MODIFIED RATIO CUM PRODUCT TYPE EXPONENTIAL ESTIMATOR OF POPULATION MEAN IN STRATIFIED RANDOM SAMPLING

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ABSTRACT
In this article, we proposed a novel dual to ratio cum product type exponential estimator in stratified random sampling. The bias and mean square error up to the first degree of approximation of the proposed estimator have been obtained. The proposed estimator has been compared with the unbiased estimator in stratified random sampling, combined ratio and product estimators, dual to combined ratio, and product estimators. Our results showed a great improvement in terms of relative efficiency. Also, the results are supported by empirical studies.

KEYWORDS: Ratio and Product Estimator, Population Mean, Bias, Mean Square Error, Efficiency.

MSC: 62D05

RESUMEN
En este artículo proponemos un estimador novedoso dual para el estimador acumulativo del tipo producto exponencial en el muestreo estratificado. El sesgo y el error cuadrático medio son obtenidos desarrollando en series hasta el primer grado de aproximación del estimador propuesto. Ese es comparado con los estimadores: insesgado en el muestreo estratificado, razón combinada, producto, dual de la razón combinada y del producto. Nuestros resultados mostraron una gran incremento en términos de la relativa eficiencia. También los resultados son soportado empíricamente.

PALABRA CLAVE: Estimador de Razón y Producto, Media Poblacional, Sesgo, Error Cuadrático Medio, , Eficiencia.

1. INTRODUCTION:

Bahl and Tuteja (1991) visualize ratio and product type exponential estimators using an exponential function. These estimators were studied in stratified random sampling by Singh et al. (2008): Singh et al. (2009) visualize ratio-cum-product type exponential estimator for population mean. Using Srivenkataramana (1980) transformation on auxiliary variables Tailor et al. (2013) obtained dual to Singh et al. (2008) Exponential ratio type estimators in stratified random sampling. Tailor et al. (2012) Dual to ratio and product type exponential estimators of finite population mean. Tailor and Chouhan (2013) proposed ratio-cum-product type exponential estimator of the population mean in stratified random sampling. Lone et al. (2014a) Generalized Ratio-Cum-Product Type Exponential Estimator in Stratified Random Sampling. Tailor et al. Improved ratio-cum-product type exponential estimators for a ratio of two population means in sample surveys. Lakhkar Khan & Javid Shabbir (2017) Generalized exponential type ratio-cum ratio estimators of population mean in a ranked set and stratified ranked set sampling. Hatice Oncel Cekim & Cem Kadilar (2018) New families of unbiased estimators in stratified random sampling. Tolga Zaman & Cem Kadilar (2020) On estimating the population mean using an auxiliary character in stratified random sampling.
Consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of N distinct and identifiable units, which is divided into L strata of sizes $N_h(h=1,2,\ldots,L)$: Let Y is a study variable and x and z be two auxiliary variables taking values y_{hi} , x_{hi}, and z_{hi} respectively where $i=1,2,3,\ldots N_h$ the auxiliary variate is correlated with z is assumed to

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be negatively correlated with the study variable y. To estimate the population mean with some desirable properties based on a stratified random sample. Then we define

$\bar{Y}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi}=\frac{1}{N}\sum_{h=1}^{L} N_h\ \bar{Y}_h=\sum_{h=1}^{L} W_h\bar{Y}_h$: Population mean of the study variate y.

$\bar{X}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi}=\sum_{h=1}^{L} W_h\bar{X}_h$: Population mean of the auxiliary variate x.

$\bar{Z}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} z_{hi}=\sum_{h=1}^{L} W_h\bar{z}_h$: Population mean of the auxiliary variate z.

Our problem is to estimate population mean of $\bar{Y}$ of the study variate y using stratified random sampling Similarly, unbiased estimators of population mean $\bar{X}$ of the auxiliary variate x and population mean $\bar{Z}$ of the auxiliary variate z are defined respectively as:

$\bar{x}_{st}=\sum_{h=1}^{L} W_h\bar{x}_h,\ \bar{z}_{st}=\sum_{h=1}^{L} W_h\bar{z}_h$

Where

$$\bar{x}_h=\frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}\ ,\ \bar{z}_h=\frac{1}{n_h}\sum_{i=1}^{n_h} z_{hi}$$

are the means of the sample taken from h^{th} stratum for the auxiliary variate x and z respectively.

