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A NEW APPROACH TO MULTILEVEL PROGRAMMING PROBLEM WITH MULTI-CHOICE PARAMETERS

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ABSTRACT
This article models a multilevel programming problem with multichoice parameters (MMCP) in which objective functions are linear fractional with cost coefficients of the objective functions being multichoice parameters. The multichoice parameters are replaced with Lagrange Interpolating polynomial by using transformation technique and the solution is determined by fuzzy programming approach to determine a compromise or satisfactory solution of the transformed problem. Approximating the objective functions by interpolating polynomials converts MMCP into mixed integer quadratic fractional programming problem. Finally, an algorithm based on fuzzy programming is proposed to determine a solution which satisfies both decision makers of problem. A numerical example is exhibited to evince the algorithm using Lingo 17.0 software.

KEYWORDs*:* Multilevel programming; Linear fractional programming; Fuzzy programming; Lagrange's transformation; Compromise solution; Multi-choice parameters.

MSC*:* 90C20; 90C26: 90C10; 60J27; 60J28;90C59

RESUMEN
Este articulo modela un problema de Programación Multinivel con parámetros de múltiple selección (MMCP) en el que las funciones objetivo son fraccionales lineales donde los coeficientes de costo de la función objetivo son de múltiple selección. Los parámetros de múltiple selección son reemplazados por un polinomio de Interpolación de Lagrange usando la técnica de transformación y la solución es determinada mediante un enfoque de Programación Borrosa para determinar un compromiso o una satisfactoria solución del transformado problema. Aproximando las funciones objetivo por polinomios de interpolación convierte MMCP en un problema de Programación Entero Mixto . Finalmente, un algoritmo basado en Programación Borrosa es propuesto pra determinar una solución que satisfaga ambos decisores del problema. Un ejemplo numérico es exhibido para evidenciar el algoritmo usando el software Lingo 17.0.

PALABRAS CLAVE Programación Multinivel; Programación Lineal; Programación Fraccional; Programación Borrosa; Lagrange transformación; Solución de Compromiso solución; Parámetros de Múltiple Selección.

1.INTRODUCTION

For a mathematical programming problem, the parameters which form parametric space of the problem are decided by the decision maker (DM) and value of these parameters are tackled by the experts. In routine practice, these values are real constants, but in most of real-life situations, these parameters are considered as either random variable or fuzzy variable. Multilevel Programming has been drawing considerable attention from scientific community in recent years. Many problems need to be modelled as multilevel programs as a consequence of which new efficient methods have been evolved. Multilevel Programming Problems are characterized by planner at a certain hierarchical level determining his/her own objective and constraint space by successive levels partially under cooperation. Multilevel organizations consist of interactive decision-making units within a hierarchical structure. The execution of decisions is hierarchical/sequential, starting from the top and then moving to lower levels. The decision maker at each level tries to maximize its own benefits, but is affected by decisions of decision makers at other levels through externalities. The decision maker at first level optimizes his/her decision first and for a given choice of the variables under its control, the lower-level decision makers optimize their objectives. However, each of the decision makers can influence/control other level decisions to achieve his objectives. The hierarchical nature of problem is reflected by the order imposed on the choice of decision taken. The leader at first level governs information of followers' objectives and constraints, while followers optimize their objectives after

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Leader's strategy is declared. The decision makers at same level execute their decisions cooperatively i.e. For decentralized systems, where there are more than one decision makers at same level, the objectives are given same level of preference for given choice of upper-level decision maker.

A general Multilevel Decentralized Programming Problem (MDPP) is concerned with decentralized systems in a hierarchical organization with two/ more objectives at the same level. There are numerous methods available to optimize Multilevel Programming Problems. Multilevel Programs have been tackled by many researchers in several fields ranging from economics to transportation engineering. MLPP has also been used to model real life problem involving multiple decision makers. These problems include genetic algorithms [9,23,24], traffic signal optimization problems [19], structural design problems [20]. Applications of almost all of the extreme point algorithms have been applied to determine the solution of a linear MLPP. Every linear multilevel programming problem with a finite optimal solution possesses an important feature viz. the optimal solution is attained at an extreme point of the constraint set. This result was first established by Candler et. al. [10]. Later on, Bialas et. al. [8] proved the result under the assumption that the constraint region is bounded and convex polyhedron. Although this problem is very relevant in practice, there are few methods available and their quality is hard to determine. Fractional programming (FP) which has been being used as an important planning tool for the past four decades is applied for a lot of disciplines such as engineering, business, finance, economics etc. FP is generally used for modeling real life problem which has one or more than one objective as a ratio of two functions such as profit/loss, inventory/sales, actual cost/standard cost, output/employee etc. Fractional programs arise in various contexts viz in investment problems, the firm wants to select a number of projects on which money is to be invested so that the ratio of the profits to the capital invested is maximum subject to the total capital available and other economic requirements which may be assumed to be linear. If price per unit depends linearly on output and capital is a linear function then problem is reduced to a linear fractional program. Example of linear fractional programming was first solved by Isbell where algorithm generates a sequence of linear programs whose solutions converge to solution of the fractional program in a finite number of iterations. Since then, several methods of solutions have been proposed. Charnes et.al. have shown that a solution of the problem can be obtained by solving at most two ordinary linear programs. Algorithms based on the parametric form of the problem has been developed by Jagannathan.

