GENERALIZED REGRESSION-CUM-EXPONENTIAL MEAN ESTIMATOR USING CONVENTIONAL AND NON-CONVENTIONAL AUXILIARY PARAMETERS

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ABSTRACT
The present research article emphasizes on proposing an improved generalized regression-cum-exponential (GRE) estimator to achieve better efficiency for estimating the mean. The proposed GRE estimator is based on optimal use of the available known conventional and non-conventional parameters of the auxiliary variable such as coefficient of skewness, coefficient of kurtosis, median, quartile deviation, Downton's scale method and probability weighted moments. The expressions of bias and mean square error of the proposed estimator are obtained under large sample approximation to study their properties. The optimal condition for obtaining the minimum mean square error of the proposed estimators is determined up to the first order of approximation. Theoretical as well as empirical comparisons have elaborately been presented to exhibit the efficiency of suggested estimators over the conventional and other promising relevant estimators. The performances of suggested GRE estimators over the well-known discussed contemporary estimators in the text have also been confirmed through a simulation study.

KEYWORDS: Mean; exponential; bias; mean square error; auxiliary parameters.

MSC: 62D05

RESUMEN
Este articulo de investigación enfatiza la proposición de un estimadores mejorado generalizado del tipo regresión-cum-exponencial (GRE) para obtener más eficiencia en la estimación. El $GRE -$ estimador se basa en el uso optimal de conocidos parámetros de la variable auxiliar conocidos o no , tales como los coeficientes de deformación ,apuntamiento , mediana, desviación cuartílico , el método de escala Downton y momentos ponderados de la probabilidad. Las expresiones del sesgo y el error cuadrático medio del propuesto estimador son obtenidos bajo la aproximación para muestras grandes. La óptima condición para obtener el mínimum del error cuadrático medio es determinada. Comparaciones teóricas y empíricas han sido elaboradas y presentadas para exhibir la eficiencia de los sugeridos estimadores sobre los convencionales y otros promisorios y relevantes. El comportamiento de los sugeridos GRE -estimadores sobre otros bien conocidos contemporáneamente, en el texto han sido también confirmados a través de los estudios simulación.

PALABRASCLAVE: Media; exponencial; sesgo; el error cuadrático medio ; parámetros auxiliares.

1. INTRODUCTION AND LITERATURE REVIEW

Sample surveys are always very useful in many fields like engineering, agriculture, medical science, industry etc. and enhancing the efficiency of the estimators to estimate the unknown parameters is a big challenge to the researchers. Cochran (1940) took a new step in this order to enhance the efficiency of the usual mean estimator by suggesting ratio estimator with the help of positively correlated auxiliary variable. On the contrary, Robson (1957) proposed product estimator for the negatively correlated auxiliary variable. Then using the theory of introducing auxiliary character(s) for efficiency enhancement, various types of estimators like regression, generalized, chain etc. are developed [see review of Tripathi et al. (1994), Bouza et al. (2013) and Swain (2013)]. Exponential function has been firstly used by Bahl and Tuteja (1991) to propose exponential ratio and product type estimators for estimation of population mean using the auxiliary information. Research endeavors of Grover and Kaur (2011), Shabbir and Gupta (2011), Solanki and Singh (2013), Bouza et al. (2017), Singh et al. (2014), Ekpenyong and Enang (2015), Kadilar (2016), Sinha and Kumar (2017), Prasad (2020), Unal and Kadilar (2021), Zaman and Kadilar

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(2021) and Sinha and Bharti (2021, 2022a, 2022b) can be considered as notable contributions in estimation of mean under different sampling techniques.

Let (y, x) be the study and auxiliary variables. Let us consider a population $P = (P_1, P_2, \ldots, P_N)$ having N units and take a sample of n units through SRSWOR (simple random sampling without replacement) from this population for estimation of population mean. Let y_i, x_i denote the i^{th} units of study and auxiliary variables. Let $\bar{Y}$ and $\bar{X}$ denote the population mean of study and auxiliary variables whereas $\bar{y}\left(=\frac{1}{n}\sum_{i=1}^{n} y_i\right)$ and $\bar{x}\left(=\frac{1}{n}\sum_{i=1}^{n} x_i\right)$ are the sample means of study and auxiliary variables.

Some of the conventional estimators taken from the literature with their variance (V) or mean square error (M) up to the first order of approximation are given in the following table.

S. No.	Estimators	Mean square errors
(i)	Usual mean per unit estimator $\bar{y}$	$V(\bar{y}) = \lambda \bar{Y}^2 C_y^2$
(ii)	Ratio estimator $\bar{y}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$	$M(\bar{y}_R) = \lambda \bar{Y}^2 \left(C_y^2 + C_x^2 - 2\rho_{xy} C_y C_x\right)$
(iii)	Product estimator $\bar{y}_P = \bar{y}\frac{\bar{x}}{\bar{X}}$	$M(\bar{y}_P) = \lambda \bar{Y}^2 \left(C_y^2 + C_x^2 + 2\rho_{xy} C_y C_x\right)$
(iv)	Regression estimator $\bar{y}_{Reg} = \bar{y} + \beta(\bar{X} - \bar{x})$	$\left[M\left(\bar{y}_{Reg}\right)\right]_{min} = \lambda \bar{Y}^2 C_y^2 \left(1 - \rho_{xy}^2\right)$ at $\beta_{opt} = \rho_{xy}\frac{S_y}{S_x}$
(v)	Generalized estimator $\bar{y}_G = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}$, where α is an unknown constant.	$[M(\bar{y}_G)]_{min} = \lambda \bar{Y}^2 C_y^2 \left(1 - \rho_{xy}^2\right)$ at $\alpha_{opt} = -\rho_{xy}\frac{C_y}{C_x}$