2. ESTIMATORS IN LITERATURE:

In this part, we consider some exisiting estimators of the finite population mean in the literature. The variance and MSE's of all the estimators computed are derived using a first order of approximation.

Hansen et al. (1946) envisaged a combined ratio estimator for the population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_{RC}=\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right) \qquad (2.1)$$

In the presence of a negative correlation between the study variate y and the auxiliary variate z , is preferred the combined product estimator defined as

$$\hat{\bar{Y}}_{PC}=\bar{y}_{st}\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) \qquad (2.2)$$

Variance of unbiased estimators $\bar{y}_{st},\bar{x}_{st},\bar{z}_{st}$ are respectively given as

$$V(\bar{y}_{st})=\sum_{h=1}^{L} W_h^2\gamma_h S_{yh}^2 \qquad (2.3)$$

$$V(\bar{x}_{st})=\sum_{h=1}^{L} W_h^2\gamma_h S_{xh}^2 \qquad (2.4)$$

and

$$V(\bar{z}_{st})=\sum_{h=1}^{L} W_h^2\gamma_h S_{zh}^2 \qquad (2.5)$$

The biases and mean squared errors of the combined ratio estimator $\hat{\bar{Y}}_{RC}$ and the combined product estimator $\hat{\bar{Y}}_{PC}$ are

$$B\left(\hat{\bar{Y}}_{RC}\right)=\frac{1}{\bar{X}}\sum_{h=1}^{L} W_h^2\gamma_h(R_1S_{xh}^2-S_{yxh}) \qquad (2.6)$$

$$B\left(\widehat{\overline{Y}}_{PC}\right) = \frac{1}{\bar{Z}}\sum_{h=1}^{L} W_h^2\gamma_h(R_2^2 S_{zh}^2 + S_{yzh})$$

$$MSE\left(\widehat{\overline{Y}}_{RC}\right) = \sum_{h=1}^{L} W_h^2\gamma_h(S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}) \quad (2.8)$$

$$MSE\left(\widehat{\overline{Y}}_{PC}\right) = \sum_{h=1}^{L} W_h^2\gamma_h(S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh}) \quad (2.9)$$

Where $R_1 = \frac{\bar{Y}}{\bar{X}}$ and $R_2 = \frac{\bar{Y}}{\bar{Z}}$.

Bahl and Tuteja (1991) developed ratio and product type exponential estimators for population mean $\bar{Y}$ in simple random sampling as

$$\widehat{\overline{Y}}_{Re} = \bar{y}\, exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right), \quad (2.10)$$

And

$$\widehat{\overline{Y}}_{Pe} = \bar{y}\, exp\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}}\right), \quad (2.11)$$

Singh et al. (2008) studied Bahl and Tuteja (1991) estimators in stratified random sampling and propose

$$\widehat{\overline{Y}}_{Re}^{ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{X}_h - \bar{x}_h)}{\sum_{h=1}^{L} W_h(\bar{X}_h + \bar{x}_h)}\right] \quad (2.12)$$

And

$$\widehat{\overline{Y}}_{Pe}^{ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{z}_h - \bar{Z}_h)}{\sum_{h=1}^{L} W_h(\bar{z}_h + \bar{Z}_h)}\right] \quad (2.13)$$

Tailor et al. (2013) used Srivenkataramana (1980) transformation and obtained dual to Singh et al. (2008) ratio and product type exponential estimators $\widehat{\overline{Y}}_{Re}^{ST}$, and $\widehat{\overline{Y}}_{Pe}^{ST}$ as

$$\widehat{\overline{Y}}_{Re}^{*ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{x}^*_h - \bar{X}_h)}{\sum_{h=1}^{L} W_h(\bar{x}^*_h + \bar{X}_h)}\right] \quad (2.14)$$

And

$$\widehat{\overline{Y}}_{Pe}^{*ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{Z}_h - \bar{z}^*_h)}{\sum_{h=1}^{L} W_h(\bar{Z}_h + \bar{z}^*_h)}\right] \quad (2.15)$$

Where $\bar{x}^*_h = \frac{\bar{X}_h N_h - \bar{x}_h n_h}{N_h - n_h}$, $\bar{z}^*_h = \frac{\bar{Z}_h N_h - \bar{z}_h n_h}{N_h - n_h}$ are unbiased estimators of population mean $\bar{X}$ and $\bar{Z}$ respectively

