AN EXPONENTIAL APPROACH FOR ESTIMATING POPULATION MEAN USING TWO AUXILIARY VARIABLES IN STRATIFIED RANDOM SAMPLING

Housila P. Singh, Anita Yadav and Surya K. Pal
School of Studies in Statistics, Vikram University, Ujjain-456010, India

ABSTRACT

In this paper we have considered the problem of estimating the population mean $\bar{Y}$ of the study variable y using information on auxiliary variable *x* in stratified random sampling. A class of estimators has been proposed. The bias and mean squared error have been obtained up to first degree of approximation. Optimum condition is obtained in which the proposed class of estimators has least mean squared error. We have also compared the proposed class of estimators with some existing estimators. An empirical study is carried out in support of the present study.

KEYWORDS: Auxiliary variate; Study variable; Bias; Mean squared error; Stratified random sampling.

MSC: 62D05.

RESUMEN

En este paper hemos considerado el problema de estimar la media de la población $\bar{Y}$ de la variable de estudio y usando información sobre la auxiliar *x* en el muestreo aleatorio estratificado. Una clase de estimadores ha sido propuesta. El sesgo y el error cuadrático medio han sido obtenidos hasta el primer grado de aproximación. Una condición de óptimo es obtenida en la que la clase propuesta posee el menor error cuadrático medio. También hemos comparado la propuesta clase de estimadores con algunos de los existentes estimadores. Un estudio empírico es llevado a cabo para dar soporte al presente estudio .

PALABRAS CLAVE: Variable auxiliar; Variable de estudio; Sesgo; Error cuadrático medio; Muestreo aleatorio estratificado.

1. INTRODUCTION

The problem of estimating the population mean in the presence of an auxiliary variable has been widely discussed in finite population sampling literature. Out of many ratio, product and regression methods of estimation are good examples in this context. A large amount of work has been carried out by various authors for estimating population mean $\bar{Y}$ of the study variable y using auxiliary information in simple random sampling. For instance see Singh, H.P. (1986), Singh, S. (2003), Pal et al. (2018, 2020) and the references cited therein. Since simple random sampling has its limitations that it is suitable when population is homogeneous. When population is heterogeneous stratified random sampling is used in which whole population is divided into homogeneous groups called strata and a sample of predetermined size is drawn from each stratum. Stratified random sampling is also useful when estimates of sub-groups are also required. Diana (1993), Kadilar and Cingi (2003), Singh and Vishwakarma (2008), Koyuncu and Kadilar (2009), Singh et al. (2013), Malik and Singh (2017) and Pal et al. (2020) proposed estimators in stratified random sampling and discussed their properties under large sample approximation. However, sometimes it is better to use information on two auxiliary variables rather than one auxiliary variable for the estimation of finite population mean.

Singh and Kumar (2012) , Tailor and Chouhan (2014), Lu and Yan (2014) and Singh et al. (2018) have suggested improved estimators of population mean using two auxiliary variables alongwith their properties in stratified random sampling.In this paper we have suggested a generalized ratio-cum-product estimator of finite population mean in stratified random sampling and its properties are studied.

2. NOTATIONS AND DEFINITIONS

Consider the finite population $(U = U_1, U_2, ..., U_N)$ of N units. Let the population be heterogeneous. It is divided into L non-overlapping homogeneous groups (i.e.in L strata) each of size N_h (h=1, 2,…, L)such that $N = \sum_{h=1}^{L} N_h$. It is desired to estimate the population mean $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$ (with $W_h = \frac{N_h}{N}$ and $\bar{Y}_h$ being the mean of the hth stratum) using information on two auxiliary variables x and z. Assume that a simple random sample of size n_h is drawn using simple random sampling without replacement (SRSWOR) scheme from hth stratum such that the total sample size $n = \sum_{h=1}^{L} n_h$.

In what follows we shall use the following notations throughout the paper.

$\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} y_{hi}$: The h^{th} stratum population mean for the study variable y,

$\bar{X}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$: The h^{th} stratum population mean for the auxiliary variable x,

$\bar{Z}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} z_{hi}$: The h^{th} stratum population mean for the auxiliary variable z,

$\bar{Y} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} y_{hi} = \frac{1}{N} \sum_{h=1}^{L} N_h \bar{Y}_h = \sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean of the study variate y,

$\bar{X} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} x_{hi} = \frac{1}{N} \sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of the auxiliary variate x,

$\bar{Z} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} z_{hi} = \frac{1}{N} \sum_{h=1}^{L} W_h \bar{Z}_h$: Population mean of the auxiliary variate z,

$\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi}$: Sample mean of the study variate y for hth stratum,

$\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi}$: Sample mean of the auxiliary variate x for hth stratum,

$\bar{z}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} z_{hi}$: Sample mean of the auxiliary variate z for hth stratum,

$W_h = \frac{N_h}{N}$: Stratum weight of hth stratum.

$R_1 = \frac{\bar{Y}}{\bar{X}}, R_2 = \frac{\bar{Y}}{\bar{Z}}, \gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h} \right), V_0 = \sum_{h=1}^{L} \gamma_h W_h^2 S_{yh}^2,\ V_1 = \sum_{h=1}^{L} \gamma_h W_h^2 S_{xh}^2, V_2 = \sum_{h=1}^{L} \gamma_h W_h^2 S_{zh}^2$,

$V_{01} = \sum_{h=1}^{L} \gamma_h W_h^2 S_{yxh}, V_{02} = \sum_{h=1}^{L} \gamma_h W_h^2 S_{yzh},\ V_{12} = \sum_{h=1}^{L} \gamma_h W_h^2 S_{xzh},\ S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y})^2$,

$S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X})^2, S_{zh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (z_{hi} - \bar{Z})^2,\ S_{yxh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y})(x_{hi} - \bar{X})$,

$S_{yzh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y})(z_{hi} - \bar{Z}),\ S_{xzh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X})(z_{hi} - \bar{Z})$, Usual unbiased estimators of population means $\bar{Y}$, $\bar{X}$ and $\bar{Z}$ in stratified random sampling are defined respectively as

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h \text{ , } \bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h \text{ , and } \bar{z}_{st} = \sum_{h=1}^{L} W_h \bar{z}_h \text{ .}$$

