A REDEFINED CLASS OF RATIO ESTIMATORS USING AUXILIARY INFORMATION FOR THE ESTIMATION OF POPULATION MEAN UNDER SIMPLE RANDOM SAMPLING SCHEME

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ABSTRACT
The present paper advocates the improved estimation of population mean through a new class of estimators using the known informations on an auxiliary variable under the Simple Random Sampling Scheme. The existing estimators of population mean have been shown as the members of the proposed class. The Bias and Mean Squared Error (MSE) of the suggested class are derived upto the first order of approximation. The values of Bias and the minimum MSE are obtained by optimizing the chracterizing scalar. The MSE of the suggested estimator has also been compared with the MSEs of the existing estimators for the comparison of these estimators. Some of the new members of the suggested family are also presented in the paper which are more efficient than the existing ones emperically which has been shown using a natural population.

KEYWORDS: Study Variable, Auxiliary Variables, Simple Random Sampling, Constants, MSE, PRE
MSC: 62D05

RESUMEN
El presente artículo aboga por una mejor estimación de la media de la población a través de una nueva clase de estimadores que utilizan la información conocida sobre una variable auxiliar bajo el esquema de muestreo aleatorio simple. Los estimadores existentes de la media de la población se muestran como los miembros de la clase propuesta. El sesgo y el error cuadrático medio (MSE) de la clase sugerida se derivan hasta el primer orden de aproximación. Los valores de Bias y el MSE mínimo se obtienen optimizando el escalar caracterizador. El MSE del estimador sugerido también se ha comparado con los MSE de los estimadores existentes para la comparación de estos estimadores. Algunos de los nuevos miembros de la familia sugerida también se presentan en el documento, que son más eficientes que los existentes, que se ha demostrado con una población natural.

PALABRAS CLAVE: Variable de estudio, Variables Auxiliares, Muestreo Aleatorio Simple, Constantes, MSE, PRE

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1. INTRODUCTION

The study variable combined with the auxiliary information provides considerably efficient estimation of the population parameters. Cochran (1940) was the first who used the auxiliary information in development of ratio estimator. Ratio estimator is effective when there is positive correlation between the study and auxiliary variable otherwise in case of negative correlation product estimator would be the wiser choice. The parameters of the auxiliary variable such as the mean, median, variance and the coefficient of variation or the combinations of the parameters of auxiliary variable is used as the components of the concerned estimator. So many researchers have studied on the auxiliary variable and the ratio estimators and tried to obtain the minimum as well as the unbiased estimators. As a result we end up having a lot of estimators. If some are useful for one situation others are useful for other situations. We don't have any universal estimator. Different estimators are useful in different situations. Considering all of them is a tuff job for us so we have considered some of them in our literature which are similar to the situation we are studying on.

Upadhyaya and Singh (1991), Kadilar and Cingi (2003), Al-Omari et al.(2009) suggested new improved ratio estimators. Bahl and Tuteja (1991) proposed exponential type of estimators. Jeelani et al.(2013) used the linear combination of coefficient of skewness and quartile deviation for enhanced estimation. Jerajuddin and Kishun (2016) suggested the use of sample size selected from the population to modify the ratio estimator. His estimator did not require the auxiliary information. Singh, Tailor and Kakran (2004) used the power transformation for the improvement. Singh and Tailor(2003), Sisodia and Dwivedi (1981), Yan and Tian (2010), Subramani and Kumarpandiyan (2012), Yadav et al.(2019) took the use of various characteristics of auxiliary information and their combinations to obatain the efficient estimators. Encouraged by these researchers we proposed a generalized class of estimators which has more precision than the existing competing estimators.

We have considered the mean estimation and tried to obtain the efficient estimator under the simple random sampling scheme. Let from the finite population (X, Y) of size N, a bivariate sample $(x_i, y_i); i = 1, 2, \ldots, n$ of size n is taken using SRSWOR scheme. The sample means $\bar{x}$ and $\bar{y}$ are unbiased estimators of population means $\overline{X}$ and $\overline{Y}$ respectively. If we ignore the property of unbiasedness, we would obtain the Mean Square Error much lower than the usual variance. Though we have obtained the biasedness of our proposed estimator yet we focused on minimizing our Mean Square Error (MSE).

