EFFECT OF HUMAN ERRORS ON AN INVENTORY MODEL UNDER TWO WAREHOUSE ENVIRONMENT

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ABSTRACT
The surety of production of all the goods in the standard inventory model to be of good quality comes out to be an impractical assumption. Though, the observation in the practical environment shows the possibility of occurrence of errors in the controlled batch. The collection of these items is generally done at the time of the process of selection and selling is done in a single batch when the process of selection ends. Consideration of human errors of $Type\ I$ and $Type\ II$ in this model is done when the inspector shows tiredness and ignorance at the time of the process of inspection. The purpose of this article is to carry out the evaluation of the storage value and inventory decision. Thus, in this paper, the effect of human factors upon an inventory model under the two-warehouse environment are bought into consideration assuming that the cost of stock rent warehouse (RW) is greater than own warehouse (OW) because of the better preservation facilities in RW. The validation of results has been done by utilizing numerical based examples. The sensitivity analysis has also been presented.

KEYWORDS: Two warehouse; defective items; shortages; human factors.

MSC:90B05

RESUMEN
La seguranza de la producción de todos los bienes futuros de buena calidad en un estándar modelo de inventario es una importante asunción. Además, la observación en un ambiente práctico muestra la posibilidad de que ocurran errores en el batch controlado. La recolección de estos ítems es generalmente hecho al mismo tiempo que se procede a la selección y venta en un batch cuando el proceso de selección termina. Consideraciones de los humanos errores de $Tipo\ I$ y $Tipo\ II$ son hechos en el modelo cuando el inspector da señales de cansancio e ignorancia cuando se realiza el proceso de inspección. El propósito de este articulo es desarrollar la evaluación del valor de almacenaje y la decisión de inventario. Así que , en este paper, el efecto de humanos factores en el modelo de inventario en un ambiente de "dos-almacenes" son tomados en consideración asumiendo que el costo del stock rentando un almacén (RW) ha madurado y que usar el propio (OW), dadas la mejores facilidades de preservación en RW. La validación de los resultados fue efectuada usando ejemplos numéricos. Un anális de sensibilidad también es presentado.

PALABRAS CLAVE: Dos almacenes, ítems defectuosos; carencias ; humanos factores.

1. INTRODUCTION

In the real world, every industry wants to get more and more profit with the small investment. Besides this, customer's satisfaction is also a big issue. Every business industry tries to attract the customers by giving them good quality of product. Generally, it is assumed that the quality of product is 100% perfect but practically this is not possible certain percentage of items may be imperfect due to some errors in production process. Everyone is trying to improve the quality of the product so that they can get more customers. Practically, it is not possible that all the products are produced perfectly. In the production process, some of the items might be defected, some human errors will be made. More shortages can also be there due to a huge amount of demand. Until now, a basic economic order quantity (EOQ) model is generated by assuming an ideal situation which is not applicable in the reality. Porteus [25] modified the basic inventory model with the effect of defective items. In the inventory model Rosenblatt and Lee [27] assumed that the defective items can be instantaneously reworked at the fixed cost. In this model time assumed as in exponential form. Salameh

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and Jaber [29] developed a EOQ/EPQ model, considering some of the items are not perfect in the nature. These types of items were sold at a discounted price. Further Cárdenas-Barrón [2] corrected the formula of Salameh and Jaber [29].

However, many of the researchers ignored the effect of human error. Human errors can be possible due to any reason like exhaustion, sickness and unawareness. Initially, Raouf et al. [26] suggested that mistakes could also be made by an inspector. Khan et al. [15] established the ordering policy of retailer with human errors. Khan et al. [17] talked about the issue of management in supply chain with human errors and learning in the manufacture. One such model, Mandeep et al. [32] was developed by considering human errors and defective items whereas Jaber et al. [16] investigated two type of human errors for EOQ/EPQ models.

In the past, the space for storing any items was limited but as the number of the customer's increases, the demand increases, as a result more stock is needed to fulfil the demand. Hence, more space is required for the storage of the stock. For this, industries use two kind of storages: one is own warehouse (OW), it has limited capacity and the other one is the rented warehouse (RW), it contains unlimited capacity of the stock. As the result, the holding cost of rent warehouse is always greater than the holding cost of own warehouse. In the last few years, a lot of research has been done in this area. This kind of problem was first proposed by Hartley [10]. After that, Sarma [31] generalized this model by using shipment cost from RW to OW. Das, Maity, and Maiti, [6] proposed a two-warehouse inventory model with supply chain. Hsieh, Dye and Ouyang [11] studied a two warehouses inventory model by optimizing the total cost for deteriorating items.