3. SOME EXISTING ESTIMATORS

When the population means $(\bar{X}, \bar{Z})$ of auxiliary variables (x, z) are known the classical combined ratio and product estimators to estimate population mean $\bar{Y}$ in stratified random sampling are defined as

$$\hat{\bar{Y}}_{Rc} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right), \tag{3.1}$$

$$\hat{\bar{Y}}_{Pc} = \bar{y}_{st}\left(\frac{\bar{z}_{st}}{\bar{Z}}\right), \tag{3.2}$$

The bias and mean squared error of the classical combined ratio $\hat{\bar{Y}}_{RC}$ and product estimators $\hat{\bar{Y}}_{PC}$ are given as

$$B\left(\hat{\bar{Y}}_{RC}\right) = \frac{1}{\bar{X}} \sum_{h=1}^{L} W_h^2 \gamma_h \left(R_1 S_{xh}^2 - S_{yxh}\right) = \frac{1}{\bar{X}}\left(R_1 V_1 - V_{01}\right), \tag{3.3}$$

$$B\left(\hat{\bar{Y}}_{PC}\right) = \frac{1}{\bar{Z}} \sum_{h=1}^{L} W_h^2 \gamma_h S_{yzh} = \left(\frac{1}{\bar{Z}}\right) V_{02} \text{ ,} \tag{3.4}$$

$$MSE\left(\hat{\bar{Y}}_{RC}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + R_1^2 S_{xh}^2 + 2R_1 S_{yxh}\right) = \left(V_0 - 2R_1 V_{01} - R_1^2 V_1\right), \tag{3.5}$$

$$MSE\left(\hat{\bar{Y}}_{PC}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh}\right) = \left(V_0 + 2R_2 V_{02} - R_2^2 V_2\right) \tag{3.6}$$

Singh et al. (2008) have suggested following exponential ratio and product type estimators in the stratified random sampling as

$$t_{\mathrm{Re}} = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right), \tag{3.7}$$

$$t_{Pe} = \bar{y}_{st} \exp\left(\frac{\bar{z}_{st} - \bar{Z}}{\bar{z}_{st} + \bar{Z}}\right), \tag{3.8}$$

Using known population means $(\bar{X}, \bar{Z})$ of two auxiliary variables (x, z) Tailor et al. (2012), defined the ratio- product estimator to estimate the population mean $\bar{Y}$ of the study variable y in stratified random sampling as

$$\hat{\bar{Y}}_{RP}^{ST} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) \text{ ,} \tag{3.9}$$

Singh and Kumar (2012) proposed some ratio-cum-product type exponential estimators for population mean using two auxiliary variables (x, z) as

$$t_1 = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right) \exp\left(\frac{\bar{Z} - \bar{z}_{st}}{\bar{z}_{st} + \bar{Z}}\right) \tag{3.10}$$

$$t_2 = \bar{y}_{st} \exp\left(\frac{\bar{x}_{st} - \bar{X}}{\bar{X} + \bar{x}_{st}}\right) \exp\left(\frac{\bar{z}_{st} - \bar{Z}}{\bar{z}_{st} + \bar{Z}}\right) \tag{3.11}$$

$$t_3 = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right) \exp\left(\frac{\bar{z}_{st} - \bar{Z}}{\bar{z}_{st} + \bar{Z}}\right) \tag{3.12}$$

$$t_4 = \bar{y}_{st} \exp\left(\frac{\bar{x}_{st} - \bar{X}}{\bar{X} + \bar{x}_{st}}\right) \exp\left(\frac{\bar{Z} - \bar{z}_{st}}{\bar{z}_{st} + \bar{Z}}\right) \tag{3.13}$$

It is to be mentioned that Tailor and Chouhan (2014) have revisited the estimator

$$t_{RPe} = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right) \exp\left(\frac{\bar{z}_{st} - \bar{Z}}{\bar{z}_{st} + \bar{Z}}\right)$$

due to Singh and Kumar (2012).

Up to the first degree of approximation, the mean squared error of t_{Re} and t_{Pe} are obtained as:

$$MSE(t_{\mathrm{Re}}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4} R_1^2 S_{xh}^2 - R_1 S_{yxh} \right] = \left(V_0 + \frac{1}{4} R_1^2 V_1 - R_1 V_{01} \right), \tag{3.14}$$

$$MSE(t_{Pe}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4} R_2^2 S_{zh}^2 - R_1 S_{yzh} \right] = \left(V_0 + \frac{1}{4} R_1^2 V_1 + R_1 V_{01} \right), \tag{3.15}$$

$$\begin{aligned} MSE\left(\hat{\bar{Y}}_{RP}^{ST}\right) &= \sum_{h=1}^{L} \gamma_h W_h^2 \left[S_{yh}^2 + R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2R_1 S_{yxh} + 2R_2 S_{yzh} - 2R_1 \; R_2 S_{xzh} \right] \\ &= [V_0 + R_1^2 V_1 + R_2^2 V_2 - 2R_1 V_{01} + 2R_2 V_{02} - 2R_1 R_2 V_{12}], \end{aligned} \tag{3.16}$$

$$\begin{aligned} MSE(t_1) &= \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4}\left(R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 + 2R_1 R_2 S_{xzh}\right) - R_1 S_{yxh} - R_2 S_{yzh} \right] \\ &= [V_0 + (1/4)\{R_1^2 V_1 + R_2^2 V_2 + 2R_1 R_2 V_{12}\} - R_1 V_{01} - R_2 V_{02}], \end{aligned} \tag{3.17}$$

$$\begin{aligned} MSE(t_2) &= \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4}\left(R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 + 2R_1 R_2 S_{xzh}\right) + R_1 S_{yxh} + R_2 S_{yzh} \right] \\ &= [V_0 + (1/4)\{R_1^2 V_1 + R_2^2 V_2 + 2R_1 R_2 V_{12}\} + R_1 V_{01} + R_2 V_{02}], \end{aligned} \tag{3.18}$$

$$\begin{aligned} MSE(t_3) = MSE(t_{RPe}) &= \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4}\left(R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2R_1 R_2 S_{xzh}\right) - R_1 S_{yxh} + R_2 S_{yzh} \right] \\ &= [V_0 + (1/4)\{R_1^2 V_1 + R_2^2 V_2 - 2R_1 R_2 V_{12}\} - R_1 V_{01} + R_2 V_{02}], \end{aligned} \tag{3.19}$$

$$\begin{aligned} MSE(t_4) &= \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4}\left(R_1^2 S_{xh}^2 + R_2^2 S_{zh}^2 - 2R_1 R_2 S_{xzh}\right) + R_1 S_{yxh} - R_2 S_{yzh} \right] \\ &= [V_0 + (1/4)\{R_1^2 V_1 + R_2^2 V_2 - 2R_1 R_2 V_{12}\} + R_1 V_{01} - R_2 V_{02}], \end{aligned} \tag{3.20}$$

