A GAME THEORETIC APPROACH: IMPACT OF LEARNING ON THE OPTIMAL ORDERING POLICIES FOR IMPERFECT QUALITY ITEMS

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ABSTRACT

The supply chain management is outlined as amalgamation of business processes aligned to operational management and marketing problems like information sharing, supply chain coordination and inventory control. In recent decades, researchers have effectively worked on seeking optimal policies under supply chain management for obtaining practical and powerful outcomes. The outcome has ensued a generous body of work intended to contribute distantly using game theory to exemplify the nature of problems surfaced to basic and complex supply chains. In this paper,a new concept is attempted like game theoretic approach with learning effect under supply chain model and this model is developed to support the dealing between the performers (players) of the supply chain, supplier and purchaser, presented by non-cooperative game approach. Defective items are considered in this model and after the inspection process these items are sold at discounted price. It is also assumed that demand is sensitive to promotion expenses cost and buyer's price without trade credit. Results are determined by the non-cooperative Stackelberg game theoretical approach. Finding suggest that buyer's income is more due to the learning effect than seller in this model. Both the players get benefitted in case of leadership position. In the last, numerical visuals with sensitivity scrutiny are presented to support the theory of this paper.

KEYWORDS: Supply chain, Learning curve, Imperfect quality items, Non-cooperative games, Game theory

MSC: 91A99

RESUMEN

El manejo de la cadena de suministro es presentada como una amalgama de procesos alineados con problemas del manejo operacional y de marketing, como la compartimentación de información, coordinación de la cadena de suministro y el control de inventarios. En recientes décadas, los investigadores han trabajado efectivamente en la búsqueda de políticas óptimas bajo el manejo de la cadena de suministro para obtener resultados prácticos y potentes. Como resultado se provee un marco de trabajo que intenta contribuir en conectarse, en cierto modo, con la Teoría de Juegos para ejemplificar la naturaleza de los problemas que aparecen inmersos en lo básico de complejas cadenas de suministros. En este paper, un nuevo a concepto es presentado, desde la Teoría de Juegos con efectos de aprendizaje , bajo un modelo de cadena de suministro y, este modelo es desarrollado para modelar la relación entre los "performers" (jugadores) de la cadena de suministros, el abastecedor y el tramitador, presentados desde el enfoque dado por un Juego No-Cooperativo. Los ítems defectuosos son considerados en este modelo y después del proceso de inspección estos ítems son vendidos con un precio de descuento. También se asume que la demanda es sensible a la inversión en promoción y el precio del comprador sin trámite de crédito. Los resultados son determinados mediante el enfoque teórico dado por un Juego No-Cooperativo de Stackelberg. Los hallazgos sugieren que los ingresos del comprador son más influenciados por el efecto de aprendizaje, que por la venta. Ambos jugadores son beneficiados por la posición del liderazgo. Finalmente , una visualización numérica con escrutinio sensible es presentada para dar soporte a la teoría presentada en este paper.

PALABRAS CLAVE: Cadena de suministro, curva de aprendizaje, calidad imperfecta de los ítems, Juegos No-Cooperativos, Teoría de Juegos

1. INTRODUCTION

The study of knowledge concentrated on the performance of particular subjects. These studies exposed that the moment in time necessary to make a job faded at a diminishing rate as knowledge with deed improved and this occurrence is called learning. We can say that learning includes style of progress of presentation that comes concerning at outcomes of practice. The secreted information obtained through learning effects becomes essential to support decision-making. The occurrence of learning from failure is indisputable but corporations that do it tremendously irregular. Learning involves in many fields like that in commerce field managers within the hue majority of organizations really wanted to help their businesses study from disasters to support future performance such as products design, constructions companies, financial services etc. Learning occurrence from day by day activity is work that how to gain experience as distributers learns from each order of his customer and from lead time of delivered goods between supplier and buyer. In many situations manufactures and their companies had to develop many hours after evaluations and sometimes distributers observe that there is no actual exchange from these painstaking efforts, the reason behind is managers are not thinking in the right way. Game theories are numerical/logical device and recognize the impact of learning on supply chain structure with imperfect items. Generally, a supply chain model can be express as "a system of suppliers, manufacturers, and customers where materials flow downstream from suppliers to customers and the information flows in both directions "Ganeshan (1999).The behavior of learning curve has been suggested by Wright (1936) and formulated in the form of quantitative shape as well as resulted in the hypothesis of the learning curve and different to the excess of literature on LC (learning curves) there is a scarcity of the literature review on forgetting curves. Hammer (1957) discussed on the logical revise of learning curves as a means of involving work standards. Nadler and Smith (1963) explained the development of structure of mathematical model (design) for the minimization of inventory cost. Baloff (1966) discussed about the mathematical behavior of the learning theory (learning slope varied widely and also explained the justification outcomes of a practical come up to carry out the parameters of LC by urbanized skill and provisional studied in the data of learning).This scarcity of study has been credited almost certainly to the sensible difficulties occupied in obtaining information regarding the stage of forgetting as a function of time Levin et al. (1989). The rate of learning has been suggested by Cunningham (1980) with using of special type of facts, i.e. composed learning rates reported in 15 diverse U.S. industries in the years (1860-1978) and Thomas and Dutton (1984) justified learning rates under distribution in 108 forms. Argote and Epple (1990) discussed about the factor by which the rate of learning varies in different situation which is the major factor in research field. Salameh et al. (1993) considered a limited manufacture stock form (Production inventory model) with the outcome of human knowledge and also discussed variable demand rate and learning in time to optimize the cost. Jaber et al. (1996) to explain the theory of forgetting using manufacture breaks, learning curve and discussed optimal manufacture amount and minimize the whole stock price. Jaber et al. (1996) has been worked on assuming the optimal lot sizing using the condition of bounded learning cases and focuses on optimal production quantity and minimization of total inventory cost with LCs. Jaber et al. (1997) discussed on a revise of learning and forgetting theory focuses author on the comparison of different type of model such as VRVF, VRIF and LFCM. An EOQ mathematical model has been prepared by Jaber et al. (1995) for the lot sizing problem under the learning concept where shortages are allowed. Jaber et al. (2008) derived an ordering policy model for imperfect quality items with percentage defective per lot reduces according to the LC (learning Curve). Jaber and Bonney (2003) considered the lot shape with the theory of learning -forgetting in managed system and in manufactured goods excellence and focused on minimize production time, reduces rework process and optimal production quantity. Balkhi (2003) discussed on maximum manufacture lot volume for decaying items and shortage case (partially backordered) material with time unreliable order and rot rates with the help of learning effects. Jaber et al. (2004) presented on learning curve for processes generating defects required reworks authors consider, generating rate defects is stable and modification of Wright, learning curve how to defective items can be rework by this model. Khan et al. (2010) considered an ordering policy for defective quality things using inspection process and maximize production and minimize the cost of production.

