

A GAME THEORY SOLUTION TO THE DILEMMA OF LEGAL UTILITARIANISM VERSUS ETHICS USING NEUTROSOPHIC AND DEONTIC LOGIC

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ABSTRACT

This research addresses the tension between legal utilitarianism and the protection of fundamental rights in state security contexts, taking as a case study the prolonged state of emergency in El Salvador. The relevance of the topic is evident in the growing debate over the legitimacy of security policies that prioritize effectiveness and the reduction of violence, but that simultaneously weakens constitutional guarantees and threaten democratic stability. This issue is crucial in Latin America, where states face structural challenges of violence and governability. The primary objective of this research is to examine the viability of a robust utilitarian framework within state security policies, while critically assessing its inherent tensions with principles of justice, equity, and individual agency. To achieve this objective, a model based on the operators of deontic logic is designed, evaluated with a triple-valued neutrosophic semantics of truth (T), indeterminacy (I), and falsity (F), to capture the contradictions that may exist in the evaluations. To resolve the dilemma, we use Nash equilibrium with mixed strategies for a non-cooperative game with three actors: the State, the Legal System, and the Citizenry. The legal systems of El Salvador and Ecuador are compared.

KEYWORDS: Legal Utilitarianism, Fundamental Rights, Deontic Operators, Deontic Logic; Neutrosophic Logic, Non-Cooperative Game Theory, Nash Equilibrium, Mixed Strategies.

MSC: 03B42, 03B52, 91A10, 91B06, 91B10.

RESUMEN

Esta investigación aborda la tensión entre el utilitarismo jurídico y la protección de los derechos fundamentales en contextos de seguridad estatal, tomando como estudio de caso el prolongado estado de emergencia en El Salvador. La relevancia del tema es evidente en el creciente debate sobre la legitimidad de las políticas de seguridad que priorizan la eficacia y la reducción de la violencia, pero que simultáneamente debilitan las garantías constitucionales y amenazan la estabilidad democrática. Este problema es crucial en América Latina, donde los Estados enfrentan desafíos estructurales de violencia y gobernabilidad. El objetivo principal de esta investigación es examinar la viabilidad de un marco utilitarista robusto dentro de las políticas de seguridad estatal, evaluando críticamente sus tensiones inherentes con los principios de justicia, equidad y agencia individual. Para lograrlo, se diseña un modelo basado en operadores de lógica deóntica, evaluado con una semántica neutrosófica de tres valores de veracidad (T), indeterminación (I) y falsedad (F), para capturar las contradicciones que puedan existir en las evaluaciones. Para resolver el dilema, se utiliza el equilibrio de Nash con estrategias mixtas en un juego no cooperativo con tres actores: el Estado, el Sistema Jurídico y la Ciudadanía. Se comparan los sistemas legales de El Salvador y Ecuador.

PALABRAS CLAVES: Utilitarismo jurídico, Derechos fundamentales, Operadores deónticos, Lógica deóntica, Lógica neutrosófica, Teoría de juegos no cooperativa, Equilibrio de Nash, Estrategias mixtas.

1. INTRODUCCIÓN

Legal utilitarianism justifies the legitimacy of actions based on their contribution to collective well-being, assessed through the principle of utility proposed by Bentham, which guides actions toward maximizing happiness. John Stuart Mill expanded this view by introducing a qualitative distinction between types of pleasure. This doctrine has influenced political philosophy, economics, and institutional design, although it has been criticized for its potential incompatibility with the protection of fundamental rights and human dignity. Objections include the reduction of well-being to subjective pleasure and the instrumentalization of the individual.

For Jeremy Bentham, rule utilitarianism involves evaluating laws and decisions according to their ability to produce the greatest possible well-being. John Stuart Mill argues that actions are right to the extent that they tend to promote happiness, and wrong to the extent that they tend to produce the opposite. Thus, he introduced into the understanding of legal utilitarianism the idea that the normative function of law should be guided by utility, understood as the promotion of collective well-being. On the other hand, Amartya Sen conceives utilitarianism as a

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basis for social evaluation based on the sum of individual satisfactions, but recognizes that this basis is vulnerable to problems of equity, rights, and individual agency, which is why he criticizes this legal category for ignoring non-utilitarian dimensions of well-being, such as freedom and justice.

As a result of the systematization of the theoretical positions analyzed, this study assumes that legal utilitarianism constitutes a normative doctrine that bases the validity of norms on their capacity to maximize collective well-being, based on the evaluation of their consequences for general happiness. However, its application requires incorporating limits linked to equity, rights, and individual agency, considered irreducible elements of a just conception of the legal order.

The security approach adopted by El Salvador, under the leadership of Nayib Bukele, can be analyzed from the perspective of classical utilitarianism proposed by Jeremy Bentham in his work “Introduction to the Principles of Morals and Legislation”. According to Bentham, nature has placed humanity under the rule of two sovereigns: pain and pleasure. They govern everything we do, everything we say, everything we think. He argued that the purpose of all legislation should be the maximization of the general welfare, understood as the greatest happiness for the greatest number of people. In this context, the Salvadoran security strategy seeks to reduce the social pain generated by gang violence, even at the cost of temporarily restricting certain individual rights. This utilitarian approach is reflected in concrete results, such as the notable reduction in homicides in 2023, where the rate was 2.4 per 100,000 inhabitants, the lowest in recent Latin American history.

President Bukele has defended this model as pragmatic. Despite improved security and positive economic prospects, El Salvador also faces criticism for the erosion of the rule of law, the concentration of power, and a lack of transparency in policy implementation, raising questions about the sustainability and long-term consequences of this utilitarian approach.

This research begins with the general question: To what extent can the application of utilitarian principles in security policies weaken the ethical foundations of law without guaranteeing justice, equity, and respect for individual agency? This formulation problematizes the use of utilitarian criteria as a basis for state decisions in highly conflictive contexts. We aim to determine under what conditions of citizenship, legality, and the state a society under a utilitarian regime can be produced.

