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MULTI-CHOICE DECISION-MAKING APPROACH IN UNCERTAIN MULTIVARIATE STRATIFIED SAMPLING UNDER TWO-STAGE RANDOMIZED RESPONSE MODEL

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ABSTRACT
This paper extends a multi-choice decision-making approach for a stochastic randomized response model with two stages in a multivariate stratified sampling problem. In the proposed problem enumeration cost follows a normal distribution and budget parameter presented in multiple choices. Binary variables are used to handle the multiple choices of the budget parameter in the proposed problem. Further, utilizing the chance constraint approach, the probabilistic constraint has been transformed into a deterministic two-stage randomized response problem. Compromise optimum allocation of samples within each stratum is derived by using Goal Programming, Chebychev Approximation, and Chebychev Goal Programming techniques. For demonstration of the proposed model, a numerical illustration has been presented in the study.

KEYWORDS: Chance Constraint Programming, Multi-Choice programming, Stratified sampling, Two-Stage randomized response, Transformation technique.

MSC: 62D05, 90C70

RESUMEN
Este paper extiende un enfoque de toma de decisión para un modelo de rsepuestas aleatorizadas multi-selección estocástica de dos etapas para el problema del muestreo estratificado multivariado. En el propuesto problema el costo de enumeración sigue una distribución normal y el parámetro de presupuesto se presenta con múltiples alternativas. Variables binarias son usadas para manejar las múltiples alternativas del parámetro de presupuesto en el problema. Además , se utiliza un enfoque de restricciones restringidas, la restricción probabilística ha sido transformada en un problema bietápico de respuestas aleatorizadas determinísticas. Una afijación óptima de compromiso para las muestras de los estratos es derivada usando Programación por Metas, Aproximación de Chebychev y Programación por Metas de Chebychev. Para la demostración del modelo propuesto, una ilustración numérica ha sido presentada en el estudio.

PALABRAS CLAVE: Programación por Restricciones Restringidas, Programación Multi-seleción, Muestreo estratificado, Problema bietápico de respuestas aleatorizadas , Técnica de transformación.

1. INTRODUCTION

In a structured survey interview, a randomized response research method is used. Randomized Response (RR) method is likewise a gadget suspected to enhance the accessibility of positive reaction identifying with delicate and sensitive issues on which information is hard to get. For reasons of humility, the dread of being thought extremist, or just a hesitance to trust insider facts to outsiders, numerous people endeavor to dodge certain inquiries put to them by questioners. To take out sly answers predisposition by diminishing the pace of non-reaction keeping the respondent's secrecy (Warner, 1965) developed Randomized response technique, which was further modified by Greenberg et al. (1969) permits respondents to answer to sensitive questions (for example, criminal conduct or sexuality) while looking after privacy. Chance concludes, obscure to the questioner, whether the inquiry is to be addressed honestly, or "yes", paying little mind to reality.
Mangat and Singh (1990) suggested a two-stage RR model based on Warner's model. Hong et al. (1994) suggested a stratified RR technique. Various other RR techniques are also considered such as Chang and Huang

(2001) used RR for the estimation of the proportion of a qualitative character, Chaudhuri (2001) used RR to estimate a sensitive proportion from a complex survey, Moors (1971) formulated the randomized response model to optimize the unrelated questions, Padmawar and Vijayan (2000) revisited the randomized response method and Singh (2002) proposed a new stochastic randomized response method that evokes more prominent participation from the respondents and can be made more efficient by selecting specific parameters. Some recent work on two-stage sampling and programming problems are due to Khan et al. (2006) who obtained optimum allocation in two-stage and stratified two-stage sampling for multivariate surveys, Khan et al. (2008) used dynamic programming to determine the optimum strata boundary points and Khan et al. (2012a) obtained compromise allocation in two-stage and stratified two-stage sampling designs for multivariate study. Shahid and Hussain (2016) presents the problem of the reliable data procuring on stigmatizing variable. For efficient estimation of average and sensitivity level, they proposed the two optional randomized response models.

In numerous uses of the RR procedure, more than one sensitive issues are under examination i.e.multiple delicate inquiry settings are to be thought off. In such cases, an allocation called “Compromise allocation" is used to obtain optimum allocation for all the characteristics in some sense.

In the sample survey problems, the uncertainties are inherent as only a part of the populace is measured and because of non-sampling errors. There are several approaches to model uncertainty in mathematical programming, such as fuzzy programming, stochastic programming, and multi-choice programming, etc. A probability distribution on the parameters usually characterizes uncertainty. Stochastic multivariate sample allocation problem in stratified sample surveys has been studied by many authors namely Kozak (2006), Diaz-Garcia and Cortez (2008), Diaz-Garcia and Gary-Tapia (2007), Bakhshi et al. (2010), Khan et al. (2011), Khan et al. (2012b), Ali et al. (2013), Gupta et al. (2013a, 2013b), Khan et al. (2016), Gupta and Bari (2017), Gupta et al. (2020), Haq et al. (2020), Zhang et al. (2020), Nasseri and Bavandi (2021) among others.