Singh (1967) utilized information on the population mean of two auxiliary variates i.e. $\bar{X}$ and $\bar{Z}$ and suggested a ratio-cum-product estimator for population mean $\bar{Y}$ in simple random sampling as

$$\hat{\hat{\bar{Y}}}_{RP} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{z}}{\bar{Z}}\right) \tag{2.16}$$

Tailor et al. (2012) re-defined Singh (1967) estimator $\hat{\hat{\bar{Y}}}_{RP}$ in stratified random sampling and proposed

$$\hat{\hat{\bar{Y}}}_{RP}^{ST} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) \tag{2.17}$$

Singh et al. (2009) defined the ratio-cum-product type exponential estimator in simple random sampling

$$\hat{\hat{\bar{Y}}}_{RPe} = \bar{y}\, exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) exp\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}}\right) \tag{2.18}$$

Tailor and Chouhan (2013) studied $\hat{\hat{\bar{Y}}}_{RPe}$ in stratified random sampling and suggested

$$\hat{\hat{\bar{Y}}}_{RPe}^{ST} = \bar{y}_{st}\, exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right) exp\left(\frac{\bar{z}_{st} - \bar{Z}}{\bar{z}_{st} + \bar{Z}}\right)\ , \tag{2.19}$$

Or

$$\hat{\hat{\bar{Y}}}_{Re}^{ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{X}_h - \bar{x}_h)}{\sum_{h=1}^{L} W_h(\bar{X}_h + \bar{x}_h)}\right] exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{z}_h - \bar{Z}_h)}{\sum_{h=1}^{L} W_h(\bar{z}_h + \bar{Z}_h)}\right] \tag{2.20}$$

3. THE SUGGESTED ESTIMATOR

Using the transformation $x_i^* = \frac{N\bar{X} - n\bar{x}}{N-n}$, Srivenkataramana (1980) obtained the dual to classical ratio estimator

$$\hat{\hat{\bar{Y}}}_{R}^{*} = \bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right) \tag{3.1}$$

Using the same transformation i.e. $\bar{x}^*{}_h = \frac{N_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}$ and $\bar{z}^*{}_h = \frac{N_h\bar{Z}_h - n_h\bar{z}_h}{N_h - n_h}$ on auxiliary variates x and z a dual to Tailor and Chouhan (2013) suggested estimators are

$$\hat{\hat{\bar{Y}}}_{RPe}^{*ST} = \left(\sum_{h=1}^{L} W_h\bar{y}_h\right) exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{X}_h - \bar{x}^*{}_h)}{\sum_{h=1}^{L} W_h(\bar{X}_h + \bar{x}^*{}_h)}\right] exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{z}^*{}_h - \bar{Z}_h)}{\sum_{h=1}^{L} W_h(\bar{z}^*{}_h + \bar{Z}_h)}\right]$$

$$\hat{\hat{\bar{Y}}}_{RPe}^{*ST} = \left(\sum_{h=1}^{L} W_h\bar{y}_h\right) exp\left[\frac{\sum_{h=1}^{L} W_h\bar{X}_h - \sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\bar{X}_h + \sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}\right)}\right]$$

$$exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h - n_h\bar{z}_h}{N_h - n_h}\right) - \sum_{h=1}^{L} W_h\bar{Z}_h}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h - n_h\bar{z}_h}{N_h - n_h}\right) + \sum_{h=1}^{L} W_h\bar{Z}_h}\right]$$

$$= \left(\sum_{h=1}^{L} W_h\bar{y}_h\right) exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h + n_h\bar{x}_h - N_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h - n_h\bar{x}_h + N_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}\right)}\right]$$

$$exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h + n_h\bar{Z}_h - N_h\bar{Z}_h - n_h\bar{z}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h - n_h\bar{Z}_h + N_h\bar{Z}_h - n_h\bar{z}_h}{N_h - n_h}\right)}\right]$$

$$= \left(\sum_{h=1}^{L} W_h \bar{y}_h\right) exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{n_h\bar{x}_h - n_h\bar{X}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h - n_h\bar{x}_h + N_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}\right)}\right]$$

$$exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{n_h\bar{Z}_h - n_h\bar{z}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h - n_h\bar{x}_h + N_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}\right)}\right]$$