Jaber et al. (2010) discussed on, how to develop a merger of average dispensation time process give way with respect to the number of batches and planned the consequence of unreliable the learning curve parameters in manufacture and revise for developed model. Anzanello and Foglitto (2011) presented on the different kind of the application of learning formulation and author focuses on how to use this model in different mathematical form. A formulation of inventory representation has been presented by Konstantaras et al. (2011) for the maximization of construction with shortage and inspection under defective things. An ordering policy model has been formulated by Jaggi et al. (2013) for the defective feature things with credit financing policy under shortages and construction process.

Glock and Jaber et al. (2013) considered a manufacture stock model with LC and FC "learning and forgetting" theory in manufacture and also discussed how much minimization of the number of batches of a lot from production to subsequently order length. Tsair et al. (2014) discussed on lot size policies in EPQ models under the learning curve production costs with trade credit. Givi et al. (2015) explained different kind of mathematical model for the employee consistency (reliability) under the fatigue and learning theory. Sangal and Rani (2016) discussed the working policy of a fuzzy concept model with shortages under the learning effect. Sangal et al. (2017) proposed effect of learning with non-instant deteriorating model. Jaggi, Tiwari and Goel (2017) exposed the policy of trade credit financing in different inventory ordering strategies for non-instantaneous deteriorating things and concluded that demand is a function of selling fewer than two storage amenities. Rani et al. (2018) used the concept of green supply chain with learning effect for non-instantaneous deteriorating inventory model. Tiwari et al. (2018) proposed a combined store and pricing model for decaying items with ending dates and partial backlogging under two level trade credit policies in the provided sequence. Patro et al. (2018) proposed a fuzzy EOQ model for decaying things with defective quality using balanced reduction under the impact of learning. Jayaswal et al. (2019) discussed the learning phenomenon on seller ordering strategy for defective quality articles with permissible delay in payment. Jayaswal et al. (2019) found out the impact of learning on inventory -policies with defective quality and decaying things under the trade financing strategy. Yadav et al. (2019) analyzed the behavior of learning on best strategy of source chain followers for defective quality things: Game theory concepts. In practical implications, the components of a stoke model are indeterminate, inaccurate and the purpose to get maximum size length is difficult as it is a non-stochastic indistinguishable managerial process. This paper has established supply chain models of imperfect quality items with learning effect by non-cooperative theoretic approach. It has been considered that demand is sensitive to selling price and marketing expenditure of the buyer. The impact of LC is also being shown on the different parameters and profit function of two partners of the supply chain. Further, the S-shape logistic learning curve is considered to be in the shape and established to good fit the data $P(n) = a/(g + S^{bn})$, where a, b and g are positive constants and known as effective parameters, n is the cumulative number of lots and$p(n)$ is the percentage defective per batch n. The non-cooperative correlation is assumed in two different cases scenarios: Seller-Stackelberg and Buyer-Stackelberg. The author contribution table is given below.

Table 1.1. Contribution of different authors in the related field

Author(s)	**Supply chain model**	**Inspection**	**Learning effect**	**Non-cooperative game**	**Non-cooperative game with learning effect**
Wright (1936)			√		
Baloff (1966)			√		
Cunningham (1980)			√		
Argoteet al. (1990)			√		
Jaber and Bonney (1996)	√		√		
Eroglu and Ozdemir (2006)	√	√			
Salameh and Jaber (2000)	√	√			
Jaber et al. (2008)	√	√	√		
Jaber and Bonney (2003)	√	√	√		
Jaber and Guiffrida (2004)	√	√	√		
Khan et al. (2010)	√				
Jaberet al. (2010)	√		√		
Anzanello and Fogliatto (2011)	√		√		
Konstantaras et al. (2012)	√	√	√		
Jaggi et al. (2013)	√	√			
Jaber et al. (2013)	√	√	√		
Abad and Jaggi (2003)	√		√	√	
Esmaeili et al. (2009)	√			√	
Present paper	√	√	√	√	√

1.1 LEARNING CURVE

Some authors discussed the impact of the learning shape. Some form of learning shape is proposed by Wright (1936), Jordon (1958) and Carlson (1973) and mathematically as well graphically shown below. The figure explains three unlike phases, where the first phase is called incipient or segment stage, the second is called learning phase and the last phase is called the maturity phase.