A multi-choice programming problem is a problem where the RHS goals (accessibility or prerequisite vectors) of certain requirements are multi-choice. For each limitation, there may exist a different number of goals, out of which precisely one is to be picked. Goals selection ought to be in such a way that the blend of decisions for every requirement ought to give an ideal answer for the objective function. There may exist more than one mix, which will give an ideal arrangement. Multi-choice linear programming problems have been discussed earlier by Ravindran et al. (1987), Hiller and Lieberman (1990), Chang (2007, 2008), Biswal and Acharya (2006, 2011), Roy and Mahapatra (2014), Pradhan and Biswal (2017), Patro et al. (2018), and recently several researchers focus on the application of multi-choice linear programming some of them are Rout et al. (2020), Nasseri and Bavandi (2020), and Belay and Acharya (2021).

This paper considers a RR model with two stages and a multi-objective non-linear programming problem (MONLPP) has been formulated. The MONLPP involves random variables following normal distribution and multiple choices for the right hand side budget parameter in cost constraint. Binary variables and some additional constraints are used to handle the multi-choice nature of budget parameter and chance constrained programming is used to transform probabilistic cost constraint into deterministic one. The compromise allocations are derived by three different techniques viz. Goal Programming (GP), Chebyshev Approximation (CA) and Chebyshev Goal programming (CGP). To validate the proposed model a numerical illustration based on the secondary data has been employed and all the mathematical models are solved by an optimization software LINGO-17. A detailed flow of methodology adopted in this paper is presented in Figure 1 below.

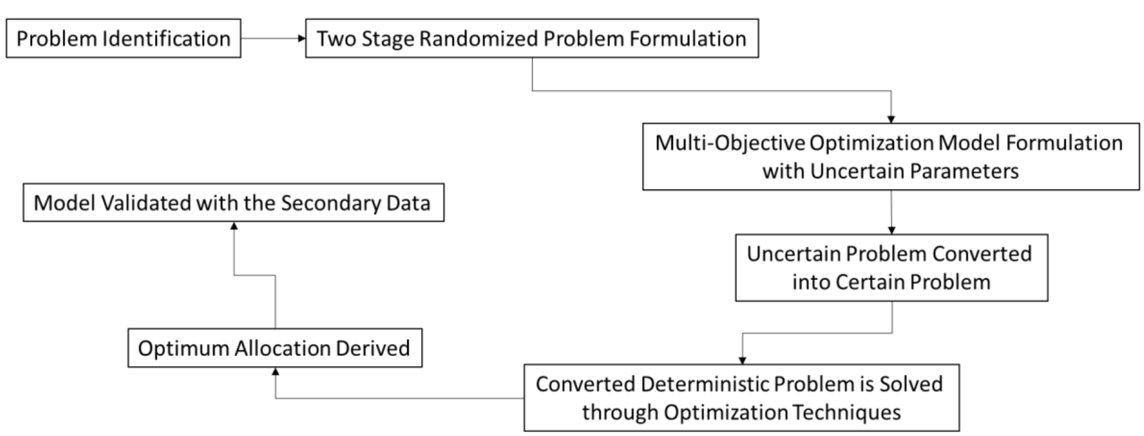

Figure 1: Methodology Flow Chart

2. TWO STAGE RANDOMIZED RESPONSE MODEL

The initial notations and model has been considered from Ghufran et al. (2013).

Notations:

Stratum size:-$N_h, h = 1,2,\dots,L$ size of h^{th} stratum

Populace:- $N = \sum_{h=1}^{L} N_h$

Stratum weights:- $W_h = \frac{N_h}{N}$

Sample size:- n_h sample size of h^{th} stratum and $n = \sum_{h=1}^{L} n_h$ is the total sample size for the stratum h.

A stratified populace of size N has considered and partitioned into L disjoint strata. In the first stage an individual respondent in the sample is told to utilize the randomization gadget R_{1h} which comprises of the accompanying two explanations:

(i) "I belong to the sensitive group" & (ii) "Go to the randomization device R_{2h} in the second stage" with known probabilities M_h & $(1 - M_h)$ respectively.

Respondents are advised in the second stage to use the randomization device R_{2h} comprises of the accompanying two explanations:

(i) "I belong to the sensitive group" & (ii) "I do not belong to the sensitive group" with known probabilities P_h & $(1 - P_h)$ respectively.