In this context, the authors hypothesize that the application of utilitarian criteria in security policies maximizes collective well-being but undermines essential principles of law by omitting considerations of equity, rights, and individual agency. The relationship between public utility and normative legitimacy is, therefore, a central object of analysis. Furthermore, we study whether a combination of factors exists that makes the implementation of a utilitarian regime feasible.

To this end, we rely on a comparative analysis of the Penal Code of El Salvador and the Comprehensive Organic Penal Code of Ecuador, using the comparative law method and a philosophical-legal perspective.

To accomplish these objectives, we apply neutrosophic logic to the deontic operators of Obligation (O), Permission (P), and Prohibition (F) [3][11][19][20]. Specifically, we evaluate the legal aspects measured from the perspective of various legal frameworks, within a utilitarian state and a rule of law state. We then compare both outcomes from a game-theoretic perspective with the Nash equilibrium solution for mixed strategies [4][15].

First of all, let us recall that deontic logic (from the Greek *deon*, meaning “what is due” or “what is necessary”) focuses on normative statements, such as “what must be done” or “what is not permitted” [8]. It does not deal with what is true or false, but rather with what must be valid or invalid. Deontic logic has applications in ([7][10][22]):

1. Law: To analyze legal norms, legal duties, and individual rights.
2. Ethics: To formalize moral judgments about what should or should not be done.
3. Artificial intelligence: In systems that must make ethical or legal decisions (e.g., self-driving cars).
4. Philosophy: To study what should be versus what is.

Neutrosophic logic is dedicated to evaluating a proposition primarily through propositional calculus based on a triplet of values: one of truth (T), another of indeterminacy (I), and a third of falsity (F) [13]. In this way, indeterminacy is explicitly incorporated, due not only to neutrality but also to the lack of knowledge or information, the presence of contradictions, inconsistencies, and paradoxes, among others. All of this is part of the daily routine of legal processes, where the objectivity of the facts is affected by the divergent criteria of the parties involved in litigation. Usually, each of the parties expresses their points of view, which are contrary to each other, and where the administration of justice must be carried out to the extent of several criteria.

On the other hand, game theory is the mathematical theory designed to model conflict situations. Legal science is, par excellence, the science of the search for equilibrium and justice [2][14]. Therefore, it can be modeled with the help of game theory. Non-cooperative games in particular model conflicts where the parties involved have opposing interests and do not cooperate.

In summary, in this paper, we propose to study the theoretical and practical opposition that arises from the implementation of either utilitarianism or the rule of law as part of a country's regime. We also incorporate the type of authoritarian state into our analysis. We specifically evaluate legal systems that are fundamentally opposed, namely the Salvadoran and Ecuadorian legal systems. To do so, we use a logical-mathematical model devised by the authors, which includes neutrosophic logic to capture the uncertainty in valuations, deontic operators used to represent legal terms, and non-cooperative game theory to determine the equilibrium position in this contradiction. Specifically, we determine what type of citizenship, state, and legal system leads to a Nash equilibrium.

The paper is divided into a Related Works section, which explains fundamental concepts of legal science, such as utilitarianism versus ethics, and summarizes the operators of deontic logic, neutrosophic logic, and the Nash solution to non-cooperative games. The Study section proposes a logical-mathematical model of the problem and a solution to this dilemma. Finally, the article concludes.

2. RELATED WORK

This section contains the main concepts of the theories we propose in this article. These are: utilitarianism and the rule of law in legal sciences; operators of deontic logic and neutrosophic logic; non-cooperative game theory; and Nash equilibrium.

2.1. Utilitarianism versus Rule of Law

In the analysis of the Salvadoran regime of exception, a utilitarian justification of state policies, especially in matters of security, is observed. This practice is based on Bentham's perspective, which defends the normative value of utility as a legislative guide, exploring the idea that the normative and administrative decisions adopted by the State respond to a utilitarian logic. The paper analyzes whether the justification of measures is predominantly based on consequentialist or merely deontological criteria. This logic raises important tensions, ranging from the instrumental effectiveness of public policies to the application of human rights.

Regarding the restriction of rights, the intensity, duration, and scope of the restrictions imposed on fundamental rights through the state of emergency are assessed, especially in relation to personal liberty, due process, and the presumption of innocence. Although Article 29 of El Salvador's Constitution authorizes the suspension of certain rights in exceptional contexts, its repeated application (such as in March 2022, when it was amended more than 20 times) has transformed it from a temporary measure for isolated cases under specific circumstances into a permanent and structural practice. In this context, an imbalance between security and rights is identified, where the prolongation of the state of emergency erodes the rule of law and weakens democratic guarantees.

When exploring the perspective of normative coherence, this exceptional regime reveals a significant dissociation between the legal framework of guarantees and current government practices, resulting from the instrumental application of legal utilitarianism. Furthermore, the lack of effective controls by the legislative body and the erosion of judicial independence affect procedural coherence, generating a deficit in legal and democratic legitimacy, as Mill suggests.

The state of emergency implemented in El Salvador, conceived from a utilitarian perspective, reflects a notable failure to recognize individual autonomy as a legal and moral value. This shortcoming is evident in practices such as arbitrary detentions and preemptive criminalization, particularly targeting vulnerable communities. Their members, as minorities, are prone to being instrumentalized as means to achieve collective ends, rather than being recognized as legal subjects with inherent dignity and the ability to choose. This situation demonstrates how individuals can be reduced to mere instruments of a state logic oriented toward efficiency, to the detriment of their fundamental rights.

The state of emergency in El Salvador demonstrates a discrepancy between the guarantee-based model established by the 1983 Constitution and its practical implementation. Although the constitutional text recognizes non-derogable rights, even in emergency contexts, and establishes clear limits on the exercise of power, the utilitarian approach currently adopted—this transformation, without a formal reform of the constitutional text—has resulted in a de facto model of governance with authoritarian features, which erodes essential pillars of the rule of law, such as constitutional supremacy, judicial independence, and the principle of non-regression in human rights matters.