Sometimes, demand becomes more than the stock. Thus, in this situation the quantity of items demanded by the customers is greater than quantity supplied. In this case, shortages occur. Salameh and Jaber [29] model further extended by Wee [39] by using shortages. There is another extension of Salameh and Jaber [29] model shortages were allowed which was proposed by Eroglu and Ozdemir [7]. They suggested that there exists a quantity of items with good quality which can accomplish existing demand as well as backorders while screening. They concluded increment in the proportion of imperfect items in a lot will lead to decrement in total profit per unit time.

This paper proposed two warehouse inventory model with the concept of human errors of $Type\ I$ and $Type\ II$ and defective items.

Table 1. Influence of different authors

Authors	Defective items	Two warehouses	Shortage	Human error
Sarma (1987)		✓		
Zhang and Gerchak (1990)	✓			
Goswami and Chaudhari (1992)	✓	✓		
Pakkala and Achary	✓	✓		
Wee (1993)				
Bhunia and Maiti (1998)	✓	✓		
Jaber and Bonney (1998)				✓
Salameh and Jaber (2000)	✓			
Cardenas-Barron (2000)	✓			
Goyal and Cardenas-Barron (2002)	✓			
Duffuaa and Khan (2002)				✓
Yang (2004)		✓	✓	
Papachristos and Konstantaras (2006)	✓		✓	
Eroglu and Ozdemir (2007)	✓		✓	
Das, Maity and Maity (2007)		✓		
Jaber and Goyal (2008)			✓	✓
Chung et al. (2009)	✓	✓		
Khan et al. (2011)			✓	✓

Khan et al. (2012)			✓	✓
Jaggi and Mittal (2013)	✓		✓	
Khan et al. (2014)			✓	✓
Zhou et al. (2015)			✓	✓
Panda et al. (2018)		✓	✓	
Mittal et al. (2018)			✓	✓
Jaggi et al. (2018)	✓	✓	✓	
This model	✓	✓	✓	✓

Present paper proposed an inventory model with the perception of human errors of $Type\ I$ and $Type\ II$ containing not so good quality items in two warehouses when shortages happen. The inventory model accounts the impact of defective items into two warehouses on total profit from retailer to supplier.

2. ASSUMPTIONS

1. Human errors are considered in the inspection process.
2. Items which are of perfect quality are considered as imperfect item ($Type\ I\ error$) and imperfect item as perfect ($Type\ II\ error$) during inspection. (Shin et al., 2018)
3. A known, constant, and continuous demand rate is considered.
4. Instantaneous replenishment rate.
5. Inventory to be sold in single batch only.
6. Proportion of defective items ρ follows a uniform distribution$[\alpha, \beta]$ where $[0 \le \alpha \le \beta \le 1]$ (Wee et al., 2007)
7. The shortages are allowed and fully backlogged.

3. NOTATION

μ	lot size units/cycle
ω	fixed space of OW units/cycle
$\mu - \omega$	unlimited space in RW units/cycle
R	demand rate unit / year
j	variable cost/ unit
K	ordering cost / order
ρ	proportion of defective items in μ
α_1	Type I error represented by random variable
α_2	Type II error represented by random variable
$p1$	defective items (in percentage) witnessed by the buyer during screening process
$f(\rho)$	probability density function of ρ
$f(\alpha_1)$	probability density function of α_1
$f(\alpha_2)$	probability density function of α_2
$f(p1)$	probability density function of $p1$
s	selling price of items with good quality
v	selling price of defective items,$v < j$
b	backordering cost per unit
h_r, h_ω	holding cost for the items in RW and OW, respectively, $h_r \ge h_\omega$
d	cost for screening
t_r	screening rate in RW
t_ω	screening rate in OW
t_1	time when all the inventory used up of RW
t_2	time when all the inventory used up of OW
t_3	time period of shortages
B	backorder quantity in units
x	rate of screening
T	length of a cycle
μ^*_{opt}	optimal lot size
$TP(B, \mu)$	total profit per cycle
$TR(B, \mu)$	total sale revenue

