (in Year)

I_p	rate of interest charged by the supplier per unit time
I_{p1}	higher interest charged by the supplier during progressive period ($I_{p1} > I_p$) per dollar per year
I_e	rate of interest earned by the retailer per dollar per year
I_{e1}	higher interest earned by the retailer per dollar per year
t_d	time length during which the item has no deterioration (in year)
$I_{01}(t)$	the inventory level at time t when there is no deterioration (in year)
$I_{02}(t)$	the inventory level at time t when there is deterioration (in year)
Q	the retailer's maximum order quantity
TC_i	the total average costs (in dollar)
T	The length of replenishment cycle (in Year)

4. MODEL FORMULATION

In this part of the document the picture demonstrates the replenishing problem of an inventory model for a single non-instantaneous item that deteriorates. In the beginning, Q volume figures tap into the inventory system. Given that not immediate items are being considered, no deterioration occurs during the period [0, t_d] and consequently, the inventory is only reduced responding to demand. Further, during the time period [t_d, T], stock loss is caused by demand and decline collective impact, reaching null at time T. The model's achievement over the whole stage [0, T] was illustrated below.

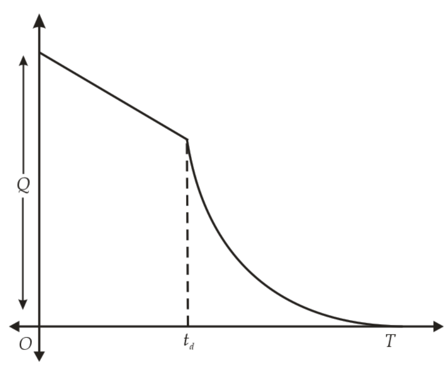


Figure 1 – Representation of Inventory Model

The diff. equations that describe the inventory level at any time t over the period [0, T] are given by:

$$\frac{dI_{01}(t)}{dt} = -D \qquad 0 < t < t_d \tag{4.1}$$

$$\frac{dI_{02}(t)}{dt} = -D - \alpha\, I_{02}(t) \qquad t_d < t < T \tag{4.2}$$

By solving the above equation with boundary conditions

$I_{01}(t) = Q$ at $t = 0$, $I_{01}(t) = I_{02}(t)$ at $t = t_d$ and $I_{02}(t) = 0$ at $t = T$, we get

$$\Rightarrow \quad I_{01}(t) = -D(t) + Q$$

$$\Rightarrow \quad Q = I_{01}(t) + D(t)$$

Similarly, by solving equation (4.2) with appropriate boundary conditions,
$I_{01}(t) = I_{02}(t)$ at $t = t_d$ and $I_{02}(t) = 0$ at $t = T$ we have,

$$I_{01}(t) = D\left[\frac{1}{\alpha}\left(e^{\alpha(T-t_d)} - 1\right) + t_d - t\right] \tag{4.3}$$

$$I_{02}(t) = \frac{D}{\alpha}\left[e^{\alpha(T-t)} - 1\right], \qquad t_d < t < T \tag{4.4}$$

Considering continuity of $I(t)$ at $t = t_d$, it follows from equation (4.3) and (4.4) that

$$I_{01}(t) = I_{02}(t) = \frac{D}{\alpha}\left[e^{\alpha(T-t_d)} - 1\right] \tag{4.5}$$

Now putting the value of $I_{01}(t)$ in eqn. $Q = I_{01}(t) + D(t)$, we get

$$Q = D\left[\frac{1}{\alpha}\left(e^{\alpha(T-t_d)} - 1\right) + t_d\right] \qquad \text{(By expanding the exponential term)}$$

$$\Rightarrow Q = D\left[T + \frac{\alpha(T-t_d)^2}{2}\right] \tag{4.6}$$

Now, the total cost per cycle includes the following components:
1. Ordering cost per cycle = A
2. Purchasing Cost = Qc

$$Qc = c\,D\left[T + \frac{\alpha(T-t_d)^2}{2}\right]$$

3. Holding cost (*h*) per cycle

$$= h\left(\int_0^{t_d} I_{01}(t)\,dt + \int_{t_d}^{T} I_{02}(t)\,dt\right) = h\,D\left[t_d\left(T - \frac{t_d}{2}\right) + \frac{(T-t_d)^2}{2}(1 + \alpha\,t_d)\right] \tag{4.7}$$

In this model we consider that the provider provides the dealer a trade credit period *M* to pay the loan amount. If at the specified credit time, the retailer was unable to repay the credit amount, the supplier will charge interest at that time. In view of the distinctive situations there may emerges a few cases which are examined in section 5.

5. CASE ANALYSIS

As M is the trade credit period offered by the supplier, depending on the position of M, there may arise following cases:

Case 1: $0 < M < t_d$

Case 2: $t_d < M < T$

Case 3: $M > T$

Let's discuss these cases in detail.

5.1. Case 1: $0 < M < t_d$

In this case, the supplier offers credit period *M* which lies before the point from where deterioration starts *i.e.* t_d. Hence, the retailer has the option to settle the credit amount *i.e.* Qc amount to the supplier at $t = M$ and need not to pay any interest during this period.

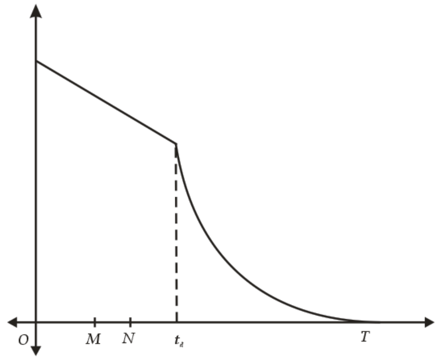


Figure – 2 Graphical representation for Case 1

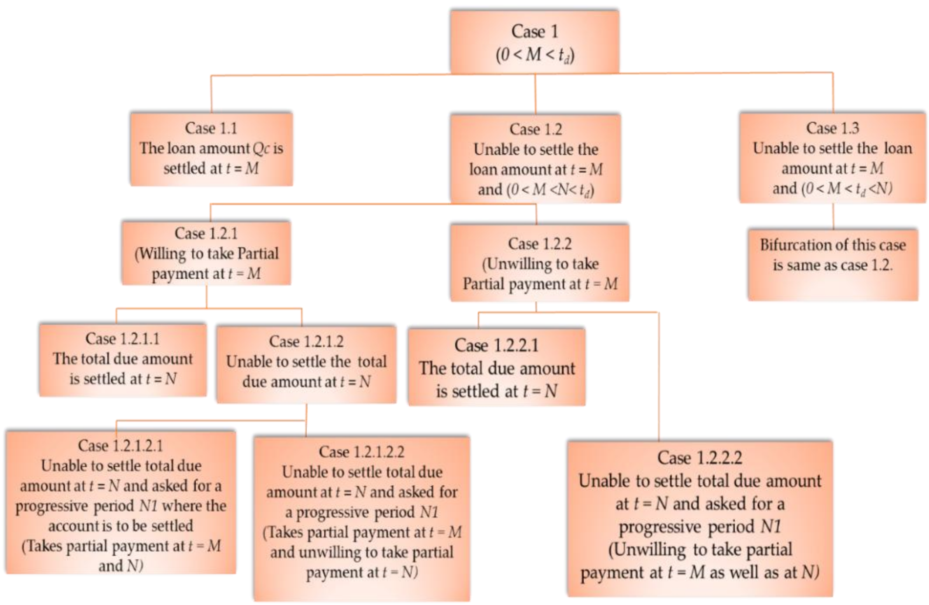


Figure – 3 Graphical representation of Case 1 bifurcation

In between, the retailer generates revenue continuously (due to sales and interest earned) is given by,