There are some real-life problems where these parameters are multi-choice type i.e., there exists a set of choices for a given parameter, out of which only one is to be selected in order to optimize the objective function. Such mathematical programming problems are known as multichoice programming problems (MCP) (Acharya et. al. 2015)[2]. The multichoice programming problem belongs to the non-convex mathematical programming problem. MCP problem was introduced by Healey (1964). Multichoices can occur for any parameter in a mathematical programming problem. In a MMCP, we find combination of values of parameters so that it optimizes the objectives. MMCP is used in real-life decision-making problems, e.g., appointing a new security personnel, implementing a new policy for a country or community selecting new car, new house etc. However, formulating large and complex systems invariably needs decomposing the system into a number of smaller subsystems, each with its own goals and constraint parameters. The interconnections among the subsystems might take on many forms, but one of the most common form is the hierarchical structure in which a particular level decision-maker controls or co-ordinates his/her lower-level DMs. This type of decomposed system is called multi-level system. In a decision-making problem, if there are two or more levels present having one DM in each level to take decision, then the problem is called multilevel programming problem (MLPP). Multilevel optimization problems are described as mathematical programs which possess a subset of variables controlled by them to be the optimal solution of other programs which are parametrized by the remaining variables. In many decision-making situations, there arise many planning problems that can be represented by multilevel programs.

This methodology presents a multilevel linear fractional programming problem where the cost coefficients of the objective functions at each of the levels being multi-choice parameters. The multichoice parameters are then replaced by polynomial approximation using Lagrange’s interpolating polynomials. The problem under consideration is transformed into a fuzzy programming problem by defining the membership functions for the leader, the followers and the variables controlled by the leader. A satisfactory solution is obtained after this fuzzy programming problem is solved. Solution procedure uses the concepts of interpolating polynomials to tackle the multi-choice parameters of the problem and transform the problem into a standard mathematical programming problem. Then the transformed multilevel mixed integer programming problem is solved by using fuzzy max-min decision model, which generates Pareto optimal solution for the original problem directly. This is the main advantage of the proposed approach for solving MMCP problems. Due to the presence of the integer variable and interpolating polynomials the transformed model becomes mixed integer nonlinear programming problem. Incorporating the traditional approach for MMCP, size of the problem becomes large and complex. But using the above methodology, the problem requires less computational work and efforts.

2. THEORETICAL BACKGROUND

Multilevel linear fractional programs have been used to represent various decision making situations. For instance, indices such as inventory/sales, profit/cost output/employees, play a very important role in evaluating economic activities. Therefore, models that can effectively handle such fractional objectives are preferred for such type of problems. Multilevel linear fractional programming (MLFP) problems are studied by a few approaches which have appeared in Kornbluth et. al. (1981); Lai (1996); Luhandjula (1984); Sakawa et. al. (1983). Luhandjula (1984) proposed a linguist approach to multi objective linear fractional programming by introducing linguistic variables to represent linguistic aspirations of the decision makers. The model of the problem constructed with fuzzy data due to FP approaches [Luhandjula (1984); Sakawa et. al. (1983)] used to solve MLFP problems have a difficulty in computation. In the framework of fuzzy decision, Bellman and Zadeh (1970) ; Sakawa et. al. (1983) presented a fuzzy programming approach for solving multi objective linear fractional programming problem by combined use of the bisection method and the phase one of simplex methods of linear programming. Multi-level multi-objective linear or non-linear programming problems are new combination problems in the field of multi-level (or multi- objective) decision making problems. Ibrahim (2009) proposed Fuzzy goal programming algorithm for solving decentralized bi-level multi-objective programming problems. Again, Ibrahim (2010) proposed a fuzzy goal programming approach to Solve multilevel multiobjective linear programming problems. Eren et. al.(2013) used fuzzy multiobjective linear programming approach for optimizing a closed-loop supply chain network. Some of these traditional techniques, which give accurate results are computationally expansive and become inefficient for a large domain.
We consider the following t-level multilevel linear fractional programming problem (MFPP) [4]:

$$\max_{x_1 x_2 .. x_t} f_1(x_1, x_2, \ldots x_t) = \max_{x_1 x_2 .. x_t} \frac{c_{11}x_1 + c_{12}x_2 + \ldots + c_{1t}x_t}{d_{11}x_1 + d_{12}x_2 + \ldots + d_{1t}x_t}$$

where $x_2, x_3 \ldots x_t$ solve

(MFPP)
$$\max_{x_1 x_2 .. x_t} f_2(x_1, x_2, \ldots x_t) = \max_{x_1 x_2 .. x_t} \frac{c_{21}x_1 + c_{22}x_2 + \ldots + c_{2t}x_t}{d_{21}x_1 + d_{22}x_2 + \ldots + d_{2t}x_t} \quad \text{for a given } x_1$$

:

:

$$\max_{x_1 x_2 .. x_t} f_t(x_1, x_2, \ldots x_t) = \max_{x_1 x_2 .. x_t} \frac{c_{t1}x_1 + c_{t2}x_2 + \ldots + c_{tt}x_t}{d_{t1}x_1 + d_{t2}x_2 + \ldots + d_{tt}x_t} \quad \text{for a given } x_1, x_2\ x_3 \ldots x_{t-1}$$

subject to

$$A_1x_1 + A_2\, x_{2+\ldots} + A_t\, x_t \le b$$
$$x_1, x_2, x_3 \ldots x_t \quad \ge 0$$

where $S = \{(x_1, x_2, x_3 \ldots x_t) : A_1\, x_1 + A_2\, x_{2+\ldots} + A_t\, x_t \le b\}$

$x_1 \in \mathcal{R}^{n_1},\ x_2 \in \mathcal{R}^{n_2}, \ldots x_t \in \mathcal{R}^{n_t}$

$d_{11}x + d_{12}y + d_{13}z + \beta_1 > 0$, $d_{11}x + d_{12}y + d_{13}z + \beta_1 > 0$ and $d_{31}x + d_{32}y + d_{33}z + \beta_3 > 0$ $\forall (x,y,z) \in S$
the multilevel linear fractional programming problem considered above, each of the objective functions at each level is linear fractional in nature and hence they are both pseudoconcave and pseudoconvex. Therefore, it admits of extreme point optimal solution in S. The methodology of fuzzy mathematical programming was introduced by Tanaka [20] in the framework of fuzzy decision of Bellman et.al. [7]. Later on, a different approach based on fuzzy programming was introduced to optimize linear programming with several objectives at same level was introduced by Zimmermann [24]. Recent studies convey that Mohammed [18] analyzed some new fuzzy programming forms by making use of the concept of

conventional goal programming approach which was further studied by Kamal et. al. [13] as also by Pal et. al. [19]. Many authors studied fractional programming with multichoice parameters [6,8,9,10,11]. Lee et. al. [14] proposed an interactive fuzzy approach to multilevel decision making problems. Mohammed [18] proposed a fuzzy max-min decision model to tackle the multilevel non-linear multiple objective programming problems. The fixed charge bilevel transportation problem was studied by Arora et. al. [3,4]. The multichoice programming problem (MCP) is a special class of non-convex mathematical programming problems which was introduced by Healy Jr.[12]. MCPP has many applications in decision making problems in the field of combinatorics & integer programming [16]. Acharya et. al. [2] prepared a case study of a garment manufacture company demonstrating applications of MMCP. Liao [15] applied multi-choice model to the management systems.
This article presents a multilevel linear fractional programming problem with multichoice parameters (MMCP). The objective functions at each of the levels have multichoice cost coefficients. The methodology uses the concept of interpolating polynomials which converts MMCP into a mixed integer quadratic fractional programming problem and finally, an algorithm based on fuzzy programming approach is proposed to find a solution which satisfies each of the decision makers of the problem. Thereafter, Chang [11] and Liao [15] introduced different solution techniques by transforming the problem with multi-choice parameters into a mixed integer programming problem.

3. METHODOLOGY TO SOLVE A MULTICHOICE PROGRAMMING PROBLEM

Multiple choices for a parameter are available to the decision makers in a multichoice programming problem. The model presented here accommodates appropriate resources from a given set of multiple resources. In this paper, we deal with the objective functions which have multichoice cost coefficients at all levels. For this, interpolating polynomial approximation is used and the problem with multichoice parameters is reduced to a mixed integer programming problem solution to which is determined by fuzzy solution approach to achieve the satisfactory/compromise solution which satisfies all the decision makers. The transformed multilevel mixed integer programming problem is solved by using fuzzy max-min decision model, which generates Pareto optimal solution for the original problem directly. This is the main advantage of the proposed approach for solving MMCP problems. Due to the presence of the integer variable and interpolating polynomials the transformed model becomes mixed integer nonlinear programming problem. Incorporating the traditional approach for MMCP, size of the problem becomes large and complex. But using the above methodology, the problem requires less computational work and efforts.

4. MATHEMATICAL FORMULATION

Mathematically, a multilevel multichoice linear fractional programming (MMCP) problem is formulated as

(MMCP) $$\max_{x_1 x_2 .. x_t} f_1 = \frac{\left(c_{11}^{(1)} c_{11}^{(2)} \ldots\ldots c_{11}^{(t_1)}\right)}{\left(d_{11}^{(1)} d_{11}^{(2)} \ldots\ldots d_{11}^{(t_1)}\right)} x_1 + \frac{\left(c_{12}^{(1)} c_{12}^{(2)} \ldots\ldots c_{12}^{(t_2)}\right)}{\left(d_{12}^{(1)} d_{12}^{(2)} \ldots d_{12}^{(t_2)}\right)} x_2 + \ldots + .\frac{\left(c_{1n}^{(1)} c_{1n}^{(2)} \ldots\ldots c_{1n}^{(t_n)}\right)}{\left(d_{1n}^{(1)} d_{1n}^{(2)} \ldots d_{1n}^{(t_n)}\right)} x_n$$