Bahl and Tuteja (1991) proposed ratio and product type exponential estimators using the exponential function to estimate the population mean by means of the known population mean of the auxiliary variable $(\bar{X})$. Further, Shabbir and Gupta (2011), Grover and Kaur (2011), Kadilar (2016) suggested different modified exponential estimators for estimation of population mean following the strategy of Bahl and Tuteja (1991), which are given by

S. No.	Estimators	Mean square errors
(vi)	Bahl and Tuteja Ratio Estimator $(\bar{y}_{BT})_R = \bar{y}\ exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	$[M(\bar{y}_{BT})_R] = \lambda \bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} - \rho_{xy} C_y C_x\right)$
(vii)	Bahl and Tuteja Product Estimator $(\bar{y}_{BT})_P = \bar{y}\ exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right)$	$[M(\bar{y}_{BT})_P] = \lambda \bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} + \rho_{xy} C_y C_x\right)$
(viii)	Shabbir and Gupta Estimator $\bar{y}_{SG} = [k_1\bar{y} + k_2(\bar{X} - \bar{x})]exp\left[\frac{\bar{\mathcal{A}}-\bar{a}}{\bar{\mathcal{A}}+\bar{a}}\right]$ where $\bar{\mathcal{A}} = \bar{X} + N\bar{X}$ and $\bar{a} = \bar{x} + N\bar{X}$	$[M(\bar{y}_{SG})]_{min} = \bar{Y}^2 \left\{1 - \frac{\lambda C_x^2}{4(1+N)^2} - \frac{\left(1-\frac{\lambda C_x^2}{8(1+N)^2}\right)^2}{1+\lambda C_y^2(1-\rho_{xy}^2)}\right\}$ for $(k_1)_{opt} = \frac{1-\frac{\lambda C_x^2}{8(1+N)^2}}{1+\lambda C_y^2(1-\rho_{xy}^2)}$ and $(k_2)_{opt} = \frac{\bar{Y}}{\bar{X}}\left[\frac{1}{2(1+N)} - k_1\left\{\frac{1}{1+N} - \rho_{xy}\frac{C_y}{C_x}\right\}\right]$.
(ix)	Grover and Kaur Estimator $\bar{y}_{GK} = [k_1\bar{y} + k_2(\bar{X} - \bar{x})]exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	$[M(\bar{y}_{GK})]_{min} = \frac{\lambda \bar{Y}^2 C_y^2(1-\rho_{xy}^2)}{1+\lambda C_y^2(1-\rho_{xy}^2)} - \frac{\lambda^2 \bar{Y}^2 C_x^2\left\{4C_y^2(1-\rho_{xy}^2)+\frac{C_x^2}{4}\right\}}{16\{1+\lambda C_y^2(1-\rho_{xy}^2)\}}$ for $(k_1)_{opt} = \frac{1-\frac{\lambda C_x^2}{8}}{1+\lambda C_y^2(1-\rho_{xy}^2)}$ and $(k_2)_{opt} = \frac{\bar{Y}\left\{-C_x+\frac{\lambda C_x^3}{4}-\frac{\lambda\rho_{xy}C_y C_x^2}{4}+2\rho_{xy}C_y+\lambda C_x C_y^2\right\}}{2S_x\{1+\lambda C_y^2(1-\rho_{xy}^2)\}}$.
(x)	Kadilar Estimator $\bar{y}_K = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}\ exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	$[M(\bar{y}_K)]_{min} = \lambda \bar{Y}^2 C_y^2 \left(1 - \rho_{xy}^2\right)$ for $\alpha_{opt} = \frac{C_x - 2\rho_{xy}C_y}{2C_x}$.

On the contrary, using the non-conventional measure of auxiliary variable, Subzar et al. (2016) and Subzar et al. (2017) have proposed some modified ratio type estimators using population deciles, median, correlation coefficient and coefficient of variation of the auxiliary variable. Abid et al. (2016) studied the application of non-conventional location parameters to suggest enhanced mean ratio estimators for estimating population mean.

Following Subzar et al. (2018), Yadav et al. (2021) suggested a new family of estimators using known non-conventional parameters of auxiliary variable along with some conventional one's as

$$\bar{y}_{Yi} = \bar{y}_{Reg}\left[\alpha\left(\frac{\bar{X}a_i + b_i}{\bar{x}a_i + b_i}\right) + (1-\alpha)\left(\frac{a_i(\bar{X}-\bar{x})}{a_i(\bar{X}+\bar{x})+2b_i}\right)\right] \quad (1)$$

where α is a characterizing constant used for minimizing the MSE of $\bar{y}_{Yi}$. a_i and b_i are the parameters of the secondary variate that include the coefficient of skewness, kurtosis, correlation and variation and other conventional and non-conventional parameters of the auxiliary variable.

Yadav et al. (2021) shown in his literature that the estimators proposed by Yadav and Zaman (2021) converts into a particular member of $\bar{y}_{Yi}$ for $\alpha = 1$ while estimators suggested by Subzar et al. (2016), Abid et al. (2016) and Subzar et al. (2017) come to be the particular member of $\bar{y}_{Yi}$ for $\alpha = 0$ under specific values of a_i and b_i [see Yadav et al. (2021) for the considered parameters/constants of a_i and b_i].