$TC(B,\mu)$	total cost

4. MATHEMATICAL MODELLING

At the beginning a lot size of μ units come into the inventory system at time $t = 0$. It is assumed that from this lot size ω units are stored in the own warehouse (OW) and $(\mu - \omega)$ units are stored in the rent warehouse (RW). The rent warehouse has better preserving facilities than own warehouse which implies that the cost of holding the stock in rent warehouse is higher than that of own warehouse. The behavior of the inventory model is illustrated in Figure 1 and Figure 2. Due to certain reasons such as improper transport, low labor skills, low quality of raw material, among others, the production process manufactures some goods that are not of high-quality. Due to this, a screening process must be conducted with the rate of x units per unit time when the complete lot enters the inventory. It is presumed that each lot μ contains an ρ percent of imperfect items. Thus, the lot μ has $p1\mu$ defective items and $(1 - p1)\mu$ non-defective items. After the end of screening phase in RW in the interval $[t_r, t_1]$ all the demand fulfilled from RW at to the inventory level of RW reduces to zero therefore the upcoming demand is now fulfilled from OW till the time t_1. At t_1 the inventory level reduces to zero in OW, too during t_2 to t_3 is the time period in which shortages occur. However, during inspection process, inspector by mistake selects perfect item as imperfect(defective) ($Type\ I$ error) and imperfect item as perfect ($Type\ II$ error). A joint expected total cost including human errors in the supply chain is obtained. Inspection is done with fixed rate. α_1 items are incorrectly categorized as imperfect from the (1 – α) α_1 perfect items, and α_2 items are incorrectly acknowledged as perfect items from the α(1 – α_2) imperfect items. Thus, the total imperfect items as detected by the inspector are α_e and equal to (1 – α) α_1 + α(1 – α_2). α, α_1, α_2 and α_e are assumed random variables.

Thus, the expected value of the total imperfect items is expressed as

$$E[\alpha_e] = (1 - E[\alpha])E[\alpha_1] + E[\alpha](1 - E[\alpha_2]) \tag{1}$$

The expected time $E[T]$ between two shipments is equal to

$$E[T] = (1 - E[\alpha_e])Q / D. \tag{2}$$

About the computation of TP(B, μ)there are two cases to occur.

Case 1: If $t_1 \geq t_\omega$ this case has been explained by Figure 1.

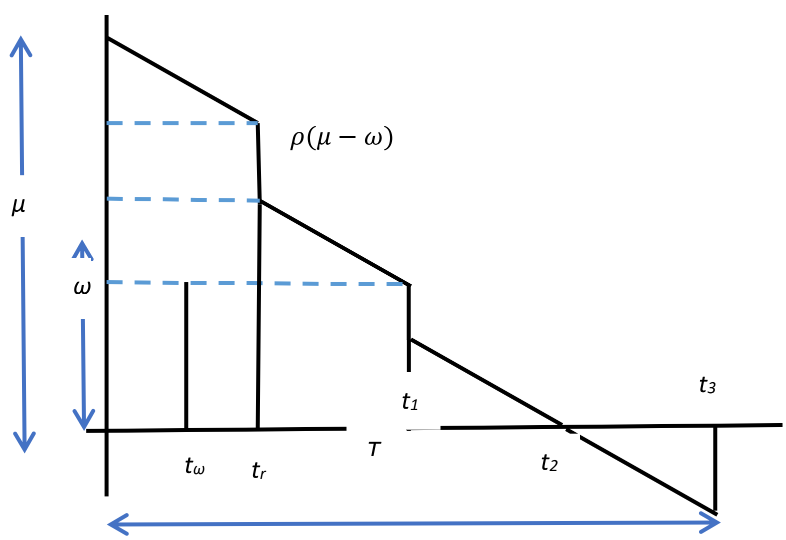


Fig. 1 Case $t_1 \geq t_\omega$

Then, the whole revenue is defined as $TR(\rho,\mu)$ and the total cost is denoted as $TC(B,\mu)$. The $TR(\rho,\mu)$ is the sum of sales of perfect items and the defective items.