1.1. Notations

- N : Size of the population
- n : Size of the sample
- ${}^NC_n$: Number of possible samples of size n from the population of size N
- Y : Study Variable
- X : Auxiliary Variable
- M_y, M_x : Medians
- $\overline{Y}, \overline{X}$: Population means
- $\bar{y}, \bar{x}$: Sample means
- ρ : Correlation Coefficient between X and Y
- β : Regression Coefficient of Y on X

- S_y^2, S_x^2 : Population Mean Squares
- S_{yx} : Covariance between X and Y
- C_y, C_x : Coefficients of Variation
- $Bias(\cdot)$: Bias of the estimator
- $V(\cdot)$: Variance of the estimator
- $\beta_{1(x)}$: Coffecient of Skewness
- $\beta_{2(x)}$: Coeffecient of Kurtosis
- $Q_{1(x)}$: First Quartile
- $Q_{3(x)}$: Third Quartile
- QD : Quartile Deviation
- $Q_{a(x)}$: Quartile Average
- $Q_{r(x)}$: Quartile Range
- TM : Tri Mean
- $MSE(\cdot)$: Mean Squared Error of the Estimator
- $PRE(\bar{y}, t)$: Percentage Relative Efficiency of the proposed estimator with respect to the SRS mean

1.2. Formulae

- $\lambda = \frac{1-f}{n}$
- $f = \frac{n}{N}$
- $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$
- $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $V(\bar{y}) = \lambda \overline{Y}^2 C_y^2$
- $V(\bar{x}) = \lambda \overline{X}^2 C_x^2$
- $C_y = \frac{S_y}{\overline{Y}}$
- $C_x = \frac{S_x}{\overline{X}}$
- $S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \overline{Y}\right)\left(X_i - \overline{X}\right)$
- $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \overline{Y}\right)^2$
- $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \overline{X}\right)^2$
- $\rho = \frac{S_{yx}}{S_x S_y}$
- $d = \rho\frac{C_y}{C_x}$
- $QD = \frac{Q_3 - Q_1}{2}$
- $Q_{a(x)} = \frac{Q_1 + M_x + Q_3}{3}$
- $Q_{r(x)} = Q_3 - Q_1$
- $TM = \frac{Q_1 + 2M_x + Q_3)}{4}$

2. LITERATURE REVIEW OF EXISTING ESTIMATORS

There are a number of estimators which have been developed till date for the elevated estimation of population mean under simple random sampling without replacement (SRSWOR) scheme. Variour estimators from the literature are considered for the review and are presented in the Table-1 along with their Mean Squared Errors and corresponding constants up the approximation of order one.

3. PROPOSED CLASS OF ESTIMATORS

Inspired by the literature presented by Yadav et al. (2019), we suggest an improved class of ratio type estimators for the enhanced estimation of population mean of primary variable using information on secondary variable under SRSWOR as follows:

$$t = k\bar{y}\left(\frac{ab\overline{X} + cd}{ab\bar{x} + cd}\right) \tag{3.1}$$

Table 1: **Literature Review**

SNo	Estimators	MSE/Variance	Constants
1	$t_0 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ Sample Mean	$\lambda \overline{Y}^2 C_y^2$	-
2	$t_1 = \bar{y}\left(\frac{\overline{X}}{\bar{x}}\right)$ Cochran(1940)	$\lambda \overline{Y}^2 \left(C_y^2 + C_x^2 - 2C_{yx}\right)$	-
3	$t_2 = \bar{y}\exp\left(\frac{\overline{X}-\bar{x}}{\overline{X}+\bar{x}}\right)$ Bahl and Tuteja(1991)	$\lambda \overline{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x\right)$	-
4	$t_3 = \bar{y}\left(\frac{\overline{X}+C_x}{\bar{x}+C_x}\right)$ Sisodia and Dwivedi (1981)	$\lambda \overline{Y}^2 \left(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_{yx}\right)$	$\theta_3 = \frac{\overline{X}}{\overline{X}+C_x}$
5	$t_4 = \bar{y}\left(\frac{\overline{X}C_x+\beta_{2(x)}}{\bar{x}C_x+\beta_{2(x)}}\right)$ Upadhyaya and Singh(1999)	$\lambda \overline{Y}^2 \left(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_{yx}\right)$	$\theta_4 = \frac{\overline{X}C_x}{\overline{X}C_x+\beta_{2(x)}}$
6	$t_5 = \bar{y}\left(\frac{\overline{X}\beta_{2(x)}+C_x}{\bar{x}\beta_{2(x)}+C_x}\right)$ Upadhyaya and Singh(1999)	$\lambda \overline{Y}^2 \left(C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_{yx}\right)$	$\theta_5 = \frac{\overline{X}\beta_{2(x)}}{\overline{X}\beta_{2(x)}+C_x}$

where constant k is suitably chosen such that the MSE of the suggested estimator is minimum and the constants a, b, c, d are either the constants or the known parameters of the auxiliary variable. Also (a, b, c, d) are free to take those real and parametric values which makes the MSE minimum.