To obtain the bias and mean squared error of the suggested estimator $\hat{\hat{Y}}_{RPe}^{*ST}$, we write $\bar{y}_h = \bar{Y}_h(1+\varepsilon_{0h}), \bar{x}_h = \bar{X}_h(1+\varepsilon_{1h}), \bar{z}_h = \bar{Z}_h(1+\varepsilon_{2h})$, such that

$$E(\varepsilon_{0h}) = E(\varepsilon_{1h}) = E(\varepsilon_{2h}) = 0,\ E(\varepsilon_{0h}^2) = \lambda_h C_{yh}^2, E(\varepsilon_{1h}^2) = \lambda_h C_{xh,}^2 E(\varepsilon_{2h}^2) = \lambda_h C_{zh}^2,$$

$$E(\varepsilon_{0h}\varepsilon_{1h}) = \lambda_h \rho_{yxh} C_{yh} C_{xh\,,}\ E(\varepsilon_{0h}\varepsilon_{2h}) = \lambda_h \rho_{yzh} C_{yh} C_{zh\,,} \text{and } E(\varepsilon_{1h}\varepsilon_{2h}) = \lambda_h \rho_{xzh} C_{xh} C_{zh\,,}$$

Now the suggested dual to ratio-cum-product type exponential estimators in terms of $e_i's$ are expressed as

$$\hat{\hat{Y}}_{RPe}^{*ST} = \left(\sum_{h=1}^{L} W_h \bar{Y}_h\,(1+\varepsilon_{0h})\right) exp\frac{\sum_{h=1}^{L} W_h\gamma_h(\bar{X}_h(1+\varepsilon_{1h}) - \bar{X}_h))}{\sum_{h=1}^{L} W_h\left(\frac{2N_h\bar{X}_h - n_h(\bar{X}_h + \bar{X}_h(1+\varepsilon_{1h}))}{N_h - n_h}\right)}$$

$$exp\frac{\sum_{h=1}^{L} W_h n_h\left(\frac{\bar{Z}_h - \bar{Z}_h(1+\varepsilon_{2h})}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{2N_h\bar{X}_h - n_h(\bar{X}_h + \bar{X}_h(1+\varepsilon_{1h}))}{N_h - n_h}\right)}$$

$$= \bar{Y}\left(1 + \frac{\sum_{h=1}^{L} W_h\bar{Y}_h\,\varepsilon_{0h}}{\bar{Y}}\right) exp\left[\frac{\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\varepsilon_{1h}}{2\bar{X} - \sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\varepsilon_{1h}}\right] exp\left[\frac{-\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\varepsilon_{2h}}{2\bar{Z} - \sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\varepsilon_{2h}}\right]$$

$$= \bar{Y}\left(1 + \frac{\sum_{h=1}^{L} W_h\bar{Y}_h\,\varepsilon_{0h}}{\bar{Y}}\right) exp\left[\frac{\frac{\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\varepsilon_{1h}}{\bar{X}}}{2 - \frac{\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\varepsilon_{1h}}{\bar{X}}}\right] exp\left[\frac{-\frac{\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\varepsilon_{2h}}{\bar{Z}}}{2 - \frac{\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\varepsilon_{2h}}{\bar{Z}}}\right]$$

The proposed estimator $\hat{\hat{Y}}_S$ can be written in terms of $e_i's$ as:

$$\hat{\hat{Y}}_{RPe}^{*ST} = \bar{Y}(1+\varepsilon_0) exp\left(\frac{\varepsilon_1}{2-\varepsilon_1}\right) exp\left(\frac{-\varepsilon_2}{2-\varepsilon_2}\right)$$

Where,

$$\gamma_h = \frac{n_h}{N_h - n_h},\ \varepsilon_0 = \frac{1}{\bar{Y}}\sum_{h=1}^{L} W_h\gamma_h\bar{Y}_h\,\varepsilon_{0h}\ ,\ \varepsilon_1 = \frac{1}{\bar{X}}\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\,\varepsilon_{1h}\ , \varepsilon_2 = \frac{1}{\bar{Z}}\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\,\varepsilon_{2h}$$