Yadav et al. (2021) also shown that the different estimators proposed by Abid et al. (2016), Subzar et al. (2017) and Subzar et al. (2018) can be expressed by a family of estimators as

$$\bar{y}_{fi} = \bar{y}_{Reg}\left(\frac{\bar{X}a_i + b_i}{\bar{x}a_i + b_i}\right) \quad (2)$$

and in the similar way, the estimators suggested by Singh and Yadav (2020) can be generalized as

$$\bar{y}_{gi} = \bar{y}_{Reg}\left(\frac{a_i(\bar{X}-\bar{x})}{a_i(\bar{X}+\bar{x})+2b_i}\right) \quad (3)$$

The mean square errors of $\bar{y}_{fi}$, $\bar{y}_{gi}$ and $\bar{y}_{Yi}$ up to the first order of approximation are respectively given by

$$M(\bar{y}_{fi}) = \lambda\bar{Y}^2\left[R_i^2 C_x^2 + C_y^2(1-\rho_{xy}^2)\right], \quad (4)$$

$$M(\bar{y}_{gi}) = \lambda\bar{Y}^2\left[R_i^2 \frac{C_x^2}{4} + C_y^2(1-\rho_{xy}^2)\right], \quad (5)$$

and $$[M(\bar{y}_{Yi})]_{min} = \lambda\bar{Y}^2\left[C_y^2(1-\rho_{xy}^2)\right], \quad (6)$$

where $R_i = \frac{\bar{Y}a_i}{\bar{X}a_i + b_i}$.

After comparing the mean square errors of $\bar{y}_{fi}$, $\bar{y}_{gi}$ and $\bar{y}_{Yi}$, a remarkable observation is equal minimum mean square error of regression and the estimator suggested by Yadav et al. (2021).

Many times in sample survey, auxiliary variable is not normally distributed especially when it covers large area of investigation with moderate number of units. As a result, the different available known parameters of the auxiliary variable like coefficient of skewness and kurtosis, median, quartile deviation, Downton's scale method and probability weighted moments can be utilized to improve the efficiency of the estimate. In this manuscript, an effort has been made to suggest an improved Generalized Regression-cum-Exponential (GRE) estimator for acquiring better efficiency of the estimate using conventional and non-conventional parameters of auxiliary variate and their properties have been elaborated. In this manuscript an attempt has been made to suggest an improved Generalized Regression-cum-Exponential (GRE) estimator to achieve better efficiency of estimation using conventional and non-traditional parameters of auxiliary variable and its properties are studied.

2. SUGGESTED ESTIMATORS

Inspired by the application of conventional and non-conventional auxiliary parameters, the goal of this manuscript is to develop various improved regression-cum-exponential estimators, which give efficient estimate and do not belong to the Yadav et al. (2021) family of estimators.

The proposed improved regression-cum-exponential estimators are as follows:

$$(\bar{y}_{sugg})_1 = [A\bar{y} + B(\bar{X}-\bar{x})]\, exp\left[\frac{(\beta_1\bar{X}-M_d)-(\beta_1\bar{x}-M_d)}{(\beta_1\bar{X}-M_d)+(\beta_1\bar{x}-M_d)}\right],$$

$$(\bar{y}_{sugg})_2 = [A\bar{y} + B(\bar{X}-\bar{x})]\, exp\left[\frac{(\beta_1\bar{X}-Q.D.)-(\beta_1\bar{x}-Q.D.)}{(\beta_1\bar{X}-Q.D.)+(\beta_1\bar{x}-Q.D.)}\right],$$

$$(\bar{y}_{sugg})_3 = [A\bar{y} + B(\bar{X}-\bar{x})]\, exp\left[\frac{(\beta_1\bar{X}-D)-(\beta_1\bar{x}-D)}{(\beta_1\bar{X}-D)+(\beta_1\bar{x}-D)}\right],$$

$$(\bar{y}_{sugg})_4 = [A\bar{y} + B(\bar{X}-\bar{x})]\, exp\left[\frac{(\beta_1\bar{X}-S_{pw})-(\beta_1\bar{x}-S_{pw})}{(\beta_1\bar{X}-S_{pw})+(\beta_1\bar{x}-S_{pw})}\right],$$

$$(\bar{y}_{sugg})_5 = [A\bar{y} + B(\bar{X}-\bar{x})]\, exp\left[\frac{(\beta_2\bar{X}-M_d)-(\beta_2\bar{x}-M_d)}{(\beta_2\bar{X}-M_d)+(\beta_2\bar{x}-M_d)}\right],$$

$$\left(\bar{y}_{sugg}\right)_6 = [A\bar{y} + B(\bar{X} - \bar{x})]\, exp\left[\frac{(\beta_2\bar{X} - Q.D.) - (\beta_2\bar{x} - Q.D.)}{(\beta_2\bar{X} - Q.D.) + (\beta_2\bar{x} - Q.D.)}\right],$$

$$\left(\bar{y}_{sugg}\right)_7 = [A\bar{y} + B(\bar{X} - \bar{x})]\, exp\left[\frac{(\beta_2\bar{X} - D) - (\beta_2\bar{x} - D)}{(\beta_2\bar{X} - D) + (\beta_2\bar{x} - D)}\right],$$

and $$\left(\bar{y}_{sugg}\right)_8 = [A\bar{y} + B(\bar{X} - \bar{x})]\, exp\left[\frac{(\beta_2\bar{X} - S_{pw}) - (\beta_2\bar{x} - S_{pw})}{(\beta_2\bar{X} - S_{pw}) + (\beta_2\bar{x} - S_{pw})}\right].$$

Here, A and B are the unknown constants used for optimizing the error terms. The different parameters involved in the suggested estimators are as follows-

β_1 =Coefficient of skewness of auxiliary variable x,

β_2 =Coefficient of kurtosis of auxiliary variable x,

M_d =Median of auxiliary variable x,

$Q.D.$ (Quartile deviation of auxiliary variable) $= \frac{Q_3 - Q_1}{2}$,

where Q_1 =First quartile and Q_3 =Third quartile.