Utilitarianism seeks to maximize the well-being of the greatest number of people. In El Salvador, this logic has been applied especially in public security policies under Nayib Bukele's government. Although there are ethical criticisms, some positive results include:

- Crime reduction: Measures such as the state of emergency and mass arrests have significantly reduced rates of violence and homicide.
- Greater perception of security: Many citizens report feeling safer in their communities, which has improved the quality of life in areas previously dominated by gangs.

- Popular support: The policies have generated high levels of approval, as they are perceived as effective for collective well-being in the short term.
- From a utilitarian perspective, these benefits can be considered morally justifiable if the overall result is a significant improvement in the lives of the majority, even if certain individual rights are sacrificed.

On the other hand, the rule of law is a fundamental principle where everyone—citizens, officials, businesses, and the state itself—is subject to fair laws, applied impartially and transparently. In Ecuador, this concept has its ups and downs.

Among the positive aspects are:

- Constitutional recognition of rights: Ecuador guarantees fundamental rights such as life, equality, freedom of expression, and the rights of indigenous peoples.
- Environmental protection: It was the first country to legally recognize the rights of nature, which has promoted conservation policies.
- Solid legal framework: The Constitution establishes clear procedures for the creation and implementation of laws, which strengthens institutional legitimacy.

Some negative aspects are:

- Institutional corruption: 64% of Ecuadorians believe that most politicians are corrupt, which weakens confidence in the judicial system.
- Weak enforcement: Although rights are guaranteed on paper, their implementation is limited by a lack of resources, corruption, and bureaucracy.
- Citizen insecurity: In 2021, one in four Ecuadorians was a victim of crime, and more than half felt unsafe in their neighborhood.
- Distrust in justice: Only 43.8 % believe that the courts guarantee fair trials.

That is to say, in general terms, it cannot be clearly established that one system is more effective than the other; each of them has both positive and negative results. Some related studies in Argentina, Colombia, Spain, and the United States can be found in [5][12][16][18].

2.2. Deontic Logic and Neutrosophic Logic

The details of Deontic Logic will be discussed below. Deontic Logic is the branch of formal logic that primarily uses and symbolizes linguistic material related to the normative function for the analysis of reasoning and the formal structures of communication [23]. Therefore, it does not exclusively use language related to the informative function.

The normative function of language is a species of directive function, and includes the language of legal norms, but not only this, but also that of other types of norms. Therefore, deontic logic can also be used in religion, in morality (whether social, familial, or otherwise), and in any other type of normative phenomenon.

In this logic, the elements of propositional logic remain; these are [23]:

1. An atomic proposition denoted by lowercase Latin letters, e.g., p , q , r , ...
2. A unary negation operator for an atomic or compound proposition $\neg p$ meaning “not p ”.
3. The conjunction that means that two propositions, whether atomic or compound, are fulfilled at the same time, $p \wedge q$ which symbolizes “ p and q .”
4. The disjunction that means that at least one of two propositions is fulfilled, whether atomic or compound, $p \vee q$, which symbolizes “ p or q ”.
5. The implication means that if one is true, then the other is true, whether these are atomic or compound $p \rightarrow q$, which symbolizes “ p implies q ” or “if p then q ”.

Added to this are deontic modalities, such as the modality O that means obligation and that appears before a proposition as Op if it symbolizes that “ p is obligatory,” where p is not just any proposition, but one that indicates a behavior.

As for bivalent semantics where 0 means false and 1 means true, it is necessary that for negation $\neg p$ and $\neg Op$ the same truth table is satisfied, but not for Op and $O\neg p$, since although when Op is true then $O\neg p$ is false, in the case of Op is false, $O\neg p$ may be either false or true.

On the other hand, $Op \vee Oq$ has the same truth table as $p \vee q$; however, $O(p \vee q)$ has the following truth table, which is different:

Op	Oq	$O(p \vee q)$
1	1	0
1	0	0
0	1	0
0	0	0 or 1

Table 1. Truth table for the obligatory modality of disjunction. Source [24].

Like disjunction, implication $Op \rightarrow Oq$ satisfies the same truth table as $p \rightarrow q$; however, it is not the same as that of $O(p \rightarrow q)$. It satisfies the truth values given in Table 2.

p	Oq	$O(p \rightarrow q)$
1	1	1
1	0	1
0	1	0
0	0	1

Table 2. Truth table for the obligatory implication modality. Source [24].

Therefore, the following reasoning is valid:

$Op \vee Oq$	$Op \rightarrow Oq$	$O(p \rightarrow q)$
$\neg Op$	Op	$\neg Oq$
Oq	Oq	$\neg p$

The last of them is the *modus tollens* ([24]).

Bi-implication is still defined as in propositional logic with the formula $p \leftrightarrow q := (p \rightarrow q) \wedge (q \rightarrow p)$.

Other modalities can be defined besides the O of obligation. One is the P for permission, the other is the F for prohibition.

Added to this are the axioms and deductions of propositional calculus, for example, *Modus Ponens* and *Modus Tollens*, when propositions with modalities are not taken into account.

Below, we recall the fundamental concepts of neutrosophic logic.

Definition 1 ([17]). Consider a universe of discourse X . A *Neutrosophic Set* (NS) A is defined by three membership functions:

$$u_A(x), r_A(x), v_A(x) : X \rightarrow]^{-}0, 1^{+}[,$$

which are constrained by the condition:

$$^{-}0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^{+} \text{ for every element } x \in X.$$

These functions, denoted by $u_A(x)$, $r_A(x)$, and $v_A(x)$, map to standard or non-standard subsets of $]^{-}0, 1^{+}[$ and represent the degrees of truth-membership, indeterminacy-membership, and falsity-membership of the element x to the set A , respectively.

