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IMPROVED RATIO AND PRODUCT ESTIMATORS USING FUNCTION OF QUARTILES IN RANKED SET SAMPLING

Nitu Mehta (Ranka)*[1] and V.L.Mandowara**
*Dep. of Agricultural Economics & Management, University of Agriculture and Technology, India.
****** Rtd.Professor, University College of Science, M. L. Sukhadia University, Rajasthan, India

ABSTRACT
This paper deals with some new improved ratio and product estimators of finite population mean using the quartiles and its function of the auxiliary variable in Ranked Set Sampling (RSS). It has been shown that this method is highly beneficial to the estimation based on Simple Random Sampling (SRS). The bias and mean squared error of the proposed estimators with large sample approximation are derived. As a result, it is found that the suggested estimators are more efficient than the estimators in simple random sampling. A numerical example is also carried out to demonstrate the merits of the proposed estimators using RSS over the usual estimators in SRS.

KEYWORDS: Ratio estimator, product estimator, ranked set sampling, population mean, auxiliary variable, Quartiles, inter quartile range, quartile average, bias, mean squared error.

RESUMEN
Este paper trata de algunos estimadores de razón y producto mejorados para la media poblacional usando los cuartiles y funciones de una auxiliar variable en el Muestreo por Rangos Ordenados (Ranked Set Sampling , RSS). Se demuestra que este método es muy superior a la estimación basada en el Muestreo Simple Aleatorio (Simple Random Sampling , SRS). El sesgo y el Error Cuadrático Medio de las propuestas para una aproximación basada en muestras grandes son derivados. Como resultado, se halló que los sugeridos estimadores son más eficientes que los de SRS. Un ejemplo numérico es llevado a acabo para demostrar los méritos de los propuestos estimadores usando RSS respecto a los usuales en SRS.

PALABRAS CLAVE: Estimador de razón, estimador producto, muestreo por rangos ordenados, media de la población , auxiliar variable, Cuartiles, recorrido inter cuartílico , cuartil average, sesgo, error cuadrático medio

1. INTRODUCTION

In survey sampling, a researcher often comes across supplementary information supplied by auxiliary variables, which are correlated with the study variable and he takes advantage of this to devise methods of estimation to obtain more precise estimators of finite population parameters. When the correlation between study variable and auxiliary variable is positive (high), the ratio method of estimation is used for estimating the population mean. On the other hand, if the correlation is negative, the product method of estimation is used. In the past, a number of modified estimators suggested with known coefficient of variation, coefficient of scenes and coefficient of kurtosis of auxiliary variable have used for improving the efficiency of the estimators of finite population mean. However there is no attempt is made to use the known value of the population quartiles and their functions of the auxiliary variable to ratio and product estimators. Further we know that the value of quartiles and their functions are unaffected and robustness by the extreme. There are three quartiles called, first quartile, second quartile and third quartile. The second quartile is equal to the median. The first quartile is also called lower quartile and is denoted by Q_1 . The third quartile is also called upper quartile and is denoted by Q_3 . The inter-quartile range is another range used as a measure of the spread. The difference between upper and lower quartiles, which is called the inter-quartile range, also indicates the dispersion of a data set. The formula for inter-quartile range is $Q_r = Q_3 - Q_1$.

[1] Corresponding author: nitumehta82@gmail.com

The semi-quartile range is another measure of spread. It is calculated as one half of the difference between Q_3 and Q_1. The formula for semi-quartile range is $Q_d = (Q_3 - Q_1)/2$. Another measure has been suggested in this paper and named as Quartile average and defined $Q_a = (Q_3 + Q_1)/2$.

The classical ratio and product estimators given by Cochran (1940) and Murthy (1964) respectively for estimating the population mean $\bar{Y}$ respectively are defined as

$$\bar{y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) , \tag{1.1}$$

$$\bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) , \tag{1.2}$$

Al- Omari et al. (2009) have suggested the following ratio estimators using the known information of the auxiliary variable such as first quartile Q_1 and third quartile Q_3 as

$$\bar{y}_{MR1} = \bar{y}\left[\frac{\bar{X} + Q_1}{\bar{x} + Q_1}\right] \tag{1.3}$$

$$\bar{y}_{MR2} = \bar{y}\left[\frac{\bar{X} + Q_3}{\bar{x} + Q_3}\right] \tag{1.4}$$

Utilizing the information on known inter quartile range, semi inter quartile range and semi inter quartile average of the auxiliary variable, Subramani & Kumarapandiyan (2012) have introduced the following ratio type estimators respectively

$$\bar{y}_{MR3} = \bar{y}\left[\frac{\bar{X} + Q_r}{\bar{x} + Q_r}\right] \tag{1.5}$$

$$\bar{y}_{MR4} = \bar{y}\left[\frac{\bar{X} + Q_d}{\bar{x} + Q_d}\right] \tag{1.6}$$

$$\bar{y}_{MR5} = \bar{y}\left[\frac{\bar{X} + Q_a}{\bar{x} + Q_a}\right] \tag{1.7}$$

Motivated by Al- Omari et al. (2009), Subramani & Kumarapandiyan (2012), Subramani and Ajith S (2017) developed product estimators using the known quartiles and its function for the population mean $\bar{Y}$ as

$$\bar{y}_{MP1} = \bar{y}\left[\frac{\bar{x} + Q_1}{\bar{X} + Q_1}\right] \tag{1.8}$$

$$\bar{y}_{MP1} = \bar{y}\left[\frac{\bar{x} + Q_3}{\bar{X} + Q_3}\right] \tag{1.9}$$

$$\bar{y}_{MP3} = \bar{y}\left[\frac{\bar{x} + Q_r}{\bar{X} + Q_r}\right] \tag{1.10}$$

$$\bar{y}_{MP4} = \bar{y}\left[\frac{\bar{x} + Q_d}{\bar{X} + Q_d}\right] \tag{1.11}$$

$$\bar{y}_{MP5} = \bar{y}\left[\frac{\bar{x} + Q_a}{\bar{X} + Q_a}\right] \tag{1.12}$$

