# BAYESIAN PREDICTION IN EXPONENTIAL DISTRIBUTION WITH RANDOM SAMPLE SIZE UNDER MULTIPLY TYPE-II CENSORING

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**ABSTRACT**
The prediction of future observations can tell us at an early stage of testing how costly the testing is and whether actions should be taken to redesign the test. This paper deals with the Bayes interval prediction of the future ordered observations in a multiply type-II censored sample from an exponential distribution. The analysis will depend mainly on assumption that the sample size n is fixed as well as random variable. The Binomial and Poisson distributions are used for describing the random variable n. Numerical example is cited to illustrate the procedure and simulation study is carried out to study the performance of the process.

**KEY WORDS:** Future failures, Gamma prior, Non informative prior, Survival function, Confidence interval.

**MSC:** 62F15, 62F30, 62N01

**RESUMEN**
La predicción de futuras observaciones puede decirnos, en una etapa temprana de la prueba, cuan costosa es y las acciones que deben ser tomadas para rediseñar el test. Este paper trata de la predicción Bayesiana de la predicción de las futuras observaciones, ordenadas en una muestra censurada múltiple del tipo-II de una distribución exponencial. El análisis dependerá, principalmente de la asunción de que el tamaño n de la muestra es fija o una variable aleatoria. Se usan las distribuciones Binomial y Poisson para describir la variable aleatoria n. Un ejemplo numérico es citado para ilustrar el procedimiento y el estudio de simulación es desarrollado para estudiar el comportamiento del proceso.

**KEY WORDS:** Fallos futuros , Gamma prior, prior No-informativa, function de sobrevida , intervalo de Confianza.

## 1. INTRODUCTION

One of the most important problems in life testing is predicting future failures on the basis of available observed failure times. The prediction of future ordered observations shows how long a sample of units might run till all fail in life testing. The use of classical approach needs to construct some pivotal statistic to construct confidence interval of the future failure times, but sometimes to get the exact distribution of such a statistic is mathematically intractable and hence it is impossible to obtain exact predictive points for it by analytical methods. Usually Monte Carlo Markov Chain sampling is used in such situation. However, now a day a Bayes predictive approach is receiving much attention.

In life testing experiments the test is terminated either at predetermined time is observed (Type-I censoring) or at a predetermined number of failures observed (Type-II censoring). Such censoring schemes may be from left or right. Sometimes left and right censoring appears together, this is known as doubly censoring. Furthermore, if mid censoring arises amongst the doubly censoring in the type II censoring scheme, the scheme is known as multiply type-II censoring, see Sing and Kumar(2007), Shah and Patel(2008).

The exponential distribution received more emphasis in life testing experiment as a life time model due to its interesting properties. Several researchers have studied Bayesian prediction in the case of exponential distribution , among them are e.g. Al-Hussaini and Jaheen (1999), Al-hussaini (1999). In all these references, the sizes for old or future samples were taken fixed. The problem of Statistical inference when the sample size is a random variable is very important in practice. Samples of random size occur frequently in a natural manner. In some applications it is almost impossible to have a fixed sample size because some observations are always missing for various reasons, in medical or biological experiments some patients of the sample under consideration may left or die during the experiments. Prediction problem based on random sample size from exponential distribution have been treated by Lingappaiah (1986). The prediction is made in a future

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sample based on earlier (k-r) samples when sample size fixed as well as random. Abdel-Atq et al (2003) considered a Bayesian prediction for exponential distribution with change point under fixed and random sample size to predict failure times of second sample. All such works are based on simply under type-II censoring scheme.
This paper is concerned with the Bayesian predictive interval for the remaining ordered failure times based on the multiply type-II censored data when sample size is random. When the population is very large or infinite Poisson distribution is used and for the finite population binomial distribution is used as the model for the variable sample size. The prediction intervals are constructed under informative (gamma) prior as well as non informative prior suggested by Martz and Waller(1982) for the parameter of the exponential distribution. A simulation study is carried out to check the performance of the new setup with fixed sample sized.