$$\max_{x_1 x_2 .. x_t} f_2 = \frac{\left(c_{21}^{(1)} c_{21}^{(2)} \ldots\ldots c_{21}^{(t_1)}\right)}{\left(d_{21}^{(1)} d_{21}^{(2)} \ldots\ldots d_{21}^{(t_1)}\right)} x_1 + \frac{\left(c_{22}^{(1)} c_{22}^{(2)} \ldots\ldots c_{22}^{(t_2)}\right)}{\left(d_{22}^{(1)} d_{22}^{(2)} \ldots d_{22}^{(t_2)}\right)} x_2 + \ldots + .\frac{\left(c_{2n}^{(1)} c_{2n}^{(2)} \ldots\ldots c_{2n}^{(t_n)}\right)}{\left(d_{2n}^{(1)} d_{2n}^{(2)} \ldots d_{2n}^{(t_n)}\right)} x_n \text{ for a given } x_1$$

:

$$\max_{x_1 x_2 .. x_t} f_t = \frac{\left(c_{t1}^{(1)} c_{t1}^{(2)} \ldots\ldots c_{t1}^{(t_1)}\right)}{\left(d_{t1}^{(1)} d_{t1}^{(2)} \ldots\ldots d_{t1}^{(t_1)}\right)} x_1 + \frac{\left(c_{t2}^{(1)} c_{t2}^{(2)} \ldots\ldots c_{t2}^{(t_2)}\right)}{\left(d_{t2}^{(1)} d_{t2}^{(2)} \ldots d_{t2}^{(t_2)}\right)} x_2 + \ldots + .\frac{\left(c_{tn}^{(1)} c_{tn}^{(2)} \ldots\ldots c_{tn}^{(t_n)}\right)}{\left(d_{tn}^{(1)} d_{tn}^{(2)} \ldots d_{tn}^{(t_n)}\right)} x_n \text{ for a given } x_1, x_2 \ldots x_{t-1}$$

where $X = (x_1, x_2 \ldots x_n) \in S^*$.
Here, $S^* = \{X \mid AX = b\}$ is non-empty and bounded.
We have, l_j (j = 1, ..., n) are multichoices for the jth parameters c_i^j and d_i^j;
m_k (k =1,...n) are multichoices for k-th parameters c_i^k and d_i^k and e_r (r=1,2,……n) are the multichoices for r-th parameters c_i^r and d_i^r: $A \in R^{mxn}$, $b \in R^{mx1}$;
c_{1i}^j ,d_{1i}^j (i= 1,..., n, j= 1,..., t_1)∈ R ; c_{2i}^j ,d_{2i}^j (i= 1,.... n, j=1,..., t_2) ∈ R;….
c_{ti}^j ,d_{ti}^j (i=1...n, j=1...t_n) ∈R; $\left(d_{1j}^{(1)} d_{1j}^{(2)} \ldots \ldots d_{1j}^{(t_j)}\right) > 0$, j =1,2...n;
$\left(d_{1j}^{(1)} d_{1j}^{(2)} \ldots \ldots d_{1j}^{(t_j)}\right) > 0$, j = 1,2...n; $\left(d_{2j}^{(1)} d_{2j}^{(2)} \ldots \ldots d_{2j}^{(t_j)}\right) > 0$, j = 1,2...n;
$\left(d_{1j}^{(1)} d_{1j}^{(2)} \ldots \ldots d_{1j}^{(t_j)}\right) > 0$. j = 1,2...n.

4.1. Solution technique

To solve MMCP, we incorporate approximation by Lagrange's interpolating polynomials [5]. The solutions so obtained after solving the upper level and lower level problems are different since the objective function at all the levels are conflicting in nature despite of having partial cooperation among them. Since the leader controls the decision variable x, therefore to achieve the satisfactory or compromise solution for all the decision makers, the leader has to produce the range for x. Further, it is assumed that for each response of the Leader, each of the followers' have a unique response.

4.1.1. Interpolation

In order to find polynomial $P_n(x)$ for which deg $(P_n) \leq n$, passing through the points $(x_0,y_0),(x_1,y_1)$ $\ldots,(x_n,y_n)$, the solution is given by Lagrange's interpolation formula mentioned below:

$P_n(x) = y_0 L_0(x) + y_1 L_1(x) + \ldots\ldots.. + y_n L_n(x)$

where

$$L_k(x) = \prod_{i=k} \frac{(x-x_i)}{(x_k-x_i)} \qquad k = 0, 1, \ldots, n; \tag{4.1}$$

Here, in this formula, each such function is a polynomial of degree n. Also,

$$L_k(x_i) = \begin{cases} 1 & i = k \\ 0 & i \neq k \end{cases}$$

Using these properties, it follows that the formula

$$P_n(x) = y_0 L_0(x) + y_1 L_1(x) + \ldots\ldots.. + y_n L_n(x)$$

must satisfy the interpolating problem in order to find a solution to

$\deg(P_n) \leq n$ and $P_n(x_i) = y_i$; I = 0,1..,n.