D =Downton's method $= \frac{2\sqrt{\pi}}{N(N-1)}\sum_{i=1}^{N}\left(i - \frac{N+1}{2}\right)x_{(i)}$

and S_{pw} =Probability weighted moments $= \frac{\sqrt{\pi}}{N^2}\sum_{i=1}^{N}(2i - N - 1)x_{(i)}$.

In order to study the properties of the suggested estimators, we have combined all the suggested estimators $\left(\bar{y}_{sugg}\right)_i; i = 1,2,\dots,8$ and an improved generalized regression-cum-exponential (GRE) estimator is proposed as

$$\left(\bar{y}_{sugg}\right)_i = [A\bar{y} + B(\bar{X} - \bar{x})]\, exp\left[\frac{(a_i\bar{X} - b_i) - (a_i\bar{x} - b_i)}{(a_i\bar{X} - b_i) + (a_i\bar{x} - b_i)}\right] \quad ; i = 1,2,\dots 8 \tag{7}$$

where a_i and b_i are the known population parameters of auxiliary variable $(\beta_1, \beta_2, M_d, Q.D., D, S_{pw})$.

To calculate the bias and mean square error of suggested generalized exponential estimator, following assumptions are considered

$$\frac{\bar{y} - \bar{Y}}{\bar{Y}} = \varepsilon_0, \qquad \frac{\bar{x} - \bar{X}}{\bar{X}} = \varepsilon_1 \tag{8}$$

such that $E(\varepsilon_0) = E(\varepsilon_1) = 0$, $E(\varepsilon_0^2) = \lambda C_y^2$, $E(\varepsilon_1^2) = \lambda C_x^2$ and $E(\varepsilon_0\varepsilon_1) = \lambda\rho_{xy}C_yC_x$. (9)

where $\lambda = \frac{1}{n} - \frac{1}{N}$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$, $C_x^2 = \frac{S_x^2}{\bar{X}^2}$ and ρ_{xy} is the correlation coefficient between y and x.

Simplifying the estimator given in equation (7) under the assumptions mentioned in (8), the suggested generalized exponential estimator can be reduced in

$$\left(\bar{y}_{sugg}\right)_i = A\bar{Y} - A\bar{Y}\Theta_i\varepsilon_1 + \frac{3}{2}A\bar{Y}\Theta_i^2\varepsilon_1^2 + A\bar{Y}\varepsilon_0 - A\bar{Y}\Theta_i\varepsilon_0\varepsilon_1 - B\bar{X}\varepsilon_1 + B\bar{X}\Theta_i\varepsilon_1^2, \tag{10}$$

where $\Theta_i = \frac{a_i\bar{X}}{2(a_i\bar{X} - b_i)}$.

Taking the expectation on both sides of equation (10) and using the results given in equation (9), we get

$$E\left(\bar{y}_{sugg}\right)_i = A\bar{Y} + \frac{3}{2}\lambda A\bar{Y}\Theta_i^2C_x^2 - \lambda A\bar{Y}\Theta_i\rho_{xy}C_yC_x + \lambda B\bar{X}\Theta_iC_x^2. \tag{11}$$

Subtracting $\bar{Y}$ from both sides of equation (11), we get the bias of suggested generalized exponential estimator as

$$B\left(\bar{y}_{sugg}\right)_i = (A - 1)\bar{Y} + \frac{3}{2}\lambda A\bar{Y}\Theta_i^2C_x^2 - \lambda A\bar{Y}\Theta_i\rho_{xy}C_yC_x + \lambda B\bar{X}\Theta_iC_x^2. \tag{12}$$

From equation (10), the mean square error of the estimator $\left(\bar{y}_{sugg}\right)_i$ up to the first order of approximation $[\mathcal{O}\{n^{-1}\}]$ is given by

$$M\left(\bar{y}_{sugg}\right)_i = E\left\{\left(\bar{y}_{sugg}\right)_i - \bar{Y}\right\}^2$$

or
$$\begin{aligned} M\left(\bar{y}_{sugg}\right)_i = {} & (A - 1)^2\bar{Y}^2 + 4\lambda A^2\bar{Y}^2\Theta_i^2C_x^2 + \lambda A^2\bar{Y}^2C_y^2 + \lambda B^2\bar{X}^2C_x^2 - 3\lambda A\bar{Y}^2\Theta_i^2C_x^2 \\ & -4\lambda A^2\bar{Y}^2\Theta_i\rho_{xy}C_yC_x + 2\lambda A\bar{Y}^2\Theta_i\rho_{xy}C_yC_x + 4\lambda AB\bar{X}\bar{Y}\Theta_iC_x^2 - 2\lambda B\bar{X}\bar{Y}\Theta_iC_x^2 \\ & -2\lambda AB\bar{X}\bar{Y}\rho_{xy}C_yC_x. \end{aligned} \tag{13}$$