The number of imperfect items presents in each batch is assumed by an S-shape logistic learning curve.

$$P(n) = \frac{a}{g + s^{bn}}, a > 0,\ g > 0,$$ n is the shipment and b is the learning rate.

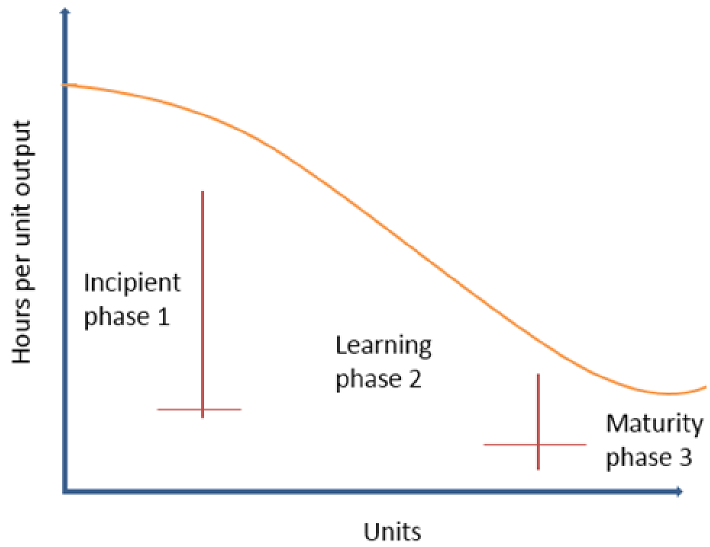


Figure 1.1.1 Three phase of learning curve

The impact of shipment on the imperfect items is shown by graph in the Figure 1.1.2.

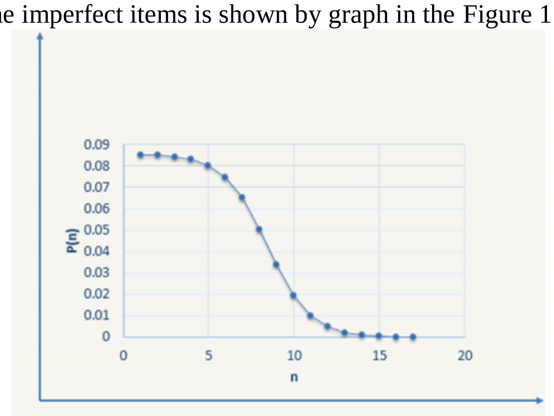


Figure 1.1.2 Impact of n on $P(n)$

2. NOTATIONS AND ASSUMPTIONS

2.1. Notations

c_b Seller's selling price (decision variable of seller) ($/unit)
M Marketing expenditure cost (decision variable of buyer) ($/unit)
p_b Buyer's selling price (decision variable of buyer) ($ / unit)
y_n Order quantity determined by buyer for nth lot (decision variable of buyer) (in units), where $n \geq 1$

Parameters

A_b Set-up cost for buyer ($ / order)
A_s Set-up cost for seller ($/order)
H_b Holding cost of inventory ($/ unit time)
I Percent of holding cost of inventory ($/ unit)

$p(n)$ Percentage of defective items which follow the LCs.
C Seller's purchasing cost (\$/unit)
λ Screening rate (unit per year) ($D < \lambda$)
$\mathrm{c_s}$ Defective cost per unit items (\$/ year) ($c_s < c_b$)
β Demand function's marketing expenditure elasticity ($0 < \beta < 1,\ \beta + 1 < e$)
e Price elasticity for demand function ($e > 1$)
$\mathrm{s_c}$ Screening cost (\$/units)
t_n Screening time, $t_n = y_n/\lambda$ (years)
k Scaling constant for demand rate($k > 0$)
D Demand (unit/year), $= kp_b{}^{-e}M^{\beta}$
T_n Cycle time in years for the buyer, $T_n = y_n(1 - P(n)/D$
$T_n{}^*$ Cycle time in years for the seller, $T_n{}^* = y_n/D$
$T_n{}^{**}$ Cycle time in years for the Seller − Stackelberg model , $T_n{}^{**} = \mathrm{Max}(T_n, T_n{}^*)$

2.2. Assumptions

1. Demand is presumed as function of selling price p_b and marketing expenditure M.
2. Time horizon is finite.
3. Shortages are not permitted.
4. It is considered that there is no carrying cost for seller (lot-to-lot strategy rule).
5. It is considered that there is some imperfect items' percentage in each individual lot (Salameh and Jaber, 2000).
6. It is presumed that the demand rate is less than the screening rate (Jaggi et al., 2013).

7. Defective items' percentage in each individual shipment is governed by the learning curve (Jaber et al., 2008)
.

3. MATHEMATICAL MODELS

Two mathematical models are presented below:

3.1. Buyer's model

The aim of the buyer is to maximize the total profit $TP_b(p_b, M, y_n)$, where
$TP_b(p_b, M, y_n)$ = Sales income - Purchasing cost - Marketing expenditure cost-screening cost-Ordering cost – Holding cost of inventory

$$= p_b\big(1 - p(n)\big)y_n + c_s p(n)y_n - c_b y_n - My_n - s_c y_n - A_b - \left(\frac{y_n\big(1-p(n)\big)T_n}{2} + \frac{p(n){y_n}^2}{\lambda}\right)H_b$$

Put $T_n = \frac{(1-p(n))y_n}{D}$, $t = \frac{y_{n,}}{\lambda}$, $H_b = \mathrm{I}c_b$ then buyer's profit is given by

$$TP_b(p_b, M, y_n) = p_b\big(1 - p(n)\big)y_n + c_s p(n)y_n - c_b y_n - My_n - s_c y_n - A_b - \left(\frac{{y_n}^2\big(1 - p(n)\big)^2}{2D} + \frac{p(n){y_n}^2}{\lambda}\right)\mathrm{I}c_b$$