## 2. MAIN RESULTS

The exponential probability density function and its cumulative distribution function are given by

$f(x|\theta)=\theta e^{-\theta x},\quad x>0,\quad \theta>0$ and

$$F(x|\theta)=1-e^{-\theta x},\ \mathrm{x}>0 \tag{1}$$

where $\theta$ is a scale parameter.
Consider the multiply type-II censoring scheme as follow:
Out of n observations on the test first r failure timers are not observed, then k ordered failure times are observed. After that again t ordered failure times are not observed and then the test is terminated as soon as the s-th failure is observed. Thus only ordered observations available are

$x_{r+1}<x_{r+2}<\dots\dots<x_{r+k}$ and $x_{r+k+t+1}<x_{r+k+t+2}<\dots\dots<x_{s}$

Note that (i) When r = 0 and t = 0 the scheme reduces to type-II censoring scheme.
(ii) When t= 0 the scheme reduces to doubly type –II censoring.
(iii) When r = 0 and s = n the scheme reduces to middle censoring.
(iv) When r = t= 0 and s = n the scheme reduces to complete sample scheme.

The likelihood function based on such a multiply type-II censoring scheme can be written as

$$L=\frac{n!}{r!t!(n-s)!}\left(F(x_{r+1})\right)^{r}\left(F(x_{r+k+t+1})-F(x_{r+k})\right)^{t}\prod_{i=r+1}^{r+k}f(x_i)\prod_{i=r+k+t+1}^{s}f(x_i)\left(1-F(x_s)\right)^{n-s} \tag{2}$$

Using (1) , the likelihood function becomes

$$L=C\sum_{j=0}^{r}\sum_{w=0}^{t}h_{1j}h_{2w}\theta^{s-r-t}e^{-\theta\{T+jx_{r+1}+wA_1\}} \tag{3}$$

where

$$C=\frac{n!}{r!t!(n-s)!},\ h_{1j}=\binom{r}{j}(-1)^{j},\quad h_{2w}=\binom{t}{w}(-1)^{w},\quad \mathrm{T}=\sum_{i=r+1}^{r+k}x_i+\sum_{i=r+k+t+1}^{s}x_i+(n-s)x_s+tx_{r+k},$$

$$A_1=x_{r+k+t+1}-x_{r+k}$$

### 2.1 Natural Conjugate Prior

A natural conjugate prior for the parameter θ of the exponential distribution is well known to be a gamma prior, given as

$$\pi(\theta)=e^{-b\theta}\theta^{a-1}b^{a}/\Gamma a \tag{4}$$

where the hyper parameters a and b are chosen to reflect our beliefs. Applying Bayes theorem and using the likelihood (3) we get the posterior density of θ given x under conjugate prior as

$$h_1(\theta|x)=\frac{L\pi(\theta)}{\int_\theta L\pi(\theta)d\theta}$$

$$=\frac{\sum_{j=0}^{r}\sum_{w=0}^{t} h_{1j}h_{2w}\theta^{n_1-1}e^{-\theta\{H+jx_{r+1}+wA_1\}}}{\Gamma n_1\sum_{j=0}^{r}\sum_{w=0}^{t} h_{1j}h_{2w}\{H+jx_{r+1}+wA_1\}^{-n_1}},\ \theta>0,\ \text{and}\ n_1=s-r-t+a \qquad (5)$$

**2.2 Non Informative Prior**

A general non informative prior for the parameter θ may be taken as ( See: Martz and Waller (1982))

$$\pi_2(\theta)\propto\frac{1}{\theta^\beta},\theta>0,\beta>0. \qquad (6)$$

For β = 1 it reduces to Jeffery's non informative prior. Again the posterior distribution of θ can be obtained by combining this non informative prior with the likelihood function via Bayes theorem as

$$h_2(\theta|x)=\frac{\sum_{j=0}^{r}\sum_{w=0}^{t} h_{1j}h_{2w}\theta^{n_2-1}e^{-\theta\{T+jx_{r+1}+wA_1\}}}{\Gamma n_2\sum_{j=0}^{r}\sum_{w=0}^{t} h_{1j}h_{2w}\{T+jx_{r+1}+wA_1\}^{-n_2}},\ \theta>0, \qquad (7)$$

**3. BAYES PREDICTIVE INTERVAL FOR THE REMAINING (N-S) ORDER STATISTICS UNDER NATURAL CONJUGATE PRIOR**

Now we consider the predictive interval for the remaining ( n-s) failure times given the observed failure times under multiply type-II censoring of a sample of size n units whose life time distribution is exponential given in (1).
Let $x_{s+m}$ denote the failure time of (s+m)th component, m = 1, 2, … , n-s.
Set for simplicity y = $x_{s+m}$. then the conditional density of y given x is given by

$$f(y|x)=\frac{m\binom{n-s}{m}\{F(y)-F(x_s)\}^{m-1}\{1-F(y)\}^{n-s-m}f(y)}{\{1-F(x_s)\}^{n-s}},\ y>x_s \qquad (8)$$