$$U_1 = DM\,p\left(1 + \frac{1}{2}MI_e\right)$$

Based on the amount of revenue generated, the model is further divided into two subcases i.e. $U_1 \geq Qc$ and $U_1 < Qc$. If, $U_1 < Qc$, then the retailer has the privilege to settle the due amount at the next allowable credit period *N*. Here two more scenarios may arise depending on the position of *N*, *i.e.* either $N < t_d$ or $N > t_d$.
By observing the above scenarios, we named them as:

Case: 1.1. $U_1 \geq Qc$

Case: 1.2. $U_1 < Qc \ \left(\text{where } N < t_d\right)$

Case: 1.3. $U_1 < Qc \ \left(\text{where } N > t_d\right)$

Case: 1.1. $U_1 \geq Qc$

In this case, the revenue generated by the retailer is more than the purchasing cost of the total inventory i.e. $U_1 \geq Qc$. So, after paying the Qc amount to the supplier, the retailer will earn interest on the excess amount (if any) *i.e.* $U_1 - Qc$ for the period [*M*, *T*]. Again, the retailer will accumulate the revenue continuously on the sales during [*M*, *T*] period and earn interest on it.

Therefore, the total interest earned $IE_{1.1}$ by the retailer during [*M*, *T*] period is,

$IE_{1.1}$ = Interest earned on the revenue generated during [*M*, *T*] period + Higher interest earn on the excess amount if any during [*M*, *T*] period

$$IE_{1.1} = I_e \int_M^T pD.t\, dt + I_{e_1} \int_M^T \left(U_1 - Qc\right) dt = \left[D\, p \frac{\left(T+M\right) I_e}{2} + Ie_1\left(U_1 - Qc\right)\right]\left(T-M\right)$$

Thus, the total average cost $TC_{1.1}$ for the cycle is given by

$$TC_{1.1}\left(T\right) = \frac{1}{T}\left\{\begin{array}{l}\text{Ordering cost per cycle} + \text{Purchasing cost} + \text{Inventory holding} \\ \text{cost per cycle} - \text{interest earn}\end{array}\right\}$$

$$TC_{1.1}\left(T\right) = \frac{1}{T}\left\{\begin{array}{l} A + c\,D\left[T + \dfrac{\alpha\left(T-t_d\right)^2}{2}\right] + h\,D\left[t_d\left(T - \dfrac{t_d}{2}\right) + \dfrac{\left(T-t_d\right)^2}{2}\left(1+\alpha\, t_d\right)\right] \\ -\left(T-M\right)\left[D\,p\dfrac{\left(T+M\right)I_e}{2} + I_{e1}\left(D\,p\,M\left(1+\dfrac{1}{2}MI_e\right) - c\,D\left[T + \dfrac{\alpha(T-t_d)^2}{2}\right]\right)\right]\end{array}\right\}$$

Problem 1. Minimize $TC_{1.1}\left(T\right)$

Subject to $0 < M < t_d < t_R < T$

***Case**: 1.2.* $U_1 < Qc$ (where $M < N < t_d$)

In this case, two more sub-cases arise.

Case: 1.2. $U_1 < Qc$ (Where $M < N < t_d$)

The revenue generated by the retailer is less than the purchasing cost i.e. $U_1 < Qc$. Thus, the retailer has to pay (if possible) the total due amount at next allowable credit period *N*. So, there may arise two scenarios.

Case: 1.2.1. The supplier accepts the partial payment at *t* = *M* and the rest amount is to be paid at next allowable credit period *t* = *N*.

***Case**: 1.2.2.* The retailer will have to pay the full payment at *t* = *N*, due to unwillingness to accept partial payment by the supplier.

Case: 1.2.1. At *t* = *M*, the supplier agreed to accept partial payment and the rest amount $Qc - U_1$ is to be paid at *t* = *N*. As the retailer made partial payment, hence he/she must have to pay the interest for $Qc - U_1$ amount for the period [*M*, *N*].

Therefore, the total interest payable $IP_{1.2.1.}$ by the retailer during [*M*, *N*] period is,

$$IP_{1.2.1.} = I_p \int_M^N \left(Qc - U_1\right) dt = I_p \left[\mathrm{c} D\left[T + \frac{\alpha(T - t_d)^2}{2} \right] - D\ p\ M\left(1 + \frac{MI_e}{2}\right)\right][N - M]$$

and the total payable amount at $t = N$ ($N > M$) is (say) $Z = Qc - U_1 + IP_{1.2.1}$

The retailer will collect revenues during this period [M, N] (due to sales as well as interest earned on it) is given by

$$U_2 = D\left(N - M\right) p \left(1 + \frac{\left(N + M\right) I_e}{2}\right)$$

Based on the amount of revenue accumulated, there may arise two cases.

Sub Case: 1.2.1.1. $U_2 \geq Z$

Sub Case: 1.2.1.2. $U_2 < Z$

Sub Case: 1.2.1.1. $U_2 \geq Z$

In this case, the revenue generated during [M, N] period is more than the total amount to be paid *i.e.* Z. So, after paying the due amount to the supplier, the retailer will earn interest on the excess amount *i.e.* $U_2 - Z$ for the period [N, T]. Again, the retailer will accumulate the revenue continuously by sales during [N, T] period and earns interest on it.