Using the known information on inter-quartile range, semi inter quartile range, quartile average, coefficient of kurtosis and coefficient of skewness of the auxiliary variable , Srisodaphol et al (2015) suggested the following product estimators respectively

$$T_{SP1} = \bar{y}\left[\frac{\bar{x}\beta_1(x)+Q_r}{\bar{X}\beta_1(x)+Q_r}\right] \tag{1.13}$$

$$T_{SP2} = \bar{y}\left[\frac{\bar{x}\beta_1(x)+Q_d}{\bar{X}\beta_1(x)+Q_d}\right] \tag{1.14}$$

$$T_{SP3} = \bar{y}\left[\frac{\bar{x}\beta_1(x)+Q_a}{\bar{X}\beta_1(x)+Q_a}\right] \tag{1.15}$$

$$T_{SP4} = \bar{y}\left[\frac{\bar{x}\beta_2(x)+Q_r}{\bar{X}\beta_2(x)+Q_r}\right] \tag{1.16}$$

$$T_{SP5} = \bar{y}\left[\frac{\bar{x}\beta_2(x)+Q_d}{\bar{X}\beta_2(x)+Q_d}\right] \tag{1.17}$$

$$T_{SP6} = \bar{y}\left[\frac{\bar{x}\beta_2(x)+Q_a}{\bar{X}\beta_2(x)+Q_a}\right] \tag{1.18}$$

Where $\beta_1(x)$ and $\beta_2(x)$ are known value of coefficient of skewness and coefficient of kurtosis.

To the first degree of approximation the mean squared error (MSE) of the estimators $\bar{y}_{MR1}$, $\bar{y}_{MR2}$, $\bar{y}_{MR3}$, $\bar{y}_{MR4}$, $\bar{y}_{MR5}$, $\bar{y}_{MP1}$, $\bar{y}_{MP2}$, $\bar{y}_{MP3}$, $\bar{y}_{MP4}$, $\bar{y}_{MP5}$, T_{SP1}, T_{SP2}, T_{SP3}, T_{SP4}, T_{SP5} and T_{SP6} respectively are

$$MSE(\bar{y}_{MR1}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_1^2 C_x^2 - 2\lambda_1\rho_{yx}C_yC_x\right], \tag{1.19}$$

$$MSE(\bar{y}_{MR2}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_2^2 C_x^2 - 2\lambda_2\rho_{yx}C_yC_x\right], \tag{1.20}$$

$$MSE(\bar{y}_{MR3}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_3^2 C_x^2 - 2\lambda_3\rho_{yx}C_yC_x\right], \tag{1.21}$$

$$MSE(\bar{y}_{MR4}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_4^2 C_x^2 - 2\lambda_4\rho_{yx}C_yC_x\right], \tag{1.22}$$

$$MSE(\bar{y}_{MR5}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_5^2 C_x^2 - 2\lambda_5\rho_{yx}C_yC_x\right], \tag{1.23}$$

$$MSE(\bar{y}_{MP1}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_1^2 C_x^2 + 2\lambda_1\rho_{yx}C_yC_x\right], \tag{1.24}$$

$$MSE(\bar{y}_{MP2}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_2^2 C_x^2 + 2\lambda_2\rho_{yx}C_yC_x\right], \tag{1.25}$$

$$MSE(\bar{y}_{MP3}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_3^2 C_x^2 + 2\lambda_3\rho_{yx}C_yC_x\right], \tag{1.26}$$

$$MSE(\bar{y}_{MP4}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_4^2 C_x^2 + 2\lambda_4\rho_{yx}C_yC_x\right], \tag{1.27}$$

$$MSE(\bar{y}_{MP5}) = \theta\bar{Y}^2\left[C_y^2 + \lambda_5^2 C_x^2 + 2\lambda_5\rho_{yx}C_yC_x\right], \tag{1.28}$$

$$MSE(T_{SP1}) = \theta\bar{Y}^2\left[C_y^2 + \gamma_1^2 C_x^2 + 2\gamma_1\rho_{yx}C_yC_x\right], \tag{1.29}$$

$$MSE(T_{SP2}) = \theta\bar{Y}^2\left[C_y^2 + \gamma_2^2 C_x^2 + 2\gamma_2\rho_{yx}C_yC_x\right], \tag{1.30}$$

$$MSE(T_{SP3}) = \theta\bar{Y}^2\left[C_y^2 + \gamma_3^2 C_x^2 + 2\gamma_3\rho_{yx}C_yC_x\right], \tag{1.31}$$

$$MSE(T_{SP4}) = \theta\bar{Y}^2\left[C_y^2 + \gamma_4^2 C_x^2 + 2\gamma_4\rho_{yx}C_yC_x\right], \tag{1.32}$$

$$MSE(T_{SP5}) = \theta \bar{Y}^2 \left[C_y^2 + \gamma_5^2 C_x^2 + 2\gamma_5 \rho_{yx} C_y C_x\right], \tag{1.33}$$

$$MSE(T_{SP6}) = \theta \bar{Y}^2 \left[C_y^2 + \gamma_6^2 C_x^2 + 2\gamma_6 \rho_{yx} C_y C_x\right]. \tag{1.34}$$

where $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, $\rho_{yx} = \frac{S_{yx}}{S_y S_x}$, $\theta = \frac{1}{n}$(on ignoring $f = \frac{n}{N}$), $\lambda_1 = \frac{\bar{X}}{\bar{X} + Q_1}$,

$\lambda_2 = \frac{\bar{X}}{\bar{X} + Q_3}$, $\lambda_3 = \frac{\bar{X}}{\bar{X} + Q_r}$, $\lambda_4 = \frac{\bar{X}}{\bar{X} + Q_d}$, $\lambda_5 = \frac{\bar{X}}{\bar{X} + Q_a}$, $\gamma_1 = \frac{\bar{X}\beta_1(x)}{\bar{X}\beta_1(x) + Q_r}$,

$\gamma_2 = \frac{\bar{X}\beta_1(x)}{\bar{X}\beta_1(x) + Q_d}$, $\gamma_3 = \frac{\bar{X}\beta_1(x)}{\bar{X}\beta_1(x) + Q_a}$, $\gamma_4 = \frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x) + Q_r}$, $\gamma_5 = \frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x) + Q_d}$,

$\gamma_6 = \frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x) + Q_a}$, $S_y^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})^2}{N-1}$, $S_x^2 = \frac{\sum_{i=1}^{N}(x_i - \bar{X})^2}{N-1}$ and $S_{yx} = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})}{N-1}$.