Partially differentiate equation (13) with respect to A and B and equating them to zero, we get the optimum value of constants as

$$A_{opt} = \frac{2 - \lambda\Theta_i^2C_x^2}{2\{1 + \lambda C_y^2(1 - \rho_{xy}^2)\}} \tag{14}$$

and
$$B_{opt} = \frac{\bar{Y}\{2\rho_{xy}C_y - 2\Theta_iC_x + 2\lambda\Theta_i^3C_x^3 - \lambda\Theta_i^2\rho_{xy}C_yC_x^2 + 2\lambda\Theta_iC_y^2C_x(1 - \rho_{xy}^2)\}}{2\bar{X}C_x\{1 + \lambda C_y^2(1 - \rho_{xy}^2)\}} \tag{15}$$

Substituting these optimum values of constants in equation (13), the minimum mean square error of $\left(\bar{y}_{sugg}\right)_i$ is given by

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} = \frac{\bar{Y}^2\{4\lambda C_y^2(1-\rho_{xy}^2)-\lambda^2\Theta_i^4 C_x^4-4\lambda^2\Theta_i^2 C_y^2 C_x^2(1-\rho_{xy}^2)\}}{4\{1+\lambda C_y^2(1-\rho_{xy}^2)\}} \tag{16}$$

Remark: Sometimes, the optimum value of constants involve the population parameters which may or may not be known in advance, so in case of unavailability of required parameters, one may use their estimates at their places or use their values from the prior data without any loss of efficiency up to the first degree of approximation [see Koyuncu and Kadilar (2009)].

3. EFFICIENCY COMPARISONS

In order to prove that the suggested GRE estimators are efficient than the relevant estimators, their mean square errors are compared and conditions are derived for the recommendation of these estimators over the relevant existing estimators.

(a) From (16) and (i)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le V(\bar{y}), \quad \text{if} \quad \Theta_i^2 \ge 2\frac{-G \pm C_y\sqrt{C_y^2(1-\rho_{xy}^2)^2-\left(G+\frac{\rho_{xy}^2}{\lambda}\right)}}{C_x^2}.$$

(b) From (16) and (ii)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le M\left(\bar{y}_R\right) \quad \text{if} \quad \Theta_i^2 \ge 2\frac{-G \pm \sqrt{G^2-\left[\frac{\left(\rho_{xy}C_y-C_x\right)^2}{\lambda}+G\left(C_y^2+C_x^2-2\rho_{xy}C_yC_x\right)\right]}}{C_x^2}.$$

(c) From (16) and (iii)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le M\left(\bar{y}_P\right) \quad \text{if} \quad \Theta_i^2 \ge 2\frac{-G \pm \sqrt{G^2-\left[\frac{\left(\rho_{xy}C_y+C_x\right)^2}{\lambda}+G\left(C_y^2+C_x^2+2\rho_{xy}C_yC_x\right)\right]}}{C_x^2}.$$

(d) From (16) and (iv)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le \left[M\left(\bar{y}_{Reg}\right)\right]_{min} \quad \text{if} \quad (\Theta_i^2 C_x^2+2G)^2 \ge 0, \text{ which is always true.}$$

(e) From (16) and (v)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le \left[M\left(\bar{y}_G\right)\right]_{min} \quad \text{if} \quad (\Theta_i^2 C_x^2+2G)^2 \ge 0, \text{ which is always true.}$$

(f) From (16) and (vi)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le M\left(\bar{y}_{BT}\right)_R \quad \text{if} \quad \Theta_i^2 \ge 2\frac{-G \pm \sqrt{G^2-\left[\frac{\left(\rho_{xy}C_y-\frac{C_x}{2}\right)^2}{\lambda}+G\left(C_y^2+\frac{C_x^2}{4}-\rho_{xy}C_yC_x\right)\right]}}{C_x^2}.$$

(g) From (16) and (vii)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le M\left(\bar{y}_{BT}\right)_P \quad \text{if} \quad \Theta_i^2 \ge 2\frac{-G \pm \sqrt{G^2-\left[\frac{\left(\rho_{xy}C_y+\frac{C_x}{2}\right)^2}{\lambda}+G\left(C_y^2+\frac{C_x^2}{4}+\rho_{xy}C_yC_x\right)\right]}}{C_x^2}.$$

(h) From (16) and (viii)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le \left[M\left(\bar{y}_{SG}\right)\right]_{min} \quad \text{if} \quad \Theta_i^2 \ge \frac{1}{4(1+N)^2}.$$

(i) From (16) and (ix)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le \left[M\left(\bar{y}_{GK}\right)\right]_{min} \quad \text{if} \quad \Theta_i^2 \ge \frac{1}{4}.$$

(j) From (16) and (x)

$$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \le \left[M\left(\bar{y}_K\right)\right]_{min} \quad \text{if} \quad (\Theta_i^2 C_x^2+2G)^2 \ge 0, \text{ which is always true.}$$

(k) From (16) and (4)

$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \leq M\left(\bar{y}_{fi}\right)$if $\quad (\Theta_i^2 C_x^2 + 2G)^2 + 4R_i^2 C_x^2 \left(1 + \frac{G}{\lambda}\right) \geq 0$, which is always true.

(l) From (16) and (5)
$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \leq M\left(\bar{y}_{gi}\right)$ if $\quad (\Theta_i^2 C_x^2 + 2G)^2 + R_i^2 C_x^2 \left(1 + \frac{G}{\lambda}\right) \geq 0$, which is always true.