Expecting that the "Yes" or a "No" reports are made honestly for various results and M_h and P_h are set by the questioner, at that point the likelihood of a "Yes" answer in layer h is given by

$$Y_h = M_h\pi_{sh} + (1 - M_h)[P_h\pi_{sh} + (1 - P_h)(1 - \pi_{sh})]; h = 1,2,\dots,L. \quad (1)$$

furthermore, π_{sh} is the extent of respondents having a place with the delicate gathering from layer h. The most extreme probability gauge of π_{sh} is

$$\hat{\pi}_{sh} = \frac{\hat{Y}_h - (1 - M_h)(1 - P_h)}{2P_h - 1 + 2M_h(1 - P_h)}; h = 1,2,\dots,L. \quad (2)$$

where $\hat{Y}_h$ is the assessed extent of "Yes" answers which follows a binomial distribution $B(n_h, Y_h)$. It very well may be seen that the estimator $\hat{\pi}_{sh}$ is unbiased for π_{sh} with variance

$$V(\hat{\pi}_{sh}) = \frac{\pi_{sh}(1 - \pi_{sh})}{n_h} + \frac{(1 - M_h)(1 - P_h)[1 - (1 - M_h)(1 - P_h)]}{n_h[2P_h - 1 + 2M_h(1 - P_h)]^2} \quad (3)$$

Since n_h are drawn autonomously from every layer, the estimators for singular layers can be added to acquire the estimator for the entire populace. Thus an unbiased estimate of π_{sh} is given by

$$\hat{\pi}_s = \sum_{h=1}^{L} W_h\hat{\pi}_{sh}$$

Utilizing Eqn (2)

$$\hat{\pi}_s = \sum_{h=1}^{L} W_h \left[\frac{\hat{Y}_h - (1 - M_h)(1 - P_h)}{2P_h - 1 + 2M_h(1 - P_h)}\right] \quad (4)$$

3. OPTIMIZATION MODEL FORMULATION

To optimize the two stage randomized response under uncertain environment, a multi-choice stochastic model has been considered and formulated as MONLPP as in Ghufran et al. (2014):

$$\left.\begin{array}{ll} \text{Minimize} & \begin{bmatrix} V(\hat{\pi}_{s1}) \\ \vdots \\ V(\hat{\pi}_{sp}) \end{bmatrix} \\ \text{Subject to} & \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h\sqrt{n_h} \le C_0 \\ & 2 \le n_h \le N_h; h = 1,2,\dots,L \\ \text{and} & n_h \text{ are integers} \end{array}\right\} \quad (5)$$

where

$$V(\hat{\pi}_{sj}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left\{\pi_{shj}(1 - \pi_{shj}) + \frac{(1 - M_h)(1 - P_h)[1 - (1 - M_h)(1 - P_h)]}{[2P_h - 1 + 2M_h(1 - P_h)]^2}\right\}$$

In most of the situations, the cost of measurement c_h, and the travel cost t_h in several strata are assigned by the Decision-Maker (DM). But, in reality, DM itself does not know the precise values of those parameters. To deal with the uncertainty, the DM may consider these parameters as random. Thus, we assume that c_h and t_h are independently normally distributed random variables and that the multiple choices are available for the total budget. Therefore, the multi-choice non-linear probabilistic cost constraint is given as

$$P\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h\sqrt{n_h} \leq \left(C_0^{(1)}, C_0^{(2)}, \dots, C_0^{(k)}\right)\right) \geq p_0$$

where p_0 is a specified probability and $0 \leq p_0 \leq 1$.

With a probabilistic cost function, all the p variances given by (5) can be minimized simultaneously to derive the compromise allocation. Thus, the following Multi-Choice Stochastic Non-Linear Programming Problem (MCSNLPP) is solved to find the optimum compromise allocations n_h:

$$\left.\begin{array}{ll}\text{Minimize} & \begin{bmatrix} V(\hat{\pi}_{s1}) \\ \vdots \\ V(\hat{\pi}_{sp}) \end{bmatrix} \\ \text{Subject to} & P\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h\sqrt{n_h} \leq (C_0^{(1)}, C_0^{(2)}, \dots, C_0^{(k)})\right) \geq p_0 \\ & 2 \leq n_h \leq N_h;\ h = 1,2,\dots,L \\ \text{and} & n_h \text{ are integers} \end{array}\right\} \quad (6)$$

In order to solve the above MCSNLPP, probabilistic and multiple choice constraints are transformed into an equivalent deterministic form using chance constrained programming and binary integer programming as discussed below:

3.1 Chance Constrained Programming

Chance constrained programming includes constraints that are relied upon to be fulfilled distinctly in an extent of cases or with given probabilities. In chance constraint formulation, the vulnerability surface is converted into input moments, bringing about an equal deterministic optimization problem. Therefore, using the chance constrained programming, we obtain the equivalent deterministic form of problem given in Eqn. (5) as follows (Khan et al., 2011) (MCSNLPPD):

$$\left.\begin{array}{lll}\text{Minimize} & \begin{bmatrix} V(\hat{\pi}_{s1}) \\ \vdots \\ V(\hat{\pi}_{sp}) \end{bmatrix} & (i) \\ \text{Subject to} & & \\ & \left(\sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h\sqrt{n_h}\right) + & \\ & K_\alpha\sqrt{\left(\sum_{h=1}^{L} \sigma_{c_h}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{t_h}^2 n_h\right)} \leq (C_0^{(1)}, C_0^{(2)}, \dots, C_0^{(k)}) & (ii) \\ & 2 \leq n_h \leq N_h;\ h = 1,2,\dots,L & (iii) \\ \text{and} & n_h \text{ are integers} & (iv) \end{array}\right\} \quad (7)$$

The right hand side of constraint 7(ii) contains *k* number of goals out of which the goal that will minimize the objective is to be selected. To solve the problem of multi-choice parameter we use a transformation procedure for different cases in the next section.

3.2 Binary Integer Programming Technique

Binary integer programming is used to convert the multiple choice problem into an equivalent deterministic problem (Biswal and Acharya, 2009). The model MCINLPPD is formulated for maximum of two choices for any availability of budget in RHS of cost constraint function. Two cases are presented below for k=2 and 3.

Case 1: $k = 2$

For $k = 2$, RHS of constraint 7(ii) has two parameters $C_0^{(1)}, C_0^{(2)}$, out of which one has to be selected. Since the total number of elements of the set is 2, one binary variable is required. Using the binary variable $\omega^{(1)}$, the cost constraint will be:

$$C_0 = \omega^{(1)} C_0^{(1)} + (1-\omega^{(1)}) C_0^{(2)}$$

where $\omega^{(1)} = 0/1$

Case 2: $k = 3$

For $k = 3$, RHS of constraint 7(ii) has three parameters $C_0^{(1)}, C_0^{(2)}, C_0^{(3)}$, out of which one has to be selected. Since $2^1 < 3 < 2^2$, so we need two binary variables $\omega^{(1)}$ and $\omega^{(2)}$. Since 3 can be expressed as $\binom{2}{2} + \binom{2}{1}$ or $\binom{2}{1} + \binom{2}{0}$, therefore, the remaining one's (i.e. 4-3) term can be restricted by introducing additional constraint in problem (8). Using the binary variables $\omega^{(1)}$ and $\omega^{(2)}$ and introducing additional constraint, two models of the cost constraint will be:

$$Model(a): C_0 = (1-\omega^{(1)})(1-\omega^{(2)}) C_0^{(1)} + (1-\omega^{(1)})\omega^{(2)} C_0^{(2)} + \omega^{(1)}(1-\omega^{(2)}) C_0^{(3)}$$

with additional constraint

$$\omega^{(1)} + \omega^{(2)} \leq 1$$

and $\omega^{(q)} = 0/1; q = 1,2$

$$Model(b): c_0 = (1-\omega^{(1)})\omega^{(2)} C_0^{(1)} + \omega^{(1)}(1-\omega^{(2)}) C_0^{(2)} + \omega^{(1)}\omega^{(2)} C_0^{(3)}$$

with additional constraint

$$\omega^{(1)} + \omega^{(2)} \leq 1$$

and $\omega^{(q)} = 0/1; q = 1,2$ Similarly we can formulate the deterministic models for more then three choices of the parameters.

4 COMPROMISE SOLUTION USING DIFFERENT OPTIMIZATION TECHNIQUES

In this section three optimization techniques has been discussed in detail to derive the optimum allocation of the proposed MONLPP.

4.1 Goal Programming approach

Goal programming approach (GP) is a well known programming approach. Initially, the goal programming formulation was introduced by Charnes et al. (1955) and a more formal theory of goal programming is given by Charnes and Cooper (1961). Further, it has been discussed by several authors such as Varshney et al. (2011), Ghufran et al. (2014) etc. Using Goal programming approach by these authors, our GP model will be:

$$\left.\begin{array}{ll}
\text{Minimize} & \sum_{j=1}^{p} x_j \\
\text{Subject to} & \sum_{h=1}^{L} \frac{W_h^2}{n_h} \pi_{shj}(1-\pi_{shj}) + A_h - x_j \leq V(\widehat{\pi}_{sj})^* \\
& \left(\sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h \sqrt{n_h}\right) + K_\alpha \sqrt{\left(\sum_{h=1}^{L} \sigma_{c_h}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{t_h}^2 n_h\right)} \leq \left(C_0^{(1)}, C_0^{(2)}, \ldots, C_0^{(k)}\right) \\
& 2 \leq n_h \leq N_h; h = 1,2,\ldots,L; x_j \geq 0; j = 1,2,\ldots,p \\
\text{and} & n_h \text{ are integers}
\end{array}\right\} \quad (8)$$

where

$$A_h = \frac{(1-M_h)(1-P_h)[1-(1-M_h)(1-P_h)]}{[2P_h - 1 + 2M_h(1-P_h)]^2}$$

$V(\hat{\pi}_{sj})^*$ = variance at the individual optimum allocation n_j^* and $x_j (j = 1,2,\ldots,p)$ are the unknown goal variables.

4.2 Chebyshev Goal Programming approach

Among several forms of Chebyshev goal programming, we limit our coverage to just one for the solution of formulated problem (7). Chebyshev goal programming minimizes the maximum deviation from any single soft goal. Returning to our problem, one possible chebyshev goal programming model will be:

$$
\left.
\begin{array}{lll}
\text{Minimize} & \delta & \\
\text{Subject to} & & \\
& \dfrac{\left(\sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h \sqrt{n_h}\right) + K_\alpha}{\sqrt{\left(\sum_{h=1}^{L} \sigma_{c_h}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{t_h}^2 n_h\right)}} \le \left(C_0^{(1)}, C_0^{(2)}, \dots, C_0^{(k)}\right) & \\
& \delta \ge (V(\hat{\pi}_{sj}) - U_j)/(U_j - L_j) & \\
& 2 \le n_h \le N_h; h = 1,2, \dots, L & \\
& \delta \ge 0 & \\
\text{and} & n_h \text{ are integers} &
\end{array}
\right\} \quad (9)
$$

where

U_j=the worst possible value for objective j.

L_j=the best possible value for objective j.

δ=a dummy variable representing the worst deviation level.

$V(\hat{\pi}_{sj})$=the value of the j^{th} objective function.

4.3 Chebyshev Approximation

Here we apply Chebyshev Approximation to problem (7) for which we have to convert the problem into convex programming problem, so by making the transformation $n_h = \frac{1}{x_h}, h = 1,2, \dots, L$ and putting

$$a_{hj} = W_h^2 \left\{\pi_{shj}(1 - \pi_{shj}) + \frac{(1-M_h)(1-P_h)[1-(1-M_h)(1-P_h)]}{[2P_h - 1 + 2M_h(1-P_h)]^2}\right\},$$

the problem (3) is equivalent to minimizing the linear form (Khan et al., 2011)

$$
\left.
\begin{array}{ll}
\text{Minimize } Z & = x_{L+1} \\
\text{Subject to} & a_j V(\hat{\pi}_{sj}) \le x_{L+1} or a_j \sum_{h=1}^{L} a_{hj} x_h - x_{L+1} \le 0, j = 1,2, \dots, p \\
& \dfrac{\left(\sum_{h=1}^{L} \frac{\bar{c}_h}{x_h} + \sum_{h=1}^{L} \frac{\bar{t}_h}{\sqrt{x_h}}\right) + K_\alpha}{\sqrt{\left(\sum_{h=1}^{L} \frac{\sigma_{c_h}^2}{x_h^2} + \sum_{h=1}^{L} \frac{\sigma_{t_h}^2}{x_h}\right)}} \le \left(C_0^{(1)}, C_0^{(2)}, \dots, C_0^{(k)}\right) \\
\text{and} & \frac{1}{N_h} \le x_h \le \frac{1}{2}; h = 1,2, \dots, L
\end{array}
\right\} \quad (10)
$$

where a_j are the weights assigned to the variances according to their importance.

Now the problem (10) is a convex programming problem which is solved by an optimization software LINGO-17.

5 NUMERICAL ILLUSTRATION

To illustrate the computational details following example based on artificial data is given below.
The populace size N is taken to be 1000. It is estimated that for complete budget of the survey, choices 6000 or 6100 or 6200 units are available. The artificial data for four characteristics and four strata are given in Table 1. In this problem, it is assumed that measurement and travel costs are independently normally distributed random variables with known means and standard deviations which are provided in Table 1. Utilizing the values provided in Table 1, the values of A_h are obtained as $A_1 = 0.072830578, A_2 = 0.072830578, A_3 = 0.072830578 and A_4 = 0.072830578$. Let the chance constraint in 7(ii) required to be satisfied with 99 percent probability such that K_α = 0.99. The value of the standard normal variable corresponding to 99 percent confidence limit is 2.33 (by linear interpolation).