and for the exponential distribution it becomes

$$f(y|x)=m\binom{n-s}{m}\sum_{i=0}^{m-1}h_{3i}\theta e^{-\theta\{(y-x_s)(n-d+i\}},\ y>x_s \qquad (9)$$

where $h_{3i}=\binom{m-1}{i}(-1)^i,\ d = s+m-1.$

Forming the product of equations (5) and (9) and integrating over θ gives the predictive density of y given x as

$$p(y|x)=\frac{m\binom{n-s}{m}n_1\sum_{i=0}^{m-1}\sum_{j=0}^{r}\sum_{w=0}^{t}h_{3i}h_{1j}h_{2w}\{(y-x_s)(n-d+i)+H+jx_{r+1}+wA_1\}^{-(n_1+1)}}{\sum_{j=0}^{r}\sum_{w=0}^{t}h_{1j}h_{2w}\{H+jx_{r+1}+wA_1\}^{-n_1}}, \qquad y>x_s \tag{10}$$

**3.1 When sample size n has a Poisson distribution**

For infinite or very large population size we suppose that the sample size n is a Poisson random variable with probability mass function(pmf)

$$p_1(n)=\frac{e^{-\lambda}\lambda^n}{n!}, n=0,1,2,........, \lambda>0. \tag{11}$$

Using Consul (1984) and Gupta and Gupta (1984), the predictive density of y given x is obtained as

$$p_1(y|x)=\frac{\sum_{n=s+m}^{\infty}p(y|x)\,p_1(n)}{P(n\geq s+m)}, m=1,2,....,n-s. \tag{12}$$

Using (10) and (11) in (12) we get

$$p_1(y|x)=C_1 m n_1\sum_{n=s+m}^{\infty}\frac{\binom{n-s}{m}\sum_{i=0}^{m-1}\sum_{j=0}^{r}\sum_{w=0}^{t}h_{3i}h_{1j}h_{2w}\left\{\begin{matrix}(y-x_s)(n-d+i)+\\ H+jx_{r+1}+wA_1\end{matrix}\right\}^{-(n_1+1)}\lambda^n/n!}{\sum_{j=0}^{r}\sum_{w=0}^{t}h_{1j}h_{2w}\{H+jx_{r+1}+wA_1\}^{-n_1}} \tag{13}$$

where $C_1=e^{-\lambda}\left\{1-\sum_{i=0}^{s+m-1}\frac{e^{-\lambda}\lambda^i}{i!}\right\}^{-1}$

From which the predictive survival function of y is given by

$$P(y\geq t_0|x)=\int_{t_0}^{\infty}p_1(y|x)$$

$$=C_1 m\sum_{n=s+m}^{\infty}\frac{\binom{n-s}{m}\sum_{i=0}^{m-1}\sum_{j=0}^{r}\sum_{w=0}^{t}\frac{h_{3i}h_{1j}h_{2w}}{(n-d+i)}\left\{\begin{matrix}(t_0-x_s)(n-d+i)+\\ H+jx_{r+1}+wA_1\end{matrix}\right\}^{-n_1}\lambda^n/n!}{\sum_{j=0}^{r}\sum_{w=0}^{t}h_{1j}h_{2w}\{H+jx_{r+1}+wA_1\}^{-n_1}} \tag{14}$$

To obtain the lower and upper (1-α)100% prediction bounds for y, iterative numerical methods are required by defining $t_0$ from (14) for a given value of $\tau=P(y\geq t_0|x)$ with $\tau=1-\frac{\alpha}{2}$ and $\frac{\alpha}{2}$ respectively.

**3.2 When sample size has a Binomial distribution**

For a finite population size we suppose that the sample size n is binomial random variable with parameters M and p, 0 < p <1 with pmf

$$p_2(n)=\binom{M}{n}p^n q^{M-n},\ n=0,1,2,\ldots,M,\ q=1-p. \tag{15}$$

Using (10) and (15) the predictive density of y given x can be obtained in similar manner as

$$p_2(y|x)=C_2\sum_{n=s+m}^{M}\frac{\binom{n-s}{m}\sum_{i=0}^{m-1}\sum_{j=0}^{r}\sum_{w=0}^{t}h_{3i}h_{1j}h_{2w}\begin{Bmatrix}(y-x_s)(n-d+i)\\+H+jx_{r+1}+wA_1\end{Bmatrix}^{-(n_1+1)}\binom{M}{n}p^n q^{M-n}}{\sum_{j=0}^{r}\sum_{w=0}^{t}h_{1j}h_{2w}\{H+jx_{r+1}+wA_1\}^{-n_1}} \tag{16}$$

where $C_2=\dfrac{mn_1}{1-\sum_{i=0}^{s+m-1}\binom{M}{i}p^i q^{M-i}}$.