4. PROPOSED CLASS OF ESTIMATOR

We suggested a class of estimators for population mean $\bar{Y}$ of y using information on two auxiliary variables (x, z) in stratified random sampling as

$$t_e = \Omega_1 \bar{y}_{st} + \Omega_2 \bar{y}_{st} \exp\left\{\frac{\alpha(\bar{X} - \bar{x}_{st})}{(\bar{X} + \bar{x}_{st})}\right\} \exp\left\{\frac{\delta(\bar{z}_{st} - \bar{Z})}{(\bar{Z} + \bar{z}_{st})}\right\}. \tag{4.1}$$

where (Ω_1, Ω_2) are suitably chosen constants.

To obtain the bias and MSE of the proposed class of estimators t_e we write $\bar{y}_{st} = \bar{Y}(1+e_0)$, $\bar{x}_{st} = \bar{Y}(1+e_1)$, $\bar{z}_{st} = \bar{Z}(1+e_2)$ such that $E(e_0) = E(e_1) = E(e_2) = 0$ an $E(e_0^2) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L}\gamma_h W_h^2 S_{yh}^2 = \frac{V_0}{\bar{Y}^2}$,

$E(e_1^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L}\gamma_h W_h^2 S_{xh}^2 = \frac{V_1}{\bar{X}^2}$, $E(e_2^2) = \frac{1}{\bar{Z}^2}\sum_{h=1}^{L}\gamma_h W_h^2 S_{zh}^2 = \frac{V_2}{\bar{Z}^2}$, $E(e_0 e_1) = \frac{1}{\bar{X}\bar{Y}}\sum_{h=1}^{L}\gamma_h W_h^2 S_{yxh} = \frac{V_{01}}{\bar{X}\bar{Y}}$,

$E(e_0 e_2) = \frac{1}{\bar{Y}\bar{Z}}\sum_{h=1}^{L}\gamma_h W_h^2 S_{yzh} = \frac{V_{02}}{\bar{Y}\bar{Z}}$, $E(e_1 e_2) = \frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L}\gamma_h W_h^2 S_{xzh} = \frac{V_{12}}{\bar{X}\bar{Z}}$.

Expressing (4.1) in terms of e's, we have

$$t_e = \Omega_1 \bar{Y}(1+e_0) + \Omega_2 \bar{Y}(1+e_0)\, exp\left\{\frac{\alpha(\bar{X} - \bar{X})(1+e_1)}{\bar{X} + \bar{X}(1+e_1)}\right\} exp\left\{\frac{\delta(\bar{Z}(1+e_2) - \bar{Z})}{\bar{Z} + \bar{Z}(1+e_2)}\right\}$$

$$= \Omega_1 \bar{Y}(1+e_0) + \Omega_2 \bar{Y}(1+e_0)\, exp\left\{\frac{-\alpha e_1}{2+e_1}\right\} exp\left\{\frac{\delta e_2}{2+e_2}\right\}$$

$$= \Omega_1 \bar{Y}(1+e_0) + \Omega_2 \bar{Y}(1+e_0) \exp\left\{-\frac{\alpha e_1}{2}\left(1+\frac{e_1}{2}\right)^{-1}\right\} \exp\left\{\frac{\delta e_2}{2}\left(1+\frac{e_2}{2}\right)^{-1}\right\}$$

$$= \Omega_1 \bar{Y}(1+e_0) + \Omega_2 \bar{Y}(1+e_0)\left[1 - \frac{\alpha e_1}{2} + \frac{\alpha(\alpha+2)}{8}e_1^2 + \frac{\delta e_2}{2} - \frac{\alpha\delta\, e_1 e_2}{4} + \frac{\delta(\delta-2)}{8}e_2^2 - \ldots\right]$$

$$= \Omega_1 \bar{Y}(1+e_0) + \Omega_2 \bar{Y}\begin{bmatrix} 1 + e_0 - \frac{\alpha}{2}e_1 - \frac{\alpha}{2}e_0 e_1 + \frac{\delta}{2}e_2 + \frac{\delta}{2}e_0 e_2 - \frac{\alpha\delta}{4}e_1 e_2 + \frac{\alpha(\alpha+2)}{8}e_1^2 \\ + \frac{\delta(\delta-2)}{8}e_2^2 - \frac{\alpha\delta e_0\, e_1 e_2}{4} + \frac{\alpha(\alpha+2)}{8}e_0\, e_1^2 + \frac{\delta(\delta-2)}{8}e_0 e_2^2 + \ldots \end{bmatrix}, \tag{4.2}$$

Neglecting terms of e's having power greater than two and subtracting $\bar{Y}$ from both sides of (4.2) we have

$$(t_e - \bar{Y}) \cong \bar{Y}\left[\Omega_1(1+e_0) + \Omega_2\left\{\begin{matrix} 1 + e_0 - \frac{1}{2}(\alpha e_1 - \delta e_2) - \frac{1}{2}(\alpha e_0 e_1 - \delta e_0 e_2) - \frac{\alpha\delta}{4}e_1 e_2 \\ + \frac{\alpha(\alpha+2)}{8}e_1^2 + \frac{\delta(\delta-2)}{8}e_2^2 \end{matrix}\right\} - 1\right] \tag{4.3}$$