Therefore, the total interest earned $IE_{1.2.1.1.}$ by the retailer during [N, T] period is,

$IE_{1.2.1.1}$ = Interest earn on the revenue generated during [N, T] period + Higher interest earn on the excess amount if any during [N, T] period

$$IE_{1.2.1.1.} = I_{e_1} \int_N^T \left(U_2 - Z\right) dt + I_e \int_N^T D\ p\ .t\ dt$$

$$IE_{1.2.1.1} = I_{e_1} \left[\begin{array}{l} D\ p\left(N - M\right)\left(1 + \frac{\left(N + M\right) I_e}{2}\right) - cD\left[T + \frac{\alpha\left(T - t_d\right)^2}{2}\right] + D\ p\ M\left(1 + \frac{MI_e}{2}\right) \\ \qquad - I_p\left(cD\left[T + \frac{\alpha\left(T - t_d\right)^2}{2}\right] - D\ p\ M\left(1 + \frac{MI_e}{2}\right)\right)\left(N - M\right) \end{array} \right] \left(T - N\right)$$
$$+ I_e\ D\ p \frac{\left(T^2 - N^2\right)}{2}$$

Therefore, the total average cost $TC_{1.2.1.1}$ for the cycle is given by

$$TC_{1.2.1.1}\left(T\right) = \frac{1}{T} \left\{ \begin{array}{l} A + cap^{-b}\left[T + \frac{\alpha\left(T - t_d\right)^2}{2}\right] + h\ ap^{-b}\left[t_d\left(T - \frac{t_d}{2}\right) + \frac{\left(T - t_d\right)^2}{2}\left(1 + \alpha\ t_d\right)\right] \\ + I_p\left[\mathrm{c}\ D\left[T + \frac{\alpha(T - t_d)^2}{2}\right] - D\ M\left(1 + \frac{MI_e}{2}\right)\right][N - M] \\ - I_{e_1}\left[\begin{array}{l} D\ p\left(N - M\right)\left(1 + \frac{\left(N + M\right) I_e}{2}\right) - cD\left[T + \frac{\alpha\left(T - t_d\right)^2}{2}\right] + D\ p\ M\left(1 + \frac{MI_e}{2}\right) \\ \qquad - I_p\left(c\ D\left[T + \frac{\alpha\left(T - t_d\right)^2}{2}\right] - D\ p\ M\left(1 + \frac{MI_e}{2}\right)\right)\left(N - M\right) \end{array} \right]\left(T - N\right) \\ \qquad + I_e D\ p \frac{\left(T^2 - N^2\right)}{2} \end{array} \right\}$$

Problem. 2. Minimize $TC_{1.2.1.1}(T)$

Subject to $0 < M < N < t_d < t_R < T$

Case: 1.2.1.2. $U_2 < \left(Qc - U_1 + IP_{1.2.1}\right)$

In this case, the revenue generated during [*M*, *N*] period is not sufficient to settle the total due amount *Z*. So, there may again arise two sub cases.

Case: 1.2.1.2.1. The supplier accepts partial payment at $t = N$ and balance amount is to be paid at $t = N1$.

Case: 1.2.1.2.2. The retailer will have to pay the full payment at $t = N1$ due to unwillingness to take partial payment at $t = N$.

Case: 1.2.1.2.1. In this case, the supplier agreed to accept partial payment at $t = N$, and then, the balance amount along with a higher interest charge for the period [*N*, *N1*] must be paid at $t = N1$. Thus, the total amount due at $t = N1$ is (say) $Z_1 = Z - U_2 + IP_{1.2.1.2.1}$.

Since, the retailer made a partial payment at $t = N$, he/she needs to pay higher interest for the balance amount $Z - U_2$ at I_{p1} rate ($I_{p1} > I_p$) for the period [*N*, *N1*].

The full interest payable is therefore $IP_{1.2.1.2.1}$ by the retailer during [*N*, *N1*] period is,

$$IP_{1.2.1.2.1} = I_{p1} \int_{N}^{N1} \left(Qc - U_1 + IP_{1.2.1} - U_2\right) dt$$

$$= I_{p1} \left[\begin{array}{c} \left(c\,D\left[T + \dfrac{\alpha\left(T - t_d\right)^2}{2}\right] - D\,p\,M\left(1 + \dfrac{MI_e}{2}\right) \right. \\ \left. + I_p\left(c\,D\left[T + \dfrac{\alpha\left(T - t_d\right)^2}{2}\right] - D\,p\,M\left(1 + \dfrac{MI_e}{2}\right)\right)\left(N - M\right) \right) \\ -\left(D\left(N - M\right) p\left(1 + \dfrac{\left(N + M\right)I_e}{2}\right)\right) \end{array} \right] \left(N1 - N\right)$$

During the period [*N*, *N1*], the revenue generated by the retailer (due to sales and interest earned on it) is given by

$$U_3 = D\,p\left(N1 - N\right)\left(1 + \frac{\left(N1 + N\right)I_e}{2}\right)$$

In order to settle the account, the revenue generated during [*N*, *N1*] period must be equal to the total due amount *i.e.* $U_3 = Z_1$, that means

$$D\,p\left(N1 - N\right)\left(1 + \frac{\left(N1 + N\right)I_e}{2}\right) = Z - U_2 + IP_{1.2.1.2.1}$$

By solving the above eqn., we may find the value of *N1* as,

$$N1 = \frac{-\left[D\,p - I_{p1}B\right] \pm \sqrt{\left[D\,p - I_{p1}\mathrm{B}\right]^2 + \dfrac{4\,D\,p\,I_e}{2}\left[B\left(1 - I_{p1}N\right) + ap^{1-b}N\left(1 + \dfrac{NI_e}{2}\right)\right]}}{D\,p\,I_e}$$

Where,

$$B = c\,D\left(T + \frac{1}{2}al\left(T - td\right)^2\right) - k\,p\,M\left(1 + \frac{M\,ie}{2}\right)\left(1 + ip\left(N1 - M\right)\right) - k\,p\left(N1 - M\right)\left(1 + \frac{\left(N1 + M\right)ie}{2}\right)$$

After paying the due amount to the supplier, the retailer will accumulate the revenues by selling items, during [*N1*, *T*] period and earn interest continuously on it.

Therefore, the interest earned $IE_{1.2.1.2.1.}$ by the retailer during [*N1*, *T*] period is,

$$IE_{1.2.1.2.1.} = I_e\ p \int_{N1}^{T} D.(t)\,dt \ = \ I_e\ D\ p\left(\frac{T^2 - N1^2}{2}\right)$$

Therefore, the total average cost $TC_{1.2.1.2.1.}$ for the cycle is given by

$$TC_{1.2.1.2.1}(p,T) = \frac{1}{T}\left\{\begin{array}{l} A + cap^{-b}\left[T + \dfrac{\alpha(T-t_d)^2}{2}\right] + h\ ap^{-b}\left[t_d\left(T - \dfrac{t_d}{2}\right) + \dfrac{(T-t_d)^2}{2}(1+\alpha\ t_d)\right] \\ +I_{p1}\left[\begin{array}{c}\left(\begin{array}{l} c\ D\left[T + \dfrac{\alpha(T-t_d)^2}{2}\right] - D\ p\ M\left(1 + \dfrac{MI_e}{2}\right) \\ +I_p\left(c\ D\left[T + \dfrac{\alpha(T-t_d)^2}{2}\right] - D\ p\ M\left(1 + \dfrac{MI_e}{2}\right)\right)(N-M)\end{array}\right) \\ -\left(D(N-M)\,p\left(1 + \dfrac{(N+M)I_e}{2}\right)\right)\end{array}\right](N1-N) \\ -I_e D\ p\ \dfrac{(T^2 - N1^2)}{2}\end{array}\right\}$$

Problem. 3. Minimize $TC_{1.2.1.2.1.}(T)$

Subject to $0 < M < N < t_d < t_R < T$

Case: 1.2.1.2.2. In this case, the supplier didn't agree to accept partial payment at $t = N$. As there is no partial payment made at $t = N$, thus the retailer has to pay interest on *Z* amount for the period [*N*, *N1*] at a higher rate of interest *i.e.* I_{p1} rate ($I_{p1} > I_p$).