3.1. Bias and MSE

To derive the Bias and MSE for the suggested class of estimators, we define the following standard approximations as:

$$e_0 = \frac{\bar{y} - \overline{Y}}{\overline{Y}} \quad \text{and} \quad e_1 = \frac{\bar{x} - \overline{X}}{\overline{X}}$$

So,

$$\bar{y} = \overline{Y}(1 + e_0) \quad \text{and} \quad \bar{x} = \overline{X}(1 + e_1)$$

$$E(e_0) = E(e_1) = 0 \quad \text{and} \quad E(e_0 e_1) = \lambda C_{yx}$$

$$E(e_0^2) = \lambda C_y^2 \quad \text{and} \quad E(e_1^2) = \lambda C_x^2.$$

Now rewriting the proposed estimator from equation (3.1) as,

$$
\begin{aligned}
t &= k\overline{Y}(1+e_0)\left[\frac{ab\overline{X}+cd}{ab\overline{X}(1+e_1)+cd}\right] \\
&= k\overline{Y}(1+e_0)(ab\overline{X}+cd)\left[(ab\overline{X}+cd)+ab\overline{X}e_1\right]^{-1} \\
&= k\overline{Y}(1+e_0)\left[\frac{ab\overline{X}e_1}{ab\overline{X}+cd}\right]^{-1} \\
t &= k\overline{Y}(1+e_0)(1+\theta e_1)^{-1} \qquad (3.2)
\end{aligned}
$$

$$\text{Where,} \quad \theta = \frac{ab\overline{X}}{ab\overline{X}+cd}$$

Using the Taylor Series Expansion in (3.2), then multiplying its terms and retaining the terms upto the first order of approximation, we get,

$$
\begin{aligned}
t &= k\overline{Y}(1+e_0-\theta e_1-\theta e_0 e_1+\theta^2 e_1^2) \\
(t-\overline{Y}) &= \overline{Y}[k(1+e_0-\theta e_1-\theta e_0 e_1+\theta^2 e_1^2)-1] \qquad (3.3)
\end{aligned}
$$

Taking expectation on both sides of equation (3.3), we have,

$$
\begin{aligned}
E(t-\overline{Y}) &= \overline{Y}[k(1-\theta\lambda C_{yx}+\theta^2\lambda C_x^2)-1] \\
B(t) &= \overline{Y}[k(1-\theta\lambda C_{yx}+\theta^2\lambda C_x^2)-1] \qquad (3.4)
\end{aligned}
$$

Squaring the equation (3.3), we get,

$$
\begin{aligned}
(t-\overline{Y})^2 &= \overline{Y}^2[k^2(1+e_0-\theta e_1-\theta e_0 e_1+\theta^2 e_1^2)^2+1-2k(1+e_0-\theta e_1-\theta e_0 e_1+\theta^2+e_1^2)] \\
(t-\overline{Y})^2 &= \overline{Y}^2[k^2(1+e_0^2+\theta^2 e_1^2+2e_0-2\theta e_1-2\theta e_0 e_1+2\theta^2 e_1^2-2\theta e_0 e_1)+1-2k(1+e_0-\theta e_1-\theta e_0 e_1+\theta^2 e_1^2)] \\
E[(t-\overline{Y})^2] &= \overline{Y}^2[k^2(1+\lambda C_y^2+\theta^2\lambda C_x^2-4\theta\lambda C_{yx}+2\theta^2\lambda C_x^2)+1-2k(1-\theta\lambda C_{yx}+\theta^2\lambda C_x^2)] \\
MSE(t) &= \overline{Y}^2[k^2(1+\lambda C_y^2+3\theta^2\lambda C_x^2-4\theta\lambda C_{yx})+1-2k(1-\theta\lambda C_{yx}+\theta^2\lambda C_x^2)] \qquad (3.5)
\end{aligned}
$$

Applying the method of maxima and minima for obtaining the optimum value of the constant kappa. The optimum value of k which makes the MSE in equation (3.5) minimum, is given by,

$$\mathbf{k} = \frac{\mathbf{1}-\theta\lambda\mathbf{C_{yx}}+\theta^2\lambda\mathbf{C_x^2}}{\mathbf{1}+\lambda\mathbf{C_y^2}+\mathbf{3}\theta^2\lambda\mathbf{C_x^2}-\mathbf{4}\theta\lambda\mathbf{C_{yx}}} = \frac{\mathbf{A}}{\mathbf{B}} \qquad (3.6)$$

Thus the minimum value of MSE of t is given as,

$$\mathbf{M(t)_{min}} = \overline{\mathbf{Y}}^{\mathbf{2}}\left(\mathbf{1}-\frac{\mathbf{A^2}}{\mathbf{B}}\right) \qquad (3.7)$$

Putting the value of k in equation (3.4), we get the Bias as,

$$\mathbf{B(t)} = \overline{\mathbf{Y}}\left(\frac{\mathbf{A^2}}{\mathbf{B}}-\mathbf{1}\right) \qquad (3.8)$$