From which the predictive survival function of y is given by

$$P(y\ge t_0|x)=\int_{t_0}^{\infty}p_2(y|x)$$

$$=\frac{C_2}{n_1}\sum_{n=s+m}^{M}\frac{\binom{n-s}{m}\sum_{i=0}^{m-1}\sum_{j=0}^{r}\sum_{w=0}^{t}\frac{h_{3i}h_{1j}h_{2w}}{(n-d+i)}\begin{Bmatrix}(t_0-x_s)(n-d+i)+\\H+jx_{r+1}+wA_1\end{Bmatrix}^{-n_1}\binom{M}{n}p^n q^{M-n}}{\sum_{j=0}^{r}\sum_{w=0}^{t}h_{1j}h_{2w}\{H+jx_{r+1}+wA_1\}^{-n_1}} \tag{17}$$

The lower and upper (1-α)100% prediction bounds for y by using iterative numerical methods can be obtained by defining $t_0$ from (17) for a given value of $\tau=P(y\ge t_0|x)$ with $\tau=1-\dfrac{\alpha}{2}$ and $\dfrac{\alpha}{2}$ respectively.

## 4. BAYES PREDICTIVE INTERVALS FOR THE REMAINING (N-S) ORDER STATISTICS UNDER NON INFORMATIVE PRIOR

In this section we derive Bayes predictive bounds for y based on non informative prior for parameter θ. Forming a product of equations (7) and (10) and integrating over θ gives the predictive density of y given x as

$$g(y|x)=\frac{m\binom{n-s}{m}n_2\sum_{i=0}^{m-1}\sum_{j=0}^{r}\sum_{w=0}^{t}h_{3i}h_{1j}h_{2w}\{(y-x_s)(n-d+i)+T+jx_{r+1}+wA_1\}^{-(n_2+1)}}{\sum_{j=0}^{r}\sum_{w=0}^{t}h_{1j}h_{2w}\{H+jx_{r+1}+wA_1\}^{-n_2}},\quad y>x_s \tag{18}$$

### 4.1 When sample size n has a Poisson distribution

Using the method discussed in Section 3.1, the predictive density of y given x when n has Poisson distribution defined in (11) is

$$g_1(y|x) = C_1 m n_2 \sum_{n=s+m}^{\infty} \frac{\binom{n-s}{m} \sum_{i=0}^{m-1} \sum_{j=0}^{r} \sum_{w=0}^{t} h_{3i} h_{1j} h_{2w} \begin{Bmatrix} (y-x_s)(n-d+i)+ \\ T + jx_{r+1} + wA_1 \end{Bmatrix}^{-(n_2+1)} \lambda^n / n!}{\sum_{j=0}^{r} \sum_{w=0}^{t} h_{1j} h_{2w} \{H + jx_{r+1} + wA_1\}^{-n_2}} \tag{19}$$

From which the predictive survival function of y given x is given by

$$P(y \geq t_0 | x) = \int_{t_0}^{\infty} g_1(y|x)$$

$$= C_1 m \sum_{n=s+m}^{\infty} \frac{\binom{n-s}{m} \sum_{i=0}^{m-1} \sum_{j=0}^{r} \sum_{w=0}^{t} \frac{h_{3i} h_{1j} h_{2w}}{(n-d+i)} \begin{Bmatrix} (t_0-x_s)(n-d+i)+ \\ T + jx_{r+1} + wA_1 \end{Bmatrix}^{-n_2} \lambda^n / n!}{\sum_{j=0}^{r} \sum_{w=0}^{t} h_{1j} h_{2w} \{H + jx_{r+1} + wA_1\}^{-n_2}} \tag{20}$$

**4.2 When sample size n has binomial distribution**

Suppose that n has binomial distribution given in (15) then the predictive density of y given x can be obtained in similar manner as discussed in Section 3.1 as

$$g_2(y|x) = C_3 \sum_{n=s+m}^{M} \frac{\binom{n-s}{m} \sum_{i=0}^{m-1} \sum_{j=0}^{r} \sum_{w=0}^{t} h_{3i} h_{1j} h_{2w} \begin{Bmatrix} (y-x_s)(n-d+i) \\ + H + jx_{r+1} + wA_1 \end{Bmatrix}^{-(n_2+1)} \binom{M}{n} p^n q^{M-n}}{\sum_{j=0}^{r} \sum_{w=0}^{t} h_{1j} h_{2w} \{H + jx_{r+1} + wA_1\}^{-n_2}} \tag{21}$$