Therefore, the total interest payable $IP_{1.2.1.2.2}$ by the retailer during [*N*, *N1*] period is,

$$IP_{1.2.1.2.2} = I_{p1}\int_{N}^{N1}\left(Qc - U_1 + IP_{1.2.1}\right)dt$$

$$= I_{p1}\left(\begin{array}{l} c\ D\left[T + \dfrac{\alpha(T-t_d)^2}{2}\right] - D\ p\ M\left(1 + \dfrac{MI_e}{2}\right) \\ +I_p\left(c\ D\left[T + \dfrac{\alpha(T-t_d)^2}{2}\right] - D\ p\ M\left(1 + \dfrac{MI_e}{2}\right)\right)(N-M)\end{array}\right)(N1-N)$$

Hence, the final amount to be paid at $t = N1$, (say) $Z_2 = Z + IP_{1.2.1.2.2}$, where $Z = Qc - U_1 + IP_{1.2.1}$

As, the supplier unwilling to take partial payment at $t = N$, thus, the retailer will prefer to earn interest (rate of interest is higher than the normal rate) (say *W*) on the accumulated amount U_2 is given by,

$$W = I_{e_1}\ D(N-M)\,p\left(1 + \frac{(N+M)I_e}{2}\right)(N1-N)$$

During the period [*N*, *N1*], the retailer generate revenue (due to sales and interest earned on it) is given by

$$U_3 = D\ p\,(N1-N)\left(1 + \frac{(N1+N)I_e}{2}\right)$$

Therefore, the total revenue generated during [*N*, *N1*] period must be equal to the final amount to be paid *i.e.*

By solving the above equation, we get the value on N1 as,

$$N1=\frac{-X1\pm\sqrt{X1^2+2.D\,p\,I_e.(Y1)}}{D\,p\,I_e}$$

where, $X1=D\,p-ie_1\,D(N-M)\,p\left(1+\frac{N+M}{2}\right)$

$$-ip_1\left(\begin{array}{l} c\,D\left(T+\frac{\alpha(T-t_d)^2}{2}\right)-D\,p\,M\left(1+\frac{M\,ie}{2}\right) \\ +ip\left(c\,D\left(T+\frac{\alpha(T-t_d)^2}{2}\right)-D\,p\,M\left(1+\frac{M\,ie}{2}\right)\right)(N-M) \end{array}\right),$$ and

$$Y1=\left(\begin{array}{l} N\left(\begin{array}{l} ie_1\,D(N-M)\,p\left(1+\frac{(N+M)\,ie}{2}\right)-D\,p\left(1+\frac{N\,ie}{2}\right) \\ +ip_1\left(c\,D\left(T+\frac{\alpha(T-t_d)^2}{2}\right)-D\,p\,M\left(1+\frac{M\,ie}{2}\right)+ip\left(\begin{array}{l} c\,D\left(T+\frac{\alpha(T-t_d)^2}{2}\right) \\ -D\,pM\left(1+\frac{M\,ie}{2}\right) \end{array}\right)(N-M)\right) \end{array}\right) \\ +D\,p\,M\left(1+\frac{M\,ie}{2}\right)-c\,D\left(T+\frac{\alpha(T-t_d)^2}{2}\right)+(N-M)\left(\begin{array}{l} -ip\left(\begin{array}{l} c\,D\left(T+\frac{\alpha(T-t_d)^2}{2}\right) \\ +D\,pM \end{array}\right) \\ +D\,p\left(1+\frac{(N+M)\,ie}{2}\right) \end{array}\right) \end{array}\right)$$

So, after paying the due amount to the supplier at $t = N1$, the retailer will accumulate revenue during [*N1*, *T*] period and earn interest on it.

Thus, the interest earned $IE_{1.2.1.2.2.}$ by the retailer during [*N1*, *T*] period is,

$$IE_{1.2.1.2.2.}=I_e\;p\int_{N1}^{T}D.(t)\,dt=\;I_e\;D\,p\left(\frac{T^2-N1^2}{2}\right)$$

Therefore, the total average cost $TC_{1.2.1.2.2.}$ for the cycle is given by

$$TC_{1.2.1.2.2.}(T)=\frac{1}{T}\left\{\begin{array}{l} A+c\,D\left[T+\frac{\alpha(T-t_d)^2}{2}\right]+h\,D\left[t_d\left(T-\frac{t_d}{2}\right)+\frac{(T-t_d)^2}{2}(1+\alpha\,t_d)\right] \\ +I_{p1}\left[\left(\begin{array}{l} c\,D\left[T+\frac{\alpha(T-t_d)^2}{2}\right]-D\,p\,M\left(1+\frac{MI_e}{2}\right) \\ +I_p\left(c\,D\left[T+\frac{\alpha(T-t_d)^2}{2}\right]-D\,p\,M\left(1+\frac{MI_e}{2}\right)\right)(N-M) \end{array}\right)\right](N1-N) \\ -I_e D\,p\,\frac{\left(T^2-N1^2\right)}{2} \end{array}\right\}$$

Problem. 4. Minimize $TC_{1.2.1.2.2.}(T)$

Subject to $0 < M < N < t_d < t_R < T$

Case: 1.2.2. $U_1 < Qc$ **(The Supplier didn't accept partial payment)**

In this case, the revenue generated during 0 to *M* period is less than the purchasing cost of the total inventory. The supplier unwilling to take partial payment. Thus, the retailer will have to pay the full payment at *t* = *N* (the next allowable credit period).

As the supplier didn't accept a partial payment, hence the retailer must pay the interest for *Qc* amount for the period [*M*, *N*].

Therefore, the total interest payable $IP_{1.2.2.}$ by the retailer during [*M*, *N*] period is,

$$IP_{1.2.2.} = I_p \int_M^N Qc\, dt = I_p \left(\mathrm{c}\, D\left[T + \frac{\alpha(T-t_d)^2}{2} \right][N-M] \right)$$

Hence, the total amount to be paid at *t* = *N* (*N* > *M*) is (say) $Z_4 = Qc + IP_{1.2.2}$

During this period [*M*, *N*], the retailer generate revenue (due to sales and interest earned on it) is given by

$$D\left(N-M\right) p \left(1 + \frac{\left(N+M\right) I_e}{2} \right)$$

Again, the retailer earned a higher interest on U_1 for the period (*M*, *N*) as the supplier didn't accept partial payment.

Thus, the total revenue generated is given by,

$$U_4 = D\left(N-M\right) p \left(1 + \frac{\left(N+M\right) I_e}{2} \right) + U_1 + I_{e_1} \int_M^N U_1\, dt$$

Based on the amount of revenue generated, there may arise two cases.