3.2. Existing Members of the Proposed Class of Estimators

Many of the existing estimators of population mean Y are identified as the members of the proposed class for different values of (a, b, c, d) in (3.1), which are given in Table 2 as,

3.3. New Members of Proposed Class of Estimators

We have suggested some new members of the proposed class of estimators which come out to be more efficient than the existing estimators of population mean. These are given as follows,

- $t_{(1)} = k_1\bar{y}\left[\frac{\beta_{2(x)}M_x\overline{X}+\rho}{\beta_{2(x)}M_x\bar{x}+\rho}\right]$
- $t_{(2)} = k_2\bar{y}\left[\frac{\beta_{2(x)}M_x\overline{X}+\rho C_x}{\beta_{2(x)}M_x\bar{x}+\rho C_x}\right]$
- $t_{(3)} = k_3\bar{y}\left[\frac{{}^NC_n\overline{X}+M_x}{{}^NC_n\bar{x}+M_x}\right]$
- $t_{(4)} = k_4\bar{y}\left[\frac{M_x\overline{X}+\rho f}{M_x\bar{x}+\rho f}\right]$
- $t_{(5)} = k_5\bar{y}\left[\frac{TMQ_{3(x)}\overline{X}+\frac{Q_{a(x)}}{QD}}{TMQ_{3(x)}\bar{x}+\frac{Q_{a(x)}}{QD}}\right]$
- $t_{(6)} = k_6\bar{y}\left[\frac{QDQ_{a(x)}\overline{X}+\beta_{2(x)}\beta_{1(x)}}{QDQ_{a(x)}\bar{x}+\beta_{2(x)}\beta_{1(x)}}\right]$
- $t_{(7)} = k_7\bar{y}\left[\frac{M_xS_x\overline{X}+\rho\lambda}{M_xS_x\bar{x}+\rho\lambda}\right]$

4. THEORETICAL EFFICIENCY COMPARISON

Following are the conditions under which proposed class of estimators is more efficient than the existing estimators, where,

- $D = \theta(1 - \lambda\rho^2C_x^2 - \theta^2\lambda C_x^3 + 2\theta\lambda C_{yx}) - 6d$
- $D' = \theta D - B\theta_i^2 + 2B\theta_i d \quad ; \quad i = 3, \ldots, 20$

5. COMPUTATIONAL STUDY

To prove the theoretical results numerically we have considered a Natural Population with sample size 5.

Data Source : Daroga Singh and F.S. Chaudhary (1986, Page-177)

Data Details : **Study Variable :**

: Area under wheat in a region during year 1974

: **Auxiliary Variable**

: Cultivated Area under wheat in a region during year 1973

To compute the Percent Relative Efficiency (PRE) for different estimators with respect to Simple Random Sample Mean we use the following :

$$PRE = \frac{V(t_0)}{MSE(\cdot)}$$

6. RESULTS AND CONCLUSION

1. Table 1 Reviews the Existing literature. Table 2 shows the values for which the Existing estimators become special case of the proposed class of estimators. Table 3 shows the conditions for which our proposed class estimators is better than the existing estimators. Table 4 consists of parametric values of the data with which we verified our results empirically. Table 5 shows the MSE and PRE of existing and proposed class of estimators.

2. We study the Bias and MSE of the proposed class up to first order of approximation.Since Efficiency is stronger property than the unbiasedness. Hence here we prefer the biased estimator with minimum MSE instead of unbiased estimator with higher MSE.

3. From table 5 we can easily notice that the proposed class of estimators have lesser MSEs thus greater PREs which proves that our class of proposed estimators is efficient enough for the practical purposes.