$$TR(\rho,\mu) = s\mu(1 - p1) + v\, p1\, \mu \tag{3}$$

And $TC(B,\mu)$ = ordering cost + purchase cost + holding cost + backordering cost + screening cost

$$TC(B,\mu) = K + j\mu + d\mu + h_r + h_\omega\left[\omega t_\omega + \omega(t_1 - t_\omega)(1 - p1) + \frac{(\omega(1 - p1) - B)(t_2 - t_1)}{2}\right] + \frac{Bbt_3}{2} \tag{4}$$

Since
$$t_1 = \frac{(\mu-\omega)(1-p1)}{R},\ t_r = \frac{\mu-\omega}{x},\ t_\omega = \frac{\omega}{x},\ t_2 = \frac{\omega(1-p1)-B}{R},\ t_3 = \frac{B}{R},$$

Then Eq (4) is

$$TC(B,\mu) = K + j\mu + d\mu + h_r\left[\frac{1}{2R}(\mu-\omega)^2(1-p1)^2 + \frac{p1(\mu-\omega)^2}{x}\right] + h_\omega\left[\frac{\omega^2}{x} + \omega\left(\frac{(1-p1)(\mu-\omega)}{R} - \frac{\omega}{x}\right)(1-p1) + \frac{1}{2R}\big((\omega(1-p1)-B)^2 - \omega(\mu-w)(1-p1)^2 + B(1-p1)(\mu-\omega)\big)\right] + \frac{B^2 b}{2R} \tag{5}$$

$$TC(B,\mu) = K + j\mu + d\mu + h_r\left[\frac{1}{2R}(\mu-\omega)^2(1-\rho)^2 + \frac{p1(\mu-\omega)^2}{x}\right] + h_\omega\left[\frac{\omega(\mu-\omega)(1-p1)^2}{R} + \frac{\omega^2\, p1}{x} + \frac{1}{2R}\big((\omega(1-p1)-B)^2 - \omega(\mu-\omega)(1-p1)^2 + B(1-p1)(\mu-\omega)\big)\right] + \frac{B^2 b}{2R} \tag{6}$$

The total profit per unit of time, $TPU(B,\mu)$, is determined by the difference between total revenue per unit of time$\frac{TR(\rho,\mu)}{T}$ and total cost per unit of time $\frac{TC(B,\mu)}{T}$. Hence,

$$TPU(B,\mu) = \frac{TR(\rho,\mu) - TC(B,\mu)}{T} \tag{7}$$

Case 2: If $t_1 < t_\omega$ this case has been explained by Figure 2.

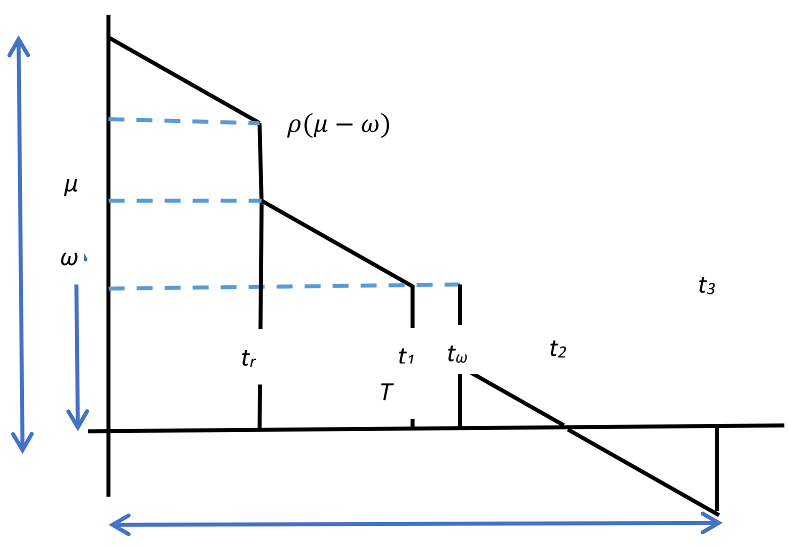


Fig. 2 Case: $t_1 < t_\omega$

The total revenue $TR(p,\mu)$ is given as in Case 1 and the total cost $TC(B,\mu)$ is given below

$$TC(B,\mu) = K + j\mu + d\mu + h_r\left[\left(\frac{1}{2}t_1(\mu-\omega)(1-p1)\right) + t_r p1(\mu-\omega)\right] + h_\omega\left[\omega t_1 + \omega p1(t_\omega - t_1) + \frac{(\omega(1-p1)-B)(t_2-t_1)}{2}\right] + \frac{Bbt_3}{2} \tag{8}$$