where

$$C_3 = \frac{m n_2}{1 - \sum_{i=0}^{s+m-1} \binom{M}{i} p^i q^{M-i}} \tag{22}$$

From which the predictive survival function of y given x is obtained as

$$P(y \geq t_0 | x) = \int_{t_0}^{\infty} g_2(y|x)$$

$$= \frac{C_3}{n_2} \sum_{n=s+m}^{M} \frac{\binom{n-s}{m} \sum_{i=0}^{m-1} \sum_{j=0}^{r} \sum_{w=0}^{t} \frac{h_{3i} h_{1j} h_{2w}}{(n-d+i)} \begin{Bmatrix} (t_0-x_s)(n-d+i)+ \\ \mathrm{T} + jx_{r+1} + wA_1 \end{Bmatrix}^{-n_2} \binom{M}{n} p^n q^{M-n}}{\sum_{j=0}^{r} \sum_{w=0}^{t} h_{1j} h_{2w} \{H + jx_{r+1} + wA_1\}^{-n_2}} \tag{23}$$

The lower and upper (1-α)100% prediction bounds for y by using iterative numerical methods can be obtained by defining $t_0$ from (20) and (23) for a given value of $\tau = P(y \geq t_0 | x)$ with $\tau = 1 - \frac{\alpha}{2}$ and $\frac{\alpha}{2}$ respectively for the cases discussed in Section 4.1 and 4.2 both.

**5. REAL EXAMPLE**

In this section the effect of random sample sizes on Bayes predictive confidence interval of first future failure after the termination of the test is studied. The results are compared with the results obtained on the basis of a fixed sample size. An example given by Lawless(1982. pp 138, 3.11( Type-B)) which represent failure times in minutes, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress is used. The ordered failure times of a sample of size 12 are

12.3, 21.8, 24.4, 28.6, 43.2, 46.9, 70.7, 75.3, 95.5, 98.1, 138.6, 151.9

We consider the following multiply Type **II** censoring scheme for the above data:

-, 21.8, 24.4, 28.6, 43.2, 46.9, -, 75.3, 95.5, 98.1, -, -

Here, we suppose that 12 items are put on the test and the experimenter failed to observe the first and seventh failure times, so these observations are censored and the last two observations are censored since the experimentation was stopped as soon as the tenth failure occurred. Our interest is to calculate the Bayes predictive confidence interval for the first ( $x_{(11)}$) of the remaining n-s lifetimes under informative as well as non informative priors. Tables 1 and 2 summarize the 95% Bayes predictive confidence interval for $x_{(11)}$ obtained from the equations (14), (17), (20) and (23). The length of the confidence intervals are given in the brackets. The actual value of $x_{(11)}$ is 138.6.

**TABLE: 1** Table of 95% Bayes predictive confidence interval for fixed sample size and random sample size with binomial distribution.

| Prior Parameters | | | Fixed Sample Size Confidence Interval ( Length) | Random sample size with binomial distribution. Confidence Interval ( Length) | | |
|---|---|---|---|---|---|---|
| a | b | β | | M=25, p = 0.95 | M=30, p = 0.90 | M=50, p = 0.70 |
| 2 | 100 | - | (98.8319, 221.0149) (122.1830) | (98.3859,146.0969) (47.7110) | (98.3719, 143.6419) (45.2700) | (98.3519, 140.3049 (41.9530) |
| 2 | 200 | - | (98.9399, 239.0749) (140.1350) | (98.4019, 148.7139) (50.3120) | (98.3839, 145.7649) (47.3810) | (98.3609, 141.7599 (43.3990) |
| 2 | 300 | - | (99.0469, 257.0729) (158.0260) | (98.4169, 151.3309) (52.9140) | (98.3969, 147.8889) (49.4920) | (98.3689, 143.2169 (44.8480) |
| 2 | 800 | - | (99.581, 346.5969) (247.0149) | (98.4949, 164.4129) (65.9180) | (98.4599, 158.5119) (60.0520) | (98.4119, 150.5069 (52.0950) |
| 4 | 800 | - | (99.3689, 306.1309) (206.7619) | (98.4379, 153.6489) (55.2110) | (98.4079, 148.7079) (50.3000) | (98.3669, 142.0059 (43.6390) |
| 5 | 800 | - | (99.2839, 290.4309) (191.1469) | (98.4159, 149.4719) (65.9180) | (98.3879, 144.9039) (46.5160) | (98.3499, 138.7079 (40.3580) |
| - | - | 2 | (98.9369, 246.4659) (147.5290) | (98.4609, 161.9959) (63.5350) | (98.4459, 159.2199) (60.7740) | (98.4249, 155.4469 (57.0220) |
| - | - | 4 | (99.1809, 302.0519) (202.8709) | (98.5649, 185.5879) (87.0230) | (98.5449, 181.7739) (83.2290) | (98.5169, 176.5939 (78.0770) |
| - | - | 6 | (99.6209, 419.9029) (320.2819 | (98.7519, 235.5919) (136.8400) | (98.7229, 229.5769) (130.8540) | (98.6849, 221.4159 (122.7310) |