Such that $E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0$ and

$$E(\varepsilon_0^2) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L}\lambda_h W_h S_{yh}^2,,\ E(\varepsilon_1^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L}\lambda_h W_h S_{xh}^2,\ E(\varepsilon_2^2) = \frac{1}{\bar{Z}^2}\sum_{h=1}^{L}\lambda_h W_h S_{zh}^2,$$

$$E(\varepsilon_0\varepsilon_1) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L}\lambda_h W_h S_{yxh\,,},\ E(\varepsilon_0\varepsilon_2) = \frac{1}{\bar{Y}\bar{Z}}\sum_{h=1}^{L}\lambda_h W_h S_{yzh\,,},\ E(\varepsilon_1\varepsilon_2) = \frac{1}{\bar{Z}\bar{X}}\sum_{h=1}^{L}\lambda_h W_h S_{xzh\,,}$$

Then are correct the approximations

$$\hat{\hat{Y}}_{Pro}^{*ST} \cong \bar{Y}(1+\varepsilon_0)\left(1 + \frac{\varepsilon_1}{2} + \frac{\varepsilon_1^2}{4} + \frac{\varepsilon_1^2}{8}\right)\left(1 - \frac{\varepsilon_2}{2} - \frac{\varepsilon_2^2}{4} + \frac{\varepsilon_2^2}{8}\right)$$

$$\hat{\hat{Y}}_{Pro}^{*ST} \cong \bar{Y}\left(1 - \frac{\varepsilon_1}{2} - \frac{\varepsilon_1^2}{8} + \frac{\varepsilon_2}{2} - \frac{\varepsilon_1\varepsilon_2}{2} + \frac{3\varepsilon_2^2}{8} + \varepsilon_0 - \frac{\varepsilon_0\varepsilon_1}{2} + \frac{\varepsilon_0\varepsilon_2}{2}\right)$$

$$\hat{\hat{Y}}_{Proe}^{*ST} - \bar{Y} \cong \bar{Y}\left(\varepsilon_0 - \frac{\varepsilon_1}{2} + \frac{\varepsilon_2}{2} - \frac{\varepsilon_1^2}{8} + \frac{3\varepsilon_2^2}{8} - \frac{\varepsilon_1\varepsilon_2}{2} - \frac{\varepsilon_0\varepsilon_1}{2} + \frac{\varepsilon_0\varepsilon_2}{2}\right) \qquad (3.2)$$

Up to the first-order approximation, the bias and MSE of the suggested estimator are obtained as

$$B\left(\hat{\bar{Y}}_{Pro}^{*ST}\right) \cong \bar{Y}\sum_{h=1}^{L} {W_h}^2\lambda_h\left[-\frac{1}{8}R_1S_{xh}^2+\frac{3}{8}\gamma_h^2S_{xh}^2-\frac{1}{2}\gamma_hS_{yxh}+\frac{1}{2}\gamma_hS_{yzh}+\frac{1}{2}\frac{\gamma_hS_{xzh}}{\bar{Z}}\right] \quad (3.3)$$

And

$$MSE\left(\hat{\bar{Y}}_{Pro}^{*ST}\right) \cong \sum_{h=1}^{L} {W_h}^2\lambda_h\left[S_{yh}^2+\frac{\gamma_h^2}{4}\begin{pmatrix}(R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1R_2S_{xzh}) \\ -\gamma_h(R_1S_{yxh}+R_2S_{yzh})\end{pmatrix}\right] \quad (3.4)$$

4. EFFICIENCY COMPARISONS:

In this section, the efficiency of the proposed estimator is studied. It is compared with the usual unbiased estimator, combined ratio and product estimators and dual to combined ratio and product estimators , Singh et al. (2008) ratio , product type exponential estimators, dual to Singh et al. (2008) estimators given by Tailor et al. (2013) and Tailor and Chouhan (2013) estimator.

Note that

$$V(\bar{y}_{st}) = \sum_{h=1}^{L} W_h^2\gamma_hS_{yh}^2 \quad (4.1)$$

$$MSE\left(\hat{\bar{Y}}_{RC}\right) = \sum_{h=1}^{L} W_h^2\lambda_h(S_{yh}^2+R_1^2S_{xh}^2-2R_1S_{yxh}) \quad (4.2)$$

$$MSE\left(\hat{\bar{Y}}_{PC}\right) = \sum_{h=1}^{L} W_h^2\lambda_h(S_{yh}^2+R_2^2S_{zh}^2+2R_2S_{yzh}) \quad (4.3)$$

$$MSE(\hat{\bar{Y}}_{RC}^{*}) = \sum_{h=1}^{L} W_h^2\lambda_h(S_{yh}^2+R_1^2\gamma_h^2S_{xh}^2-2R_1\gamma_hS_{yxh}) \quad (4.4)$$