Definition 2 ([17]). Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A over X is expressed as:

$$A = \{(x, u_A(x), r_A(x), v_A(x)) : x \in X\} \quad (1)$$

where the functions $u_A, r_A, v_A : X \rightarrow [0,1]$ are defined such that $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all $x \in X$. These functions assign the truth-membership, indeterminacy-membership, and falsity-membership degrees of $x \in A$. A *Single-Valued Neutrosophic Number* (SVNN) can be succinctly represented by the triplet $A = (a, b, c)$, where $a, b, c \in [0,1]$ and $0 \leq a + b + c \leq 3$.

Within the framework of neutrosophic logic, the *evaluation* of a proposition p is defined as [3]:

$$v_N(p) = (t, i, f) \quad (2)$$

Such that $(t, i, f) \subseteq [0, 1]^3$, and specifically $(t, i, f) \in [0, 1]^3$. Here, t means the truth-degree, i the indeterminacy-degree, and f the falsity-degree of the proposition p .

For a given universe X , let $x = (T_x, I_x, F_x)$, and $y = (T_y, I_y, F_y)$ be two elements of an SVN. A partial order relationship $x \leq_N y$ is established if and only if $T_x \leq T_y$, $I_x \geq I_y$, and $F_x \geq F_y$. The resulting structure (X, \leq_N) is a partially ordered set (poset). Furthermore, the algebraic structure (L, \wedge, \vee) forms a lattice, as it constitutes a triple direct product of lattices, with the *meet* and *join* operations defined as [9]:

$$x \wedge y = (\min\{T_x, T_y\}, \max\{I_x, I_y\}, \max\{F_x, F_y\})$$

$$x \vee y = (\max\{T_x, T_y\}, \min\{I_x, I_y\}, \min\{F_x, F_y\}).$$

It can be readily demonstrated that this lattice is complete.

It is pertinent to note that this definition of the order relation \leq_N remains applicable to interval-valued neutrosophic sets upon the substitution of point-valued operators with their interval-valued counterparts.

Two special elements are identified within this structure: $0_N = (0, 1, 1)$, which acts as the infimum, and $1_N = (1, 0, 0)$, which acts as the supremum for any SVN under the order \leq_N .

For any two neutrosophic sets A and B , three fundamental operations are defined as follows [9]:

1. Conjunction (Intersection): $A \cap B = A \wedge B$,
2. Disjunction (Union): $A \cup B = A \vee B$,
3. Complement (Negation): $A^C = (F_A, 1 - I_A, T_A)$.

Definition 3. A *neutrosophic norm* or *n-norm* N_n is a mapping [9]:

$$N_n: ([^{-0}, 1^+[x]^{-0}, 1^+[x]^{-0}, 1^+])^2 \rightarrow [^{-0}, 1^+[x]^{-0}, 1^+[x]^{-0}, 1^+]$$

defined for $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$ as:

$$N_n(x(T_x, I_x, F_x), y(T_y, I_y, F_y)) = (N_n T(x, y), N_n I(x, y), N_n F(x, y)),$$

where $N_n T$, $N_n I$, and $N_n F$ denote the resulting truth, indeterminacy, and falsity membership degrees of the conjunction of x and y . For all x, y, z in the domain, the mapping N_n must fulfill the following axioms:

1. Boundary Conditions: $N_n(x, 0_N) = 0_N$ and $N_n(x, 1_N) = x$,
2. Commutativity: $N_n(x, y) = N_n(y, x)$,
3. Monotonicity: If $x \leq_N y$, then $N_n(x, z) \leq_N N_n(y, z)$,
4. Associativity: $N_n(N_n(x, y), z) = N_n(x, N_n(y, z))$.

Definition 4. A *neutrosophic conorm* or *n-conorm* N_c is a mapping [9]:

$$N_c: ([^{-0}, 1^+[x]^{-0}, 1^+[x]^{-0}, 1^+])^2 \rightarrow [^{-0}, 1^+[x]^{-0}, 1^+[x]^{-0}, 1^+]$$

defined for $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$ as:

$$N_c(x(T_x, I_x, F_x), y(T_y, I_y, F_y)) = (N_c T(x, y), N_c I(x, y), N_c F(x, y)),$$

where $N_c T$, $N_c I$, and $N_c F$ denote the resulting truth, indeterminacy, and falsity membership degrees of the disjunction of x and y . For all x, y, z in the domain, the mapping N_c is required to satisfy these axioms:

1. Boundary conditions: $N_c(x, 0_N) = x$ and $N_c(x, 1_N) = 1_N$.
2. Commutativity: $N_c(x, y) = N_c(y, x)$.
3. Monotonicity: If $x \leq_N y$, then $N_c(x, z) \leq_N N_c(y, z)$.
4. Associativity: $N_c(N_c(x, y), z) = N_c(x, N_c(y, z))$.

Definition 5. A *single-valued neutrosophic negator* is a unary operation $N_N: [0, 1]^3 \rightarrow [0, 1]^3$ that is decreasing and meets the boundary conditions [9]:

1. $N_N(0_N) = 1_N$,
2. $N_N(1_N) = 0_N$.

This negator is classified as involutive if $N_N(N_N(x)) = x$ holds for every $x \in [0, 1]^3$.

The following neutrosophic negators, drawn from the literature, are considered herein. For an SVN element $A = (T_A, I_A, F_A)$ [9]:

Involutive Negators:

1. $N_N((T_A, I_A, F_A)) = (1 - T_A, 1 - I_A, 1 - F_A)$,
2. $N_N((T_A, I_A, F_A)) = (1 - T_A, I_A, 1 - F_A)$,
3. $N_N((T_A, I_A, F_A)) = (F_A, I_A, T_A)$ and
4. $N_N((T_A, I_A, F_A)) = (F_A, 1 - I_A, T_A)$.