Taking expectation of both sides of (4.3) we get the bias of the estimator t_e to the first degree of approximation as

$$B(t_e) = \bar{Y}\left[\Omega_1 + \Omega_2\left\{\begin{matrix} 1 + \frac{\alpha(\alpha+2)}{8}\frac{1}{\bar{X}^2}\sum_{h=1}^{L}\gamma_h W_h^2 S_{xh}^2 + \frac{\delta(\delta-2)}{8}\frac{1}{\bar{Z}^2}\sum_{h=1}^{L}\gamma_h W_h^2 S_{zh}^2 \\ - \frac{\alpha\delta}{2}\frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L}\gamma_h W_h^2 S_{xzh} - \frac{1}{2\bar{Y}}\left(\frac{\alpha}{\bar{X}}\sum_{h=1}^{L}\gamma_h W_h^2 S_{yxh} - \frac{\delta}{\bar{Z}}\sum_{h=1}^{L}\gamma_h W_h^2 S_{yzh}\right) \end{matrix}\right\} - 1\right] \tag{4.4}$$

Squaring both sides of (4.3) we have

$$(t_e - \bar{Y})^2 = \bar{Y}^2\left[\begin{matrix} 1 + \Omega_1^2(1 + 2e_0 + e_0^2) \\ + \Omega_2^2\left\{\begin{matrix} 1 + 2e_0 - \alpha e_1 + \delta e_2 + e_0^2 + \frac{1}{4}(\alpha^2 e_1^2 + \delta^2 e_2^2 - 2\alpha\delta e_1 e_2) - (\alpha e_0 e_1 \\ - \delta e_0 e_2) - \frac{\alpha\delta}{2}e_1 e_2 + \frac{\alpha(\alpha+2)}{4}e_1^2 + \frac{\delta(\delta-2)}{4}e_2^2 - (\alpha e_0 e_1 - \delta e_0 e_2) \end{matrix}\right\} \end{matrix}\right.$$

$$+2\Omega_1\Omega_2\left\{\begin{array}{l}1+e_0+e_0+e_0^2-\frac{1}{2}(\alpha e_1-\delta e_2)-\frac{1}{2}(\alpha e_0e_1-\delta e_0e_2)\\-\frac{1}{2}(\alpha e_0e_1-\delta e_0e_2)-\frac{\alpha\delta}{4}e_1e_2+\frac{\alpha(\alpha+2)}{8}e_1^2+\frac{\delta(\delta-2)}{8}e_2^2\end{array}\right\}$$

$$\left.-2\Omega_1(1+e_0)-2\Omega_2\left\{\begin{array}{l}1+e_0-\frac{1}{2}(\alpha e_1-\delta e_2)-\frac{1}{2}(\alpha e_0e_1-\delta e_0e_2)\\-\frac{\alpha\delta}{4}e_1e_2+\frac{\alpha(\alpha+2)}{8}e_1^2+\frac{\delta(\delta-2)}{8}e_2^2\end{array}\right\}\right]\right] \tag{4.5}$$

Taking expectation of both sides of (4.5) we get the mean squared error (MSE) of the estimator 't_e' to the first degree of approximation as

$$MSE(t_e)=\bar{Y}^2[1+\Omega_1^2A_1+\Omega_2^2A_{2(\alpha,\delta)}+2\Omega_1\Omega_2A_{3(\alpha,\delta)}-2\Omega_1-2\Omega_2A_{4(\alpha,\delta)}], \tag{4.6}$$

where $A_1=\left\{1+\frac{1}{\bar{Y}^2}\sum_{h=1}^{L}\gamma_hW_h^2S_{yh}^2\right\}=\left(1+\frac{V_0}{\bar{Y}^2}\right)$,

$$A_{2(\alpha,\delta)}=\left[1+\frac{V_0}{\bar{Y}^2}+\frac{\alpha(\alpha+1)}{2}\frac{V_1}{\bar{X}^2}+\frac{\delta(\delta-1)}{2}\frac{V_2}{\bar{Z}^2}-2\left\{\alpha\frac{V_{01}}{\overline{YX}}-\delta\frac{V_{02}}{\overline{YZ}}\right\}-\alpha\delta\frac{V_{12}}{\overline{XZ}}\right],$$

$$A_{3(\alpha,\delta)}=\left[1+\frac{V_0}{\bar{Y}^2}+\frac{\alpha(\alpha+2)}{8}\frac{V_1}{\bar{X}^2}+\frac{\delta(\delta-2)}{8}\frac{V_2}{\bar{Z}^2}-\alpha\frac{V_{01}}{\overline{YX}}+\delta\frac{V_{02}}{\overline{YZ}}-\frac{\alpha\delta}{4}\frac{V_{12}}{\overline{XZ}}\right],$$

$$A_{4(\alpha,\delta)}=\left[1+\frac{\alpha(\alpha+2)}{8}\frac{V_1}{\bar{X}^2}+\frac{\delta(\delta-2)}{8}\frac{V_2}{\bar{Z}^2}-\frac{\alpha}{2}\frac{V_{01}}{\overline{YX}}+\frac{\delta}{2}\frac{V_{02}}{\overline{YZ}}-\frac{\alpha\delta}{4}\frac{V_{12}}{\overline{XZ}}\right].$$

4.1. Optimum Choice of the Weights (Ω_1,Ω_2) and the Minimum MSE 't_e'

Differentiating (4.6) partially with respect to Ω_1 and Ω_2 and equating then to zero we have

$$\begin{bmatrix}A_1 & A_{3(\alpha,\delta)}\\A_{3(\alpha,\delta)} & A_{2(\alpha,\delta)}\end{bmatrix}\begin{bmatrix}\Omega_1\\\Omega_2\end{bmatrix}=\begin{bmatrix}1\\A_{4(\alpha,\delta)}\end{bmatrix} \tag{4.7}$$

Solving (4.7) we get the optimum values of (Ω_1,Ω_2) as

$$\left.\begin{array}{l}\Omega_{10}=\frac{\Delta_1}{\Delta}\\\Omega_{20}=\frac{\Delta_2}{\Delta}\end{array}\right\}, \tag{4.8}$$

where $\Delta=(A_1A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}^2)$

$$\Delta_1=\begin{vmatrix}1 & A_{3(\alpha,\delta)}\\A_{4(\alpha,\delta)} & A_{2(\alpha,\delta)}\end{vmatrix}=[A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}A_{4(\alpha,\delta)}]$$

and

$$\Delta_2=\begin{vmatrix}A_1 & 1\\A_{3(\alpha,\delta)} & A_{4(\alpha,\delta)}\end{vmatrix}=[A_1A_{4(\alpha,\delta)}-A_{3(\alpha,\delta)}].$$

Putting (4.8) in (4.6) we get the resulting minimum MSE of the proposed estimator t_e as

$$\min.MSE(t_e)=\bar{Y}^2\left[1-\frac{\Delta_1}{\Delta}-\frac{A_{4(\alpha,\delta)}\Delta_2}{\Delta}\right]=\bar{Y}^2\left[1-\frac{\left(A_{2(\alpha,\delta)}-2A_{3(\alpha,\delta)}A_{4(\alpha,\delta)}+A_1A_{4(\alpha,\delta)}^2\right)}{\left\{A_1A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}^2\right\}}\right] \tag{4.9}$$

Thus we established the following theorem.