The literature on Ranked set sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Ranked set sampling was first suggested by McIntyre (1952) to increase the efficiency of estimator of population mean. Kadilar et al. (2009) used this technique to improve ratio estimator given by Prasad (1989) and Bouza (2008) has modified the product estimators. Mehta and Mandowara (2016) proposed a modified ratio-cum-product estimator using information on coefficient of variation and co-efficient of kurtosis of auxiliary variable in RSS. Here we shall improve ratio and product estimators given by Al- Omari et al. (2009), Subramani & Kumarapandiyan (2012), Subramani and Ajith S (2017) and Srisodaphol et al (2015) respectively by using quartiles and their functions in RSS based on auxiliary variable.

2. RATIO AND PRODUCT ESTIMATORS IN RANKED SET SAMPLING

In Ranked set sampling (RSS), m independent random sets, each of size m are selected with equal probability and with replacement from the population. The members of each random set are ranked with respect to the characteristic of the study variable or auxiliary variable. Then, the smallest unit is selected from the first ordered set and the second smallest unit is selected from the second ordered set. By this way, this procedure is continued until the unit with the largest value chosen from the m^{th} set. This cycle may be repeated r times, so $mr = (n)$ units have been measured during this process.

When we rank on the auxiliary variable, let $(y_{[i]}, x_{(i)})$ denote a i^{th} judgment ordering in the i^{th} set for the study variable and i^{th} set for the auxiliary variables.

Samawi and Mutlak (1996) and Bouza (2008) defined the ratio and product estimators for the population mean are

$$\bar{y}_{R,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right), \tag{2.1}$$

$$\bar{y}_{P,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}}{\bar{X}}\right). \tag{2.2}$$

Where $\bar{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$, $\bar{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$ are the ranked set sample means for variables y and x respectively.

To the first degree of approximation the mean squared error (MSE) of the estimators $\bar{y}_{R,RSS}$ and $\bar{y}_{P,RSS}$, respectively are

$$MSE(\bar{y}_{R,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\} - \{W_{y[i]} - W_{x(i)}\}^2\right]$$
(2.3)
$$MSE(\bar{y}_{P,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x\} - \{W_{y[i]} + W_{x(i)}\}^2\right]. \quad (2.4)$$

Al- Omari et al. (2009) proposed new ratio-type estimators for $\bar{Y}$ using RSS, when the quartiles Q_1 and Q_3 of auxiliary variable are known as

$$\bar{y}_{MR1,RSS} = \bar{y}_{[n]}\left[\frac{\bar{X} + Q_1}{\bar{x}_{(n)} + Q_1}\right] \quad (2.5)$$

$$\bar{y}_{MR2,RSS} = \bar{y}_{[n]}\left[\frac{\bar{X} + Q_3}{\bar{x}_{(n)} + Q_3}\right] \quad (2.6)$$

To the first degree of approximation the mean squared error (MSE) of the estimators $\bar{y}_{MR1,RSS}$ and $\bar{y}_{MR2,RSS}$, respectively are

$$MSE(\bar{y}_{MR1,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \lambda_1^2 C_x^2 - 2\lambda_1\rho_{yx}C_yC_x\} - \{W_{y[i]} - \lambda_1 W_{x(i)}\}^2\right], \quad (2.7)$$

$$MSE(\bar{y}_{MR2,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \lambda_2^2 C_x^2 - 2\lambda_2\rho_{yx}C_yC_x\} - \{W_{y[i]} - \lambda_2 W_{x(i)}\}^2\right]. \quad (2.8)$$

3. SUGGESTED NEW RATIO ESTIMATORS BASED ON RANKED SET SAMPLING

Motivated by Subramani & Kumarapandiyan (2012), we propose new ratio-type estimators for $\bar{Y}$ using the information on known inter quartile range, semi inter quartile range and semi inter quartile average of the auxiliary variable in RSS are given as

$$\bar{y}_{MM1,RSS} = \bar{y}_{[n]}\left[\frac{\bar{X} + Q_r}{\bar{x}_{(n)} + Q_r}\right] \quad (3.1)$$

$$\bar{y}_{MM2,RSS} = \bar{y}_{[n]}\left[\frac{\bar{X} + Q_d}{\bar{x}_{(n)} + Q_d}\right] \quad (3.2)$$

$$\bar{y}_{MM3,RSS} = \bar{y}_{[n]}\left[\frac{\bar{X} + Q_a}{\bar{x}_{(n)} + Q_a}\right] \quad (3.3)$$

To obtain bias and MSE of the estimator $\bar{y}_{MM1,RSS}$, we put $\bar{y}_{[n]} = \bar{Y}(1+\varepsilon_0)$ and $\bar{x}_{(n)} = X(1+\varepsilon_1)$ so that

$$E(\varepsilon_0) = E(\varepsilon_1) = 0 \;;\; V(\varepsilon_0) = E(\varepsilon_0^2) = \frac{V(\bar{y}_{[n]})}{\bar{Y}^2} = \frac{1}{mr}\frac{1}{\bar{Y}^2}\left[S_y^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{y[i]}^2\right] = \left[\theta C_y^2 - W_{y[i]}^2\right]$$

Similarly, $V(\varepsilon_1) = E(\varepsilon_1^2) = \left[\theta C_x^2 - W_{x(i)}^2\right]$ and $Cov(\varepsilon_0, \varepsilon_1) = E(\varepsilon_0, \varepsilon_1) = \dfrac{Cov(\bar{y}_{[n]}, \bar{x}_{(n)})}{\bar{X}\bar{Y}}$