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A APPENDIX

R CALCULATION

```
# Secondry data(From Daroga and Singh)
# Calculation of parameters
Y¡-c(50,149,284,381,278,111,
634,278,112,355,99,498,111,6,339,80,
105,27,515,249,85,221,133,144,103,
175,335,219,62,79,60,100,141,263)
X¡-c(70,163,320,440,250,125,558,
254,101,359,109,481,125,5,427,78,75,
45,564,238,92,247,134,131,129,190,
363,235,73,62,71,137,196,255)
# mean
y¡-mean(Y)
x¡-mean(X)
# standard deviation
sy¡-sqrt(var(Y))
sx¡-sqrt(var(X))
# coeffcient of variation
cy¡-sy/y
cx¡-sx/x
# median
mx¡-median(X)
my¡-median(Y)
# correlation coefficient
r¡-cor(X,Y)
cyx¡-r∗ cy∗ cx
library(moments)
# coefficient of skewness b1(beta 1)
be1¡-skewness(X)ˆ 2
# coefficient of kurtosis b2( beta 2)
be2¡-kurtosis(X)+3
N=34
n=5
f=n/N
l=(1-f)/n
Ncn=choose(N,n)
summary(X)
# quartile deviation
qd=(q3-q1)/2
# interquartile range
qr=q3-q1
# quartile average
qa=(q1+mx+q3)/3
# tri mean
tm¡-(q1+2∗ mx+q3)/4
# # Review of literature
# Sample mean
m0¡-l∗ yˆ 2∗ cyˆ 2
# Cochran (1940)
m1¡-l∗ yˆ 2∗ (cyˆ 2+cxˆ 2-(2∗ cyx))
# Bahl and Tuteja(1991)
m2¡-l∗ yˆ 2∗ (cyˆ 2+(cxˆ 2/4)-cyx)
# Function of MSE calculation
m¡-function(c){
mse=l∗ yˆ 2∗ (cyˆ 2+(cˆ 2∗ cxˆ 2)-(2∗ c∗ cyx))
print(mse)
}
# Sisodia and Dwivedi(1981)
c3¡-x/(x+cx)
m(c3)
# # [1] 154.5255
# Upadhyaya and Singh (1999)........
c4¡-(x∗ cx)/((x∗ cx)+be2)
c5¡-(x∗ be2)/((x∗ be2)+cx)
m(c4)
m(c5)
# Singh and Tailor(2003)......
c6¡-x/(x+r)
m(c6)
# singh et al.(2004)........
c7¡-x/(x+be2)
m(c7)
# Al-Omari et al. (2009).
c8¡-x/(x+q1)
m(c8)
```

```
# # Al-Omari et al. (2009)
c9¡-x/(x+q3)
m(c9)
# Yan and Tian(2010)
c10¡-x/(x+be1)
c11¡-(x∗ be1)/((x∗ be1)+be2)
c12¡-(x∗ cx)/((x∗ cx)+be1)
m(c10)
m(c11)
m(c12)
# Subramani and Kumarpandiyan(2012a)
c13¡-x/(x+mx)
c14¡-(x∗ cx)/((x∗ cx)+mx)
m(c13)
m(c14)
# Subramani and Kumarpandiyan(2012b)
c15¡-x/(x+qr)
c16¡-x/(x+qd)
c17¡-x/(x+qa)
m(c15)
m(c16)
m(c17)
# Jeelani et al.(2013).............
c18¡-(x∗ be1)/((x∗ be1)+qd)
m(c18)
# Jerajuddin and Kishun(2016)....
c19¡-x/(x+n)
m(c19)
# Yadav et al. (2019).....
c20_1¡-(be2∗ mx∗ x)/((be2∗ mx∗ x)+(r))
c20_2¡-(be2∗ mx∗ x)/((be2∗ mx∗ x)+(r∗ cx))
c20_3¡-(be1∗ mx∗ x)/((be1∗ mx∗ x)+(r))
c20_4¡-(be1∗ mx∗ x)/((be1∗ mx∗ x)+(r∗ cx))
c20_5¡-(n∗ x)/((n∗ x)+(r))
c20_6¡-(n∗ x)/((n∗ x)+(cx))
c20_7¡-(n∗ x)/((n∗ x)+(cx∗ r))
c20_8¡-(n∗ r∗ x)/((n∗ r∗ x)+(cx))
c20_9¡-(n∗ cx∗ x)/((n∗ cx∗ x)+(r))
m(c20_1)
m(c20_2)
m(c20_3)
m(c20_4)
m(c20_5)
m(c20_6)
m(c20_7)
m(c20_8)
m(c20_9)
# Proposed Estimators.............
mse¡-function(th){
A=1-(th∗ l∗ r∗ cy∗ cx)+(thˆ 2∗ l∗ cxˆ 2)
B=1+(l∗ cyˆ 2)+(thˆ 2∗ l∗ cxˆ 2)-(4∗
th∗ l∗ r∗ cy∗ cx)+(2∗ thˆ 2∗ l∗ cxˆ 2)
M=yˆ 2∗ (1-(Aˆ 2/B))
print(M)
}
th1=((be2∗ mx∗ x)/((be2∗ mx∗ x)+(r)))
mse(th1)
th2=((be2∗ mx∗ x)/((be2∗ mx∗ x)+(r∗ cx)))
mse(th2)
th3=(Ncn∗ x)/((Ncn∗ x)+(mx))
mse(th3)
th4=(mx∗ x)/((mx∗ x)+(f∗ r))
mse(th4)
th5=(tm∗ q3∗ x)/((tm∗ q3∗ x)+(qa/qd))
mse(th5)
th6=(qd∗ qa∗ x)/((qd∗ qa∗ x)+(be2∗ be1))
mse(th6)
th7=(mx∗ sx∗ x)/((mx∗ sx∗ x)+(r∗ l))
mse(th7)
PRE¡-function(me){
PRE¡-(m0/me)∗ 100
print(PRE)
}
```