$$MSE(\hat{\bar{Y}}_{PC}^{*}) = \sum_{h=1}^{L} W_h^2\lambda_h(S_{yh}^2+R_2^2\gamma_h^2S_{zh}^2+2R_2\gamma_hS_{yzh}) \quad 4.5)$$

$$MSE\left(\hat{\bar{Y}}_{Re}^{ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h(S_{yh}^2+\frac{1}{4}R_1^2S_{xh}^2-R_1S_{yxh}) \quad (4.6)$$

$$MSE\left(\hat{\bar{Y}}_{Pe}^{ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h(S_{yh}^2+\frac{1}{4}R_2^2S_{zh}^2+R_2S_{yzh}) \quad (4.7)$$

$$MSE\left(\hat{\bar{Y}}_{Re}^{*ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h(S_{yh}^2+\frac{1}{4}R_1^2\gamma_h^2S_{xh}^2-R_1\gamma_hS_{yxh}) \quad (4.8)$$

$$MSE\left(\hat{\bar{Y}}_{Pe}^{*ST}\right) = \sum_{h=1}^{L} W_h^2 \lambda_h (S_{yh}^2 + \frac{1}{4} R_2^2 \gamma_h^2 S_{zh}^2 - R_2 \gamma_h S_{yzh}) \quad (4.9)$$

$$MSE\left(\hat{\bar{Y}}_{RPe}^{ST}\right) = \sum_{h=1}^{L} {W_h}^2 \lambda_h \left[S_{yh}^2 + \frac{1}{4}\begin{pmatrix}(R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2R_1R_2 S_{xzh}) \\ -\left(R_1 S_{yxh} + R_2 S_{yzh}\right)\end{pmatrix}\right] \quad (4.10)$$

The comparisons showed that the proposed dual to ratio cum product type exponential estimator of the population mean $\hat{\bar{Y}}_{Pro}^{*ST}$ would have a smaller MSE than

(i) $V(\bar{y}_{st})$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\frac{\gamma_h^2}{4}\begin{pmatrix}(R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2R_1R_2 S_{xzh}) \\ -\gamma_h\left(R_1 S_{yxh} + R_2 S_{yzh}\right)\end{pmatrix}\right] < 0 \quad (4.11)$$

(ii) $\hat{\bar{Y}}_{RC}$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\begin{matrix}\frac{1}{4} R_1^2 S_{xh}^2 (\gamma_h^2 - 4) + \frac{\gamma_h^2}{4}(R_2^2 S_{zh}^2 - 2R_1R_2 S_{xzh}) \\ -R_1 S_{yxh}(\gamma_h - 2) + \gamma_h R_2 S_{yzh}\end{matrix}\right] < 0 \quad (4.12)$$

(iii) $\hat{\bar{Y}}_{PC}$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\begin{matrix}\frac{1}{4} R_2^2 S_{zh}^2 (\gamma_h^2 - 4) + \frac{\gamma_h^2}{4}(R_1^2 S_{xh}^2 - 2R_1R_2 S_{xzh}) \\ -R_2 S_{yzh}(\gamma_h - 2) + \gamma_h R_1 S_{yxh}\end{matrix}\right] < 0 \quad (4.13)$$

(iv) $\hat{\bar{Y}}_{RC}^{*}$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\begin{matrix}\frac{\gamma_h^2}{4}(-3R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2R_1R_2 S_{xzh}) \\ +\gamma_h\left(R_1 S_{yxh} + R_2 S_{yzh}\right)\end{matrix}\right] < 0 \quad (4.14)$$

(v) $\hat{\bar{Y}}_{PC}^{*}$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\begin{matrix}\frac{\gamma_h^2}{4}(R_1^2 S_{xh}^2 - 3S_{zh}^2 - 2R_1R_2 S_{xzh}) \\ -\gamma_h\left(R_1 S_{yxh} + R_2 S_{yzh}\right)\end{matrix}\right] < 0 \quad (4.15)$$

(vi) $\hat{\bar{Y}}_{Re}^{ST}$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\begin{matrix}\frac{1}{4} R_1^2 S_{xh}^2 (\gamma_h^2 - 1) + \frac{\gamma_h^2}{4}(R_2^2 S_{zh}^2 - 2R_1R_2 S_{xzh}) \\ -R_1 S_{yxh}(\gamma_h - 2) + \gamma_h R_2 S_{yzh}\end{matrix}\right] < 0 \quad (4.16)$$