Table 1: Data for four characteristics and four strata

h	N_h	W_h	π_{sh1}	π_{sh2}	π_{sh3}	π_{sh4}	M_h	P_h	$E(c_h)$	$V(c_h)$	$E(t_h)$	$V(t_h)$

1	81	0.0808	0.28	0.33	0.40	0.62	0.8	0.7	15	3	10	2
2	343	0.3434	0.48	0.53	0.35	0.22	0.8	0.7	20	5	13	1.5
3	455	0.4546	0.68	0.73	0.55	0.82	0.8	0.7	30	6.5	10	1
4	121	0.1212	0.88	0.93	0.75	0.32	0.8	0.7	18	2.5	12	3

Using the data given in Table 1 and procedure discussed in sections 2.1 & 2.2, the following Multi-Objective Non-Linear Programming Problem (MONLPP) will be solved as a non-linear programming problem (NLPP) for each objective function.

$$\left.\begin{array}{l}
\text{Minimize } \frac{0.001791658449}{n_1}+\frac{0.038022161}{n_2}+\frac{0.06002072}{n_3}+\frac{0.00262104527}{n_4} \\
\text{Subject to } (15n_1+20n_2+30n_3+18n_4+10\sqrt{n_1}+13\sqrt{n_2}+10\sqrt{n_3}+12\sqrt{n_4}) \\
+2.33\sqrt{(3n_1^2+5n_2^2+6.5n_3^2+2.5n_4^2)+(2n_1+1.5n_2+n_3+3n_4)} \\
\leq 6000(1-\omega^{(1)})\omega^{(2)}+6100\omega^{(1)}(1-\omega^{(2)})+6200\omega^{(1)}\omega^{(2)} \\
z^{(1)}+z^{(2)}\leq 1; 2\leq n_1\leq 81; 2\leq n_2\leq 343; 2\leq n_3\leq 455; 2\leq n_4\leq 121 \\
\text{and } n_1,n_2,n_3,n_4 \text{ integers.}
\end{array}\right\} \tag{11}$$

Solving the problem in Eqn. (11) has been solved by an optimization software LINGO-17 and the optimal allocation is derived as follows:
$n_1=21, n_2=84, n_3=88, n_4=25$ with $V(\pi_{sh1})=0.001324857$.

$$\left.\begin{array}{l}
\text{Minimize } \frac{0.001918966929}{n_1}+\frac{0.037963199}{n_2}+\frac{0.055784166}{n_3}+\frac{0.00202612295}{n_4} \\
\text{Subject to } (15n_1+20n_2+30n_3+18n_4+10\sqrt{n_1}+13\sqrt{n_2}+10\sqrt{n_3}+12\sqrt{n_4}) \\
+2.33\sqrt{(3n_1^2+5n_2^2+6.5n_3^2+2.5n_4^2)+(2n_1+1.5n_2+n_3+3n_4)} \\
\leq 6000(1-\omega^{(1)})\omega^{(2)}+6100\omega^{(1)}(1-\omega^{(2)})+6200\omega^{(1)}\omega^{(2)} \\
z^{(1)}+z^{(2)}\leq 1; 2\leq n_1\leq 81; 2\leq n_2\leq 343; 2\leq n_3\leq 455; 2\leq n_4\leq 121 \\
\text{and } n_1,n_2,n_3,n_4 \text{ integers.}
\end{array}\right\} \tag{12}$$

Solving the problem in Eqn. (12) has been solved by an optimization software LINGO-17 and the optimal allocation is derived as follows:
$n_1=23, n_2=88, n_3=86, n_4=22$ with $V(\pi_{sh2})=0.001255583$.

$$\left.\begin{array}{l}
\text{Minimize } \frac{0.002042358225}{n_1}+\frac{0.03541605}{n_2}+\frac{0.066199888}{n_3}+\frac{0.003824110406}{n_4} \\
\text{Subject to } (15n_1+20n_2+30n_3+18n_4+10\sqrt{n_1}+13\sqrt{n_2}+10\sqrt{n_3}+12\sqrt{n_4}) \\
+2.33\sqrt{(3n_1^2+5n_2^2+6.5n_3^2+2.5n_4^2)+(2n_1+1.5n_2+n_3+3n_4)} \\
\leq 6000(1-\omega^{(1)})\omega^{(2)}+6100\omega^{(1)}(1-\omega^{(2)})+6200\omega^{(1)}\omega^{(2)} \\
z^{(1)}+z^{(2)}\leq 1; 2\leq n_1\leq 81; 2\leq n_2\leq 343; 2\leq n_3\leq 455; 2\leq n_4\leq 121 \\
\text{and } n_1,n_2,n_3,n_4 \text{ integers.}
\end{array}\right\} \tag{13}$$