Case: 1.2.2.1. $U_4 \geq \left(Qc + IP_{1.2.2}\right)$

Case: 1.2.2.2. $U_4 < \left(Qc + IP_{1.2.2}\right)$

Case: 1.2.2.1. $U_4 \geq \left(Qc + IP_{1.2.2}\right)$

In this case, the total revenue generated during [*M*, *N*] period is more than the total amount to be paid *i.e.* $U_4 \geq Z_4$. So, after paying the due amount to the supplier, the retailer will earn interest on the excess amount if any *i.e.* $U_4 - Z_4$ for the period [*N*, *T*]. The retailer also accumulates revenue continuously on the sales during [*N*, *T*] period and earn interest on it.

Thus, the total interest earned $IE_{1.2.2.1.}$ by the retailer during [*N*, *T*] period is,

$IE_{1.2.2.1}$ = Interest earn on the revenue generated during [*N*, *T*] period + Higher interest earn on the excess amount if any during [*N*, *T*] period on $U_4 - Z_4$.

$$IE_{1.2.2.1} = I_{e_1} \int_N^T \left(U_4 - Z_4\right) dt + I_e\, p \int_N^T D.t\, dt$$

$$= I_{e_1} \left[\begin{array}{l} D\, p\left(N-M\right)\left(1 + \frac{\left(N+M\right)I_e}{2}\right) + D\, p\, M\left(1 + \frac{MI_e}{2}\right) + I_{e_1} D\, p\, M\left(1 + \frac{MI_e}{2}\right)\left(N-M\right) \\ \quad - c\, D\left[T + \frac{\alpha\left(T-t_d\right)^2}{2}\right] - I_p\left(\mathrm{c}D\left[T + \frac{\alpha(T-t_d)^2}{2}\right][N-M]\right) \end{array} \right]\left(T-N\right) + I_e D\, p\, \frac{\left(T^2 - N^2\right)}{2}$$

Therefore, the total average cost $TC_{1.2.2.1}$ for the cycle is given by

$$TC_{1.2.2.1}(T)=\frac{1}{T}\left\{\begin{array}{l} A+c\,D\left[T+\frac{\alpha\left(T-t_d\right)^2}{2}\right]+h\,D\left[t_d\left(T-\frac{t_d}{2}\right)+\frac{\left(T-t_d\right)^2}{2}\left(1+\alpha\,t_d\right)\right]+I_p\left(\mathrm{c}D\left[T+\frac{\alpha(T-t_d)^2}{2}\right][N-M]\right)-I_{e_1}\left[\begin{array}{l} D\,p\left(N-M\right)\left(1+\frac{\left(N+M\right)I_e}{2}\right)+I_{e_1}D\,p\,M\left(1+\frac{MI_e}{2}\right)\left(N-M\right) \\ -c\,D\left[T+\frac{\alpha\left(T-t_d\right)^2}{2}\right]-I_p\left(\mathrm{c}D\left[T+\frac{\alpha(T-t_d)^2}{2}\right][N-M]\right) \end{array}\right]\left(T-N\right) \\ +I_e\,D\,p\frac{\left(T^2-N^2\right)}{2} \end{array}\right\}$$

Problem. 5. Minimize $TC_{1.2.2.1}(T)$

Subject to $0<M<N<t_d<t_R<T$

Case: 1.2.2.2. $U_4<\left(Qc+IP_{1.2.2}\right)$

In this case, the total revenue generated during [*M*, *N*] period is insufficient to settle the total amount to be paid *i.e.* $U_4<Z_4$. As the supplier did not accept any partial payment, thus, the retailer will have to pay the full amount at *t* = *N*1, which we have to find out.

As the supplier didn't accepts partial payment, hence the retailer must pay the higher interest for $Z_4=Qc+IP_{1.2.2}$ amount for the period [*N*, e*N1*].

Thus, the total interest payable $IP_{1.2.2.2}$ by the retailer during [*N*, *N1*] period is,

$$IP_{1.2.2.2.}=I_{p1}\left(c\,D\left(T+\frac{\alpha\left(T-t_d\right)^2}{2}\right)\left(1+I_p\left(N-M\right)\right)\right)\left(N1-N\right)$$

Hence, the total amount to be paid at *t* = *N1* (*N1* > *N*) is (say) $Z_5=Qc+IP_{1.2.2}+IP_{1.2.2.2.}$

During this period [*N*, *N1*] the retailer generate revenue (due to sales and interest earned on it) is given by

$$D\left(N1-N\right)p\left(1+\frac{\left(N1+N\right)I_e}{2}\right)$$

Again, the retailer earns a higher interest on U_5 for the period (*N*, *N1*) as no partial payment made.

Thus, the total revenue generated is given by,

$$U_5=D\left(N1-N\right)p\left(1+\frac{\left(N1+N\right)I_e}{2}\right)+U_4+I_{e_1}\int_N^{N1}U_4\,dt$$

Again, the revenue generated during [*N*, *N1*] period must be equal to the final amount to be paid *i.e.* $U_5=Z_5$

By solving this, we get *N1* as,

$$N1=\frac{-X\pm\sqrt{X^2+2D\,p\,I_e.Y}}{D\,p\,I_e}$$

Where $X=\left[I_{e_1}\left[\begin{array}{l} D\,p\left(N-M\right)\left(1+\frac{\left(N+M\right)I_e}{2}\right) \\ +I_{e_1}D\,p\,M\left(1+\frac{MI_e}{2}\right)\left(N-M\right) \end{array}\right]-I_{p1}\left(\begin{array}{l} c\,D\left(T+\frac{\alpha\left(T-t_d\right)^2}{2}\right). \\ .\left(1+I_p\left(N-M\right)\right) \end{array}\right)+D\,p\right]$, and

$$Y = -I_e N\left[D\,p\left(N-M\right)\left(1+\frac{\left(N+M\right)I_e}{2}\right)+I_{e_1}D\,p\,M\left(1+\frac{MI_e}{2}\right)\left(N-M\right)\right]+$$

$$I_p\left[c\,D\left(T+\frac{\alpha\left(T-t_d\right)^2}{2}\right).\left(N-M\right)\right]-I_{p_1}N\left(c\,D\left(T+\frac{\alpha\left(T-t_d\right)^2}{2}\right).\left(1+I_p\left(N-M\right)\right)\right)$$

$$+D\,p\left[N+\frac{N^2 I_e}{2}\right]$$

After paying the total due amount at t = $N1$, the retailer accumulates revenues continuously on the sales during [*N1*, *T*] period and earn interest on it.