(vii) $\hat{\bar{Y}}_{Pe}^{ST}$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\begin{matrix}\frac{1}{4} R_2^2 S_{zh}^2 (\gamma_h^2 - 1) + \frac{\gamma_h^2}{4}(R_1^2 S_{xh}^2 - 2R_1R_2 S_{xzh}) \\ +R_2 S_{yzh}(\gamma_h - 2) + \gamma_{hR_1 S_{yxh}}\end{matrix}\right] < 0 \quad (4.17)$$

(viii) $\hat{\bar{Y}}_{Re}^{*ST}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h \left[\frac{\gamma_h^2}{4}(R_2^2 S_{zh}^2 - 2R_1R_2S_{xzh}) - \gamma_h R_2 S_{yzh}\right] < 0 \quad (4.18)$$

(ix) $\widehat{\bar{Y}}_{Pe}^{*ST}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h \left[\frac{\gamma_h^2}{4}(R_1^2 S_{xh}^2 - 2R_1R_2S_{xzh}) - \gamma_h R_1 S_{yxh}\right] < 0 \quad (4.19)$$

(x) $\widehat{\bar{Y}}_{PRe}^{*ST}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h \left[\frac{1}{4}(R_1^2 S_{xh}^2 - 2R_1R_2S_{xzh})(\gamma_h^2 - 1) - (\gamma_h - 1)\left(R_1S_{yxh} - R_2S_{yzh}\right)\right] < 0 \quad (4.20)$$

5. EMPIRICAL STUDIES

5.1. Study data sets in the related literature

A well-known data set used in the literature of survey sampling for evaluating the performance of procedures. It is used for illustrating how large is the efficiency of our proposal. A description of the populations is given below.

Population [Source: Murthy (1967), p. 228]

y : Output

x: Fixed capital

z: Number of workers

Table 5.1:. Murty`s Data

Constant	Stratum I	Stratum II
N_h	05	05
n_h	2	2
$\bar{Y}_h$	1925	3115.60
$\bar{X}_h$	214.40	333.80
$\bar{Z}_h$	51.80	60.60
S_{yh}	615.92	340.38
S_{xh}	74.87	66.35
S_{zh}	0.75	4.84
S_{yxh}	39360.68	22356.50
S_{yzh}	-411.20	-1536.00
S_{xzh}	38.08	287.92

The value of the needed parameters are given in table 5.2.

Table 5.2: Mean square errors **,** Percent relative efficiencies of $V(\bar{y}_{st})$, $\widehat{\bar{Y}}_{RC}$, $\widehat{\bar{Y}}_{PC}$ $\widehat{\bar{Y}}_{RC}^{*}$, $\widehat{\bar{Y}}_{PC}^{*}$, $\widehat{\bar{Y}}_{Re}^{ST}$, $\widehat{\bar{Y}}_{Pe}^{ST}$, $\widehat{\bar{Y}}_{Re}^{*ST}$ $\widehat{\bar{Y}}_{Pe}^{*ST}$ and $\widehat{\bar{Y}}_{PRe}^{*ST}$ for $\widehat{\bar{Y}}_{Pro}^{*ST}$

Estimators	MSE	PRE's
$V(\bar{y}_{st})$	74282.40	100
$\widehat{\bar{Y}}_{RC}$	23675.66	313.75
$\widehat{\bar{Y}}_{PC}$	63863.01	116.32
$\widehat{\bar{Y}}_{RC}^{*}$	17193.81	432.03
$\widehat{\bar{Y}}_{PC}^{*}$	60031.03	123.74
$\widehat{\bar{Y}}_{Re}^{ST}$	42705.76	173.94
$\widehat{\bar{Y}}_{Pe}^{ST}$	68818.23	107.94
$\widehat{\bar{Y}}_{Re}^{*ST}$	31614.91	234.96

$\hat{\bar{Y}}_{Pe}^{*ST}$	66353.19	111.95
$\hat{\bar{Y}}_{PRe}^{*ST}$	30336.68	244.86
$\hat{\bar{Y}}_{Pro}^{*ST}$	16341.60	454.56

Note that the proposal outperforms in more than a 107% the best of its competitors

5.2. Study of the effectiveness of acupuncture intervention in the recovering of athletes

Commonly to make decisions on the effectiveness of a treatment is to first nominate subjectively the worthwhile of clinical treatment effects. The measurement processes include in the consideration of patients' perceptions on both the benefits and costs of the treatment. In traditional Chinese medicine theory, needling or acupuncture could activate the meridian and create a "de-qi" sensation. Acupuncture has been proven effective in various musculoskeletal disorders including muscle soreness, which accelerated the recovery of the symptoms of exercise-induced , see Chang et al., (2019). A study on the effectiveness of acupuncture intervention after intense exercise for alleviating the effect of extreme exercises. It was developed with a population of amateurs athletes.