Since
$$t_1 = \frac{(\mu-\omega)(1-p1)}{R}, t_r = \frac{\mu-\omega}{x}, t_\omega = \frac{\omega}{x}, t_2 = \frac{\omega(1-p1)-B}{R}, t_3 = \frac{B}{R},$$

Then Eq (8) is

$$TC(B,\mu) = K + j\mu + d\mu + h_r\left[\frac{1}{2R}(\mu-\omega)^2(1-p1)^2 + \frac{p1(\mu-\omega)^2}{x}\right] + h_w\left[\frac{\omega(\mu-\omega)(1-p1)}{R} + \omega\, p1\left(\frac{\omega}{x} - \frac{(1-p1)(\mu-\omega)}{R}\right) + \frac{1}{2R}\left((\omega(1-p1)-B)^2 - \omega(\mu-\omega)(1-p1)^2 + B(1-p1)(\mu-\omega)\right)\right] + \frac{B^2 b}{2R} \quad (9)$$

$$TC(B,\mu) = K + j\mu + d\mu + h_r\left[\frac{1}{2R}(\mu-\omega)^2(1-p1)^2 + \frac{p1(\mu-\omega)^2}{x}\right] + h_\omega\left[\frac{\omega(\mu-\omega)(1-p1)^2}{R} + \frac{\omega^2\, p1}{x} + \frac{1}{2R}\left((\omega(1-p1)-B)^2 - \omega(\mu-\omega)(1-p1)^2 + B(1-p1)(\mu-\omega)\right)\right] + \frac{B^2 b}{2R} \quad (10)$$

The total profit per unit of time, $TPU(B,\mu)$, is

$$TPU(B,\mu) = \frac{TR(\rho,\mu) - TC(B,\mu)}{T} \quad (11)$$

Since eq. (4) and eq. (8) are same so on merging these equations

$$TC(B,\mu) = K + j\mu + d\mu + h_r\left[\frac{1}{2R}(\mu-\omega)^2(1-p1)^2 + \frac{p1(\mu-\omega)^2}{x}\right] + h_\omega\left[\frac{\omega(\mu-\omega)(1-p1)^2}{R} + \frac{\omega^2\, p1}{x} + \frac{1}{2R}\left((\omega(1-p1)-B)^2 - \omega(\mu-\omega)(1-p1)^2 + B(1-p1)(\mu-\omega)\right)\right] + \frac{B^2 b}{2R} \quad (12)$$

total profit, $TPU(B,\mu)$

$$TPU(B,\mu) = R\left[s - v + \mathrm{h}_r\frac{(\mu-\omega)^2}{xu} + \mathrm{h}_\omega\frac{\omega^2}{x\mu}\right] + \left(\frac{R}{1-p1}\right)\left(v - \frac{\mathrm{K}}{\mu} - j - d - \mathrm{h}_r\frac{(\mu-\omega)^2}{x\mu} - \mathrm{h}_\omega\frac{\omega^2}{x\mu}\right) - \mathrm{h}_r(\mu-\omega)^2\left(\frac{1-p1}{2\mu}\right) - \mathrm{h}_\omega\left[\frac{\omega(\mu-\omega)(1-p1)}{2\mu} + \frac{\omega^2(1-p1)}{2\mu} - \left(\frac{\omega B}{\mu}\right) + \frac{B(\mu-\omega)}{2\mu}\right]$$
(13)