(m) From (16) and (6)
$\left[M\left(\bar{y}_{sugg}\right)_i\right]_{min} \leq [M(\bar{y}_{Yi})]_{min}$ if $\quad (\Theta_i^2 C_x^2 + 2G)^2 \geq 0$, which is always true.
where $\quad G = C_y^2\left(1 - \rho_{xy}^2\right)$.

4. EMPIRICAL WORK

To illustrate and validate the theoretical outcomes and comparisons with all the competing estimators, an empirical work has been carried out with real data sets. However, it has been theoretically proven that the proposed GRE estimators are always more efficient than the recently suggested estimators of Yadav et al. (2021), but to further elaborate the findings some relevant members of $\bar{y}_{fi}$, $\bar{y}_{gi}$ and $\bar{y}_{Yi}$ are considered by assigning the different values of a_i and b_i in (2), (3) and (1).

***Population I*:**
{source Murthy (1967, Pg. 228)**}** Here, we consider the data on output (y) and the number of workers (x) of 80 factories in a region. The different parameters for this population are as follows:

$$N = 80 \quad \bar{Y} = 5182.6375 \quad \bar{X} = 285.1250 \quad S_y = 1835.65852 \quad S_x = 270.42945$$
$$n = 20 \quad M_d = 148 \quad Q.D. = 179.375 \quad D = 247.824 \quad S_{pw} = 244.726$$
$$\rho = 0.915 \quad \beta_1 = 1.628 \quad \beta_2 = 3.581$$

Population II:
{source Khare and Sinha (2012)**}** Here, we consider the data on number of cultivators (y) and Total population (x) of 109 of villages of Baria (Urban) Police station, Champua Tahsil, District-Orissa, India from Census Handbook of Orissa, 1981 published by Government of India. The different parameters for this population are as follows:

$$N = 109 \quad \bar{Y} = 100.55 \quad \bar{X} = 255.97 \quad S_y = 73.54357 \quad S_x = 155.25044$$
$$n = 30 \quad M_d = 220.55 \quad Q.D. = 96.62 \quad D = -16.1156 \quad S_{pw} = -15.9673$$
$$\rho = 0.717 \quad \beta_1 = 2.1627 \quad \beta_2 = 5.8483$$

Using the theoretical results discussed in the previous section, the mean square error (MSE) and percent relative efficiency (PRE) of the estimators are calculated and given in Table 1 and 2 for population I and II respectively. The PRE of suggested GRE $\left(\bar{y}_{sugg}\right)_{(.)}$ and relevant existing $(\bar{y}_{exis})$ estimators with respect to usual mean per unit estimator $(\bar{y})$ has been calculated by

$$PRE = \frac{V(\bar{y})}{MSE(\bar{y}_{exis/sugg})} \times 100.$$

Table 1: MSE, PRE and optimum value of constants involved in the estimators (for population I)

Estimator	MSE	PRE	Optimum value of Constants
$\bar{y}$	126362	100%	
$\bar{y}_R$	413231	30.6%	
$\bar{y}_P$	1654670	7.6%	
$\bar{y}_{Reg}$, $\bar{y}_{Yi}$	20568.5	614.3%	$\beta_{opt} = 6.211$
$\bar{y}_G$	20568.5	614.3%	$\alpha_{opt} = -0.342$
$(\bar{y}_{BT})_R$	43274.4	292%	
$(\bar{y}_{BT})_P$	662493	19.1%	
$\bar{y}_{SG}$	20552.7	614.8%	$(k_1)_{opt} = 0.999,\ (k_2)_{opt} = 6.094$
$\bar{y}_{GK}$	19902.2	634.9%	$(k_1)_{opt} = 0.995,\ (k_2)_{opt} = -2.818$
$\bar{y}_K$	20568.5	614.3%	$\alpha_{opt} = 0.158$
$\bar{y}_{f11}$ $\quad a_{11} = 1,\ b_{11} = \beta_1$	916397	13.7%	$R_{11} = 18.074$
$\bar{y}_{f13}$ $\quad a_{13} = 1,\ b_{13} = M_d$	413226	30.6%	$R_{13} = 11.966$
$\bar{y}_{f16}$ $\quad a_{16} = \beta_2,\ b_{16} = M_d$	711757	17.8%	$R_{16} = 15.876$
$\bar{y}_{f40}$ $\quad a_{40} = 1,\ b_{40} = D$	279909	45.1%	$R_{40} = 9.724$