Theorem 4.1: To the first degree of approximation, $MSE(t_e) \geq \bar{Y}^2\left[1-\frac{\left(A_{2(\alpha,\delta)}-2A_{3(\alpha,\delta)}A_{4(\alpha,\delta)}+A_1A_{4(\alpha,\delta)}^2\right)}{\left\{A_1A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}^2\right\}}\right]$

with equality holding if $\Omega_1 = \frac{\Delta_1}{\Delta}$ and $\Omega_2 = \frac{\Delta_2}{\Delta}$.

5. SOME SPECIAL CASES AND EFFICIENCY COMPARISON

Special Case I

Let us consider the case $\Omega_1 + \Omega_2 = 1$ in (4.1). Then the estimator $t_{e(1)}$ reduces to the class of estimators for $\bar{Y}$ as

$$t_{e(1)} = \Omega_1\bar{y}_{st} + (1-\Omega_1)\exp\left\{\frac{\alpha(\bar{X}-\bar{x}_{st})}{(\bar{X}+\bar{x}_{st})}\right\}\exp\left\{\frac{\delta(\bar{Z}-\bar{z}_{st})}{(\bar{Z}+\bar{z}_{st})}\right\}. \tag{5.1}$$

Putting $\Omega_2 = (1-\Omega_1)$ in (4.6) we get the *MSE* of the estimators $t_{e(1)}$ to the first degree of approximation as

$$MSE(t_{e(1)}) = \bar{Y}^2[1+\Omega_1^2\{A_1+A_{2(\alpha,\delta)}-2A_{3(\alpha,\delta)}\}-2\Omega_1\{A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}+1-A_{4(\alpha,\delta)}\}+A_{2(\alpha,\delta)}-2A_{4(\alpha,\delta)}]$$
$$=\bar{Y}^2[1+A_{2(\alpha,\delta)}-2A_{4(\alpha,\delta)}+\Omega_1^2\{A_1+A_{2(\alpha,\delta)}-2A_{3(\alpha,\delta)}\}-2\Omega_1\{1+A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}-A_{4(\alpha,\delta)}\}]. \tag{5.2}$$

The MSE of $t_{e(1)}$ at (5.2) is minimized for

$$\Omega_1 = \frac{\left\{1+A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}-A_{4(\alpha,\delta)}\right\}}{\left\{A_1+A_{2(\alpha,\delta)}-2A_{3(\alpha,\delta)}\right\}} = \Omega_{10}^*. \tag{5.3}$$

Inserting (5.3) in (5.2) we get the minimum *MSE* of the subclass of estimators $t_{e(1)}$ as

$$min.MSE(t_{e(1)}) = \bar{Y}^2\left[1+A_{2(\alpha,\delta)}-2A_{4(\alpha,\delta)}-\frac{\left\{1+A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}-A_{4(\alpha,\delta)}\right\}^2}{\left\{A_1+A_{2(\alpha,\delta)}-2A_{3(\alpha,\delta)}\right\}}\right]. \tag{5.4}$$

Thus we established the following theorem.

Theorem 5.1: To the first degree of approximation,

$$MSE(t_{e(1)}) \geq \bar{Y}^2\left[1+A_{2(\alpha,\delta)}-2A_{4(\alpha,\delta)}-\frac{\left\{1+A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}-A_{4(\alpha,\delta)}\right\}^2}{\left\{A_1+A_{2(\alpha,\delta)}-2A_{3(\alpha,\delta)}\right\}}\right],$$

with equality holding if $\Omega_1 = \Omega_{10}^*$.

From (4.9) and (5.4) we have

$$min.MSE(t_{e(1)}) - min.MSE(t_e) = \frac{H_1^2}{H_2H_3}, \tag{5.5}$$

where

$H_1 = [A_1(A_2-A_4)-(A_2-A_3)-A_3(A_3-A_4)]$, $H_2 = [A_1+A_{2(\alpha,\delta)}-2A_{3(\alpha,\delta)}]$, $H_3 = [A_1A_{2(\alpha,\delta)}-A_{3(\alpha,\delta)}^2]$.

Expression (5.5) clearly indicates that the proposed family of estimators $t_{e(1)}$ is inferior to the proposed class of estimators 't'.

Special Case II

If we set $(\Omega_1,\Omega_2) = (\Omega_1, 0)$ in (4.1) then t_e reduces to the estimator

$$t_{es} = \Omega_1\bar{y}_{st} \tag{5.6}$$

for population mean $\bar{Y}$. The estimator t_{es} is Searls (1964) type estimator in stratified random sampling. Inserting $(\Omega_1,\Omega_2) = (\Omega_1, 0)$ in (4.4) and (4.6) respectively we get the bias and *MSE* of t_{es} as

$$B(t_{es}) = (\Omega_1 - 1)\bar{Y} \tag{5.7}$$

and

$$MSE(t_{es}) = \bar{Y}^2\left(1 + \Omega_1^2 A_1 - 2\Omega_1\right). \tag{5.8}$$

The $MSE(t_{es})$ is minimized for

$$\Omega_1 = (1/A_1). \tag{5.9}$$

Putting (5.9) in (5.8) we get the minimum *MSE* of t_{es} as

$$min.MSE(t_{es}) = \bar{Y}^2 \frac{(A_1 - 1)}{A_1}. \tag{5.10}$$

From (4.9) and (5.10) we have

$$min.MSE(t_{es}) - min.MSE(t_e) = \frac{\bar{Y}^2\left\{A_{3(\alpha,\delta)} - A_1 A_{4(\alpha,\delta)}\right\}^2}{A_1\left\{A_1 A_{2(\alpha,\delta)} - A_{3(\alpha,\delta)}^2\right\}}, \tag{5.11}$$

which is positive.