**TABLE: 2** Table of 95% Bayes predictive confidence interval for random sample size with Poisson distribution.

| Prior Parameters | | | Random sample size with Poisson distribution. Confidence Interval ( Length) | | | |
|---|---|---|---|---|---|---|
| a | b | β | λ = 20 | λ = 30 | λ = 35 | λ = 40 |
| 2 | 100 | - | (98.4249, 159.3189) (60.8940) | (98.1019, 141.6099) (43.5080) | (98.1019, 135.0709) (36.9690) | (98.1019, 125.2649 (27.1630) |
| 2 | 200 | - | (98.4469, 164.8509) (66.4070) | (98.1019, 143.6279) (45.5260) | (98.1019, 136.5569) (38.4550) | (98.1019, 126.2789 (28.1770) |
| 2 | 300 | - | (98.4679, 170.4099) (71.9410) | (98.1019, 145.6489) (47.5470) | (98.1019, 138.0449) (39.9430) | (98.1019, 127.2929 (29.1910) |
| 2 | 800 | - | (98.576, 198.4239) (99.8470) | (98.1019, 155.7989) (57.6970) | (98.1019, 145.4949) (47.3930) | (98.1019, 132.3629 (34.2610) |
| 4 | 800 | - | (98.5080, 182.2409) (83.7320) | (98.1019, 146.4829) (48.3810) | (98.1019, 137.9329) (39.8310) | (98.1019, 127.0289 (28.9270) |
| 5 | 800 | - | (98.4819, 175.9499) (77.4680) | (98.1019, 142.8639) (44.7620) | (98.1019, 134.9869) (36.8850) | (98.1019, 124.9379 (26.8360) |
| - | - | 2 | (98.5029, 176.5809) (78.0780) | (98.1019, 158.3819) (60.2800) | (98.1019, 155.3889) (57.2870) | (98.1019, 151.5759 (53.4740) |

| | | | | | | |
|---|---|---|---|---|---|---|
| - | - | 4 | (98.6189, 205.5219) (106.9030) | (98.1019, 180.6109) (82.5090) | (98.1019, 176.4699) (78.3680) | (98.1019, 171.0719 (72.9700) |
| - | - | 6 | (98.8269, 266.6419) (167.8150 | (98.1019, 227.7069) (129.6050) | (98.1019, 221.0839) (122.9820) | (98.1019, 212.0879 (113.9860) |

## 6. SIMULATION STUDY

In this section a simulation study is considered. A random sample of size 20 is generated 500 times from the exponential distribution with mean $\theta = 0.5$ under two types of multiply type –II censoring schemes viz: (A) (r=0, k = 1, t = 0, s = 18) i.e. usual type-II censoring and (B) (r = 3, k = 6, t = 3, s = 18). In both the censoring schemes the experiment was terminated as soon as the 18$^{th}$ failure was observed. The Bayes predictive confidence intervals for $x_{(19)}$, the first failure time after the termination of the test in case of informative and non informative priors are calculated for each of the 500 samples and the average vales of such results are demonstrated in the Tables 3 and 4 for scheme (A) and in Tables 5 and 6 for scheme (B). The length of the confidence intervals are given the brackets. The simulated value of the $x_{(19)}$ (Average of 500 generated values of $x_{(19)}$) came out 4.5144.