We assume demand function, $D = kp_b{}^{-e}M^{\beta}$
Buyer's profit per cycle is given by

$$TP^c{}_b(p_b, M, y_n) = \left[\frac{TP_b(p_b, M, y_n)}{T_n}\right]$$

$$\frac{D}{\big(1 - p(n)\big)y_n}\left[p_b\big(1 - p(n)\big)y_n + c_s p(n)y_n - c_b y_n - My_n - s_c y_n - A_b - \left(\frac{{y_n}^2\big(1 - p(n)\big)^2}{2D} + \frac{p(n){y_n}^2}{\lambda}\right)\mathrm{I}c_b\right]$$

$$= p_b D + \frac{1}{(1-p(n))}\left[c_s p(n)D - c_b D - MD - s_c D - \frac{A_b D}{y_n} - \left(\frac{y_n(1-p(n))^2}{2} + \frac{p(n) y_n D}{\lambda}\right) Ic_b\right]$$

We assume demand function is $D = kp_b^{-e} M^{\beta}$

$$TP^c{}_b(p_b, M, y_n) = \mathrm{k} p_b^{-e+1} M^{\beta} + \frac{\mathrm{k}\, p_b^{-e} M^{\beta}}{(1-p(n))}\left[c_s p(n) - c_b - s_c - M - \frac{A_b}{y_n} - \frac{p(n) y_n}{\lambda} Ic_b\right] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))} Ic_b\right)$$

(3.1)

Now aim is to find the maximum values of p_b, M and y_n to optimize the worth $[\mathrm{TP^c}_{\mathrm{b}}(p_b, \mathrm{M}, y_n)]$, for this, we equate first derivative of equation (3.1) with respect to p_b to zero.

$\frac{\partial [TP^c{}_b(p_b, M, y_n)]}{\partial p_b} = 0$, yields

$$p_b = \frac{e}{(e-1)(1-p(n))}\left[M + c_b + s_c - c_s p(n) + \frac{A_b}{y_n} + \frac{Ic_b p(n) y_n}{\lambda}\right] \quad (3.2)$$

The profit function of the player buyer, $[TP^c{}_b(p_b, M, y_n)]$ is pseudo concave with respect to p_b for constants M and y_n (Yadav et al., 2018)

Substituting the value of p_b into equation (3.1) and then resultant equation is

$$[TP^c{}_b(p_b(M), M, y_n(M))] = \frac{\mathrm{K}}{e}\left[\frac{e}{(e-1)(1-p(n))}\left(M + c_b + s_c + \frac{A_b}{y_n} + \frac{p(n) y_n Ic_b}{\lambda} - c_s p(n)\right)\right]^{-e+1} M^{\beta} - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))} Ic_b\right) \quad (3.3)$$

Now differentiate w.r.to M, we get

$$M = \frac{\beta}{(e-\beta-1)}\left[c_b + s_c + \frac{A_b}{y_n} + \frac{Ic_b p(n) y_n}{\lambda} - c_s p(n)\right] \quad (3.4)$$

The profit function of the player buyer, $[TP^c{}_b(p_b(M), M, y_n(M)]$ is concave with respect to M for constant y_n (Yadav et al., 2018)

Substituting the value of equation (3.4) into the equation (3.2), we get

$$p_b = \frac{e}{(e-\beta-1)(1-p(n))}\left[c_b + s_c + \frac{A_b}{y_n} + \frac{Ic_b p(n) y_n}{\lambda} - c_s p(n)\right] \quad (3.5)$$

$$[TP^c(y_n)] = k\left(\frac{e}{(e-\beta-1)(1-p(n))}\left[c_b + s_c + \frac{A_b}{y_n} + \frac{Ic_b p(n) y_n}{\lambda} - c_s p(n)\right]\right)^{-e}\left(\frac{\beta}{(e-\beta-1)}\left[c_b + s_c + \frac{A_b}{y_n} + \frac{Ic_b p(n) y_n}{\lambda} - \right.\right.$$

csp(n)
βe(e−β−1)(1−pn)cb+sc+Abyn+Icbpnynλ−csp(n)+11−pncspn−sc−cb−β(e−β−1)cb+sc+Abyn+Icbpnynλ−csp(n)−Abyn−pnynλIcb−yn[1−pn2]2Icb (3.6)

The first order condition of equation (3.6) w.r.t y_n finds the constraints as follows:

$y_n{}^2 Ic_b([(1-p(n))^2]\lambda + 2Dp(n)) = 2D\lambda, \quad i.e.$

$$y_n{}^2 Ic_b([(1-p(n))^2]\lambda = 2ke^{-e}\beta^{\beta}\left(\left[c_b + s_c + \frac{A_b}{y_n} + \frac{Ic_b p(n) y_n}{\lambda} - c_s p(n)\right]\right)^{\beta-e}(e-\beta-1)^{e-\beta}(1-$$

p(n))eλAb−yn2p(n (3.7)

It's quite difficult to prove the concavity of the above expected total profit function defined in equation (3.6) analytically. Thus, expected total profit $[TP^c{}_b(p_b(M), M, y_n(M))]$ defined in equation (3.6) is concave function with respect to order quantity is shown with the help of the graph (Figure 3.1.1)