$= \dfrac{1}{\bar{X}\bar{Y}}\dfrac{1}{mr}\left[S_{yx} - \dfrac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right] = \left[\theta\rho_{yx}C_yC_x - W_{yx(i)}\right]$, where $\theta = \dfrac{1}{mr}$, $C_y^2 = \dfrac{S_y^2}{\bar{Y}^2}$, $C_x^2 = \dfrac{S_x^2}{\bar{X}^2}$,

$C_{yx} = \dfrac{Syx}{\bar{X}\bar{Y}} = \rho_{yx}C_yC_x$, $W_{x(i)}^2 = \dfrac{1}{m^2 r}\dfrac{1}{\bar{X}^2}\sum_{i=1}^{m}\tau_{x(i)}^2$, $W_{y[i]}^2 = \dfrac{1}{m^2 r}\dfrac{1}{\bar{Y}^2}\sum_{i=1}^{m}\tau_{y[i]}^2$ and

$W_{yx(i)} = \frac{1}{m^2 r}\frac{1}{\overline{X}\,\overline{Y}}\sum_{i=1}^{m}\tau_{yx(i)}$. Here we would also like to remind $\tau_{x(i)} = \mu_{x(i)} - \overline{X}$, $\tau_{y[i]} = \mu_{y[i]} - \overline{Y}$ and $\tau_{yx(i)} = (\mu_{x(i)} - \overline{X})\ (\mu_{y[i]} - \overline{Y})$.

Further to validate first degree of approximation, we assume that the sample size is large enough to get $|\varepsilon_0|$ and $|\varepsilon_1|$ as small so that the terms involving ε_0 and or ε_1 in a degree greater than two will be negligible.

The Bias and MSE of $\overline{y}_{MM1,RSS}$ can be found as follows-

$$B(\overline{y}_{MM1,RSS}) = E(\overline{y}_{MM1,RSS}) - \overline{Y}$$

Here $\overline{y}_{MM1,RSS} = \overline{Y}(1+\varepsilon_0)(1+\lambda_3\varepsilon_1)^{-1}$ where $\lambda_3 = \frac{\overline{X}}{\overline{X}+Q_r}$

Suppose $|\lambda_3\varepsilon_1| < 1$ so that $(1+\lambda_3\varepsilon_1)^{-1}$ is expandable.

$$\overline{y}_{MM1,RSS} = \overline{Y}(1+\varepsilon_0)\{1-\lambda_3\varepsilon_1+\lambda_3{}^2\varepsilon_1^2+O(\lambda_3\varepsilon_1)\}$$

(Using Taylor series expansion, where $O(\varepsilon_1)$ with power more than 2 are neglected for large power of ε_1.)

$B(\overline{y}_{MM1,RSS}) = \overline{Y}\left[\lambda_3{}^2E(\varepsilon_1^2) - \lambda_3E(\varepsilon_0\varepsilon_1)\right]$, because $E(\varepsilon_0) = E(\varepsilon_1) = 0$

$= \overline{Y}\left[\lambda_3{}^2\{\theta C_x^2 - W_{x(i)}^2\} - \lambda_3\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\}\right] \Rightarrow B(\overline{y}_{MM1,RSS}) = \overline{Y}\left[\theta(\lambda_3{}^2C_x^2 - \lambda_3\rho_{yx}C_yC_x) - \{\lambda_3{}^2W_{x(i)}^2 - \lambda_3W_{yx(i)})\}\right]$ (3.4)

Now $MSE(\overline{y}_{MM1,RSS}) = E(\overline{y}_{MM1,RSS} - \overline{Y})^2 = \overline{Y}^2E\left[\varepsilon_0 - \lambda_3\varepsilon_1 + \lambda_3{}^2\varepsilon_1^2 - 2\lambda_3\varepsilon_0\varepsilon_1\right]^2$

(Using Taylor series expansion, where $O(\varepsilon_1)$ with power more than 2 are neglected for large power of ε_1.)

$= \overline{Y}^2E\left[\varepsilon_0^2 + \lambda_3{}^2\varepsilon_1^2 - 2\lambda_3\varepsilon_0\varepsilon_1\right] = \overline{Y}^2\left[\theta C_y^2 - W_{y[i]}^2 + \lambda_{31}{}^2(\theta C_x^2 - W_{x(i)}^2) - 2\lambda_3\left(\theta\rho_{yx}C_yC_x - W_{yx(i)}\right)\right]$

$= \overline{Y}^2\left[\theta\{C_y^2 + \lambda_3{}^2C_x^2 - 2\lambda_3\rho_{yx}C_yC_x\} - \{W_{y[i]}^2 + \lambda_3{}^2W_{x(i)}^2 - 2\lambda_3W_{yx(i)}\}\right]$

$$MSE(\overline{y}_{MM1,RSS}) = \overline{Y}^2\left[\theta\{C_y^2 + \lambda_3{}^2C_x^2 - 2\lambda_3\rho_{yx}C_yC_x\} - \{W_{y[i]} - \lambda_3W_{x(i)}\}^2\right] \quad (3.5)$$

Similarly, the bias and mean squared error of the estimators $\overline{y}_{MM2,RSS}$ and $\overline{y}_{MM3,RSS}$ can be obtained respectively

$$B(\overline{y}_{MM2,RSS}) = \overline{Y}\left[\theta(\lambda_4{}^2C_x^2 - \lambda_4\rho_{yx}C_yC_x) - \{\lambda_4{}^2W_{x(i)}^2 - \lambda_4W_{yx(i)})\}\right] \quad (3.7)$$

$$MSE(\overline{y}_{MM2,RSS}) = \overline{Y}^2\left[\theta\{C_y^2 + \lambda_4{}^2C_x^2 - 2\lambda_4\rho_{yx}C_yC_x\} - \{W_{y[i]} - \lambda_4W_{x(i)}\}^2\right] \quad (3.8)$$

$$B(\overline{y}_{MM3,RSS}) = \overline{Y}\left|\theta(\lambda_5{}^2C_x^2 - \lambda_5\rho_{yx}C_yC_x) - \{\lambda_5{}^2W_{x(i)}^2 - \lambda_5W_{yx(i)})\}\right| \quad (3.9)$$

$$MSE(\overline{y}_{MM3,RSS}) = \overline{Y}^2\left[\theta\{C_y^2 + \lambda_5{}^2C_x^2 - 2\lambda_5\rho_{yx}C_yC_x\} - \{W_{y[i]} - \lambda_5W_{x(i)}\}^2\right] \quad (3.10)$$

Where $\lambda_4 = \frac{\overline{X}}{\overline{X}+Q_d}$ and $\lambda_5 = \frac{\overline{X}}{\overline{X}+Q_a}$.