4.1.2. Approximating by Interpolating polynomial the problem MMCP

Consider the problem MMCP with objective functions having multi-choice cost coefficients at all levels. In order to deal with the multichoice parameters c_j (j= 1,2…n) in the numerator at the upper level, we use Lagrange's interpolation. For this, we introduce an integer variable a_j which takes a t_j number of values (a_j = 0,1... t_j-1). We formulate a Lagrange's interpolating polynomial f (a_j) which passes through all the t_j number of points, as

$$f_{d_j}(a_j) = \frac{(a_j-1)(a_j-2)\ldots\cdot(a_j-t_j+1)}{(-1)^{t_j-1}(t_j-1)!} d_j^{(1)} + \frac{(a_j)(a_j-2)\ldots\cdot(a_j-t_j+1)}{(-1)^{t_j-2}(t_j-2)!} d_j^{(2)} + \frac{(a_j)(a_j-1)\ldots\cdot(a_j-t_j+1)}{(-1)^{t_j-3}2!(t_j-3)!} d_j^{(3)} + \ldots + \frac{(a_j-1)(a_j-2)\ldots\cdot(a_j-t_j+1)}{(t_j-1)!} d_j^{(t_j)}; \ j=1,2,\ldots\ldots.n$$

Similarly, for the other lower level problems $f_j(X)$, for $2 \leq j \leq t$ introducing the integer variables as above, one can define the Lagrange interpolating polynomials in the respective manner. On introducing interpolating polynomials in the numerator and denominator of the problem MMCP is reduced to the mixed integer quadratic fractional programming problem (MIQFP), defined as

$$\max_{x_1 x_2 .. x_t} \ f_1 = \sum_{j=1}^{n} \frac{f_{(d_j)}(a_j)}{f_{(c_j)}(b_j)} x_j$$

$$\max_{x_1 x_2 .. x_t} \ f_2 = \sum_{j=1}^{n} \frac{f_{(p_j)}(l_j)}{f_{(q_j)}(m_j)} x_j \quad \text{for a given } x_1$$

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$$\max_{x_1 x_2 .. x_t} \ f_t = \sum_{j=1}^{n} \frac{f_{(e_j)}(g_j)}{f_{(k_j)}(h_j)} x_j \quad \text{for given } x_1, x_2, \ldots x_{t-1}$$

where $X \in S^*$, $S^* = \{X \mid AX \leq b\}$. Also,

$0 \leq a_j \leq t_j$-1, $0 \leq b_j \leq t_j$-1; (4.2)

$0 \leq l_j \leq s_j$-1, $0 \leq m_j \leq s_j$-1;

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$0 \leq g_j \leq r_j$- 1, $0 \leq h_j \leq r_j$ -1;

$a_j, l_j, g_j \in Z^+ \cup \{0\}$, j =1...n; $b_j, m_j, h_j \in Z^+$, j=1...n.

Next, we aim to determine the solution of the above mentioned problem which satisfies all the decision makers and to achieve we find a compromise solution which can be obtained using fuzzy programming approach.

4.2. Fuzzy programming approach to linear fractional multilevel programming problem

Let us consider the problem MMCP. Construct the fuzzy membership functions in order to apply fuzzy programming to MMCP. Solve $f_1(X)$ subject to the constraint (5.2). Let its individual best and worst solution be f_1^b and f_1^w respectively. This data can be used to define the membership function for the upper level problem.

$$\mu(f_1(x)) = \begin{cases} 1, & \text{if } f_1(x) > f_1^b \\ \frac{f_1(x)-f_1^w}{f_1^b-f_1^w}, & \text{if } f_1^w \le f_1(x) \le f_1^b \\ 0 & \text{if } f_1(x) < f_1^w \end{cases}$$

Let λ_1 be the minimum acceptable degree of satisfaction for the upper level problem $f_1(x)$. Again, let f_1^b and f_1^w be the best and worst solutions of $f_j(x)$; j = 2,3...t.

Next, let us construct the membership function for $f_j(x)$, j=2,3,…..t as

$$\mu(f_j(x)) = \begin{cases} 1 & \text{if } f_j(x) > f_j^b \\ \frac{f_1(x)-f_1^w}{f_1^b-f_1^w}, & \text{if } f_j^w \le f_j(x) \le f_j^b \quad \text{for } j=2,3\ldots t \\ 0 & \text{if } f_j(x) < f_j^w \end{cases}$$

Let λ_j be the minimum degree of satisfaction acceptable for the follower problems $f_j(x)$; j=2,3,…t. The solutions so obtained after solving the upper level and lower level problems are different since the objective function at all the levels are conflicting in nature despite of having partial cooperation among them. Since the leader controls the decision variable x, therefore to achieve the satisfactory or compromise solution for all the decision makers, the leader has to produce the range for x. Let x_1 and x_2 be the maximum and minimum tolerance limits for x.

Let us now define the membership function for x as

$$\mu(x) = \begin{cases} \frac{x-(x^F-x_2)}{x_2} & (x^F - x_2) \le x \le x^F \\ \frac{(x^F+x_1)-x}{x_1} & x^F \le x \le (x^F + x_1) \end{cases} \tag{4.3}$$

Let λ_3 be the minimum acceptable degree of satisfaction of the decision variable x.