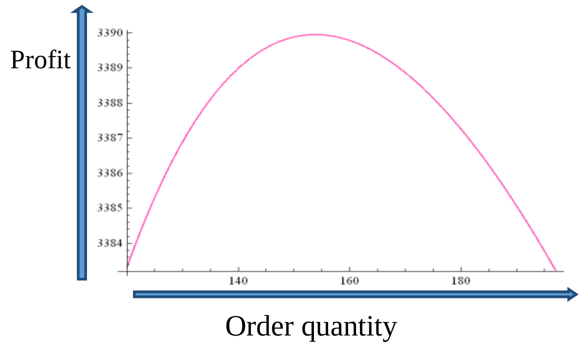


Figure 3.1.3 Plot of Buyer's profit function with respect to order quantity

3.2. Seller's model

Seller's Profit = Sales Revenue - Purchasing Cost - Ordering cost

$TP_s(c_b) = c_b y_n - C y_n - A_s$

Seller's cycle length, $T_n{}^* = \frac{y_n}{D}$

Seller's profit per cycle is given by,

$$\mathrm{TP}^c{}_s(c_b) = \frac{D}{y_n}(c_b y_n - C y_n - A_s)$$

$$= k p_b{}^{-e} M^{\beta}\left(c_b - C - \frac{A_s}{y_n}\right) \qquad (3.8)$$

Seller's profit is zero at $c_{b_0} = C + \frac{A_s}{y_n}$

Since the seller would prefer,

$c_{b_0} > C + \frac{A_s}{y_n}$, as always desires to have positive profit.

or we can write,

$$\boldsymbol{c_b} = F c_{b_0} = F\left(C + \frac{A_s}{y_n}\right) \text{ for some, } F > 1 \qquad (3.9)$$

i.e. the optimal value for c_b is the highest value agreed upon by seller and buyer through negotiation.

3.3. The non-cooperative Stackelberg game theory approach

Generally, non-cooperative and cooperative supply chain models are clogged to the cases where the buyer and seller together maintains symmetric information with their business strategies. At symmetric information setup, players carry in-depth knowledge about each other's activities. In real scenerio, there may be rational behaviour between supply chain partners and they hide information from each other.

3.3.1.The seller-Stackelberg model

Here in mathematical representation, seller is considered as the dominant player. The aim of the seller is to exploit his profit on the basis of given decision variables. The problem is,

$$Max\ E\,[\mathrm{TP}^c{}_s(c_b)] = \frac{D}{y_n}(c_b y_n - C y_n - A_s) = k p_b{}^{-e} M^{\beta}\left(c_b - C - \frac{A_s}{y_n}\right) \qquad (3.10) \text{ subject to}$$

$$M = \frac{\beta}{(e-\beta-1)}\left[c_b + s_c + \frac{A_b}{y_n} + \frac{I c_b p(n) y_n}{\lambda} - c_s p(n)\right] \qquad (3.11)$$

$$p_b = \frac{e}{(e-\beta-1)(1-p(n))}\left[c_b + s_c + \frac{A_b}{y_n} + \frac{I c_b p(n) y_n}{\lambda} - c_s p(n)\right] \qquad (3.12)$$

Constraints will be

$$y_n{}^2 I c_b([(1-p(n))^2)\lambda = 2ke^{-e}\beta^{\beta}\left(\left[c_b + s_c + \frac{A_b}{y_n} + \frac{I c_b p(n) y_n}{\lambda} - c_s p(n)\right]\right)^{\beta-e}(e-\beta-1)^{e-\beta}(1-$$

$p(n)\ )e\lambda Ab{-}yn2p(n)$ (3.13)

Cycle length, $T_n{}^{**} = max(T_n, T_n{}^*)$

By using equation (3.11) and equation (3.12) and with the constraints (3.13) on equation (3.10). The resultant equation can be solved using software Mathematica 9.0 and can found the value of the decision variable.

3.3.2. The buyer-Stackelberg model

Here in mathematical representation, purchaser acts as the dominant. The purchaser maximizes his profit on the basis of decision variables,

$$Max\ E[TP^c{}_b(p_b, M, y_n)] = \mathrm{k}p_b{}^{-e+1}M^{\beta} + \frac{\mathrm{k}\, p_b{}^{-e}M^{\beta}}{(1-p(n))}\left[c_s p(n) - c_b - s_c - M - \frac{A_b}{y_n} - \frac{p(n)y_n}{\lambda}Ic_b\right] - \left(\frac{y_n(1-p(n))^2}{2(1-p(n))}Ic_b\right) \qquad (3.14)$$

Subject to

$$\text{At } c_b = F(C + \frac{A_s}{y_n}) \qquad (3.15)$$

By using equation (3.15) on equation (3.14) the problem converts into a non-linear (non-constrained) function. This can be solved using software Mathematica 9.0 and can found the value of the decision variables.

4. NUMERICAL EXAMPLES

Example 1

Effect of learning on the decision variables in seller- Stackelberg game model has been shown in the given example. Input parameters has been taken from two papers Esmaeili et al. (2009) and Jaber et al. (2008).
$C = \$1.5$ units, $A_b = \$38$, $A_s = \$40$, $I = 0.1, k = 36080, F = 1.8.$, $\lambda = 175200$ unit/year, $c_s = \$3.5, \beta = Le, e = 1.7, L = 0.088$, $Sc = \$0.035, I = 0.38, F = 1.8, a = 40, b = 1, n = 1, g = 999, s = 2.99\ \ p(n) = 0.03997.$
Equation (3.10) gives the results, $y_n = 153$ units and $c_b = \$4.0628$.Equations (3.4) and (3.5) produces the results, $p_b = \$13.542$ and $M = \$1.1427$. The seller's profit, $[\mathrm{TP}^c{}_s] = \$1010.80$ and the buyer's profit, $[\mathrm{TP}^c{}_b] = \$3382.64$