4. PROPOSED IMPROVED PRODUCT ESTIMATORS BASED ON RANKED SET SAMPLING

Motivated by Subramani and Ajith S (201), we suggest improved product estimators using the known quartiles and its function for the population mean $\overline{Y}$ as

$$\overline{y}_{MM4,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)} + Q_1}{\overline{X} + Q_1}\right] \quad (4.1)$$

$$\bar{y}_{MR5,RSS} = \bar{y}_{[n]}\left[\frac{\bar{x}_{(n)} + Q_3}{\bar{X} + Q_3}\right] \quad (4.2)$$

$$\bar{y}_{MM6,RSS} = \bar{y}_{[n]}\left[\frac{\bar{x}_{(n)} + Q_r}{\bar{X} + Q_r}\right] \quad (4.3)$$

$$\bar{y}_{MM7,RSS} = \bar{y}_{[n]}\left[\frac{\bar{x}_{(n)} + Q_d}{\bar{X} + Q_d}\right] \quad (4.4)$$

$$\bar{y}_{MM8,RSS} = \bar{y}_{[n]}\left[\frac{\bar{x}_{(n)} + Q_a}{\bar{X} + Q_a}\right] \quad (4.5)$$

The bias and MSE of $\bar{y}_{MMi,RSS}$ (i = 4, 5, 6, 7, 8) are given by

$$B(\bar{y}_{MMi,RSS}) = E(\bar{y}_{MMi,RSS}) - \bar{Y}$$

Here $\bar{y}_{MMi,RSS} = \bar{Y}(1+\varepsilon_0)(1+\lambda_i\varepsilon_1)$

$B(\bar{y}_{MMi,RSS}) = \bar{Y}[\lambda_i E(\varepsilon_0\varepsilon_1)]$, because $E(\varepsilon_0) = E(\varepsilon_1) = 0 \Rightarrow B(\bar{y}_{MMi,RSS}) = \bar{Y}\left[\lambda_i\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right]$ (4.6)

Now

$$MSE(\bar{y}_{MMi,RSS}) = E(\bar{y}_{MMi,RSS} - \bar{Y})^2 = \bar{Y}^2 E\left[\varepsilon_0 + \lambda_i\varepsilon_1 + \lambda_i\varepsilon_0\varepsilon_1\right]^2$$

$$= \bar{Y}^2 E\left[\varepsilon_0^2 + \lambda_i^2\varepsilon_1^2 + 2\lambda_i\varepsilon_0\varepsilon_1\right]$$

$$= \bar{Y}^2\left[\theta C_y^2 - W_{y[i]}^2 + \lambda_i^2(\theta C_x^2 - W_{x(i)}^2) + 2\lambda_i\left(\theta\rho_{yx}C_yC_x - W_{yx(i)}\right)\right]$$

$$\Rightarrow MSE(\bar{y}_{MMi,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \lambda_i^2C_x^2 + 2\lambda_i\rho_{yx}C_yC_x\} - \{W_{y[i]} + \lambda_iW_{x(i)}\}^2\right] \quad (4.7)$$

Here $\lambda_i = \dfrac{\bar{X}}{\bar{X} + m_i}$, i = 4,5,6,7,8 $m_4 = Q_1$, $m_5 = Q_3$, $m_6 = Q_r$, $m_7 = Q_d$, $m_8 = Q_a$.

Bias and MSE of the estimators $\bar{y}_{MM4,RSS}$, $\bar{y}_{MM5,RSS}$, $\bar{y}_{MM6,RSS}$, $\bar{y}_{MM7,RSS}$ and $\bar{y}_{MM8,RSS}$ defined in (4.1) to(4.5) can be obtained by substituting the values of λ_i, $(i = 4,5,6,7,8)$ in (4.6) and (4.7) are

$$B(\bar{y}_{MM4,RSS}) = \bar{Y}\left[\lambda_4\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \quad (4.8)$$

$$MSE(\bar{y}_{MM4,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \lambda_4^2C_x^2 + 2\lambda_4\rho_{yx}C_yC_x\} - \{W_{y[i]} + \lambda_4W_{x(i)}\}^2\right] \quad (4.9)$$

$$B(\bar{y}_{MM5,RSS}) = \bar{Y}\left[\lambda_5\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \quad (4.10)$$

$$MSE(\bar{y}_{MM5,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \lambda_5^2C_x^2 + 2\lambda_5\rho_{yx}C_yC_x\} - \{W_{y[i]} + \lambda_5W_{x(i)}\}^2\right] \quad (4.11)$$

$$B(\bar{y}_{MM6,RSS}) = \bar{Y}\left[\lambda_6\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \quad (4.12)$$

$$MSE(\bar{y}_{MM6,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \lambda_6^2C_x^2 + 2\lambda_6\rho_{yx}C_yC_x\} - \{W_{y[i]} + \lambda_6W_{x(i)}\}^2\right] \quad (4.13)$$

$$B(\bar{y}_{MM7,RSS}) = \bar{Y}\left[\lambda_7\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \quad (4.14)$$

$$MSE(\bar{y}_{MM7,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \lambda_7^2C_x^2 + 2\lambda_7\rho_{yx}C_yC_x\} - \{W_{y[i]} + \lambda_7W_{x(i)}\}^2\right] \quad (4.15)$$

$$B(\bar{y}_{MM8,RSS}) = \bar{Y}\left[\lambda_8\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \quad (4.16)$$

$$MSE(\bar{y}_{MM8,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \lambda_8^2C_x^2 + 2\lambda_8\rho_{yx}C_yC_x\} - \{W_{y[i]} + \lambda_8W_{x(i)}\}^2\right] \quad (4.17)$$