Since ρ is a random variable that follows a uniform distribution with known p.d.f. f(ρ) then the expected value of total profit $ETPU(B,u)$ is

$$ETPU(B,\mu) = R\left[s - v + \mathrm{h}_r\frac{(\mu-\omega)^2}{x\mu} + \mathrm{h}_\omega\frac{\omega^2}{x\mu}\right] + E\left(\frac{1}{1-p1}\right)R\left[v - \frac{\mathrm{K}}{\mu} - j - d - \mathrm{h}_r\frac{(\mu-\omega)^2}{x\mu} - \mathrm{h}_\omega\frac{\omega^2}{x\mu}\right] - \mathrm{h}_r(\mu-\omega)^2\left(\frac{1-E(p1)}{2\mu}\right) - \mathrm{h}_\omega\left[\frac{\omega(\mu-\omega)\left(1-E(p1)\right)}{2\mu} + \frac{\omega^2\left(1-E(p1)\right)}{2\mu} - \left(\frac{\omega B}{\mu}\right) + \frac{B(\mu-\omega)}{2\mu}\right]E\left(\frac{1}{1-p1}\right)\left(\frac{\mathrm{h}_\omega B^2}{2\mu} - \frac{B^2 b}{\mu}\right) \quad (14)$$

Optimal lot size is

$$\mu^* = \sqrt{\omega^2 + \frac{\mathrm{K} + \frac{B^2 b}{R} + h_2\left(\frac{p1\omega^2}{x} + \frac{B^2}{2R} - \frac{3\omega B(1-p1)}{2R}\right)}{h_1\left(\frac{(1-p1^2)}{2R} + \frac{p1}{x}\right)}} \quad (15)$$

Our aim is to calculate the optimal order quantity for both the warehouses μ^*, and optimal shortages B^* , from these we get expected total profit $ETPU(B,\mu)$, therefore the necessary condition for expected profit to be optimal are $\frac{\partial ETPU(B,\mu)}{\partial\mu}$ and $\frac{\partial ETPU(B,\mu)}{\partial B}$which can be calculated by the eq. (13)
and setting the result to zero, one has

$$\frac{\partial ETPU(B,\mu)}{\partial B} = -h_\omega\left(\frac{-\omega}{\mu}+\frac{1}{2}\frac{\mu-\omega}{\mu}\right) - E\left(\frac{1}{1-p1}\right)\left(\frac{h_\omega B}{\mu}-\frac{2Bb}{R\mu}\right)=0 \tag{16}$$

$$\frac{\partial ETPU(B,\mu)}{\partial \mu} = R\left(\frac{2h_r(\mu-\omega)}{x\mu}-\frac{h_r(\mu-\omega)^2}{x\mu^2}-\frac{h_\omega\omega^2}{x\mu^2}\right)+E\left(\frac{1}{1-p1}\right)R\left(\frac{K}{\mu^2}-\frac{2h_r(\mu-\omega)}{x\mu}+\frac{h_r(\mu-\omega)^2}{x\mu^2}+\frac{h_\omega\omega^2}{x\mu^2}\right)- \frac{h_r(\mu-\omega)(1-E(p1))}{\mu}+\frac{h_r(\mu-\omega)^2(1-E(p1))}{2\mu^2}-h_\omega\left(\frac{1}{2}\frac{\omega(1-E(p1))}{\mu}-\frac{1}{2}\frac{\omega(\mu-\omega)(1-E(p1))}{\mu^2}-\frac{1}{2}\frac{\omega^2(1-E(p1))}{\mu^2}+\frac{\omega B}{\mu^2}+\frac{B}{2\mu}- \frac{B(\mu-\omega)}{2\mu^2}\right)-E\left(\frac{1}{1-p1}\right)\left(-\frac{h_\omega B^2}{2\mu^2}+\frac{B^2 b}{\mu^2}\right)=0 \tag{17}$$

Solution of Eqs. (16) and (17) for B* and μ* respectively is done by using Maple. Taking the second derivative, one ha

$$\frac{\partial^2 ETPU(B,\mu)}{\partial B^2} = -E\left(\frac{1}{1-p1}\right)\left(\frac{h_\omega}{\mu}-\frac{2b}{\mu}\right) \tag{18}$$