Thus we have the inequality :

$$min.MSE(t_e) \le min.MSE(t_{es}) \tag{5.12}$$

It is well known under stratified random sampling that

$$\mathrm{var}(\bar{y}_{st}) = MSE(\bar{y}_{st}) = \bar{Y}^2(A_1 - 1). \tag{5.13}$$

From (5.10) and (5.13) we have

$$MSE(\bar{y}_{st}) - min.MSE(t_{es}) = \frac{\bar{Y}^2(A_1 - 1)^2}{A_1}, \tag{5.14}$$

which is non-negative.

Thus we have the inequality :

$$min.MSE(t_{es}) \le MSE(\bar{y}_{st}). \tag{5.15}$$

Combining the inequalities (5.12) and (5.15) we get that

$$min.MSE(t_e) \le min.MSE(t_{es}) \le MSE(\bar{y}_{st}). \tag{5.16}$$

It follows from (5.16) that the proposed class of estimator t is more efficient than the estimators $\bar{y}_{st}$ and t_{es} .

Special Case III

If we put $(\Omega_1, \Omega_2) = (0, \Omega_2)$ in (4.1) then we get the class of estimators for population mean $\bar{Y}$ as

$$t_{e(2)} = \Omega_2 \bar{y}_{st} \exp\left\{\frac{\alpha(\bar{X} - \bar{x}_{st})}{(\bar{X} + \bar{x}_{st})}\right\} \exp\left\{\frac{\delta(\bar{z}_{st} - \bar{Z})}{(\bar{Z} + \bar{z}_{st})}\right\}. \tag{5.17}$$

Inserting $(\Omega_1, \Omega_1) = (0, \Omega_2)$ in (4.4) and (4.6) we get the bias and *MSE* of the proposed class of estimators $t_{e(2)}$ to the first degree of approximation respectively as

$$B(t_{e(2)}) = \bar{Y}\left[\Omega_2\left\{1 + \frac{\alpha(\alpha+2)}{8}\frac{V_1}{\bar{X}^2} + \frac{\delta(\delta-2)}{8}\frac{V_2}{\bar{Z}^2} - \frac{\alpha\delta}{2}\frac{V_{12}}{\overline{XZ}} - \frac{\alpha V_{01}}{2\overline{YX}} + \frac{\delta V_{02}}{2\overline{YZ}}\right\} - 1\right] \tag{5.18}$$

and

$$MSE(t_{e(2)}) = \bar{Y}^2[1 + \Omega_2^2 A_{2(\alpha,\delta)} - 2\Omega_2 A_{4(\alpha,\delta)}]. \tag{5.19}$$

We note that the biases and mean squared errors of the estimators t_j ($j =$ 1 to 4) due Singh and Kumar (2012) and the estimator recently revisited by Tailor and Chouhan (2014) and other members belonging to the subclass of estimators $t_{e(2)}$ can be obtained just by putting the values of the scalars $(\Omega_2, \alpha, \delta)$ in (5.18) and (5.19) respectively.

The $MSE\left(t_{e(2)}\right)$ is minimum when

$$\Omega_2 = \frac{A_{4(\alpha,\delta)}}{A_{2(\alpha,\delta)}} = \Omega_{20}^{*} \ (say). \tag{5.20}$$

Substitution of (5.20) in (5.19 yields the minimum MSE of $t_{e(2)}$ as

$$\min .MSE\left(t_{e(2)}\right) = \bar{Y}^2 \left(1 - \frac{A_{4(\alpha,\delta)}^2}{A_{2(\alpha,\delta)}}\right). \tag{5.21}$$

Now we established the following theorem.

Theorem 5.2: To the first degree of approximation,

$$MSE\left(t_{e(2)}\right) \geq \bar{Y}^2 \left(1 - \frac{A_{4\alpha,\delta}^2}{A_{2(\alpha,\delta)}}\right)$$

with equality holding if $\Omega_2 = \Omega_{20}^{*}$

From (4.9) and (5.21) we have

$$\min .MSE\left(t_{e(2)}\right) - \min .MSE\left(t_e\right) = \frac{\bar{Y}^2 \left\{A_{2(\alpha,\delta)} - A_{3(\alpha,\delta)} A_{4(\alpha,\delta)}\right\}^2}{A_{2(\alpha,\delta)} \left\{A_1 A_{2(\alpha,\delta)} - A_{3(\alpha,\delta)}^2\right\}} \tag{5.22}$$

which is positive.

Thus from (5.22) we have the inequality:

$$\min .MSE\left(t_e\right) \leq \min .MSE\left(t_{e(2)}\right). \tag{5.23}$$

We consider another class of estimators for $\bar{Y}$ as

$$t_{e(3)} = \bar{y}_{st}\, exp\left\{\frac{\alpha\left(\bar{X} - \bar{x}_{st}\right)}{\left(\bar{X} + \bar{x}_{st}\right)}\right\} exp\left\{\frac{\delta\left(\bar{z}_{st} - \bar{Z}\right)}{\left(\bar{Z} + \bar{z}_{st}\right)}\right\} \tag{5.24}$$

which can be obtained from (4.1) by putting $\left(\Omega_1, \Omega_2\right) = (0,1)$ or from (5.17) by putting $\Omega_2 = 1$.

Inserting $\Omega_2 = 1$ in (5.18) and (5.19) we get the bias and *MSE* of the subclass of estimators $t_{e(3)}$ to the first degree of approximation respectively as

$$B\left(t_{e(3)}\right) = \bar{Y}\left[\frac{\alpha(\alpha+2)}{8}\frac{V_1}{\bar{X}^2} + \frac{\delta(\delta-2)}{8}\frac{V_2}{\bar{Z}^2} - \frac{\alpha\delta}{2}\frac{V_{12}}{\bar{X}\bar{Z}} - \frac{\alpha V_{01}}{2\bar{Y}\bar{X}} + \frac{\delta V_{02}}{2\bar{Y}\bar{Z}}\right], \tag{5.25}$$

and

$$MSE\left(t_{e(3)}\right) = \bar{Y}^2 \left[1 + A_{2(\alpha,\delta)} - 2A_{4(\alpha,\delta)}\right]. \tag{5.26}$$

From (5.21) and (5.26) we have

$$MSE\left(t_{e(3)}\right) - \min .MSE\left(t_{e(2)}\right) = \frac{\bar{Y}^2 \left(A_2 - A_4\right)^2}{A_2}, \tag{5.27}$$

which is positive.