Adapting the estimators in (1.13) to (1.18), given by Srisodaphol et al (2015) and utilizing the known values of $\beta_1(x)$, $\beta_2(x)$ and function of quartiles (Q_r, Q_d, Q_a), we propose the following six more reasonable transformations for x and corresponding estimators for $\overline{Y}$, which are as follows.

$$T_{MM1,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_1(x)+Q_r}{\overline{X}\beta_1(x)+Q_r}\right] \tag{4.18}$$

$$T_{MM2,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_1(x)+Q_d}{\overline{X}\beta_1(x)+Q_d}\right] \tag{4.19}$$

$$T_{MM3,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_1(x)+Q_a}{\overline{X}\beta_1(x)+Q_a}\right] \tag{4.20}$$

$$T_{MM4,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_2(x)+Q_r}{\overline{X}\beta_2(x)+Q_r}\right] \tag{4.21}$$

$$T_{MM5,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_2(x)+Q_d}{\overline{X}\beta_2(x)+Q_d}\right] \tag{4.22}$$

$$T_{MM6,RSS} = \overline{y}_{[n]}\left[\frac{\overline{x}_{(n)}\beta_2(x)+Q_a}{\overline{X}\beta_2(x)+Q_a}\right] \tag{4.23}$$

Theorem.1 The Bias and Mean Square Error (MSE) of the Proposed Sequence of Estimators, $T_{MMi,RSS}$ $(i = 1,2,3)$ on the first degree of approximation are given by

$$B(\, T_{MMi,RSS}) = E(\, T_{MMi,RSS}) - \overline{Y} = \overline{Y}\left[\gamma_i\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \tag{4.24}$$

$$MSE(\, T_{MMi,RSS}) = E(\, T_{MMi,RSS} - \overline{Y})^2 = \overline{Y}^2\left[\theta\{C_y^2 + \gamma_i^2C_x^2 + 2\gamma_i\rho_{yx}C_yC_x\} - \{W_{y[i]} + \gamma_iW_{x(i)}\}^2\right] \tag{4.25}$$

Proof: Here $T_{MMi,RSS} = \overline{Y}(1+\varepsilon_0)(1+\gamma_i\varepsilon_1)$, $(i = 1,2,3)$ (4.26)

Where $\gamma_i = \dfrac{\overline{X}\beta_1(x)}{\overline{X}\beta_1(x)+u_i}$, i = 1, 2, 3, $u_1 = Q_r, u_2 = Q_d, u_3 = Q_a$.

For $i = 1$ and $\gamma_1 < 1$, now expanding the terms of (4.26) and taking expectations, we get

$B(\, \overline{y}_{MM1,RSS}) = \overline{Y}[\gamma_1 E(\varepsilon_0\varepsilon_1)]$, because $E(\varepsilon_0) = E(\varepsilon_1) = 0 \Rightarrow B(\, \overline{y}_{MM1,RSS}) = \overline{Y}\left[\gamma_1\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right]$ (4.27)

Now

$$MSE(\, \overline{y}_{MM1,RSS}) = E(\, \overline{y}_{MM1,RSS} - \overline{Y})^2 = \overline{Y}^2 E\left[\varepsilon_0 + \gamma_1\varepsilon_1 + \gamma_1\varepsilon_0\varepsilon_1\right]^2 = \overline{Y}^2\left[\theta C_y^2 - W_{y[i]}^2 + \gamma_1^{\,2}(\theta C_x^2 - W_{x(i)}^2) + 2\gamma_1\left(\theta\rho_{yx}C_yC_x - W_{yx(i)}\right)\right]$$

$$MSE(\, T_{MM1,RSS}) = \overline{Y}^2\left[\theta\{C_y^2 + \gamma_1^2C_x^2 + 2\gamma_1\rho_{yx}C_yC_x\} - \{W_{y[i]} + \gamma_1W_{x(i)}\}^2\right] \tag{4.28}$$

Similarly, we can prove that the Bias and MSE of $T_{MM2,RSS}$ and $T_{MM3,RSS}$ defined in (4.19) and (4.20) would be obtained by substituting the values of γ_i, $(i = 2,3)$ in (4.24) and (4.25), respectively as

$$B(\, T_{MM2,RSS}) = \overline{Y}\left[\gamma_2\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \tag{4.29}$$

$$MSE(\, T_{MM2,RSS}) = \overline{Y}^2\left[\theta\{C_y^2 + \gamma_2^2C_x^2 + 2\gamma_2\rho_{yx}C_yC_x\} - \{W_{y[i]} + \gamma_2W_{x(i)}\}^2\right] , \tag{4.30}$$

$$B(\, T_{MM3,RSS}) = \overline{Y}\left[\gamma_3\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \tag{4.31}$$

$$MSE(\, T_{MM3,RSS}) = \overline{Y}^2\left[\theta\{C_y^2 + \gamma_3^2C_x^2 + 2\gamma_3\rho_{yx}C_yC_x\} - \{W_{y[i]} + \gamma_3W_{x(i)}\}^2\right] . \tag{4.32}$$

Similarly, the Bias and Mean Square Error (MSE) of the new proposed product Estimators, $T_{MMi,RSS}$ $(i = 4,5,6)$ defined in (4.21) to (4.23) can be found as follow

$$B(T_{MM4,RSS}) = \bar{Y}\left[\gamma_4\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \quad (4.33)$$

$$MSE(T_{MM4,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \gamma_4^2C_x^2 + 2\gamma_4\rho_{yx}C_yC_x\} - \{W_{y[i]} + \gamma_4 W_{x(i)}\}^2\right], \quad (4.34)$$

$$B(T_{MM5,RSS}) = \bar{Y}\left[\gamma_5\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \quad (4.35)$$

$$MSE(T_{MM5,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \gamma_5^2C_x^2 + 2\gamma_5\rho_{yx}C_yC_x\} - \{W_{y[i]} + \gamma_5 W_{x(i)}\}^2\right], \quad (4.36)$$

$$B(T_{MM6,RSS}) = \bar{Y}\left[\gamma_6\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\}\right] \quad (4.37)$$

$$MSE(T_{MM6,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + \gamma_6^2C_x^2 + 2\gamma_6\rho_{yx}C_yC_x\} - \{W_{y[i]} + \gamma_6 W_{x(i)}\}^2\right]. \quad (4.38)$$

Where $\gamma_i = \dfrac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x) + u_i}$, i = 4, 5, 6, $u_4 = Q_r$, $u_5 = Q_d$, $u_6 = Q_a$.