Example 2

The impact of learning on the decision variables in buyer-Stackelberg game model has been shown in this example. The parameters are same as defined in example-1except $c_s = 2.5$.Equation (3.14) gives the results,$p_b = \$9.3135$,M = $\$0.789$ and $y_n = 427$ units Equations (3.8) generates the results, $c_b = \$2.868$. seller's profit, $[\mathrm{TP}^c{}_s] = \$999.576$ and buyer's profit, $[\mathrm{TP}^c{}_b] = \$4071.90$.
Results indicate that in Buyer-Stackelberg model, the value of M, c_b and p_b are less than Seller-Stackeberg model. Low selling price charged to the customer by purchaser and higher profit gained by the purchaser shows that he is better off in the second model. Purchaser is improved off when he is manager. As demand is more this results the higher profit of buyer.
In the Seller-Stackelberg model, the high seller's selling price results the more gain in the profit to the seller. The numerical example shows that seller got higher profit when he is leader and less when he is follower.

5. SENSITIVITY ANALYSIS

In this section, sensitivity analysis has been performed on the basis of key parameters to determine the robustness of the model. It has been shown that how learning rate affects the different decision variables and profit of the players.

5.1. Effect of learning on the player's profit

Table 5.1.2. Impact of learning rate on the profit in Seller-Stackelberg model

No. of shipment n	$b = 1$		$b = 1.8$		$b = 2.6$	
	Buyer profit	Seller Profit	Buyer profit	Seller Profit	Buyer profit	Seller Profit
1	3382.64	1010.80	3382.30	1010.13	3381.47	1010.26

2	3382.17	1010.16	3378.80	1010.66	3364.05	1012.99
3	3380.71	1010.37	3360.79	1013.53	3317.86	1021.42
4	3376.76	1010.98	3325.97	1019.83	3307.05	1023.66
5	3366.98	1012.51	3309.80	1023.08	3306.31	1023.81

Table 5.1.3. Impact of learning rate on the profit in Buyer-Stackelberg model

No. of shipment n	$b = 1$		$b = 1.8$		$b = 2.6$	
	Buyer profit	Seller Profit	Buyer profit	Seller Profit	Buyer profit	Seller Profit
1	4071.90	999.576	4072.04	999.648	3381.47	1010.26
2	4072.10	999.647	4073.49	1000.24	3364.05	1012.99
3	4072.69	999.800	4081.05	1003.59	3317.86	1021.42
4	4074.34	1000.62	4096.44	1010.08	3307.05	1023.66
5	4078.41	1002.36	4103.86	1013.11	3306.31	1023.81

Table 5.1.2 and Table 5.1.3 shows the impact of learning on the seller and buyer's profit in two different models. When the learning rate increases from 1.0 to 2.6 in the Seller-Stackeberg model (seller behaves as a leader and buyer as a follower). The profit of the seller slightly increases. Whereas in Buyer-Stackeberg model (buyer acts as a leader and seller act as a follower) buyer's profit increases as learning rate increases from 1.0 to 2.6.

5.2. Effect of learning on different parameters

5.2.1. Seller-Stackelberg

Table 5.2.1.4. The effect of learning rate on the parameter p_b, M, Q, C_b, $[\mathrm{TP}^c{}_s]$and, $[\mathrm{TP}^c{}_b]$ with learning rate $b = 1$

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Buyer expected profit $[\mathrm{TP}^c{}_b]$	Seller expectedprofit $[\mathrm{TP}^c{}_s]$
1	0.0399	153	4.0628	13.542	1.1472	3382.64	1010.8
2	0.0397	153	4.0642	13.546	1.1478	3382.17	1010.16
3	0.0389	152	4.0684	13.559	1.1497	3380.71	1010.37
4	0. 0371	151	4.0799	13.595	1.1550	3376.76	1010.98
5	0.0323	150	4.1087	13.684	1.1680	3366.98	1012.51

5.2.2. Buyer-Stackelberg

Table 5.2.2.5. The effect of learning rate on the parameter p_b, M, Q, C_b, $[\mathrm{TP}^c{}_s]$and, $[\mathrm{TP}^c{}_b]$ with learning rate $b = 1$

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Buyer expectedprofit $[\mathrm{TP}^c{}_b]$	Seller expected profit $[\mathrm{TP}^c{}_s]$
1	0.0399	427	2.868	9.3135	0.7889	4071.90	999.576
2	0.0397	427	2.868	9.3133	0.7890	4072.10	999.647
3	0.0389	426	2.868	9.3127	0.7890	4072.69	999.800
4	0. 0371	425	2.869	9.3110	0.7911	4074.34	1000.62
5	0.0323	424	2.869	9.3065	0.7946	4078.41	1002.36

5.2.3. Seller-Stackelberg

Table 5.2.3.6. The effect of learning rate on the parameter p_b, M, Q, C_b, $[\mathrm{TP}^c{}_s]$and, $[\mathrm{TP}^c{}_b]$ with learning rate $b = 1.8$

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Buyer Expected profit $[\mathrm{TP}^c{}_b]$	Seller Expected profit $[\mathrm{TP}^c{}_s]$
1	0.0397	153	4.064	13.545	1.147	3382.30	1010.13
2	0.0381	152	4.074	13.576	1.152	3378.80	1010.66
3	0.0291	149	4.127	13.740	1.177	3360.79	1013.53
4	0.0107	142	4.237	14.069	1.228	3325.97	1019.83
5	0.0019	138	4.291	14.227	1.252	3309.80	1023.08