Table 2: *

Literature Review

SNo	Estimators	MSE/Variance	Constants
7	$t_6 = \bar{y}\left(\frac{\overline{X}+\rho}{\bar{x}+\rho}\right)$ Singh and Tailor(2003)	$\lambda\overline{Y}^2\left(C_y^2+\theta_6^2C_x^2-2\theta_6C_{yx}\right)$	$\theta_6 = \frac{\overline{X}}{\overline{X}+\rho}$
8	$t_7 = \bar{y}\left(\frac{\overline{X}+\beta_{2(x)}}{\bar{x}+\beta_{2(x)}}\right)$ Singh et al.(2004)	$\lambda\overline{Y}^2\left(C_y^2+\theta_7^2C_x^2-2\theta_7C_{yx}\right)$	$\theta_7 = \frac{\overline{X}}{\overline{X}+\beta_{2(x)}}$
9	$t_8 = \bar{y}\left(\frac{\overline{X}+Q_{1(x)}}{\bar{x}+Q_{1(x)}}\right)$ $t_9 = \bar{y}\left(\frac{\overline{X}+Q_{3(x)}}{\bar{x}+Q_{3(x)}}\right)$ Al-Omari et al.(2009)	$\lambda\overline{Y}^2\left(C_y^2+\theta_8^2C_x^2-2\theta_8C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_9^2C_x^2-2\theta_9C_{yx}\right)$	$\theta_8 = \frac{\overline{X}}{\overline{X}+Q_{1(x)}}$ $\theta_9 = \frac{\overline{X}}{\overline{X}+Q_{3(x)}}$
10	$t_{10} = \bar{y}\left(\frac{\overline{X}+\beta_{1(x)}}{\bar{x}+\beta_{1(x)}}\right)$ Yan and Tian(2010)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{10}^2C_x^2-2\theta_{10}C_{yx}\right)$	$\theta_{10} = \frac{\overline{X}}{\overline{X}+\beta_{1(x)}}$
11	$t_{11} = \bar{y}\left(\frac{\overline{X}\beta_{1(x)}+\beta_{2(x)}}{\bar{x}\beta_{1(x)}+\beta_{2(x)}}\right)$ Yan and Tian(2010)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{11}^2C_x^2-2\theta_{11}C_{yx}\right)$	$\theta_{11} = \frac{\overline{X}\beta_{1(x)}}{\overline{X}\beta_{1(x)}+\beta_{2(x)}}$
12	$t_{12} = \bar{y}\left(\frac{\overline{X}C_x+\beta_{1(x)}}{\bar{x}C_x+\beta_{1(x)}}\right)$ Yan and Tian(2010)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{12}^2C_x^2-2\theta_{12}C_{yx}\right)$	$\theta_{12} = \frac{\overline{X}C_x}{\overline{X}C_x+\beta_{1(x)}}$
13	$t_{13} = \bar{y}\left(\frac{\overline{X}+M_x}{\bar{x}+M_x}\right)$ Subramani and Kumarpandiyan(2012a)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{13}^2C_x^2-2\theta_{13}C_{yx}\right)$	$\theta_{13} = \frac{\overline{X}}{\overline{X}+M_x}$
14	$t_{14} = \bar{y}\left(\frac{\overline{X}C_x+M_x}{\bar{x}C_x+M_x}\right)$ Subramani and Kumarpandiyan(2012a)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{14}^2C_x^2-2\theta_{14}C_{yx}\right)$	$\theta_{14} = \frac{\overline{X}C_x}{\overline{X}C_x+M_x}$
15	$t_{15} = \bar{y}\left(\frac{\overline{X}+Q_{r(x)}}{\bar{x}+Q_{r(x)}}\right)$ Subramani and Kumarpandiyan(2012b)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{15}^2C_x^2-2\theta_{15}C_{yx}\right)$	$\theta_{15} = \frac{\overline{X}}{\overline{X}+Q_{r(x)}}$
16	$t_{16} = \bar{y}\left(\frac{\overline{X}+QD}{\bar{x}+QD}\right)$ Subramani and Kumarpandiyan(2012b)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{16}^2C_x^2-2\theta_{16}C_{yx}\right)$	$\theta_{16} = \frac{\overline{X}}{\overline{X}+QD}$