Non-Involutive Negators:

1. $N_N((T_A, I_A, F_A)) = \left(F_A, \frac{F_A + I_A + T_A}{3}, T_A\right)$ and
2. $N_N((T_A, I_A, F_A)) = \left(1 - T_A, \frac{F_A + I_A + T_A}{3}, 1 - F_A\right)$

2.3. Basic Notions of Non-Cooperative Game Theory

A game consists of a non-empty set of players, denoted by $N = \{1, 2, \dots, n\}$, a set of moves (or pure strategies) available to those players, denoted by $A = \{A_1, A_2, \dots, A_p\}$, and a specification of payoffs for each strategy combination [4][6]. Usually, two players are considered, where their payoffs are represented by a payoff matrix, one player selecting the row and the other the column. The element of the i th row and the j th column contains the utility obtained by player I (row-wise) when applying the i th strategy ($i \in \{1, 2, \dots, p\}$, $p \geq 1$), and when player II (column-wise) applies the j th strategy ($j \in \{1, 2, \dots, q\}$, $q \geq 1$) [1]. Let us call $u_{ij} = U(A_i, B_j)$ the payoff, where $U: A \times B \rightarrow \mathbb{R}$, A is the set of strategies of player I and B the set of strategies of player II.

“Maximin” and “minimax” criteria state that each player must minimize their maximum loss:

“Maximin” criterion: Player I chooses that his/her minimum possible collection is the highest.

“Minimax” criterion: Player II chooses the maximum payoff to Player I to be the lowest possible.

Definition 6 ([4][6]). A *Saddle Point* in a two-player zero-sum game is located at the (k, r) -th entry of the payoff matrix if it satisfies the condition:

$$\max_i \min_j u_{ij} = \min_j \max_i u_{ij}$$

Mixed strategies are defined as probability distributions over the set of pure strategies, where each pure strategy is assigned a specific probability.

Definition 7 ([4][6]). Consider a two-player game involving Player I and Player II. Let Player I's set of strategies be $A = \{A_1, A_2, \dots, A_p\}$, and Player II's set of strategies be $B = \{B_1, B_2, \dots, B_q\}$. A *mixed strategy* for Player I is a vector $x = (x_1, x_2, \dots, x_p) \in [0, 1]^p$ such that $\sum_{i=1}^p x_i = 1$. Similarly, a mixed strategy for Player II is a vector $y = (y_1, y_2, \dots, y_q) \in [0, 1]^q$ such that $\sum_{j=1}^q y_j = 1$. The *expected payoff* for Player I from Player II is defined as:

$$E(x, y) = \sum_{i=1}^p \sum_{j=1}^q x_i u_{ij} y_j = x^T u y,$$

where $u = (u_{ij})_{1 \leq i \leq p, 1 \leq j \leq q}$, is the *payoff matrix*.

Definition 8 ([4][6]). A *Saddle Point in mixed strategies* is a pair of vectors (x^*, y^*) that satisfies the condition:

$$\min_y \max_x E(x, y) = \max_x \min_y E(x, y) = E(x^*, y^*) \quad (3)$$

Theorem 1 (Nash, 1950) ([4][15]). Every finite non-cooperative game—that is, a game with a finite number of players and a finite set of pure strategies for each player—has at least one Nash equilibrium in mixed strategies. Formally, let $N = \{1, 2, \dots, n\}$ be a finite set of players, and let S_i be the finite set of pure strategies of player $i \in N$. Each player i chooses a mixed strategy $\sigma_i \in \Delta(S_i)$, the simplex of probability distributions over S_i . A mixed strategy profile $\sigma^* = (\sigma_1^*, \dots, \sigma_n^*)$ constitutes a Nash equilibrium if, for every player i and every alternative strategy $\sigma_i \in \Delta(S_i)$, it holds that $u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$. The proof relies on Kakutani's fixed-point theorem applied to the best-response correspondence of each player. This theorem guarantees that the three-player non-cooperative game used in this article—with players State, Legal System, and Citizenship, each endowed with a finite set of pure strategies—necessarily admits at least one Nash equilibrium in mixed strategies, which constitutes the theoretical foundation for the computational search carried out with Gambit 16.1.1 in Section 3.

3. THE STUDY

In this section, we first present the model we use to solve the problem we wish to study.

Traditional deontic logic works with operators such as:

$O(A)$: “A is obligated”,

$P(A)$: “A is prohibited”,

$F(A)$: “A is forbidden.”

Neutrosophic version extends this by allowing each deontic judgment to have three components:

t (truth): degree to which the norm is considered valid or applicable,

i (indeterminacy): degree of ambiguity, conflict, or lack of consensus,

f (falsehood): degree to which the rule is inapplicable or rejected.

For example:

$O(A) = (0.9, 0.06, 0.1)$: A is almost obligatory, with low ambiguity and low falsifiability.

$P(B) = (0.6, 0.3, 0.2)$: B is allowed, but there is quite a lot of indeterminacy.

To facilitate calculations, each triplet of neutrosophic values is converted into a single fuzzy number. Equation 4 of the score function is used for this end [21]:

$$S((t, i, f)) = \frac{2+t-i-f}{3} \quad (4)$$

A payoff function is defined for each player on a set of attitudes or strategies from the player's point of view, which evaluates their total ethical-legal value.

The triple $(T_{Oa_i}, I_{Oa_i}, F_{Oa_i})$ is the player's assessment of what is obligatory for the i th strategy; $(T_{Pa_i}, I_{Pa_i}, F_{Pa_i})$ is the same for what is permitted; $(T_{Fa_i}, I_{Fa_i}, F_{Fa_i})$ is the assessment of what is prohibited.

The model consists of three players: the State, the Legal System, and Citizens, each with three strategies related to utilitarianism and the rule of law. The objective is to evaluate how the three actors' strategies interact in terms of social, legal, and political utility. Thus, we have the following components:

Player	Strategy 1	Strategy 2	Strategy 3
State	Authoritarian	Legalist	Utilitarian
Legal System	Formalist	Guarantor	Pragmatic
Citizenship	Apathetic	Participatory	Activist

Table 3. Definition of the strategies followed by each player

We reviewed the scientific literature on each of these modalities of the State, the Legal System, and Citizenship. We analyzed with experts the assessments that could be obtained in each case regarding what is prohibited, what is permitted, and what is mandatory.