Delayed-onset muscle soreness (DOMS) is mainly caused by over training conveying to exhaustive and/or unaccustomed muscle work. The athletes manifest muscle swelling, weakness and loss of motion range. They lead to negative interference with their daily activities as well as with the athletic performance or increasing risk of injuries. See more elements in Lewis et al. (2012). DOMS commonly occurs within the initial 24 hours after strenuous or extreme exercises and reaches a peak between 24 and 72 hours. A question is if an acupuncture based treatment has a positive effect on relieving DOMS . In this case

$y(24)$: True value of DOM at 24 hours.

x: Report of DOM of DOM at 24 hours

z: Report of DOM by patients at 72 hours

$y(72)$: True value of DOM at 72 hours

Another effect of strenuous or extreme exercises is producing edemas. Muscle soreness was measured using the visual analogue scale (VAS). The atlhets marked their level of agreement to a statement on the relief after applying acupuntre along a continuous line. The researchers used thermal imaging (TI) to detect subtle changes in the temperature of the skin above the exercised muscles. Oedemas was obtained by measuring arm circumference at seven locations, computing the total , and then taking the difference of the summed circumference of affected and unaffected arms. A positive result indicates that the affected arm had a larger circumference than the unaffected arm. See a detailed discussion on this problem in Muller et al. (2019). In this case:

y: IT result after two weeks .

x: VAS initial result

z: VAS result after a week.

The study was developed with 285 amateurs athletes for providing up-to-date evidence for evaluating the relief due to using acupuncture. . The athletes were stratified considering the level of strenuous or extreme exercises in 4 groups. A description is given in the next table

Table 5.3: Data of the Strata of athletes

Constant	Stratum I	Stratum II	Stratum III	Stratum IV
N_h	72	86	90	37
n_h	7	9	9	4

The results of the Percent relative efficiencies is given in table 5.4.

Table 5.4: Percent relative efficiencies obtained in the experiment

Estimators	Percent	Relative	Efficiencies
	$y(24)$: True value of DOM at 24 hours.	$y(72)$: True value of DOM at 72 hours	y: result after two weeks
$V(\bar{y}_{st})$	100	100	100
$\hat{\hat{\bar{Y}}}_{RC}$	150,9	125,6	153,8
$\hat{\hat{\bar{Y}}}_{PC}$	104,6	102,4	109,7
$\hat{\hat{\bar{Y}}}^{*}_{RC}$	376,9	297,4	274,2
$\hat{\hat{\bar{Y}}}^{*}_{PC}$	117,0	117,9	118,1
$\hat{\hat{\bar{Y}}}^{ST}_{Re}$	114,7	116,2	119,6
$\hat{\hat{\bar{Y}}}^{ST}_{Pe}$	110,1	121,7	116,4
$\hat{\hat{\bar{Y}}}^{*ST}_{Re}$	281,1	213,3	281,2
$\hat{\hat{\bar{Y}}}^{*ST}_{Pe}$	128,2	120,6	124,2
$\hat{\hat{\bar{Y}}}^{*ST}_{PRe}$	216,4	181,5	179,2
$\hat{\hat{\bar{Y}}}^{*ST}_{Pro}$	428,1	380,9	282,7

See that $\hat{\hat{\bar{Y}}}_{PC}$ is the best competitor in all the alternatives but was outperformed for more than a 102%. $\hat{\hat{\bar{Y}}}^{*ST}_{Pro}$ is the worst competitor and the PRE of the proposed estimator is larger than 282%.

6. CONCLUSION

A Modified Ratio Cum Product Type Exponential Estimator of Population mean for Stratified Random Sampling is proposed. It has been seen that the proposed estimator performed better than the existing estimator theoretically and illustrate this fact empirically. It has been seen that the proposed estimator has maximum PRE's in comparison to other estimators.

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