**TABLE: 3** Table of 95% confidence interval for $X_{s+1}$ for fixed sample size and random sample size with binomial distribution.

| Prior Parameters | | | Fixed Sample Size Confidence Interval ( Length) | Random sample size with binomial distribution. Confidence Interval ( Length) | | |
|---|---|---|---|---|---|---|
| a | b | β | | M=30, p = 0.90 | M=50, p = 0.70 | M=100, p = 0.50 |
| 2 | 2 | - | (3.5961, 6.9791) (3.3830) | (4.1346, 5.6593) (1.5247) | (4.1904, 5.4023) (1.2119) | (4.2120, 5.2505) (1.0384) |
| 2 | 4 | - | (3.6257, 7.2067) (3.5810) | (4.2957, 5.9175) (1.6218) | (4.2677, 5.5206) (1.2529) | (4.2062, 5.2527) (1.0465) |
| 2 | 6 | - | (3.6833, 7.5085) (3.8252) | (4.2896, 5.9539) (1.6644) | (4.1888, 5.4496) (1.2607) | (4.2874, 5.3683) (1.0809) |
| 4 | 2 | - | (3.6568, 6.7615) (3.1047) | (4.1663, 5.5503) (1.3840) | (4.2182, 5.316) (1.0977) | (4.2381, 5.1780) (0.9399) |
| 6 | 2 | - | (3.6203, 6.04245) (2.8042) | (4.2625, 5.5485) (1.2860) | (4.3287, 5.3458) (1.0171) | (4.1761, 5.0175) (0.8413) |
| - | - | 2 | (3.6192, 7.4217) (3.8024) | (4.2161, 5.9806) (1.7645) | (4.1262, 5.5076) (1.3814) | (4.1647, 5.3636) (1.1989) |
| - | - | 4 | (3.7281, 8.2360) (4.5080) | (4.2512, 6.3045) (2.0532) | (4.2350, 5.8548) (1.6198) | (4.1167, 5.4818) (1.3651) |
| - | - | 6 | (3.7088, 8.9606) (5.2517) | (4.2468, 6.6608) (2.4140) | (4.2493, 6.1686) (1.9193) | (4.2139, 5.8564) (1.6424) |

**TABLE: 4** Table of 95% confidence interval for $X_{s+1}$ for random sample size with Poisson distribution.

| Prior Parameters | | | Random sample size with Poisson distribution. Confidence Interval ( Length) | | |
|---|---|---|---|---|---|
| a | b | β | $\lambda = 30$ | $\lambda = 40$ | $\lambda = 50$ |
| 2 | 2 | - | (4.1851, 5.7662) (1.5811) | (4.2503, 5.2205) (0.9702) | (4.0055, 4.6071 (0.6015) |
| 2 | 4 | - | (4.2517, 5.9160) (1.6644) | (4.2351, 5.2207) (0.9855) | (4.0946, 4.7158 (0.6212) |
| 2 | 6 | - | (4.2880, 6.0127) (1.7248) | (4.2101, 5.2115) (1.0014) | (4.1590, 4.7976 (0.6386) |
| 4 | 2 | - | (4.2271, 5.6700) (1.4429) | (4.2582, 5.1681) (0.9099) | (4.0950, 4.6467 (0.5516) |
| 6 | 2 | - | (4.2346, 5.5553) (1.3207) | (4.3229, 5.1118) (0.7889) | (4.3896, 4.9274 (0.5378) |
| - | - | 2 | (4.1928, 6.0368) (1.8440) | (4.1764, 5.2920) (1.1156) | (4.2412, 5.3825 (1.1413) |
| - | - | 4 | (4.1469, 6.2481) (2.1012) | (4.1813, 5.4614) (1.2801) | (4.2175, 5.5269 (1.3094) |
| - | - | 6 | (4.2243, 6.7422) (2.5179) | (4.1158, 5.5909) (1.4750) | (4.0711, 5.5610 (1.4899) |

**TABLE: 5** Table of 95% confidence interval for $X_{s+1}$ for fixed sample size and random sample size with binomial distribution.

| Prior Parameters | | | Fixed Sample Size Confidence Interval ( Length) | Random sample size with binomial distribution. Confidence Interval ( Length) | | |
|---|---|---|---|---|---|---|
| a | b | β | | M=30, p = 0.90 | M=50, p = 0.70 | M=100, p = 0.50 |
| 2 | 2 | - | (3.5196, 6.8695) (3.3499) | (4.2678, 6.1635) (1.8957) | (4.2463, 5.7607) (1.5144) | (4.1031, 5.3652) (1.2620) |
| 2 | 4 | - | (3.5521, 7.1156) (3.5635) | (4.2184, 6.1573) (1.9389) | (4.2214, 5.5206) (1.5425) | (4.1028, 5.3837) (1.2808) |
| 2 | 6 | - | (3.5066, 7.2027) (3.6961) | (4.2287, 6.2325) (2.0038) | (4.3329, 5.4927) (1.6046) | (4.2499, 5.5863) (1.3364) |
| 4 | 2 | - | (3.4996, 6.4510) (2.9513) | (4.2737, 5.9289) (1.6552) | (4.1805, 5.9375) (1.3122) | (4.0043, 5.0952) (1.0908) |
| 6 | 2 | - | (3.6860, 6.5467) (2.8607) | (4.1111, 5.5319) (1.4208) | (4.2123, 5.3860) (1.1737) | (4.0774, 5.0704) (0.9929) |
| - | - | 2 | (3.7942, 7.7722) (3.9780) | (4.1972, 6.4689) (2.2717) | (4.1929, 6.0364) (1.8435) | (4.4003, 6.0928) (1.6924) |
| - | - | 4 | (3.6196, 7.9962) (4.3765) | (4.2060, 6.9752) (2.7692) | (4.2055, 6.4523) (2.2468) | (4.2563, 6.2386) (1.9850) |
| - | - | 6 | (3.7067, 8.9200) (5.2133) | (4.1458, 7.6324) (3.4866) | (4.2303, 7.1227) (2.8924) | (4.2327, 6.7598) (2.5271) |