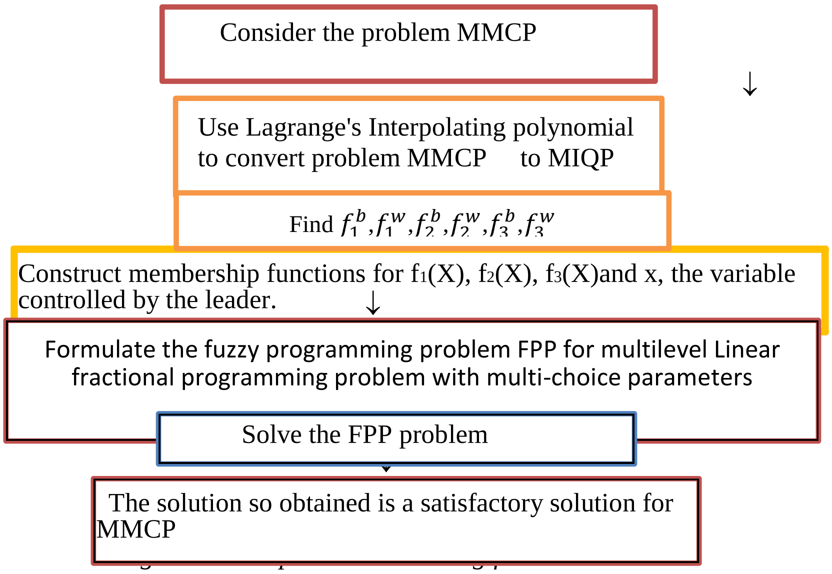
Let $\delta = \min\{\lambda_1, \lambda_2, \lambda_3\}$

Let us now generate the satisfactory solution for the problem MMCP which is a pareto optimal solution too with overall satisfaction for all decision makers, solve the following Fuzzy Programming Problem (FPP) definedas

$$\begin{aligned} & \max\ \delta \\ & \text{subject to} \\ & \mu(f_1(X)) \ge \delta \\ & \mu(f_2(X)) \ge \delta \\ & \mu(f_3(X)) \ge \delta \\ & \mu(X) \ge \delta \\ & X \in S^*. \\ & 0 \le a_J \le t_J\text{-}1,\ 0 \le b_j \le t_J\text{-}1; \\ & 0 \le l_J \le s_J\text{-}1,\ 0 \le m_j \le s_J\text{-}1; \\ & :: \\ & :: \\ & 0 \le g_J \le r_J\text{-}\ 1,\ 0 \le h_j \le r_J\ \text{-}1; \\ & a_J, l_J, g_J \in Z^+ \cup \{0\},\ j = 1....n; \\ & b_j, m_j, h_j \in Z^+,\ j=1....n;\ \delta \in [0, 1]. \end{aligned} \tag{5.4}$$

5. A METHODOLOGY TO SOLVE MULTILEVEL PROGRAMMING PROBLEM WITH MULTICHOICE PARAMETERS

Consider a multilevel linear fractional programming problem with multi-choice parameters (MMCP). In this problem, the cost coefficients of the objective functions at both levels are multi-choice. In order to solve MMCP, the objective functions at both levels are dealt using interpolating polynomials. Using Lagrange interpolation approximation, the MMCP problem is converted to a mixed integer quadratic fractional programming problem MIQP. The transformed MIQP problem is solved using fuzzy programming approach for which membership functions need to be constructed for the objective functions at all levels. The membership function is also defined for the variables controlled by the leader, by constructing membership functions. The fuzzy programming problem FPP is formulated. The FPP problem is solved to obtain a satisfactory solution for MMCP. The algorithmic development is presented below with the help of a flowchart:



6. NUMERICAL EXAMPLE

Let us consider a company which manufactures three products A, B and C. It transports the products to different markets: MKT1, MKT2, MKT3 and MKT4. The production cost consists of the lab our cost, material cost, storage, transportation expenses etc. The company targets to invest in one product in a single market. To achieve this, the company wants to determine its profit v/s cost ratio and output v/s employee ratio for an individual product. For running its production cycle, during production, the company has the following requirements:

Demand	P	Q	R	Supply
Material 1	2	0	1	20
Material 2	4	5	0	18
Material 3	1	2	2	15

Table 1

Now, the company is yielding a profit of 32 units on product A in market MKT1, 31 units in MKT2, 34 units in MKT3 and 30 units in MKT4. In addition, cost of 8 units for its products in MKT1, 12 units in MKT2, 10 units in MKT3 and 7 units in MKT4 has been incurred.

Table 2

	P1	**P2**	P3
Profit	(34,31,32,30)	(33, 28, 34)	(26, 23, 28)
Cost	(8, 11, 10, 6)	(7, 7, 6)	(8, 11, 10)
Production	(18,19,20,17)	(23, 17, 23)	(16, 14, 16)
Sale	(7, 8, 10, 6)	(8, 5, 3)	(8, 12, 11)
Inventory	(14,15,17,16)	(33,29,31)	(16, 14, 16)
No. of Labourers	(6,12,10,8)	(8,11,12)	(8, 9, 6)

Solution: Let x, y, z be the number of units of products P, Q and R respectively. Let f_1 denote the profit/cost ratio and by constructing membership functions, f_2 be output / employee's ratio and f_3 be inventory/sales ratio .