5. EFFICIENCY COMPARISON

On comparing (1.21) to (1.34) with (3.5), (3.8), (3.10), (4.9), (4.11), (4.13), (4.15), (4.17), (4.28), (4.30), (4.32), (4.34), (4.36) and (4.38) respectively, we obtained

1) $MSE(\bar{y}_{MR3}) - MSE(\bar{y}_{MM1,RSS}) = A_1 \geq 0$, where $A_1 = \left[W_{y[i]} - \lambda_3 W_{x(i)}\right]^2$

$\Rightarrow MSE(\bar{y}_{MR3}) \geq MSE(\bar{y}_{MM1,RSS})$

2) $MSE(\bar{y}_{MR4}) - MSE(\bar{y}_{MM2,RSS}) = A_2 \geq 0$, where $A_2 = \left[W_{y[i]} - \lambda_4 W_{x(i)}\right]^2$

$\Rightarrow MSE(\bar{y}_{MR4}) \geq MSE(\bar{y}_{MM2,RSS})$

3) $MSE(\bar{y}_{MR5}) - MSE(\bar{y}_{MM3,RSS}) = A_3 \geq 0$, where $A_3 = \left[W_{y[i]} - \lambda_5 W_{x(i)}\right]^2$

$\Rightarrow MSE(\bar{y}_{MR5}) \geq MSE(\bar{y}_{MM3,RSS})$

4) $MSE(\bar{y}_{MP1}) - MSE(\bar{y}_{MM4,RSS}) = A_4 \geq 0$, where $A_4 = \left[W_{y[i]} + \lambda_1 W_{x(i)}\right]^2$

$\Rightarrow MSE(\bar{y}_{MP1}) \geq MSE(\bar{y}_{MM4,RSS})$

5) $MSE(\bar{y}_{MP2}) - MSE(\bar{y}_{MM5,RSS}) = A_5 \geq 0$, where $A_5 = \left[W_{y[i]} + \lambda_2 W_{x(i)}\right]^2$

$\Rightarrow MSE(\bar{y}_{MP2}) \geq MSE(\bar{y}_{MM5,RSS})$

6) $MSE(\bar{y}_{MP3}) - MSE(\bar{y}_{MM6,RSS}) = A_6 \geq 0$, where $A_6 = \left[W_{y[i]} + \lambda_3 W_{x(i)}\right]^2$

$\Rightarrow MSE(\bar{y}_{MP3}) \geq MSE(\bar{y}_{MM6,RSS})$

7) $MSE(\bar{y}_{MP4}) - MSE(\bar{y}_{MM7,RSS}) = A_7 \geq 0$, where $A_7 = \left[W_{y[i]} + \lambda_4 W_{x(i)}\right]^2$

$\Rightarrow MSE(\bar{y}_{MP4}) \geq MSE(\bar{y}_{MM7,RSS})$

8) $MSE(\bar{y}_{MP5}) - MSE(\bar{y}_{MM8,RSS}) = A_8 \geq 0$, where $A_8 = \left[W_{y[i]} + \lambda_5 W_{x(i)}\right]^2$

$\Rightarrow MSE(\bar{y}_{MP5}) \geq MSE(\bar{y}_{MM8,RSS})$

9) $MSE(T_{SP1}) - MSE(T_{MM1,RSS}) = A_9 \geq 0$, where $A_9 = \left[W_{y[i]} + \gamma_1 W_{x_1(i)}\right]^2$

$\Rightarrow MSE(T_{SP1}) \geq MSE(T_{MM1,RSS})$

10) $MSE(T_{SP2}) - MSE(T_{MM2,RSS}) = A_{10} \geq 0$, where $A_{10} = \left[W_{y[i]} + \gamma_2 W_{x_1(i)}\right]^2$

$\Rightarrow MSE(T_{SP2}) \geq MSE(T_{MM2,RSS})$

11) $MSE(T_{SP3}) - MSE(T_{MM3,RSS}) = A_{11} \geq 0$, where $A_{11} = \left[W_{y[i]} + \gamma_3 W_{x_1(i)}\right]^2$

$\Rightarrow MSE(T_{SP3}) \geq MSE(T_{MM3,RSS})$

12) $MSE(T_{SP4}) - MSE(T_{MM4,RSS}) = A_{12} \geq 0$, where $A_{12} = \left[W_{y[i]} + \gamma_4 W_{x_1(i)}\right]^2$

$\Rightarrow MSE(T_{SP4}) \geq MSE(T_{MM4,RSS})$

13) $MSE(T_{SP5}) - MSE(T_{MM5,RSS}) = A_{13} \geq 0$, where $A_{13} = \left[W_{y[i]} + \gamma_5 W_{x_1(i)}\right]^2$

$\Rightarrow MSE(T_{SP5}) \geq MSE(T_{MM5,RSS})$

14) $MSE(T_{SP6}) - MSE(T_{MM6,RSS}) = A_{14} \geq 0$, where $A_{14} = \left[W_{y[i]} + \gamma_6 W_{x_1(i)}\right]^2$

$\Rightarrow MSE(T_{SP6}) \geq MSE(T_{MM6,RSS})$

It is easily seen that the Mean squared error of the suggested estimators given in (3.1),(3.2),(3.3), (4.1), (4.2), (4.3), (4.4), (4.5), (4.18), (4.19), (4.20), (4.21), (4.22) and (4.23) are always smaller than the estimator given in (1.5) to (1.18) respectively, because $A_1, A_2, A_3, A_4, A_5, A_6$ $A_7, A_8, A_9, A_{10}, A_{11}, A_{12}, A_{13}$ and A_{14} all are non-negative values.