$\bar{y}_{f43}$	$a_{43} = 1,\ b_{43} = S_{pw}$	282950	44.6%	$R_{43} = 9.781$
$\bar{y}_{g11}$	$a_{11} = 1,\ b_{11} = \beta_1$	244526	51.7%	$R_{11} = 18.074$
$\bar{y}_{g13}$	$a_{13} = 1,\ b_{13} = M_d$	118733	106.42%	$R_{13} = 11.966$
$\bar{y}_{g16}$	$a_{16} = \beta_2,\ b_{16} = M_d$	193366	65.3%	$R_{16} = 15.876$
$\bar{y}_{g40}$	$a_{40} = 1,\ b_{40} = D$	85403.5	148.0%	$R_{40} = 9.724$
$\bar{y}_{g43}$	$a_{43} = 1,\ b_{43} = S_{pw}$	86163.9	146.6%	$R_{43} = 9.781$
$(\bar{y}_{sugg})_1$		17962.9	703.5%	$(A)_{opt} = 0.990,\ (B)_{opt} = -6.929$
$(\bar{y}_{sugg})_2$		16723.4	755.6%	$(A)_{opt} = 0.988,\ (B)_{opt} = -8.323$
$(\bar{y}_{sugg})_3$		9633.01	1311.8%	$(A)_{opt} = 0.980,\ (B)_{opt} = -12.63$
$(\bar{y}_{sugg})_4$		10209.4	1237.7%	$(A)_{opt} = 0.980,\ (B)_{opt} = -12.39$
$(\bar{y}_{sugg})_5$		19421.2	650.6%	$(A)_{opt} = 0.993,\ (B)_{opt} = -4.324$
$(\bar{y}_{sugg})_6$		19268.4	655.8%	$(A)_{opt} = 0.993,\ (B)_{opt} = -4.694$
$(\bar{y}_{sugg})_7$		18887.5	669%	$(A)_{opt} = 0.992,\ (B)_{opt} = -5.484$
$(\bar{y}_{sugg})_8$		18821.2	671.4%	$(A)_{opt} = 0.992,\ (B)_{opt} = -5.606$

Table 2: MSE, PRE and optimum value of constants involved (for population II)

Estimator		MSE	PRE	Optimum value of Constants
$\bar{y}$		130.673	100%	
$\bar{y}_R$		65.1418	200.6%	
$\bar{y}_P$		375.917	34.8%	
$\bar{y}_{Reg},\ \bar{y}_{Yi}$		63.4955	205.8%	$\beta_{opt} = 0.340$
$\bar{y}_G$		63.4955	205.8%	$\alpha_{opt} = -0.865$
$(\bar{y}_{BT})_R$		75.4434	173.2%	
$(\bar{y}_{BT})_P$		230.831	56.6%	
$\bar{y}_{SG}$		63.0992	207.1%	$(k_1)_{opt} = 0.994,\ (k_2)_{opt} = 0.336$
$\bar{y}_{GK}$		62.9466	207.6%	$(k_1)_{opt} = 0.993,\ (k_2)_{opt} = 0.144$
$\bar{y}_K$		63.4955	205.8%	$\alpha_{opt} = -0.365$
$\bar{y}_{f11}$	$a_{11} = 1,\ b_{11} = \beta_1$	151.852	86.0%	$R_{11} = 0.389$
$\bar{y}_{f13}$	$a_{13} = 1,\ b_{13} = M_d$	89.423	146.1%	$R_{13} = 0.211$
$\bar{y}_{f16}$	$a_{16} = \beta_2,\ b_{16} = M_d$	131.757	99.2%	$R_{16} = 0.342$
$\bar{y}_{f40}$	$a_{40} = 1,\ b_{40} = D$	165.832	78.8%	$R_{40} = 0.419$
$\bar{y}_{f43}$	$a_{43} = 1,\ b_{43} = S_{pw}$	165.706	78.8%	$R_{43} = 0.419$
$\bar{y}_{g11}$	$a_{11} = 1,\ b_{11} = \beta_1$	85.585	152.7%	$R_{11} = 0.389$
$\bar{y}_{g13}$	$a_{13} = 1,\ b_{13} = M_d$	69.978	186.7%	$R_{13} = 0.211$
$\bar{y}_{g16}$	$a_{16} = \beta_2,\ b_{16} = M_d$	80.561	162.2%	$R_{16} = 0.342$
$\bar{y}_{g40}$	$a_{40} = 1,\ b_{40} = D$	89.080	146.7%	$R_{40} = 0.419$
$\bar{y}_{g43}$	$a_{43} = 1,\ b_{43} = S_{pw}$	89.048	146.7%	$R_{43} = 0.419$
$(\bar{y}_{sugg})_1$		62.6172	208.7%	$(A)_{opt} = 0.991,\ (B)_{opt} = 0.016$
$(\bar{y}_{sugg})_2$		62.8668	207.9%	$(A)_{opt} = 0.992,\ (B)_{opt} = 0.103$
$(\bar{y}_{sugg})_3$		62.9558	207.6%	$(A)_{opt} = 0.993,\ (B)_{opt} = 0.149$
$(\bar{y}_{sugg})_4$		62.9557	207.6%	$(A)_{opt} = 0.993,\ (B)_{opt} = 0.149$
$(\bar{y}_{sugg})_5$		62.8829	207.8%	$(A)_{opt} = 0.992,\ (B)_{opt} = 0.110$
$(\bar{y}_{sugg})_6$		62.9228	207.7%	$(A)_{opt} = 0.992,\ (B)_{opt} = 0.130$
$(\bar{y}_{sugg})_7$		62.9501	207.6%	$(A)_{opt} = 0.993,\ (B)_{opt} = 0.146$
$(\bar{y}_{sugg})_8$		62.9501	207.6%	$(A)_{opt} = 0.993,\ (B)_{opt} = 0.146$

5. SIMULATION STUDY

To further strengthen the performances of suggested GRE estimators along with other considered estimators over usual unbiased mean estimator $\bar{y}$, a simulation study has been carried out with bi-variate normal population. Though our suggested GRE estimators perform better than all the discussed estimators when auxiliary variable is asymmetrically distributed and highly correlated with study variable. But the objective of this study is to understand the performances of GRE estimators thus bi-variate normal population (Ω) is considered just as an illustration.