Table 3: *

Literature Review

SNo	Estimators	MSE/Variance	Constants
17	$t_{17} = \bar{y}\left(\frac{\overline{X}+Q_{a(x)}}{\bar{x}+Q_{a(x)}}\right)$ Subramani and Kumarpandiyan(2012b)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{17}^2C_x^2-2\theta_{17}C_{yx}\right)$	$\theta_{17}=\frac{\overline{X}}{\overline{X}+Q_{a(x)}}$
18	$t_{18} = \bar{y}\left(\frac{\overline{X}+n}{\bar{x}+n}\right)$ Jerajuddin and Kishun(2016)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{18}^2C_x^2-2\theta_{18}C_{yx}\right)$	$\theta_{18}=\frac{\overline{X}}{\overline{X}+n}$
19	$t_{19} = \bar{y}\left(\frac{\overline{X}\beta_{1(x)}+QD}{\bar{x}\beta_{1(x)}+QD}\right)$ Jeelani et al.(2013)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{19}^2C_x^2-2\theta_{19}C_{yx}\right)$	$\theta_{19}=\frac{\overline{X}\beta_{1(x)}}{\overline{X}\beta_{1(x)}+QD}$
20	$t_{20} = \bar{y}\left(\frac{ab\overline{X}+cd}{ab\bar{x}+cd}\right)$ $t_{20(1)} = \bar{y}\left(\frac{\beta_{2(x)}M_x\overline{X}+\rho}{\beta_{2(x)}M_x\bar{x}+\rho}\right)$ $t_{20(2)} = \bar{y}\left(\frac{\beta_{2(x)}M_x\overline{X}+\rho C_x}{\beta_{2(x)}M_x\bar{x}+\rho C_x}\right)$ $t_{20(3)} = \bar{y}\left(\frac{\beta_{1(x)}M_x\overline{X}+\rho}{\beta_{1(x)}M_x\bar{x}+\rho}\right)$ $t_{20(4)} = \bar{y}\left(\frac{\beta_{1(x)}M_x\overline{X}+\rho C_x}{\beta_{1(x)}M_x\bar{x}+\rho C_x}\right)$ $t_{20(5)} = \bar{y}\left(\frac{n\overline{X}+\rho}{n\bar{x}+\rho}\right)$ $t_{20(6)} = \bar{y}\left(\frac{n\overline{X}+C_x}{n\bar{x}+C_x}\right)$ $t_{20(7)} = \bar{y}\left(\frac{n\overline{X}+\rho C_x}{n\bar{x}+\rho C_x}\right)$ $t_{20(8)} = \bar{y}\left(\frac{n\rho\overline{X}+C_x}{n\rho\bar{x}+C_x}\right)$ $t_{20(9)} = \bar{y}\left(\frac{nC_x\overline{X}+\rho}{nC_x\bar{x}+\rho}\right)$ Yadav et al.(2019)	$\lambda\overline{Y}^2\left(C_y^2+\theta_{20}^2C_x^2-2\theta_{20}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(1)}^2C_x^2-2\theta_{20(1)}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(2)}^2C_x^2-2\theta_{20(2)}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(3)}^2C_x^2-2\theta_{20(3)}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(4)}^2C_x^2-2\theta_{20(4)}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(5)}^2C_x^2-2\theta_{20(5)}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(6)}^2C_x^2-2\theta_{20(6)}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(7)}^2C_x^2-2\theta_{20(7)}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(8)}^2C_x^2-2\theta_{20(8)}C_{yx}\right)$ $\lambda\overline{Y}^2\left(C_y^2+\theta_{20(9)}^2C_x^2-2\theta_{20(9)}C_{yx}\right)$	$\theta_{20}=\frac{ab\overline{X}}{ab\overline{X}+c.d}$ $\theta_{20(1)}=\frac{\beta_{2(x)}M_x\overline{X}}{\beta_{2(x)}M_x\bar{x}+\rho}$ $\theta_{20(2)}=\frac{\beta_{2(x)}M_x\overline{X}}{\beta_{2(x)}M_x\bar{x}+\rho C_x}$ $\theta_{20(3)}=\frac{\beta_{1(x)}M_x\overline{X}}{\beta_{1(x)}M_x\bar{x}+\rho}$ $\theta_{20(4)}=\frac{\beta_{1(x)}M_x\overline{X}}{\beta_{1(x)}M_x\bar{x}+\rho C_x}$ $\theta_{20(5)}=\frac{n\overline{X}}{n\bar{x}+\rho C_x}$ $\theta_{20(6)}=\frac{n\overline{X}}{n\bar{x}+C_x}$ $\theta_{20(7)}=\frac{n\overline{X}}{n\bar{x}+\rho C_x}$ $\theta_{20(8)}=\frac{n\rho\overline{X}}{n\rho\bar{x}+C_x}$ $\theta_{20(9)}=\frac{nC_x\overline{X}}{nC_x\bar{x}+\rho}$