Thus we have the inequality as

$$\min .MSE\left(t_{e(2)}\right) \leq MSE\left(t_{e(3)}\right). \tag{5.28}$$

It follows that the proposed subclass of estimators $t_{e(2)}$ is more efficient than the estimator $t_{e(3)}$, whatever be the values of (α, δ).

From (5.23) and (5.28) we have the inequality as

$$\min .MSE\left(t_e\right) \leq \min .MSE\left(t_{e(2)}\right) \leq \min .MSE\left(t_{e(3)}\right), \tag{5.29}$$

which follows that the proposed class of estimators t_e is better than the subclasses of estimators $t_{e(2)}$ and $t_{e(3)}$. With theoretical comparisons given above we conclude that the suggested class of estimators t_e is the best (in the sense of having least minimum *MSE*) among the estimators $\bar{y}_{st}$, t_{Re}, t_{Pe}, $t_j\left(j=1,2,3,4\right)$, $t_{e(1)}, t_{e(2)}$ and $t_{e(3)}$.

6. **NUMERICAL ILLUSTRATIONS**

To judge the merits of the suggested class of estimators' t_e ' over other estimators, we consider two natural population data sets. Descriptions of the population data sets are given below:

Population I [Source: Murthy (1967)]

y : Output, x : Fixed Capital and z : Number of worker.

$N=10$	$n=4$	$N_1=5$	$N_2=5$	$n_1=2$
$n_2=2$	$\bar{Y}_1=1925.8$	$\bar{Y}_2=315.6$	$\bar{X}_1=214.4$	$\bar{X}_2=333.8$
$\bar{Z}_1=51.80$	$\bar{Z}_2=60.60$	$S_{y1}=615.92$	$S_{y2}=340.38$	$S_{x1}=74.87$
$S_{x2}=66.35$	$S_{z1}=0.75$	$S_{z2}=4.84$	$S_{yx_1}=39360.68$	$S_{yx_2}=22356.50$
$S_{yz_1}=-411.16$	$S_{yz_2}=-1536.24$	$S_{zx_1}=-38.08$	$S_{zx2}=-287.92$	-

Population II [Source: National Horticulture Board (2010)]

y : Productivity (MT/Hectare), x : Production in'000' Tons and z : Area in '000' Hectare.

$N=20$	$n=8$	$N_1=10$	$N_2=10$	$n_1=4$
$n_2=4$	$\bar{Y}_1=1.70$	$\bar{Y}_2=3.67$	$\bar{X}_1=10.41$	$\bar{X}_2=289.14$
$\bar{Z}_1=6.32$	$\bar{Z}_2=80.67$	$S_{y1}=0.504$	$S_{y2}=1.4128$	$S_{x1}=3.53$
$S_{x2}=111.61$	$S_{z1}=1.1898$	$S_{z2}=10.819$	$S_{yx_1}=1.608$	$S_{yx_2}=144.88$
$S_{yz_1}=-0.056$	$S_{yz_2}=-7.046$	$S_{zx_1}=1.3838$	$S_{zx2}=-0.9202$	-

We have computed the percent relative efficiencies (PREs) of different estimators of population mean $\bar{Y}$ with respect to usual unbiased estimator $\bar{y}_{st}$ (for two strata in the population i.e. L=2) using the following formulae:

$$PRE\left(\hat{\bar{Y}}_{RC}, \bar{y}_{st}\right)=\frac{V_0}{\left[V_0+R_1^2V_1-2R_1V_{01}\right]}\times 100, \tag{6.1}$$

$$PRE\left(\hat{\bar{Y}}_{PC}, \bar{y}_{st}\right)=\frac{V_0}{\left[V_0+R_2^2V_2+2R_2V_{02}\right]}\times 100, \tag{6.2}$$

$$PRE\left(\hat{\bar{Y}}_{\mathrm{Re}}, \bar{y}_{st}\right)=\frac{V_0}{\left[V_0+(1/4)R_1^2V_1-R_1V_{01}\right]}\times 100, \tag{6.3}$$

$$PRE\left(\hat{\bar{Y}}_{Pe}, \bar{y}_{st}\right)=\frac{V_0}{\left[V_0+(1/4)R_2^2V_2+R_2V_{02}\right]}\times 100, \tag{6.4}$$

$$PRE\left(\hat{\bar{Y}}_{RP}^{ST}, \bar{y}_{st}\right)=\frac{V_0}{\left[V_0+R_1^2V_1+R_2^2V_2-2R_1V_{01}+2R_2V_{02}-2R_1R_2V_{12}\right]}\times 100, \tag{6.5}$$

$$PRE\left(t_1, \bar{y}_{st}\right)=\frac{V_0}{\left[V_0+(1/4)(R_1^2V_1+R_2^2V_2+2R_1R_2V_{12})-R_2V_{02}-R_1V_{01}\right]}\times 100, \tag{6.6}$$

$$PRE(t_2, \bar{y}_{st}) = \frac{V_0}{\left[V_0 + (1/4)(R_1^2 V_1 + R_2^2 V_2 + 2R_1 R_2 V_{12}) - R_2 V_{02} - R_1 V_{01}\right]} \times 100, \tag{6.7}$$

$$PRE\left(\hat{\hat{\bar{Y}}}_{RPe} \text{ or } t_3, \bar{y}_{st}\right) = \frac{V_0}{\left[V_0 + (1/4)(R_1^2 V_1 + R_2^2 V_2 - 2R_1 R_2 V_{12}) - R_1 V_{01} + R_2 V_{02}\right]} \times 100, \tag{6.8}$$

$$PRE(t_4, \bar{y}_{st}) = \frac{V_0}{\left[V_0 + (1/4)(R_1^2 V_1 + R_2^2 V_2 - 2R_1 R_2 V_{12}) + R_1 V_{01} - R_2 V_{02}\right]} \times 100, \tag{6.9}$$

$$PRE(t_e, \bar{y}_{st}) = \frac{(A_1 - 1)}{\left[1 - \frac{\left(A_{2(\alpha,\delta)} - 2A_{3(\alpha,\delta)}A_{4(\alpha,\delta)} + A_1 A_{4(\alpha,\delta)}^2\right)}{\left(A_1 A_{2(\alpha,\delta)} - A_{3(\alpha,\delta)}^2\right)}\right]} \times 100 \tag{6.10}$$

Findings are shown in Tables 6.1and 6.2.