$$\frac{\partial^2 ETPU(B,\mu)}{\partial \mu^2} = R\left(\frac{2h_r}{x\mu}-\frac{4h_r(\mu-\omega)}{x\mu^2}+\frac{2h_r(\mu-\omega)^2}{x\mu^3}+\frac{2h_\omega\omega^2}{x\mu^3}\right)+E\left(\frac{1}{1-p1}\right)R\left(-\frac{2K}{\mu^3}-\frac{2h_r}{x\mu}+\frac{4h_r(\mu-\omega)}{x\mu^2}-\frac{2h_r(\mu-\omega)^2}{x\mu^3}- \frac{2h_\omega\omega^2}{x\mu^3}\right)-\frac{h_r(1-E(p1))}{\mu}+\frac{2h_r(\mu-\omega)(1-E(p1))}{\mu^2}-\frac{h_r(\mu-\omega)^2(1-E(p1))}{\mu^3}-h_\omega\left(\frac{-\omega(1-E(p1))}{\mu^2}+\frac{\omega(\mu-\omega)(1-E(p1))}{\mu^3}+ \frac{\omega^2(1-E(p1))}{\mu^3}-\frac{2\omega B}{\mu^3}-\frac{B}{\mu^2}+\frac{B(\mu-\omega)}{\mu^3}\right)-E\left(\frac{1}{1-p1}\right)\left(\frac{h_\omega B^2}{\mu^3}-\frac{2B^2 b}{\mu^3}\right) \tag{19}$$

$$\frac{\partial^2 ETPU(B,\mu)}{\partial \mu \partial B} = -h_\omega\left(\frac{\omega}{\mu^2}+\frac{1}{2\mu}-\frac{1}{2}\frac{\mu-\omega}{\mu^2}\right)-E\left(\frac{1}{1-p1}\right)\left(\frac{-h_\omega B}{\mu^2}+\frac{2b\,B}{\mu^2}\right) \tag{20}$$

For expected total profit to be concave, the sufficient condition as

$$\left(\frac{\partial^2 ETPU(B,\mu)}{\partial \mu \partial B}\right)^2-\left(\frac{\partial^2 ETPU(B,\mu)}{\partial B^2}\right)\times\left(\frac{\partial^2 ETPU(B,\mu)}{\partial \mu^2}\right)\le 0 \tag{21}$$

And $\left(\frac{\partial^2 ETPU(B,\mu)}{\partial B^2}\right)\le 0, \left(\frac{\partial^2 ETPU(B,\mu)}{\partial \mu^2}\right)\le 0$ (22)

Proved above that the function *ETPU (B, μ)* is strictly concave with a negative-definite Hessian matrix with optimal *(B*, μ*)* values. Optimal value of *(B*, μ*)* can be derived from Eqs. (16) and (17). With the optimal B^* and μ* values known, the net profit can be derived from Eq. (14).

5. NUMERICAL EXAMPLE

Some parameters are needed for evaluate the mention inventory system and these parameters are obtained from Chung et al. (2009):
$R = 50{,}000\ unit\ /\ year$; $\omega = 800\ units/cycle$; K $= \$\,100\ /cycle$; $j = \$\,20\ /unit$; $h_R = \$\,7\ /unit/year$; $h_O = \$\,5/unit/year$; $x = 1\ unit/min$; $d = 0.5/unit$; $s = \$\,50/unit$; $v = \$\,20\ /unit$; $b = 10/unit$
In this example, the inventory model works on an 8 $hours/day$ for whole year, therefore the screening rate per year, $x = 1 * 60 * 8 * 365 = 175200\ units/year$.
The proportion of imperfect random variable, p, follows uniformly distribution with p.d.f. as

$$f(\rho) = \begin{cases} 25, & 0 \le \rho \le 0.04 \\ 0, & otherwise \end{cases}$$

From this $E(\rho) = 0.02$ and $E\left(\frac{1}{1-\rho}\right) = 1.02055$. By solving the equation (16) and (17), the optimal solution is B $*= 93.6412\ units, \mu^* = 1425.38215\ units$ the expected time periods are $T^* = 0.027\ year, {t_2}^* = 0.0121\ year,\ {t_3}^* = 0.0018\ year$ and the expected annual total profit is $\$\,1.467516 \times 10^6$

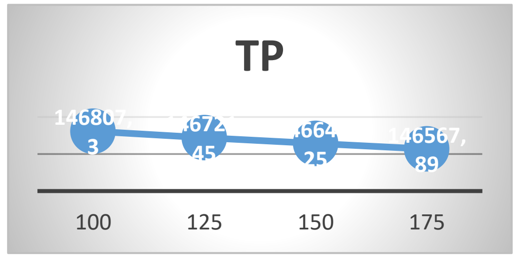
Table 2: Sensitivity analysis with different parameters of the inventory system