**TABLE: 6** Table of 95% confidence interval for $X_{s+1}$ for random sample size with Poisson distribution.

| Prior Parameters | | | Random sample size with Poisson distribution. Confidence Interval ( Length) | | |
|---|---|---|---|---|---|
| a | b | β | λ = 30 | λ = 40 | λ = 50 |
| 2 | 2 | - | (4.0950, 5.6551) (1.5601) | (3.9973, 4.9110) (0.9136) | (4.2763, 4.9111) (0.6348) |
| 2 | 4 | - | (4.1012, 5.7088) (1.6075) | (4.0946, 5.0536) (0.9590) | (4.2788, 4.9226) (0.6438) |
| 2 | 6 | - | (4.0118, 5.6659) (1.6541) | (4.2763, 5.2894) (1.0130) | (4.4217, 5.0996) (0.6779) |
| 4 | 2 | - | (3.8745, 5.2146) (1.3401) | (4.2788, 5.1543) (0.8755) | (4.3830, 4.9698) (0.5867) |
| 6 | 2 | - | (4.2225, 5.5438) (1.3212) | (4.1419, 4.9224) (0.7805) | (4.4341, 4.9753) (0.5412) |
| - | - | 2 | (4.3970, 6.3114) (1.9144) | (4.0711, 5.3139) (1.2428) | (4.3475, 5.5136) (1.1661) |
| - | - | 4 | (4.3981, 6.6080) (2.2099) | (4.1590, 5.6025) (1.4435) | (4.1419, 5.4278) (1.2859) |
| - | - | 6 | (4.2508, 6.7621) (2.5113) | (4.3475, 6.1087) (1.7612) | (4.0305, 5.5013) (1.4708) |

## 7. CONCLUSIONS

In certain circumstances some observations are always missing for various reasons, in medical, biological or zoological experiments some members of the sample under consideration may left or die during the experiments thus, random sample size arises. In analysis of such data it would be worth interesting to study the performance of the estimator under random sample size against a fixed sample size.
From the real data set as well as simulation study , we conclude that the use of a random sample size generates smaller width confidence interval than that of a fixed sample size under multiply type-II censoring and this conclusion agree with the conclusion reported by Lingappaiah(1986) based on complete sample without censoring. Also the length of the Bayes predictive confidence interval is affected by choosing the different values of parameter of the Poisson distribution ( λ) and the parameters of binomial distribution( M and p). The other conclusions observed from the Tables 1 to 6 are as follows:

(i) In case of informative prior, for fixed value of prior parameter a , the length of the confidence interval decreases with decreasing the value of the prior parameter b for fixed as well as random sample size.

(ii) Keeping the value of the prior parameter b fixed, the length of the confidence interval decreases with increasing the value of the prior parameter b for fixed as well as random sample size.

(iii) In case of non informative prior the length of the confidence interval decreases with decreasing the value of the prior parameter β for fixed as well as random sample size.

(iv) For any choice of informative or non informative prior parameters the length of the confidence interval decreases in case of sample size has binomial distribution when M increases and p decreases.

(v) The length of the confidence interval decreases for any value of the informative and non informative priors when sample size has Poisson distribution. In case of $\lambda > 30$ the length of the confidence interval becomes smaller than that of when the distribution of sample size is binomial.

(vi) In case of Multiply type-II censoring the length of the confidence interval becomes smaller than that of random sample size with binomial distribution compared to the usual type II censoring.

(vii) In case of Multiply type-II censoring the length of the confidence interval becomes smaller than that of random sample size with Poisson distribution for vary large value of parameter $\lambda(\lambda > 50)$ compared to the usual type II censoring.

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