4. NUMERICAL EXAMPLE

In this section we consider population from Cochran (1977) at page no.325, for accessing the performance of the proposed estimators. The computed values of constants and parameters of this population are given below-

$$N = 10,\ n = 3, \bar{Y} = 101.1,\quad \bar{X} = 58.8, S_x = 7.9414,\quad S_y = 15.4448,\quad C_x = 0.135,$$
$$C_y = 0.153, \rho = 0.652, \beta_1(x) = 0.236, \beta_2(x) = 2.239, Q_1 = 53, Q_3 = 61.5,\ Q_r = 8.5,$$
$$Q_d = 4.25 \text{ and } Q_a = 57.25.$$

From the above population, we took ranked set sample with size $m = 3$ and number of cycles, $r = 1$ so that $n(= mr) = 3$. To consider the performance of the various ratio and product estimators, we have computed estimated MSE and Percent relative efficiency (PRE) of all proposed estimators $\bar{y}_{MM1,RSS}$, $\bar{y}_{MM2,RSS}$, $\bar{y}_{MM3,RSS}$, $\bar{y}_{MM4,RSS}$, $\bar{y}_{MM5,RSS}$, $\bar{y}_{MM6,RSS}$, $\bar{y}_{MM7,RSS}$, $\bar{y}_{MM8,RSS}$, $T_{MM1,RSS}$, $T_{MM2,RSS}$, $T_{MM3,RSS}$, $T_{MM4,RSS}$, $T_{MM5,RSS}$ and $T_{MM6,RSS}$ in RSS with respect to usual estimators $\bar{y}_{MR3}$, $\bar{y}_{MR4}$, $\bar{y}_{MR5}$, $\bar{y}_{MP1}$, $\bar{y}_{MP2}$, $\bar{y}_{MP3}$, $\bar{y}_{MP4}$, $\bar{y}_{MP5}$, T_{SP1}, T_{SP2}, T_{SP3}, T_{SP4}, T_{SP5} and T_{SP6} in SRS are given in table1. The percent relative efficiency of $\bar{y}_{MR3}$ with respect to $\bar{y}_{MM1,RSS}$ can be computed as

$$PRE(\bar{y}_{MM1,RSS}, \bar{y}_{MR3}) = \frac{MSE(\bar{y}_{MR3})}{MSE(\bar{y}_{MM1,RSS})} \times 100\%$$

Similarly, the percent relative efficiencies of various estimators are presented in Table 1.

TABLE 1: MSE's and PRE's of all estimators using RSS and SRS

Simple random sampling		Ranked set sampling		Percent relative efficiency(PRE)
Estimators	**MSE**	**Proposed Estimators**	**MSE**	
$\bar{y}_{MR3}$	46.87884	$\bar{y}_{MM1,RSS}$	46.55235	100.7013
$\bar{y}_{MR4}$	48.07085	$\bar{y}_{MM2,RSS}$	47.70061	100.7762
$\bar{y}_{MR5}$	49.14288	$\bar{y}_{MM3,RSS}$	49.04426	100.2011
$\bar{y}_{MP1}$	145.1207	$\bar{y}_{MM4,RSS}$	144.9647	100.1076
$\bar{y}_{MP2}$	139.3824	$\bar{y}_{MM5,RSS}$	139.2458	100.0981
$\bar{y}_{MP3}$	207.2155	$\bar{y}_{MM6,RSS}$	206.8132	100.1945
$\bar{y}_{MP4}$	219.2165	$\bar{y}_{MM7,RSS}$	218.759	100.2091
$\bar{y}_{MP5}$	142.1297	$\bar{y}_{MM8,RSS}$	141.9839	100.1027
T_{SP1}	160.4744	$T_{MM1,RSS}$	160.263	100.1319
T_{SP2}	186.3092	$T_{MM1,RSS}$	185.9957	100.1685
T_{SP3}	99.97937	$T_{MM1,RSS}$	99.95131	100.0281
T_{SP4}	220.6297	$T_{MM1,RSS}$	220.1681	100.2097
T_{SP5}	226.7846	$T_{MM1,RSS}$	226.2952	100.2163
T_{SP6}	173.7758	$T_{MM1,RSS}$	173.513	100.1515

From table1, we conclude that the Mean squared error of the suggested estimators is always smaller than the usual estimators. As a result, show that all the suggested new ratio and product estimators $\bar{y}_{MM1,RSS}$, $\bar{y}_{MM2,RSS}$, $\bar{y}_{MM3,RSS}$, $\bar{y}_{MM4,RSS}$, $\bar{y}_{MM5,RSS}$, $\bar{y}_{MM6,RSS}$, $\bar{y}_{MM7,RSS}$, $\bar{y}_{MM8,RSS}$, $T_{MM1,RSS}$, $T_{MM2,RSS}$, $T_{MM3,RSS}$, $T_{MM4,RSS}$, $T_{MM5,RSS}$ and $T_{MM6,RSS}$ for the population mean using RSS are more efficient than the usual ratio product type estimators $\bar{y}_{MR3}$, $\bar{y}_{MR4}$, $\bar{y}_{MR5}$, $\bar{y}_{MP1}$, $\bar{y}_{MP2}$, $\bar{y}_{MP3}$, $\bar{y}_{MP4}$, $\bar{y}_{MP5}$, T_{SP1}, T_{SP2}, T_{SP3}, T_{SP4}, T_{SP5} and T_{SP6}.

Thus, if inter quartile range, semi inter quartile range and semi inter quartile average ,Coefficient of Skewness and Coefficient of kurtosis are known of auxiliary variable x, these proposed estimators are recommended for use in practice.

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