Thus, the total interest earned $IE_{1.2.2.2.}$ by the retailer during [*N*, *T*] period is,

$$IE_{1.2.2.2} = I_e\ p\int_N^T D.t\ dt = I_e\ D\,p\left(\frac{T^2-N1^2}{2}\right)$$

Therefore, the total average cost $TC_{1.2.2.2}$ for the cycle is given by

$$TC_{1.2.2.2.}\left(p,T\right)=\frac{1}{T}\left\{\begin{array}{l} A+c\,D\left[T+\frac{\alpha\left(T-t_d\right)^2}{2}\right]+h\,D\left[t_d\left(T-\frac{t_d}{2}\right)+\frac{\left(T-t_d\right)^2}{2}\left(1+\alpha\,t_d\right)\right] \\ +I_{p1}\left(c\,D\left(T+\frac{\alpha\left(T-t_d\right)^2}{2}\right)\left(1+I_p\left(N-M\right)\right)\right)\left(N1-N\right) \\ \qquad -I_e\ D\,p\,\frac{\left(T^2-N1^2\right)}{2} \end{array}\right\}$$

Problem. 6. Minimize $TC_{1.2.2.2}\left(T\right)$

Subject to $0<M<N<t_d<T$

Case: 1.3. $0<M<t_d<N$

In realistic condition we can't say that *N* will lies always before t_d, it may also lie after t_d. So, to check the reliability of this model we have consider another example, where *N* is greater than t_d. This case is again bifurcated into several sub-cases depending on the various scenarios.

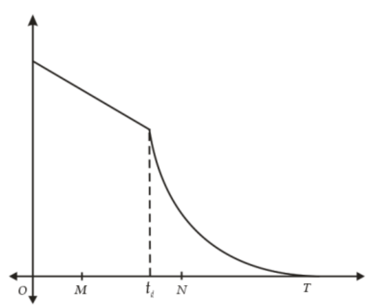


Figure – 4 Graphical representation for case 1.3

Case: 1.3.1.1. In this case the revenue generated during [*M*, *N*] period is more than the total amount to be paid *i.e.* $U_2 > Qc + IP_{1.2.1}$. The account is settled at *N,* and the retailer will earn interest from *N* to *T* on the excess amount after settling the account as well as on the sales revenue. Frome [*N*, *T*] period, the retailer earn interest on the sales revenue and on the rest amount after settling the account. As only the position of *N* is changed, the mathematical representation of the case is same as the case 1.2.1.1. Bifurcation of this case is similar to case 1.2.1.1. The sub-cases of this case 1.3.1.1 is named as Case 1.3.1.2.1, Case 1.3.1.2.2, Case 1.3.2.1 and Case 1.3.2.2 is similar to the sub-cases of Case 1.2.1.1 respectively. After studying the above cases we found that the mathematical representations for all these sub-cases are similar to the sub-cases of the previous Case 1.2. But we have changed the value of *M*, and *N* so, the average cost is also changed. We have considered this and show the outcomes in our result table 2.

5.2. Case 2: $t_d < M < T$

By changing the position of the credit period *M, i.e.* $(M > t_d)$ we observed that, the mathematical representation of different cases derived based upon the realistic condition is equivalent to the cases discussed in Case 1.

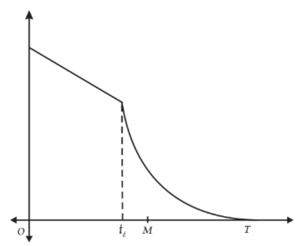


Figure – 5 Graphical representation for case 2

Bifurcation of this case is similar to Case 1.2. The subcases of this Case such as 2.1, 2.2.1.1, 2.2.1.2.1, 2.2.1.2.2, 2.2.2.1 and 2.2.2.2 is similar to the subcases of Case 1.2. respectively.
After studying the above cases we found that the mathematical representations for all these sub-cases are similar to the sub-cases of the previous Case 1.2. But we have changed the value of *M*, and *N* so, the average cost is also changed. We have considered this and show the outcomes in our result table 3.

5.3. Case 3: $T < M$

In this case, the credit period given to the retailer is more than the cycle period. Thus, in this case, the retailer didn't pay any interest to the supplier. The retailer only earn interest on the revenue generated by selling the items.

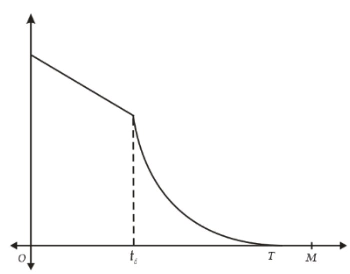


Figure – 6 Graphical representation for case 3.

Therefore, the total interest earned IE_3 by the retailer during [*0*, *M*] period is,

$$IE_3 = I_e\ p\int_0^T D.t\ dt + I_{e1}\int_T^M U_1 = I_e D\ p\left[\frac{T^2}{2}\right] + I_{e_1} D\ p\ T\left(1 + I_e\,\frac{T}{2}\right)\left[M - T\right]$$

Therefore, the total average cost TC_3 for the cycle is given by

$$TC_3(T)=\frac{1}{T}\left\{\begin{array}{c} A+c\,D\left[T+\dfrac{\alpha\left(T-t_d\right)^2}{2}\right]+h\,D\left[t_d\left(T-\dfrac{t_d}{2}\right)+\dfrac{\left(T-t_d\right)^2}{2}\left(1+\alpha\,t_d\right)\right] \\ -I_e D\,p\left[\dfrac{T^2}{2}\right]+I_{e_1} D\,p\,T\left(1+I_e\dfrac{T}{2}\right)\left[M-T\right] \end{array}\right\}$$

The derivatives of first and second order were calculated and are very complex. It is therefore very hard to mathematically demonstrate the convexity. The average cost function is highly non-linear, very complicated to solve, the convexity of the function can't be tested mathematically. Alternatively, MATHEMATICA is used for the graphic determination of the convexity of all overall cost functions.

6. SOLUTION PROCEDURE

Step 0: Input all the initial value of parameters.
Step 1: If the retailer pays full amount in $t = M$, solve the optimization problem-1 and save the result as $T_{1.1}$, $Q_{1.1}$ and $TC_{1.1}$ else go to Step-2.
Step 2: If partial payment is made at $t = M$ and full amount at $t = N$, then solve the optimization problem-2 and save the result as $T_{1.2.1.1}$, $Q_{1.2.1.1}$ and $TC_{1.2.1.1}$ else go to Step-3.
Step 3: If partial payment is made at $t = M$, then at $t = N$, and full amount at $t = N1$, then solve the optimization problem-3 and save the result as $N1_{1.2.1.2.1}$, $T_{1.2.1.2.1}$, $Q_{1.2.1.2.1}$, and $TC_{1.2.1.2.1}$ else go to step–4.
Step 4: If partial payment is made at $t = M$ but unwilling to take partial payment at $t = N$, and full amount at $t = N1$, then solve the optimization problem-4 and save the result as $N1_{1.2.1.2.1}$, $T_{1.2.1.2.1}$, $Q_{1.2.1.2.1}$ and $TC_{1.2.1.2.1}$ else go to step–5.
Step 5: If unwilling to take partial payment at $t = M$, and full amount at $t = N$, then solve the optimization problem-5 and save the result as $T_{1.2.2.1}$, $Q_{1.2.2.1}$ and $TC_{1.2.2.1}$, else go to step–6.
Step 6: If unwilling to take partial payment at $t = M$ and N, full amount at $t = N1$, then solve the optimization problem-6 and save the result as $N1_{1.2.2.1}$, $T_{1.2.2.1}$, $Q_{1.2.2.1}$ and $TC_{1.2.2.1}$
Step 7: As we also consider the cases when $N > t_d$, so, follow the same steps and solve the constrained optimization problems for all cases and store the total cost result respectively.
Step 8: The optimal solution of case 1, can be determined from the solutions of the all cases. Hence, for case 1, the optimal average total cost per unit of time is given by,