5.2.4. Buyer-Stackelberg

Table 5.2.4.7. The effect of learning rate on the parameter p_b, M, Q, C_b, $[\mathrm{TP}^c{}_s]$and, $[\mathrm{TP}^c{}_b]$ with learning rate $b = 1.8$

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Buyer Expectedprofit $[\mathrm{TP}^c{}_b]$	Seller Expectedprofit $[\mathrm{TP}^c{}_s]$
1	0.0397	427	2.868	9.3134	0.789	4072.04	999.648
2	0.0381	426	2.869	9.3118	0.790	4073.49	1000.24
3	0.0291	423	2.870	9.303	0.797	4081.05	1003.59
4	0.0107	416	2.873	9.287	0.810	4096.44	1010.08
5	0.0019	413	2.874	9.280	0.817	4103.86	1013.11

5.2.5. Seller-Stackelberg

Table 5.2.5.8. The effect of learning rate on the parameter p_b, M, Q, C_b, $[\mathrm{TP}^c{}_s]$and, $[\mathrm{TP}^c{}_b]$ with learning rate $b = 2.6$

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Buyer Expectedprofit $[\mathrm{TP}^c{}_b]$	Seller Expectedprofit $[\mathrm{TP}^c{}_s]$
1	0.0393	153	4.066	13.552	1.148	3381.47	1010.26
2	0.0307	149	4.117	13.711	1.172	3364.05	1012.99
3	0.0064	140	4.263	14.147	1.241	3317.86	1021.42
4	0.0004	138	4.300	14.254	1.257	3307.05	1023.66
5	0.00002	137	4.303	14.261	1.258	3306.31	1023.81

5.2.6 Buyer-Stackelberg

Table 5.2.6.9. The effect of learning rate on the parameter p_b, M, Q, C_b, $[TP^c{}_s]$and, $[TP^c{}_b]$ with learning rate $b = 2.6$

No. of shipment n	Defective percent in per lot $p(n)$	Order quantity $y(n)$	Selling price of the seller C_b	Selling price of the buyer p_b	Marketing expenditure M	Buyer Expected profit $[TP^c{}_b]$	Seller Expected profit $[TP^c{}_s]$
1	0.0393	427	2.868	9.313	0.789	4072.38	999.72
2	0.0307	424	2.869	9.305	0.795	4079.66	1000.90
3	0.0064	415	2.873	9.283	0.814	4100.10	1011.61
4	0.0004	413	2.874	9.278	0.818	4105.13	1013.64
5	0.00002	412	2.874	9.277	0.817	4105.48	1013.96

5.3. Effect of number of shipments and learning rate on buyer's profit in both Stackelberg models

Seller Stackelberg model

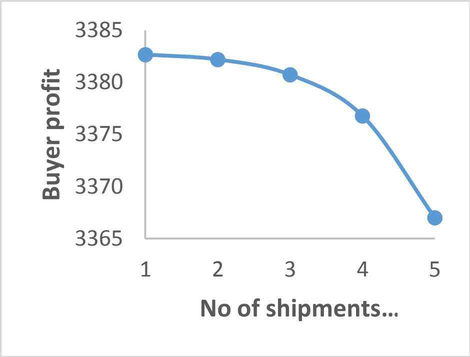


Figure 5.3.4 Effect of shipments on Buyer's Profit

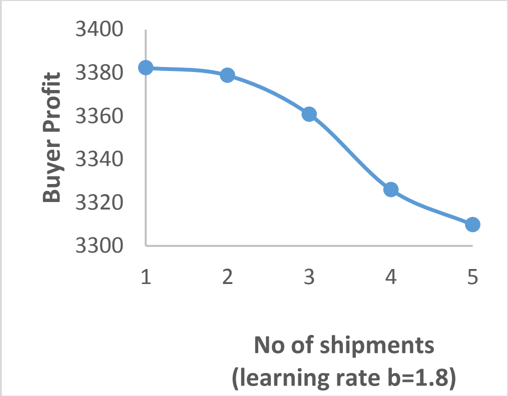


Figure 5.3.5 Effect of shipments on Buyer's Profit

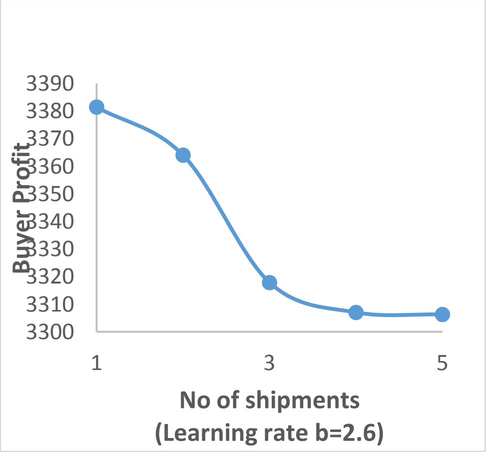


Figure 5.3.6 Effect of shipments on Buyer's Profit

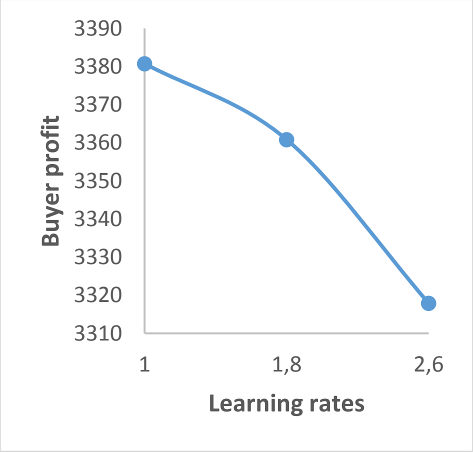
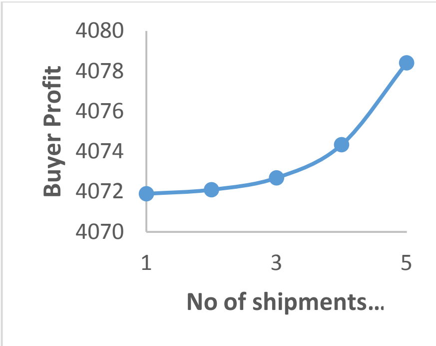