A multilevel linear fractional programming problem with multi-choice parameters is given by

$$\max_{x,y.z} f_1(x,y,z) = \frac{(34,31,32,30)}{(8,11,10,6)}x + \frac{(33,28,34)}{(7,7,6)}y + \frac{(26,23,28)}{(8,11,10)}z$$

$$\max_{x,y.z} f_2(x,y,z) = \frac{(18,19,20,17)}{(7,8,10,6)}x + \frac{(23,17,23)}{(8,5,3)}y + \frac{(16,14,16)}{(8,12,11)}z \tag{6.1}$$

$$\max_{x,y.z} f_3(x,y,z) = \frac{(14,15,17,16)}{(6,12,10,8)}x + \frac{(33,29,31)}{(8,11,12)}y + \frac{(16,14,16)}{(8,9,6)}z$$

subject to $2x + z \le 20$

$4x + 5y \le 18$

$x + 2y + 2z \le 15$

$x, y, z \ge 0$

Introduce integer variables a_J (j = 1,2,3) and b_j (j =1, 2, 3) first level; l_j (j =1,2,3) and m_j (j =1, 2, 3) in the second level; g_J and h_J (j = 1,2,3) in the third level respectively. Using interpolating polynomial approximation, the above problem reduces to

$$\max_{x,y.z} f_1(x,y,z)=\left[\frac{32-\frac{20}{3}a_1+\frac{15}{2}a_1^2-\frac{11}{6}a_1^3}{8+\frac{26}{3}b_1-\frac{11}{2}b_1^2+\frac{5}{6}m_1^3}\right]x+\left[\frac{33-7a_2+3a_2^3}{7+\frac{9}{2}b_2-\frac{5}{2}b_2^2}\right]y+\left[\frac{27+\frac{23}{2}a_3+\frac{3}{2}a_3^2}{9+\frac{7}{2}b_3-\frac{3}{2}b_3^2}\right]z$$

$$\max_{x,y.z} f_2(x,y,z)=\left[\frac{16+\frac{10}{3}l_1-\frac{1}{3}l_1^3}{9-\frac{31}{6}m_1+\frac{11}{2}m_1^2-\frac{4}{3}m_1^3}\right]x+\left[\frac{23-7l_2+3l_2^3}{8-\frac{11}{2}m_2+\frac{3}{2}m_2^2}\right]y+\left[\frac{17-\frac{17}{2}l_3\frac{3}{2}l_3^2}{9+\frac{7}{2}m_3-\frac{3}{2}m_3^2}\right]z$$

$$\max_{x,y.z} f_3(x,y,z)= x+\left[\frac{33-7g_2+3g_2^3}{9+\frac{7}{2}h_2-\frac{3}{2}h_2^2}\right]y+z$$

Subject to

$2x + z \le 20$

$4x + 5y \le 18$

$x + 2y + 2z \le 15$

$x, y, z \ge 0$

Solve the first level problem f_1, second level problem f_2, and third level problem f_3 individually using Lingo 15.0 software subject to the constraints .

The solution obtained in this way is

Max f_1 = 51.25 with x = 4.5, y= 0, z= 5.25;

Max f_2 = 31.44 with x = 0, y = 3.6, z = 3.9;

Max f_3 = 25.12 with x = 3.1, y= 0, z= 2.5.

By using the solutions obtained above, the fuzzy programming problem FPP is defined by

(FPP): max δ

subject to

$$\begin{aligned}
&\mu(f_1(x))\ge\delta\\
&\mu(f_2(x))\ge\delta\\
&\mu(f_3(x))\ge\delta\\
&\mu(y)\ge\delta\\
&\mu(z)\ge\delta\\
&0\le a_1, l_1, g_1\le 3;\quad 0\le a_2, l_2, g_2\le 2;\quad 0\le a_3, l_3, g_3\le 2;\\
&0\le b_1,m_1,h_1\le 3;\ 0\le b_2,m_2,h_2\le 2;\ 0\le a_3,l_3,g_3\le 2;\\
&a_J, l_{J,}g_J \in Z^+U\{0\},\ j=1,....n;\ b_j, m_j, h_j \in Z^+,\ j=1,....n;\ \delta\in[0, 1].
\end{aligned} \tag{6.2}$$

It can be rewritten as

max δ

subject to

$\mu(f_1(x))\ge 51.25\,\delta$

$\mu(f_2(x))\ge 31.44\,\delta$

$\mu(f_3(x))\ge 25.12\delta$

$\frac{3.9-y}{3.6}\ge\delta$

$\frac{z-3.5}{1.35}\ge\delta$

$\frac{7.5-z}{2.25}\ge\delta$

$0 \le a_1, l_1, g_1 \le 3,\ 0 \le a_2, l_2, g_2 \le 2,\ 0 \le a_3, l_3, g_3 \le 2;$

$0 \le b_1, m_1, h_1 \le 3,\ 0 \le b_2, m_2, h_2 \le 2,\ 0 \le a_3, l_3, g_3 \le 2;$

$a_J, l_{J,} g_J \in Z^{+} U\{0\}$, j =1,....n; $b_j, m_j, h_j \in Z^{+}$, j=1,....n; $\delta \in [0, 1]$.