The simulation study has been performed under the following steps using R software

(i) A bi-variate normal pseudo population (Ω) of size $N = 150$ units with parameters $(200, 30, 150, 5, 0.8)$ has been randomly generated

(ii) Bi-variate random sample using SRSWOR of size $n = 60$ units has been drawn from this pseudo population (Ω)

(iii) Obtain the values of the estimators

(iv) Replicate the procedure of (ii) and (iii) for $15{,}000$ times.

The $MSE's$ of all the estimators based on $15{,}000$ estimated values are obtained by

$$MSE\left(\bar{y}_{exis/sugg}\right) = \frac{1}{15000}\sum_{i=1}^{15000}\left(\bar{y}_{exis/sugg} - \bar{Y}\right)^2.$$

Table 3: MSE and PRE for simulated data

Estimator	MSE	PRE	Optimum value of Constants
$\bar{y}$	12.85	100%	
$\bar{y}_R$	10.26	125.24%	
$\bar{y}_P$	16.60	77.41%	
$\bar{y}_{Reg}$, $\bar{y}_{Yi}$	8.96	143.42%	$\beta_{opt} = 4.8$
$\bar{y}_G$	9.08	141.52%	$\alpha_{opt} = -3.636$
$(\bar{y}_{BT})_R$	11.41	112.62%	
$(\bar{y}_{BT})_P$	14.58	88.13%	
$\bar{y}_{SG}$	8.92	144.06%	$(k_1)_{opt} = 0.9990$, $(k_2)_{opt} = 4.7950$
$\bar{y}_{GK}$	8.94	143.74%	$(k_1)_{opt} = 0.9999$, $(k_2)_{opt} = 4.1330$
$\bar{y}_K$	9.08	141.52%	$\alpha_{opt} = -3.136$
$\bar{y}_{f11}$ $a_1 = 1$, $b_1 = \beta_1$	10.65	120.66%	$R_{11} = 0.389$
$\bar{y}_{f13}$ $a_1 = 1$, $b_1 = M_d$	9.65	133.16%	$R_{13} = 0.211$
$\bar{y}_{f16}$ $a_1 = \beta_2$, $b_1 = M_d$	10.09	127.35%	$R_{16} = 0.342$
$\bar{y}_{f40}$ $a_1 = 1$, $b_1 = D$	10.64	120.77%	$R_{40} = 0.419$
$\bar{y}_{f43}$ $a_1 = 1$, $b_1 = S_{pw}$	10.64	120.77%	$R_{43} = 0.419$
$\bar{y}_{g11}$ $a_1 = 1$, $b_1 = \beta_1$	9.65	133.16%	$R_{11} = 0.389$
$\bar{y}_{g13}$ $a_1 = 1$, $b_1 = M_d$	9.27	138.62%	$R_{13} = 0.211$
$\bar{y}_{g16}$ $a_1 = \beta_2$, $b_1 = M_d$	9.44	136.12%	$R_{16} = 0.342$
$\bar{y}_{g40}$ $a_1 = 1$, $b_1 = D$	9.65	133.16%	$R_{40} = 0.419$
$\bar{y}_{g43}$ $a_1 = 1$, $b_1 = S_{pw}$	9.65	133.16%	$R_{43} = 0.419$
$\left(\bar{y}_{sugg}\right)_1$	8.93	143.90%	$(A)_{opt} = 0.9999$, $(B)_{opt} = 4.7588$
$\left(\bar{y}_{sugg}\right)_2$	8.94	143.74%	$(A)_{opt} = 0.9999$, $(B)_{opt} = 4.3200$
$\left(\bar{y}_{sugg}\right)_3$	8.94	143.74%	$(A)_{opt} = 0.9999$, $(B)_{opt} = 4.1577$
$\left(\bar{y}_{sugg}\right)_4$	8.94	143.74%	$(A)_{opt} = 0.9999$, $(B)_{opt} = 4.1576$
$\left(\bar{y}_{sugg}\right)_5$	8.96	143.72%	$(A)_{opt} = 0.9999$, $(B)_{opt} = 3.7711$
$\left(\bar{y}_{sugg}\right)_6$	8.94	143.74%	$(A)_{opt} = 0.9999$, $(B)_{opt} = 4.1270$
$\left(\bar{y}_{sugg}\right)_7$	8.94	143.74%	$(A)_{opt} = 0.9999$, $(B)_{opt} = 4.1325$
$\left(\bar{y}_{sugg}\right)_8$	8.94	143.74%	$(A)_{opt} = 0.9999$, $(B)_{opt} = 4.1325$

6. CONCLUSIONS

Analytical study of this empirical work overall leads to a conclusion that all suggested estimators $\left(\bar{y}_{sugg}\right)_i$; $i = 1,2, \ldots 8$ are efficient over conventional and all promising estimators, including the estimators of Abid et al. (2016), Subzar et al. (2016, 17, 18), Yadav and Zaman (2021) and the most recent Yadav et al. (2021). However, the efficiency of the suggested estimators depends upon the different parameters, but the results given in Table 1 and 2 evince an important fact that the clubbed information of highly correlated asymmetrically distributed auxiliary variable increases the efficiency of suggested estimators.

Further the simulation study reveals that all the suggested GRE estimators are more or equally efficient to the existing estimators for bi-variate normally distributed variables and the small differences in the $MSE's$ are because of the replication of samples. So, on the ground of theoretical, empirical and simulation studies, the suggested GRE estimators are recommended over all the significant existing estimators and choice of one of the GRE estimators would depend upon the availability of auxiliary parameters.

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