Figure 5.3.7 Effect of learning rate on Buyer's Profit

Buyer Stackelberg model



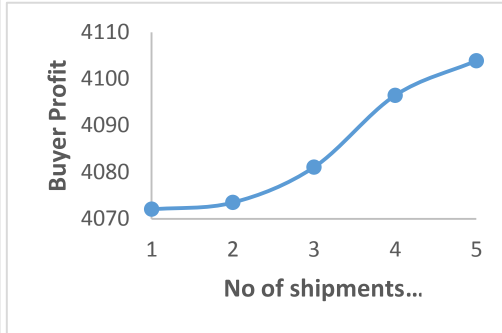
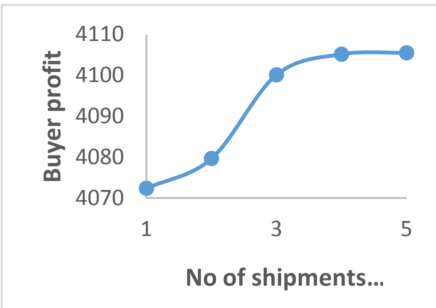


Figure 5.3.8 Effect of shipments on Buyer's Profit Figure 5.3.9 Effect of shipments on Buyer's Profit



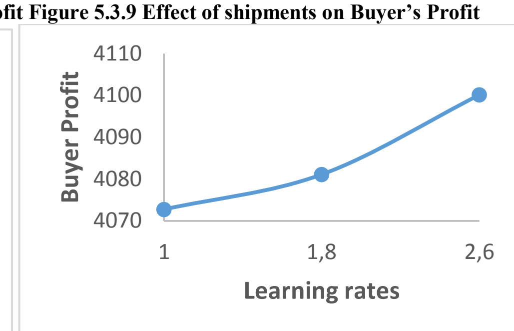


Figure 5.3.10 Effect of shipments on Buyer's Profit Figure 5.3.11Effect of learning rate on Buyer's Profit

6. OBSERVATIONS

Following are the observations:

1. Numerical example shows that buyer and seller both are is improved off when they are leader and got less profit in case of follower.
2. Table **5.1.2** and Table **5.1.3** show that as the learning rate increases from 1.0 to 2.6 with the number of shipments, the seller profit increases and buyer's profit decreases when the seller act as a leader and buyer as follower but the result is reverse in case of Buyer- Stackelberg model. It indicates that both the player gets benefitted in case of having leadership position. Buyer get more profit than seller in both the model because of learning effect.
3. In Seller-Stackelberg model, Table **5.2.1.4**, Table **5.2.3.6** and Table **5.2.5.8** shows that number of shipments increases, the order quantity decreases results in decrement in the buyer's profit whereas the seller profit increases as the selling price of the seller increases. In Buyer-Stackelberg model, Table **5.2.2.5,** Table **5.2.4.7** and Table **5.2.6.9** indicate that buyer's profit increased, and order quantity gets decreases, when increment is done in number of shipments. It shows that both the player gets benefited in case of headship position. Buyer player have more income than the seller player in both the case.
4. In Seller-Stackelberg model, Table **5.1.2,** an optimal learning rate at which buyer's profit attain maximum at $b = 1.0$. seller's get maximum profit at the optimum learning rate $b = 2.6$.
5. In the Buyer-Stackelberg model, Table **5.1.3**, results indicate that, an optimal learning rate to which buyer's profit is maximum at $b = 2.6$ whereas seller's profit is maximum at the optimum learning rate $b = 1.0$
6. Fig. **5.3.4**., fig. **5.3.5** and fig.**5.3.6**. concludes that as number of shipments increases with a given learning rate, buyer profit decreases in seller Stackeberg model in which buyer behave as a follower.
7. Fig.**5.3.7**, fig. **5.3.8** and fig. **5.3.9** illustrates that buyer profit increase as number of shipments decreases with an assumed learning rate in Buyer-Stackeberg model. In this model, buyer behave as leader.
8. Fig **5.3.10**. shows that as learning rate increases in Seller Stackelberg model, buyer profit decreases. In this model buyer act as a follower and seller behaves as a leader. Seller moves first and buyer have to follow the policies given by the seller.

9. Fig **5.3.11**. Indicates that in Buyer Stackelbeg model, as learning rate increases, buyer profit increases. In this model, buyer acts a leader and moves first, and seller play a role of follower. Seller have to follow the instruction given by buyer.

7. CONCLUSION

In this paper, some supply chain models with learning effect have been developed to establish the relation/communication among the players, the supplier and purchaser by game theoretic approach. The effect of leaning is shown on prime policies of supply chain members for faulty items. This model enhanced the order quantity and equivalent profit of buyer with some considerations. Outcome shows that the learning outcome has a magnificent effect on the reckoning of gain or losses of the supply chain. Mathematical illustrations areshown to support the theory of this paper. Result shows that buyer's profits is more than the seller in both the model due to the learning effect. Both the players get profited in case of leadership status. This model can be prolonged by considering the notion of credit period and shortages. Present model can be stretched to the model in which publicity cost can be shared by both the players. A future expansion to present model can be assume a stochastic instead of having a deterministic learning curve.

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