For each strategic combination, the experts were asked to evaluate, on a scale from 0 (not at all) to 10 (maximum), the degree of truthfulness (T), indeterminacy (I), and falseness (F) regarding how the combination represents an Obligation, Permission, or Prohibition, respectively. Thus, triples (T, I, F) were obtained. Each value of T, I, and F was then rescaled to the interval [0, 1] by dividing by 10.

The results were as follows:

Combination of Strategies			Neutrosophic Values of Deontic Operators		
State	Legal System	Citizenship	Obligated	Permitted	Forbidden
Authoritarian	Formalist	Apathetic	(0.8,0.1,0.1)	(0.3,0.1,0.6)	(0.9,0.1,0.1)
Authoritarian	Formalist	Participatory	(0.9,0.1,0.1)	(0.4,0.2,0.4)	(0.8,0.1,0.1)
Authoritarian	Formalist	Activist	(1,0,0)	(0.2,0.1,0.8)	(0.7,0.1,0.2)
Authoritarian	Guarantor	Apathetic	(0.6,0.4,0.4)	(0.5,0.5,0.5)	(0.6,0.4,0.4)
Authoritarian	Guarantor	Participatory	(0.7,0.1,0.2)	(0.6,0.4,0.4)	(0.5,0.5,0.5)
Authoritarian	Guarantor	Activist	(0.9,0.1,0.1)	(0.7,0.1,0.2)	(0.4,0.2,0.4)
Authoritarian	Pragmatic	Apathetic	(0.5,0.5,0.5)	(0.6,0.4,0.4)	(0.8,0.1,0.1)
Authoritarian	Pragmatic	Participatory	(0.7,0.1,0.2)	(0.7,0.1,0.2)	(0.7,0.1,0.2)
Authoritarian	Pragmatic	Activist	(0.9,0.1,0.1)	(0.5,0.5,0.5)	(0.6,0.4,0.4)
Legalist	Formalist	Apathetic	(0.6,0.4,0.4)	(0.4,0.2,0.4)	(0.9,0.1,0.1)
Legalist	Formalist	Participatory	(0.5,0.5,0.5)	(0.6,0.4,0.4)	(0.8,0.1,0.1)
Legalist	Formalist	Activist	(0.7,0.1,0.2)	(0.5,0.5,0.5)	(0.7,0.1,0.2)
Legalist	Guarantor	Apathetic	(0.4,0.2,0.4)	(0.6,0.4,0.4)	(0.6,0.4,0.4)
Legalist	Guarantor	Participatory	(0.3,0.1,0.6)	(0.8,0.1,0.1)	(0.5,0.5,0.5)
Legalist	Guarantor	Activist	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.4,0.2,0.4)
Legalist	Pragmatic	Apathetic	(0.3,0.1,0.6)	(0.7,0.1,0.2)	(0.7,0.1,0.2)
Legalist	Pragmatic	Participatory	(0.4,0.2,0.4)	(0.8,0.1,0.1)	(0.6,0.4,0.4)
Legalist	Pragmatic	Activist	(0.6,0.4,0.4)	(0.7,0.1,0.2)	(0.5,0.5,0.5)
Utilitarian	Formalist	Apathetic	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.8,0.1,0.1)
Utilitarian	Formalist	Participatory	(0.6,0.4,0.4)	(0.6,0.4,0.4)	(0.7,0.1,0.2)
Utilitarian	Formalist	Activist	(0.7,0.1,0.2)	(0.5,0.5,0.5)	(0.6,0.4,0.4)
Utilitarian	Guarantor	Apathetic	(0.4,0.2,0.4)	(0.6,0.4,0.4)	(0.6,0.4,0.4)

Utilitarian	Guarantor	Participatory	(0.3,0.1,0.6)	(0.8,0.1,0.1)	(0.5,0.5,0.5)
Utilitarian	Guarantor	Activist	(0.5,0.5,0.5)	(0.9,0.1,0.1)	(0.4,0.2,0.4)
Utilitarian	Pragmatic	Apathetic	(0.2,0.1,0.7)	(0.7,0.1,0.2)	(0.7,0.1,0.2)
Utilitarian	Pragmatic	Participatory	(0.3,0.1,0.6)	(0.9,0.1,0.1)	(0.6,0.4,0.4)
Utilitarian	Pragmatic	Activist	(0.4,0.2,0.4)	(0.8,0.1,0.1)	(0.5,0.5,0.5)

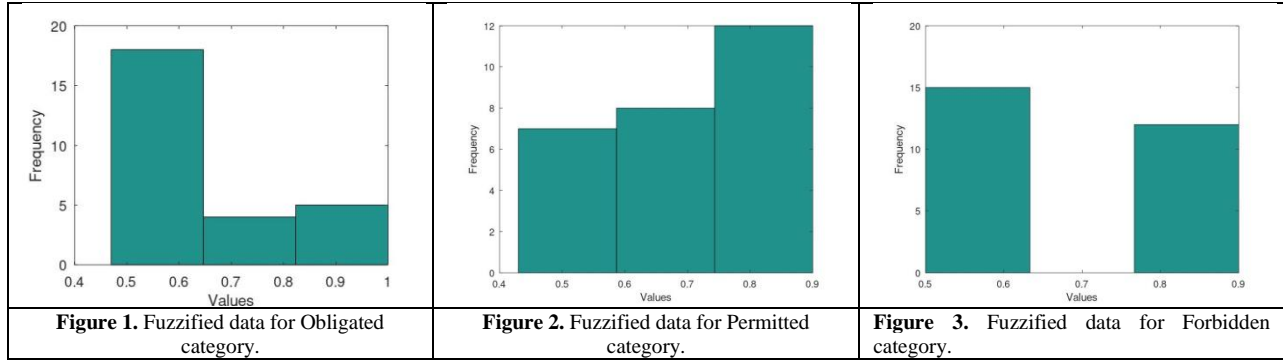
Table 4. Neutrosophic values of the deontic operators for each combination of strategies of the three players

Table 5 summarizes the values of the neutrosophic numbers fuzzified with the help of Equation 4.