$$TC^1=min\left\{\begin{array}{l} TC_{1.1},\ TC_{1.2.1.1},\ TC_{1.2.1.2.1},\ TC_{1.2.1.2.2},\ TC_{1.2.2.1},\ TC_{1.2.2.2},\ TC_{1.3.1.1},\ TC_{1.3.1.2.1}, \\ TC_{1.3.1.2.2},\ TC_{1.3.2.1}\text{ and }TC_{1.3.2.2} \end{array}\right\}$$

and the corresponding values of $N1$, T and Q as $N1^1$, T^1 and Q^1.

Proceeding in the similar way, the problems for case 2 can be solved. For case 3, the credit period is higher than the replenishment cycle. So, at $t = M$, the retailer will easily settle the credit amount. The optimal total inventory cost for case-2, case-3 and the corresponding solutions of different variables and ordered quantity are denoted as TC^2, TC^3 $\left(N1^2,\ T^2 \text{ and } Q^2\right)$ and $\left(T^3 \text{ and } Q^3\right)$ respectively. (*N1* value will stored in only those cases where, the retailer goes for the progressive credit period)
By comparing the total inventory costs for all cases, the optimal solution for the inventory system can be determined. The optimum total cost per unit of time per inventory is therefore determined by

$TC^* = min\left\{ TC^1, TC^2, TC^3 \right\}$.The corresponding values of optimal decision variables and ordered quantity for the problem is denoted by $N1^*$, T^* and Q^*.

7. NUMERICAL ANALYSIS

To demonstrate the developed model, we have following sets of examples.

In *example 1*, we assumed that, *N* lies before t_d *i.e.* $0 < M < N < t_d$ and by putting all the parameter value in the formulated cost functions we get the total average cost.

In *example 2*, we only change the value of *N*, as we assumed that *N* lies after t_d *i.e.* $0 < M < t_d < N$ and get the total average cost for the cases, sub-cases and scenarios studied under the case 1.3.

In *example 3*, we only change the value of *M* and *N*, as we assumed that *M* lies after t_d *i.e.* $t_d < M < N < T$ and get the total average cost for the cases, sub-cases and scenarios studied under the case 2.

In *example 4*, we only change the value of *M*, as we assumed that *M* is greater than *T*, and get the total average cost for the cases, sub-cases and scenarios studied under the case 3.

Table 1: Different Examples taken for numerical study

Parameters	D (*Per Year*)	A (\$)	c (\$)	p (\$)	h (\$)	I_e (\$)	I_{e1} (\$)	I_p (\$)	I_{p1} (\$)	M (In Yr)	N (In Yr)	t_d (In Yr)	α
Example 1	20000	800	40	45	8	0.07	0.08	0.09	0.11	0.01918	0.032877	0.04	0.07
Example 2	20000	800	40	45	8	0.07	0.08	0.09	0.11	0.01918	0.05	0.04	0.07
Example 3	20000	800	40	45	8	0.07	0.08	0.09	0.11	0.05	0.06	0.04	0.07
Example 4	20000	800	40	45	8	0.07	0.08	0.09	0.11	0.08		0.04	0.07

Result of example 1:

On basis of the position of M, td, N, example 1 is considered for different cases, subcases and scenarios and got the result as given in the following table.

Table 2: Results Obtained for various cases and sub cases for example 1

Case	*Subcase*	*Scenario*	*Sub scenario*	*Position of N*	*N1*	*T*	*Q*	*Average Cost*
1. (0 < M < td)	**1.1 (U1 > Qc)**	–	–	–	–	**0.0791839**	**1584.75**	**817432.78**
	1.2 (U1 < Qc)	1.2.1	1.2.1.1	N < td	–	0.0652389	1305.19	820848.56
			1.2.1.2	–	–	–	–	–
			1.2.1.2.1	N < td	0.0645	0.0726497	1453.74	818126.61
			1.2.1.2.2	N < td	0.0629	0.0708841	1418.35	818713.13
		1.2.2	1.2.2.1	N < td	–	0.0660358	1321.19	821172.22
			1.2.2.2	N < td	0.0609	0.0682823	1366.21	819550.62

It is observed that the average total cost is the lowest one for the case 1.1. Hence, the optimal solution for example 1 is as follows:

$T^* = 0.0791839$ year, $Q^* = 1584.75$ units and average total cost = 817432.78

Result of example 2:

On basis of the position of *M*, t_d, *N*, example 2 is considered for different cases, sub-cases and scenarios and got the result as given in the following table.

Table 3: Results Obtained for various cases and sub cases for example 2

Case	Subcase	Scenario	Sub scenario	Position of N	N1	T	Q	Average Cost
	1.3		1.3.1.1	td < N	–	0.0651491	1303.42	820344.7
			1.3.1.2.1	**td < N**	**0.0771**	**0.086866**	**1738.88**	**815887.78**
			1.3.1.2.2	td < N	0.064	0.072249	1445.71	817645.07
			1.3.2.1	td < N	–	0.0681356	1363.27	820933.04
			1.3.2.2	td < N	0.061	0.0682615	1365.79	818056.95

It is observed that the average total cost is the lowest for the case 1.3.1.2.1. Hence, the optimal solution for example 2 is as follows:

$T^* = 0.086866$ year, $N1^* = 0.0771$ year, $Q^* = 1738.88$ units and average total cost = 815887.78

Result of example 3:

On basis of the position of M, t_d, N, example 3 is considered for different cases, subcases and scenarios and got the result as given in the following table.