Solving the problem in Eqn. (13) has been solved by an optimization software LINGO-17 and the optimal allocation is derived as follows:
$n_1=25, n_2=78, n_3=88, n_4=29$ with $V(\pi_{sh3})=0.001419884$.

$$\left.\begin{array}{l}\text{Minimize } \frac{0.002013632209}{n_1}+\frac{0.028824123}{n_2}+\frac{0.045554438}{n_3}+\frac{0.00426626255}{n_4} \\ \text{Subject to } (15n_1+20n_2+30n_3+18n_4+10\sqrt{n_1}+13\sqrt{n_2}+10\sqrt{n_3}+12\sqrt{n_4}) \\ +2.33\sqrt{(3n_1^2+5n_2^2+6.5n_3^2+2.5n_4^2)+(2n_1+1.5n_2+n_3+3n_4)} \\ \leq 6000(1-\omega^{(1)})\omega^{(2)}+6100\omega^{(1)}(1-\omega^{(2)})+6200\omega^{(1)}\omega^{(2)} \\ z^{(1)}+z^{(2)}\leq 1; 2\leq n_1\leq 81; 2\leq n_2\leq 343; 2\leq n_3\leq 455; 2\leq n_4\leq 121 \\ \text{and } n_1,n_2,n_3,n_4 \text{ integers.}\end{array}\right\} \quad (14)$$

Solving the problem in Eqn. (14) has been solved by an optimization software LINGO-17 and the optimal allocation is derived as follows:
$n_1 = 25, n_2 = 81, n_3 = 83, n_4 = 34$ with $V(\pi_{sh4}) = 0.001110726$.
Using the individual optimum solutions and the procedures discussed in section 3, the GP model, CGP model, and CA model can be formulated as:

Goal Programming model:

$$\left.\begin{array}{l}\text{Minimize } x_1+x_2+x_3+x_4 \\ \text{Subject to } \left(\frac{0.001791658449}{n_1}+\frac{0.038022161}{n_2}+\frac{0.06002072}{n_3}+\frac{0.00262104527}{n_4}\right)-x_1\leq 0.001324857 \\ \left(\frac{0.001918966929}{n_1}+\frac{0.037963199}{n_2}+\frac{0.055784166}{n_3}+\frac{0.00202612295}{n_4}\right)-x_2\leq 0.001255583 \\ \left(\frac{0.002042358225}{n_1}+\frac{0.03541605}{n_2}+\frac{0.066199888}{n_3}+\frac{0.003824110406}{n_4}\right)-x_3\leq 0.001419884 \\ \left(\frac{0.002013632209}{n_1}+\frac{0.028824123}{n_2}+\frac{0.045554438}{n_3}+\frac{0.00426626255}{n_4}\right)-x_4\leq 0.001110726 \\ (15n_1+20n_2+30n_3+18n_4+10\sqrt{n_1}+13\sqrt{n_2}+10\sqrt{n_3}+12\sqrt{n_4}) \\ +2.33\sqrt{(3n_1^2+5n_2^2+6.5n_3^2+2.5n_4^2)+(2n_1+1.5n_2+n_3+3n_4)} \\ \leq 6000(1-\omega^{(1)})\omega^{(2)}+6100\omega^{(1)}(1-\omega^{(2)})+6200\omega^{(1)}\omega^{(2)} \\ \omega^{(1)}+\omega^{(2)}\leq 1; 2\leq n_1\leq 81; 2\leq n_2\leq 343; 2\leq n_3\leq 455; 2\leq n_4\leq 121 \\ \text{and } n_1,n_2,n_3,n_4 \text{ integers.}\end{array}\right\} \quad (15)$$