Combination of Strategies			Neutrosophic Values Fuzzification of Deontic Operators		
State	Legal System	Citizenship	Obligated	Permitted	Forbidden
Authoritarian	Formalist	Apathetic	0.87	0.53	0.90
Authoritarian	Formalist	Participatory	0.90	0.6	0.87
Authoritarian	Formalist	Activist	1	0.43	0.80
Authoritarian	Guarantor	Apathetic	0.6	0.5	0.6
Authoritarian	Guarantor	Participatory	0.80	0.6	0.5
Authoritarian	Guarantor	Activist	0.90	0.80	0.6
Authoritarian	Pragmatic	Apathetic	0.5	0.6	0.87
Authoritarian	Pragmatic	Participatory	0.80	0.80	0.80
Authoritarian	Pragmatic	Activist	0.90	0.5	0.6
Legalist	Formalist	Apathetic	0.6	0.6	0.90
Legalist	Formalist	Participatory	0.5	0.6	0.87
Legalist	Formalist	Activist	0.80	0.5	0.80
Legalist	Guarantor	Apathetic	0.6	0.6	0.6
Legalist	Guarantor	Participatory	0.53	0.87	0.5
Legalist	Guarantor	Activist	0.5	0.90	0.6
Legalist	Pragmatic	Apathetic	0.53	0.80	0.80
Legalist	Pragmatic	Participatory	0.6	0.87	0.6
Legalist	Pragmatic	Activist	0.6	0.80	0.5
Utilitarian	Formalist	Apathetic	0.5	0.5	0.87
Utilitarian	Formalist	Participatory	0.6	0.6	0.80
Utilitarian	Formalist	Activist	0.80	0.5	0.6
Utilitarian	Guarantor	Apathetic	0.6	0.6	0.6
Utilitarian	Guarantor	Participatory	0.53	0.87	0.5
Utilitarian	Guarantor	Activist	0.5	0.90	0.6
Utilitarian	Pragmatic	Apathetic	0.47	0.80	0.80
Utilitarian	Pragmatic	Participatory	0.53	0.90	0.6
Utilitarian	Pragmatic	Activist	0.6	0.87	0.5

Table 5: Neutrosophic Values fuzzifications of the deontic operators for each combination of strategies of the three players using Equation 3

In Figures 1, 2, and 3 we show the histograms of the fuzzified data for Obligated, Permitted, and Forbidden, respectively.



By conceptualizing the State, Citizenship, and the Legal System as strategic actors within a political framework, we can formulate abstract utility functions for each, grounded in three fundamental variables:

The three categories show mean values between 0.65 and 0.69, with standard deviations between 0.14 and 0.16. These results indicate medium-to-high truth values.

- O = Obligation (level of normative imposition),
- P = Permission (level of freedom of action),
- F = Forbidden (level of restriction or censorship).

Each player has different interests, but we will not assume specific ideologies or regimes. We simply model their utility based on how those three variables behave.

- State Utility (U_S):

The state seeks stability, control, and legitimacy. Its utility may depend on how effectively it enforces rules, maintains order (while keeping the forbidden in check), and allows enough to prevent dissent. We represent this by the following utility function:

$$U_S = O + (1 - F) + P \tag{5}$$

- Utility of the Legal System (U_J)

The legal system strives for normative coherence, fairness, and functionality. Its usefulness depends on how it balances what is allowed and what is forbidden, and on how clear and effective its obligations are.

$$U_J = P - F + O \tag{6}$$

- Utility of Citizenship (U_C)

Citizens seek autonomy, protection, and participation. Their usefulness depends on how freely they can act, how much they are imposed upon, and how much they are restricted.

$$U_C = P - O - F \tag{7}$$

Table 6 contains the utility triplets for the state, the legal system, and the citizenry for each possible combination of strategies, using Equations 5, 6, and 7.

Combination of Strategies			Utilities for each actor		
State	Legal System	Citizenship	State	Legal System	Citizenship
Authoritarian	Formalist	Apathetic	1.5	0.5	-1.24
Authoritarian	Formalist	Participatory	1.63	0.63	-1.17
Authoritarian	Formalist	Activist	1.63	0.63	-1.37
Authoritarian	Guarantor	Apathetic	1.5	0.5	-0.7
Authoritarian	Guarantor	Participatory	1.9	0.9	-0.7
Authoritarian	Guarantor	Activist	2.1	1.1	-0.7
Authoritarian	Pragmatic	Apathetic	1.23	0.23	-0.77
Authoritarian	Pragmatic	Participatory	1.8	0.8	-0.8
Authoritarian	Pragmatic	Activist	1.8	0.8	-1
Legalist	Formalist	Apathetic	1.3	0.3	-0.9
Legalist	Formalist	Participatory	1.23	0.23	-0.77
Legalist	Formalist	Activist	1.5	0.5	-1.1

Legalist	Guarantor	Apathetic	1.6	0.6	-0.6
Legalist	Guarantor	Participatory	1.9	0.9	-0.16
Legalist	Guarantor	Activist	1.8	0.8	-0.2
Legalist	Pragmatic	Apathetic	1.53	0.53	-0.53
Legalist	Pragmatic	Participatory	1.87	0.87	-0.33
Legalist	Pragmatic	Activist	1.9	0.9	-0.3
Utilitarian	Formalist	Apathetic	1.13	0.13	-0.87
Utilitarian	Formalist	Participatory	1.4	0.4	-0.8
Utilitarian	Formalist	Activist	1.7	0.7	-0.9
Utilitarian	Guarantor	Apathetic	1.6	0.6	-0.6
Utilitarian	Guarantor	Participatory	1.9	0.9	-0.16
Utilitarian	Guarantor	Activist	1.8	0.8	-0.2
Utilitarian	Pragmatic	Apathetic	1.47	0.47	-0.47
Utilitarian	Pragmatic	Participatory	1.83	0.83	-0.23
Utilitarian	Pragmatic	Activist	1.97	0.97	-0.23