Therefore, we obtain

(FPP) max δ

subject to

$$\left[\frac{32-\frac{20}{3}a_1+\frac{15}{2}a_1^2\frac{-11}{6}a_1^3}{8+\frac{26}{3}b_1-\frac{11}{2}b_1^2+\frac{5}{6}m_1^3}\right]x + \left[\frac{33-7a_2+3a_2^3}{7+\frac{9}{2}b_2-\frac{5}{2}b_2^2}\right]y + \left[\frac{27+\frac{23}{2}a_3+\frac{3}{2}a_3^2}{9+\frac{7}{2}b_3-\frac{3}{2}b_3^2}\right]z \ge 51.25\ \delta$$

$$\left[\frac{16+\frac{10}{3}l_1-\frac{1}{3}l_1^3}{9-\frac{31}{6}m_1+\frac{11}{2}m_1^2-\frac{4}{3}m_1^3}\right] x+\left[\frac{23-7l_2+3l_2^3}{8-\frac{11}{2}m_2+\frac{3}{2}m_2^2}\right] y + \left[\frac{17-\frac{17}{2}l_3\frac{3}{2}l_3^2}{9+\frac{7}{2}m_3-\frac{3}{2}m_3^2}\right] z \ge 31.44\ \delta$$

$$x+\left[\frac{33-7g_2+3g_2^3}{9+\frac{7}{2}h_2-\frac{3}{2}h_2^2}\right] y + z \ge 25.12\ \delta$$

$$y + 3.6\ \delta \le 3.9$$
$$z - 1.35\ \delta \ge 3.5$$
$$z + 2.25\ \delta \le 7.5$$
$$x, y, z \ge 0$$

$0 \le a_1, l_1, g_1 \le 3,\ 0 \le a_2, l_2, g_2 \le 2,\ 0 \le a_3, l_3, g_3 \le 2;$

$0 \le b_1, m_1, h_1 \le 3,\ 0 \le b_2, m_2, h_2 \le 2,\ 0 \le a_3, l_3, g_3 \le 2;$

$a_J, l_{J,} g_J,\ b_j, m_j, h_j \in Z$, j = 1,..n; $\delta \in [0, 1]$.

The optimal solution to the above problem is δ = 0.7272, x = 3.27, y = 0.981 and z = 4.88.

It must be made sure that the company wants to invest in one product for single market, so that the leader and followers can maximize their profitability ratio and output/employee ratio as well as inventory/sales ratio in that particular market.

Now, for the first decision maker(leader), we have

$$\max_{x,y.z} f_1(x,y,z) = \frac{(32,31,34,30)}{(8,12,10,7)}(3.27) + \frac{(33,29,31)}{(7,9,6)}(0.981) + \frac{(27,25,26)}{(9,11,10)}(4.88)$$

$$= (4.258, 2.58, 3.4, 4.286) + (4.714, 3.22, 5.167) + (3, 2.272, 2.6)\ldots. \quad (6.3)$$

and for the two followers we have

$$\max_{x,y.z} f_2(x,y,z) = \frac{(16,19,20,17)}{(9,8,10,7)}(3.27) + \frac{(23,19,21)}{(8,4,3)}(0.981) + \frac{(17,15,16)}{(9,11,10)}(4.88)$$

$$= (1.778, 2.375, 2, 2.429) + (2.875, 4.75, 7) + (1.889, 1.364, 1.6) \quad (6.4)$$

$$\max_{x,y.z} f_3(x,y,z) = \frac{(14,15,17,19)}{(6,12,10,7)}(3.27) + \frac{(33,29,31)}{(9,11,10)}(0.981) + \frac{(20,15,16)}{(7,9,6)}(4.88)$$

$$= (2.333, 1.25, 1.7, 2.714) + (3.667, 2.64, 3.1) + (2.86, 1.67, 2.67) \quad (6.5)$$

From (8.1),(8.2) and (8.3), we observe that the company should invest in market MKT4 for product A, in market MKT3 for product B and in market MKT1 for product C.

7. CONCLUSIONS

In this article, we present a multilevel linear fractional programming problem in which the cost coefficients of the objective functions at each of the levels are multi-choice parameters. The multi-choice parameters are replaced by polynomial approximation using Lagrange's interpolating polynomials. The problem under consideration is transformed into a fuzzy programming problem by defining the membership functions for the leader, the followers and the variables controlled by the leader. A satisfactory solution is obtained after this fuzzy programming problem is solved. Solution procedure uses the concepts of interpolating polynomials to tackle the multi-choice parameters of the problem and transform the problem into a standard mathematical programming problem. Then the transformed multilevel mixed integer programming problem is solved by using fuzzy max-min decision model, which generates Pareto optimal solution for the original problem directly. This is the main advantage of the proposed approach for solving MMCP problems. Due to the presence of the integer variable and interpolating polynomials the transformed model becomes mixed integer nonlinear programming problem. Incorporating the traditional approach for MMCP, size of the problem becomes large and complex. But using the above methodology, the problem requires less computational work and efforts.

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