Table 6.1 : PRE of the estimator $\hat{\hat{\bar{Y}}}_{RC}, \hat{\hat{\bar{Y}}}_{PC}, \hat{\hat{\bar{Y}}}_{Re}, \hat{\hat{\bar{Y}}}_{Pe}, \hat{\hat{\bar{Y}}}_{RP}^{ST}$, t_1, t_2, t_3 or $\hat{\hat{\bar{Y}}}_{RPe}$, t_4 with respect to $\bar{y}_{st}$.

Estimator	$PRE(\bullet, \bar{y}_{st})$	
	Population I	Population II
$\hat{\hat{\bar{Y}}}_{RC}$	313.75	223.85
$\hat{\hat{\bar{Y}}}_{PC}$	115.95	123.31
$\hat{\hat{\bar{Y}}}_{Re}$	173.94	359.56
$\hat{\hat{\bar{Y}}}_{Pe}$	107.94	116.92
$\hat{\hat{\bar{Y}}}_{RP}^{ST}$	346.62	288.32
t_1	158.40	199.67
t_2	66.95	40.89
t_3 or $\hat{\hat{\bar{Y}}}_{RPe}$	189.34	642.19
t_4	58.68	34.73

It is observed from Table 6.1 that in population I, the estimator $\hat{\hat{\bar{Y}}}_{RP}^{ST}$ due to Tailor et al (2012) is the best (in the sense that it has largest $PRE(\hat{\hat{\bar{Y}}}_{RP}^{ST}, \bar{y}_{st})$ =346.62% followed by the estimator t_3 (or $\hat{\hat{\bar{Y}}}_{RPe}$) proposed by Singh and Kumar (2012) and Tailor and Chouhan (2014) having the second largest $PRE\{t_3$ (or $\hat{\hat{\bar{Y}}}_{RPe}$)}=189.34%, while in population II the performance of the estimator t_3 (or $\hat{\hat{\bar{Y}}}_{RPe}$) is the best (i.e. having the largest $PRE\{t_3$ (or $\hat{\hat{\bar{Y}}}_{RPe}$)}=642.19% followed by the estimator $\hat{\hat{\bar{Y}}}_{RP}^{ST}$ having the second largest $PRE(\hat{\hat{\bar{Y}}}_{RP}^{ST}, \bar{y}_{st})$ =288.32% . Table 6.2 presents the percent relative efficiencies (PREs) of the suggested class of estimators t_e with respect to usual unbiased estimator $\bar{y}_{st}$ for different values of scalars (α, δ) .

Table 6.2: PRE of the class of estimators ‘ t_e ’ with respect to $\bar{y}_{st}$ for different values of (α, δ).

Population I			Population II		
		$PRE(t, \bar{y}\ )$			
0.75	0.25	425.19	0.50	0.25	678.09
2.00	0.50	426.14	1.00	0.50	679.76
1.00	-0.50	427.46	-0.50	-0.25	682.89
1.00	0.25	427.47	-1.00	-0.50	689.58
1.25	0.25	428.50	0.75	0.50	780.42
0.50	-0.25	428.51	-0.75	-0.50	794.05
1.50	0.25	429.05	0.50	1.00	819.09
0.75	-0.35	429.25	0.25	0.50	822.51
1.75	0.25	429.41	1.00	0.75	831.72
2.00	0.25	429.70	-0.25	-0.50	834.20
1.25	-0.50	430.54	-0.50	-1.00	842.64
1.50	-0.50	431.84	-1.00	-0.75	855.53
2.00	0.00	432.12	0.75	0.75	959.62
1.75	0.00	432.27	1.00	1.00	960.10
1.50	0.00	432.54	0.50	0.50	960.71
2.00	-0.50	432.74	0.25	0.25	963.39
0.75	-0.25	433.14	-0.25	-0.25	973.71
2.00	-0.25	433.20	-0.50	-0.50	981.48
1.75	-0.25	433.38	-0.75	-0.75	991.13
1.25	-0.05	433.42	-1.00	-1.00	1002.79
1.00	0.00	433.43	0.50	0.75	1004.41
1.50	-0.25	433.62	0.75	1.00	1024.12
1.25	-0.25	433.87	0.65	0.85	1025.16
1.00	-0.25	433.92	0.60	0.80	1025.25
0.75	-0.15	434.92	-0.50	-0.75	1035.57
0.50	-0.05	435.71	-0.75	-1.00	1069.86

Comparing the findings of Tables 6.1 and 6.2 we found that the values of PREs closed in Table 6.2 are larger than the values of PREs closed in Table 6.1 for both population data sets. It is observed from Tables 6.1 and 6.2 that there is substantial gain in efficiency by using the proposed estimator t_e over the estimators $\left(\hat{\bar{Y}}_{RP}^{ST}, t_3 \ or \ \hat{\bar{Y}}_{RPe}\right)$ due to Tailor et al (2012), Singh and Kumar (2012) and Tailor and Chouhan (2014).

In population I largest $PRE(t_e, \bar{y}_{st})$ = 435.71% is observed for (α, δ) = (0.50, -0.05) while in population II largest $PRE(t_e, \bar{y}_{st})$ =1069.86% is observed at (α, δ) = (-0.75,-1.00) which are much higher than the $PRE(\hat{\bar{Y}}_{RP}^{ST}, \bar{y}_{st})$ =346.62% in population I and the $PRE\{t_3$ (or $\hat{\bar{Y}}_{RPe}$)} =642.19% in population II.

Thus we conclude that there is enough scope of choosing the values of scalars (α, δ) in obtaining the estimators better than those earlier considered by Tailor et al (2012), Singh and Kumar (2012) and Tailor and Chouhan (2014) and hence our recommendation is in the favour of suggested class of estimators t_e .

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