Table 4 – Results Obtained for various cases and sub cases for example 3

Case	Subcase	Scenario	Sub scenario	Position of N	N1	T	Q	Average Cost
2. (td < M < T)	2.1 (U1 > Qc)	–	–	–	–	0.0885414	1772.48	815869.24
	2.2 (U1 < Qc)	2.2.1	–	–	–	–	–	–
			2.2.1.1	–	–	0.068309	1366.74	818130.82
			2.2.1.2		–	–	–	–
			2.2.1.2.1		**0.0818**	**0.0921098**	**1844.1**	**815726.48**
			2.2.1.2.2		0.0706	0.0797128	1595.36	816712.44
		2.2.2	–	–	–	–	–	–
			2.2.2.1		–	0.0697559	1395.74	818718.15
			2.2.2.2		0.061	0.0683584	1367.73	817164.15

It is observed that the average total cost is the lowest for the case 2.2.1.2.1. Hence, the optimal solution for example 3 is as follows:

$T^* = 0.0921098$ year, $N1^* = 0.0818$ year, $Q^* = 1844.1$ units and average total cost = 815726.48

Result of example 4:

On basis of the position of M when $M > T$, there exist only a single case 3.1. For this, example 4 is considered and got the result as follows:

$T^* = 0.0753388$ year, $Q^* = 1507.65$ units and average total cost = 814403.89

By observing all the above examples, we get the optimal solution at example 4, where $M > T$. But when we consider the realistic conditions, this case may not always be satisfied, because it occurs in very rare cases, when the supplier offers a credit period more than the cycle length. If, we ignore this rare case, we can observe from all the examples that the optimality occurs in case 2.2.1.2.1. where the progressive trade credit period is considered. This shows that the progressive trade credit facility is more beneficial for the retailer.

Figu ase 1.1.

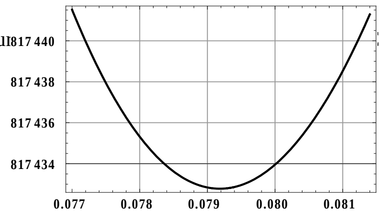


Figure – 8 Graph of average Cost versus T for Case 1.3.1.2.1.

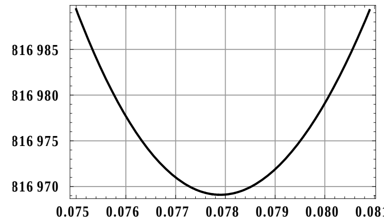


Figure – 9 Graph of average Cost versus T for Case 2.2.1.2.1.

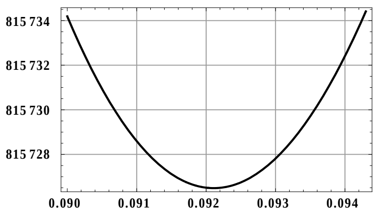
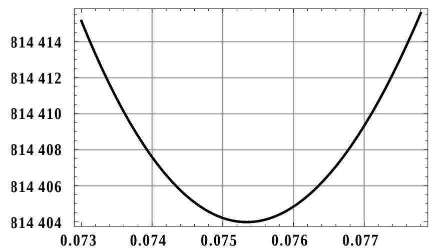


Figure – 10 Graph of average Cost versus T for Case 3.1.



8. SENSITIVITY ANALYSIS

Table – 5 Sensitivity Analysis

Changing Parameter	% Change in Parametre	N1	T	Q	Average Cost
c	–10	0.0780258	0.0976936	1956.2	735285.5
	–5	0.079949	0.0948375	1898.86	775499.43
	5	0.833912	0.0895064	1791.84	855965.7
	10	0.0849349	0.0870221	1741.99	896216.17
p	–10	0.0859445	0.0871732	1745.02	816262.49
	–5	0.0837343	0.0896488	1794.7	815984.9
	5	0.0799163	0.0945626	1893.34	815483.88
	10	0.078263	0.0970123	1942.52	815254.26
h	–10	0.0839312	0.0945754	1893.59	814979.19
	–5	0.0828103	0.0933183	1868.36	815355.31
	5	0.0806966	0.0909468	1820.75	816092.91
	10	0.0796987	0.0898268	1798.27	816454.76

Changing Parameter	% Change in Parametre	N1	T	Q	Average Cost
D	–10	0.0844492	0.0951562	1714.73	735008.24
	–5	0.0830305	0.0935653	1779.65	775371.02
	5	0.0805413	0.0907725	1908.12	856075.37
	10	0.0794423	0.089539	1971.75	896418.32
td	–10	0.0814765	0.0918069	1838.32	815858.84
	–5	0.0816012	0.0919545	1841.13	815791.5
	5	0.081872	0.0922728	1847.23	815663.81
	10	0.0820178	0.0924433	1850.51	815603.44
M	–10	0.0817141	0.0921062	1844.023	815726.61
	–5	0.0817231	0.0921081	1844.06	815726.52
	5	0.0817438	0.0921111	1844.12	815726.51
	10	0.0817551	0.0921118	1844.14	815726.58
N	–10	0.0840652	0.153796	3082.46	826043.92
	–5	0.0840469	0.153789	3081.99	825959.96
	5	0.0840213	0.153786	3081.31	825795.15
	10	0.0840151	0.153791	3081.1	825714.29
ie	–10	0.0840034	0.153757	3081.22	827363.88
	–5	0.0840185	0.153772	3081.52	826620.46
	5	0.0840457	0.153799	3082.07	825133.62
	10	0.0840607	0.153814	3082.37	824390.19
ip	–10	0.0840034	0.153757	3081.22	827363.88
	–5	0.0840185	0.153772	3081.52	826620.46
	5	0.0840457	0.153799	3082.07	825133.62
	10	0.0840607	0.153814	3082.37	824390.19
ip1	–10	0.0840034	0.153757	3081.22	827363.88
	–5	0.0840185	0.153772	3081.52	826620.46
	5	0.0840457	0.153799	3082.07	825133.62
	10	0.0840607	0.153814	3082.37	824390.19
A	–10	0.0840034	0.153757	3081.22	827363.88
	–5	0.0840185	0.153772	3081.52	826620.46
	5	0.0840457	0.153799	3082.07	825133.62
	10	0.0840607	0.153814	3082.37	824390.19
a	–10	0.0840034	0.153757	3081.22	827363.88
	–5	0.0840185	0.153772	3081.52	826620.46
	5	0.0840457	0.153799	3082.07	825133.62
	10	0.0840607	0.153814	3082.37	824390.19

To study the reliability of the model, we have analysed the sensitiveness on parameters c, p, h, I_e, I_p, I_{p1}, D, t_d, M, N, W, A, α, for the optimal policies by altering the parameters to +10%, +5%, -5% and –10% by removing a parameter and maintaining the remaining parameters intact. The results of this analysis are given in Table 5.

9. MANAGERIAL INSIGHTS

Based on the results of Table 5, we can obtain the following managerial ideas.

1) The total average inventory cost is sensitive to the demand, holding cost, deterioration rate, ordering cost, higher interest charge and increases with increment of these parameters value.
2) The total average inventory cost is sensitive to the permissible delay in payment M, N, selling price p, interest earn and decreases with increment of these parameters value.
3) The total average inventory cost is highly sensitive to the demand and ordering cost. As the value of these parameters are increases the total average inventory cost increases.

10. CONCLUSION

This paper attempts to develop an inventory model for non-instantaneous, deteriorating products, taking account of the acceptability of progressive payment delays. Various cases based on the permissible delay period offered by supplier are investigated and by using the numerical examples results are compared. This shows that the progressive trade credit facility is more beneficial for the retailer.

The suggested model can be expanded in a number of ways for further studies. This model can be extended with other types of demand. The two-level credit policy can generalize this model.

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