Table 6. Utility values for the state, the legal system, and citizens for each possible combination of strategies

According to Table 6, the mean utility for the State is 1.65 with a standard deviation of 0.25; for the Legal System, the mean is 0.65 with a standard deviation of 0.25; and for Citizenship, the mean is -0.66 with a standard deviation of 0.35. Therefore, the State's profits are always maximal, whereas those of the citizenship are negative. The results for the fuzzified utilities are plotted in the histograms as can be seen in Figures 4, 5, and 6.

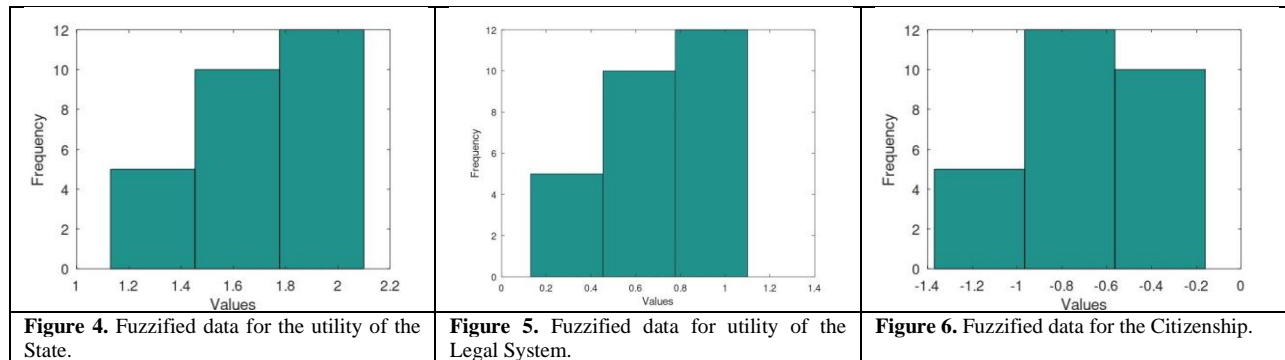


Figure 4. Fuzzified data for the utility of the State.

Figure 5. Fuzzified data for utility of the Legal System.

Figure 6. Fuzzified data for the Citizenship.

We calculated the solutions using Gambit 16.1.1 software. The first outcome was the pure equilibrium of an Authoritarian State, a Guarantor Legal System, and an Activist Citizenship. However, we consider this solution unlikely in reality because the combination of an Authoritarian State and a Guarantor of Legality is contradictory, as the state would tend to violate citizens' rights, while legality would tend to protect them. Note that the state and legality tend to be coherent. On the other hand, an Activist Citizenship would create a tense situation within the country, as it would oppose the authoritarian state.

The second option is the pure Utilitarian State equilibrium, with a Guarantee-Based Legal System and Participatory Citizenship. This combination is possible in reality, but with potential tension between the State and the Legal System. The State may be tempted to violate citizens' fundamental rights on the grounds of protecting the collective good, while the legal system would guarantee these individual rights. A participatory citizenry would make the system sustainable if it supported the State. Citizens may grow weary of crime, as in El Salvador, and therefore support a State like Bukele's, which eliminates these rights in favor of citizen security.

Another profile is a state with mixed characteristics, with 1/3 authoritarianism and 2/3 utilitarianism, with a legal system based on guarantees and participatory citizenship. This would be a similar profile to the previous one. In fact, this profile describes the current Salvadoran system well.

A final profile is the combination of a Legalist State, with Guarantee-Based Legislation and Participatory Citizenship. The characteristics of this State with this Legal System are what Ecuador currently has as a system,

combined with a citizenry that is participatory and, to some extent, also activist. This is a coherent model that should not have internal contradictions.

4. CONCLUSIONS

A comparative analysis of the penal and constitutional systems of El Salvador, Ecuador, Colombia, Spain, and Argentina demonstrates that each country adopts different ways of balancing public utility with the guarantee of rights. While Ecuador implements a guarantor model articulated with the concept of “good living,” and Spain imposes normative limits on utility through human dignity, El Salvador favors a pragmatic and functional approach, where operational effectiveness prevails over normative controls. This model has led to the normalization of the regime of exception, jeopardizing democratic equilibrium.

In this article, we modeled the deontic operators Obligation, Prohibition, and Permission, hybridized with neutrosophic logic in models of possible societies. To capture the indeterminacy of evaluations within decision-making, we used neutrosophic logic with the triple values of truth, indeterminacy, and falsity (T, I, F), all independent of each other. The deontic operators allow us to model the normative situation specific to law. We introduced equations that allow us to determine the payoff for the state, legislation, and citizens in different social models. We transformed this representation into a non-cooperative game theory problem, where we computed the Nash equilibrium for mixed strategies.

Bukele's utilitarian system in El Salvador can be sustained with any legal system, but with a participatory citizenry that supports it. A system like Ecuador's, based on guarantees, ethical principles, and with a participatory citizenry, is also sustainable. Both models were Nash equilibria, demonstrating that societies can be built with both tendencies. In future work, we will study the incorporation of other actors to deepen the study, and we will analyze models where more neutrosophic tools are present. Additionally, we recommend conducting a sensitivity analysis on the neutrosophic values assigned to the deontic operators in Table 4, verifying how reasonable variations in the triplets (T, I, F) affect the Nash equilibria obtained. This would allow evaluating the robustness of the model against the inevitable expert subjectivity inherent in the assignment of such values, thereby strengthening the internal validity of the comparative conclusions between the Salvadoran and Ecuadorian legal systems.

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