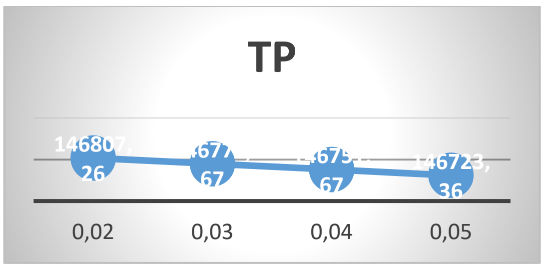
Parameter	Value	T (years)	t_1 (years)	t_2 (years)	μ (units/cycle)	B (units/cycle)	TP
ρ	0.02	0.0275	0.0118	0.0138	1407.66	97.268	146807.29
	0.03	0.0275	0.0119	0.0136	1416.53	95.44	146779.60
	0.04	0.0270	0.0120	0.0135	1425.38	93.641	146751.62
	0.05	0.0272	0.01206	0.0134	1434.22	91.87	146723.36
K	100	0.0275	0.0118	0.0138	1407.66	97.268	146807.29
	125	0.0301	0.0144	0.014	1537	84.5915	146721.45
	150	0.0325	0.0168	0.0142	1656.26	72.9014	146641.25
	175	0.0346	0.0189	0.0144	1767.49	61.9982	146566.89

v	20	0.0275	0.0118	0.0138	1407.66	97.268	146807.29
	25	0.0275	0.0119	0.0137	1407.66	97.27	147312.78
	30	0.0275	0.0119	0.0137	1407.66	97.27	147578.54
	35	0.0275	0.0119	0.0137	1407.66	97.27	148322.69
s	50	0.0275	0.0118	0.0138	1407.66	97.268	146807.29
	55	0.0275	0.0118	0.0138	1407.66	97.27	171807.67
	60	0.0275	0.0118	0.0138	1407.66	97.27	196807.53
	65	0.0275	0.0118	0.0138	1407.66	97.27	221807.18

We can clearly observe that if the cost associated with Type I and Type II error is increased the total profit of the system decrease

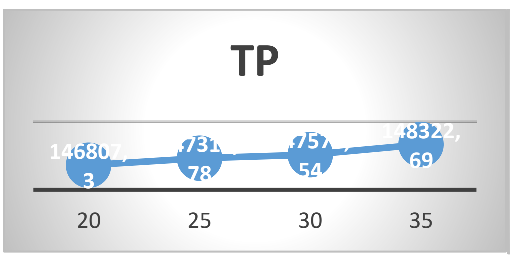
Graph 1: Effect of rate of change of human error on total profit   Graph 2: Effect of purchase cost on total profit

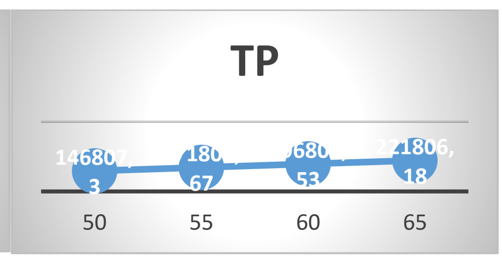




Here graph clearly proves that as increases the purchase cost the total profit of the system decreases.

Graph 3: Effect of change of selling price of defective items on total profit   Graph 4: Effect of change of selling price of good items on total price





This graph shows that as the selling price of defective items increases the total profit of system increases. The above graph shows that as the selling price of good items increases then the total profit of system increases.

6. CONCLUSION

In this model, we proposed a two-warehouse inventory model with shortages by seeing the effect of human error. In order to make this is more realistic, the shortages are allowed and are fully backlogged. This paper is very useful for retail and manufacturing trades. The purpose of this model is to find the expected total profit with the effect of Type I and Type II errors. By the assumption, as percentage of error increases the expected total profit decreases with respect to time. The significance of this model is validated with numerical examples and sensitivity analysis.

7. DIRECTIONS FOR FUTURE RESEARCH

This model can be extended in several forms. For example, it can be extended under two level and three level trade credit policy. It can be more realistic by extended this model with different types demand such as advertisement-dependent demand, ramp type demand. Further, these models can be extended under carbon emission constraints with international supply chain.

Conflict of interest: The authors confirm that there is no conflict of interest to declare for this publication.

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