$$\left.\begin{array}{l}\text{Minimize } \delta \\ \text{Subject to} \\ \delta\geq\left(\left(\frac{0.001791658449}{n_1}+\frac{0.038022161}{n_2}+\frac{0.06002072}{n_3}+\frac{0.00262104527}{n_4}\right)-0.001341307\right)/(0.00001645) \\ \delta\geq\left(\left(\frac{0.001918966929}{n_1}+\frac{0.037963199}{n_2}+\frac{0.055784166}{n_3}+\frac{0.00202612295}{n_4}\right)-0.001277130\right)/(0.000021547) \\ \delta\geq\left(\left(\frac{0.002042358225}{n_1}+\frac{0.03541605}{n_2}+\frac{0.066199888}{n_3}+\frac{0.003824110406}{n_4}\right)-0.001434843\right)/(0.000014959) \\ \delta\geq\left(\left(\frac{0.002013632209}{n_1}+\frac{0.028824123}{n_2}+\frac{0.045554438}{n_3}+\frac{0.00426626255}{n_4}\right)-0.001138720\right)/(0.000027994) \\ (15n_1+20n_2+30n_3+18n_4+10\sqrt{n_1}+13\sqrt{n_2}+10\sqrt{n_3}+12\sqrt{n_4}) \\ +2.33\sqrt{(3n_1^2+5n_2^2+6.5n_3^2+2.5n_4^2)+(2n_1+1.5n_2+n_3+3n_4)} \\ \leq 6000(1-\omega^{(1)})\omega^{(2)}+6100\omega^{(1)}(1-\omega^{(2)})+6200\omega^{(1)}\omega^{(2)} \\ \omega^{(1)}+\omega^{(2)}\leq 1; 2\leq n_1\leq 81; 2\leq n_2\leq 343; 2\leq n_3\leq 455; 2\leq n_4\leq 121 \\ \delta\geq 0; \text{ and } n_1,n_2,n_3,n_4 \text{ integers.}\end{array}\right\} \quad (16)$$

Chebyshev Approximation model:

$$
\left.\begin{array}{l}
\text{Minimize } Z = x_5 \\
\text{Subject to} \\
(0.001791658449x_1 + 0.038022161x_2 + 0.06002072x_3 + 0.00262104527x_4) - x_5 \le 0 \\
(0.001918966929x_1 + 0.037963199x_2 + 0.055784166x_3 + 0.00202612295x_4) - x_5 \le 0 \\
(0.002042358225x_1 + 0.03541605x_2 + 0.066199888x_3 + 0.003824110406x_4) - x_5 \le 0 \\
(0.002013632209x_1 + 0.028824123x_2 + 0.045554438x_3 + 0.00426626255x_4) - x_5 \le 0 \\
\left(\frac{15}{x_1} + \frac{20}{x_2} + \frac{30}{x_3} + \frac{18}{x_4} + \frac{10}{\sqrt{x_1}} + \frac{13}{\sqrt{x_2}} + \frac{10}{\sqrt{x_3}} + \frac{12}{\sqrt{x_4}}\right) \\
+2.33\sqrt{\left(\frac{3}{x_1^2} + \frac{5}{x_2^2} + \frac{6.5}{x_3^2} + \frac{2.5}{x_4^2}\right) + \left(\frac{2}{x_1} + \frac{1.5}{x_2} + \frac{1}{x_3} + \frac{3}{x_4}\right)} \\
\le 6000(1-\omega^{(1)})\omega^{(2)} + 6100\omega^{(1)}(1-\omega^{(2)}) + 6200\omega^{(1)}\omega^{(2)} \\
\omega^{(1)} + \omega^{(2)} \le 1; \frac{1}{81} \le x_1 \le \frac{1}{2}; \frac{1}{343} \le x_2 \le \frac{1}{2}; \frac{1}{455} \le x_3 \le \frac{1}{2}; \frac{1}{121} \le x_4 \le \frac{1}{2}
\end{array}\right\} \quad (17)
$$

All the three model (15-17) are solved by LINGO-17 and the compromise allocations obtained are given in Table 2.

Table 2: Compromise Allocations

	Allocations								
Approaches	n_1	n_2	n_3	n_4	$V(\pi_{sh1})$	$V(\pi_{sh2})$	$V(\pi_{sh3})$	$V(\pi_{sh4})$	Cost
GP*	23	81	88	27	0.001326437	0.001261067	0.001419939	0.001119076	6100
CA†	23	79	89	29	0.001323962	0.001260635	0.001412787	0.001111372	6100
CGP‡	17	85	86	30	0.001337995	0.001275697	0.001434035	0.001129468	6100
*GP:Goal Programming†CA:Chebyshev Approximation‡CGP:Chebyshev Goal Programming									

6. CONCLUSION

In real world problems parameters like measurement cost, travel cost, and budget can vary due to any natural or human activities. Thus, in this paper a two stage randomized response model has been formulated as a Multi-Choice Stochastic Non-Linear Programming Problem (MCSNLPP). The formulated uncertain problem is converted into deterministic Multi-Objective Non-Linear Programming problem (MONLPP) using chance constrained programming and binary integer programming. Then, the compromise optimum allocations are obtained by using Goal programming, Chebyshev Approximation and Chebyshev Goal programming optimization techniques. From the computational details given in Table 2 shows that Chebyshev Approximation technique minimizes the sampling variance for all the four characteristics and for total budget each technique gives the second choice i.e. 6100 units. Hence chebyshev approximation is the best technique amongs the three which helps the decision-maker in choosing the optimum budget for minimizing the sampling variances. Decision-maker's can utilize the proposed model and chebyshev approximation technique to take more calculative decisions.

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