Table 4: **Some Existing Members of Proposed Class of estimators t**

SNo.	**Estimators**	k	a	b	c	d
1	t_1	1	1	1	0	0
2	t_3	1	1	1	C_x	1
3	t_4	1	C_x	1	$\beta_{2(x)}$	1
4	t_5	1	$\beta_{2(x)}$	1	C_x	1
5	t_6	1	1	1	ρ	1
6	t_7	1	1	1	$\beta_{2(x)}$	1
7	t_8	1	1	1	$Q_1(x)$	1
8	t_9	1	1	1	$Q_3(x)$	1
9	t_{10}	1	1	1	$\beta_{1(x)}$	1
10	t_{11}	1	$\beta_{1(x)}$	1	$\beta_{2(x)}$	1
11	t_{12}	1	C_x	1	$\beta_{1(x)}$	1
12	t_{13}	1	1	1	M_x	1
13	t_{14}	1	C_x	1	M_x	1
14	t_{15}	1	1	1	$Q_{r(x)}$	1
15	t_{16}	1	1	1	$Q_{d(x)}$	1
16	t_{17}	1	1	1	$Q_{a(x)}$	1
17	t_{18}	1	$\beta_{1(x)}$	1	QD	1
18	t_{19}	1	1	1	n	1
20	t_{20}	1	a	b	c	d

Table 5: **Efficiency Comparison**

SNo.	**MSE(t) < MSE(·)**	**Condition**
1	$MSE(t) < V(t_0)$	$C_y^2 > \frac{\theta C_x^2 D}{B-1}$
2	$MSE(t) < MSE(t_1)$	$C_y^2 > \frac{C_x^2(\theta D - B + 2BD)}{B-1}$
3	$MSE(t) < MSE(t_2)$	$C_y^2 > \frac{C_x^2\left(\theta D - \frac{B}{4} + Bd\right)}{B-1}$
4	$MSE(t) < MSE(t_i); i = 3, \ldots 20$	$C_y^2 > \frac{C_x^2 D^{'}}{B-1}$

Table 6: **Parametric Vlues of the Population**

SNo	Information	Data Set
1	N	34
2	n	5
3	$\overline{Y}$	199.4412
4	$\overline{X}$	208.8824
5	S_y	150.215
6	S_x	150.506

Table 7: *

Parametric Vlues of the Population

SNo	Information	Data Set
7	C_y	0.7531797
8	C_x	0.7205298
9	M_y	142.5
10	M_x	150
11	ρ	0.9800867
12	C_{yx}	0.5318817
13	$\beta_{1(x)}$	0.8732281
14	$\beta_{2(x)}$	5.912272
15	f	0.1470588
16	λ	0.1705882
17	${}^{N}C_n$	278256
18	$Q_{1(x)}$	94.25
19	$Q_{3(x)}$	275.75
20	$Q_{r(x)}$	160.5
21	$Q_{a(x)}$	166.3333
22	QD	80.25
23	TM	162.25

Table 8: **MSE and PRE of Estimators**

SNo	**Estimators**	**MSE**	**PRE**
1	t_0	3849.248	100
2	t_1	153.8905	2501.29
3	t_2	1120.88	343.413
4	t_3	154.5255	2491.011
5	t_4	165.4474	2326.57
6	t_5	153.9924	2499.635

Table 9: *

MSE and PRE of Estimators

SNo	Estimators	MSE	PRE
7	t_6	154.7734	2487.021
8	t_7	161.3104	702.2823
9	t_8	548.1055	2386.237
10	t_9	1312.292	293.3224
11	t_{10}	154.6701	2364.667
12	t_{11}	162.7818	2488.682
13	t_{12}	155.0034	2483.331
14	t_{13}	841.4363	457.4616
15	t_{14}	1117.772	344.368
16	t_{15}	893.9771	430.5757
17	t_{16}	473.1776	813.4891
18	t_{17}	922.6805	417.181
19	t_{18}	535.4868	718.8315
20	t_{19}	159.8507	2408.027
21	$t_{20(1)}$	153.8915	2501.275
22	$t_{20(2)}$	153.8912	2501.279
23	$t_{20(3)}$	153.8967	2501.189
24	$t_{20(4)}$	153.895	2501.217
25	$t_{20(5)}$	154.0555	2498.612
26	$t_{20(6)}$	154.0112	2499.33
27	$t_{20(7)}$	154.0088	2499.369
28	$t_{20(8)}$	154.0137	2499.289
29	$t_{20(9)}$	154.121	2497.549
30	$\mathbf{t_{(1)}}$	152.7766	2519.527
31	$\mathbf{t_{(2)}}$	152.7763	2519.532
32	$\mathbf{t_{(3)}}$	152.7761	2519.535
33	$\mathbf{t_{(4)}}$	152.7765	2519.529
34	$\mathbf{t_{(5)}}$	152.7757	2519.542
35	$\mathbf{t_{(6)}}$	152.776	2519.537
36	$\mathbf{t_{(7)}}$	152.7756	2519.544