MULTIPLE OBJECTIVE FRACTIONAL TRANSPORTATION PROBLEM FOR BREAKABLE COMMODITY

Madhuri Jain[1]
Department of Mathematics and Statistics, Banasthali Vidyapith, Banasthali-304022, Rajasthan, India.

ABSTRACT
In the transportation business, different types of materials/items including breakable items such as units made of glass, ceramics, plastics, mud, etc. are transported from various sources to different destinations. In this paper, a multiple objective fractional transportation problem for breakable commodity is formulated and a multiple objective fractional dual is developed. An algorithm is generated to determine an initial efficient basic solution by solving the related lexicographic minimum fractional transportation problem for breakable commodity. The algorithm is supported by a real life example of Ashi India Glass Limited, India for minimizing the multiple cost for transporting glass-wrap of flat glass.

KEYWORDS: Multiple Objective, Fractional Programming, Transportation, Lexicographic, Breakable commodity.

MSC: 90B06, 90C29, 90C32.

RESUMEN
En el negocio de la transportación, diferentes tipos de materiales/ítems incluyen ítems frágiles como unidades hechas de vidrio, cerámica, plástico, barro, etc. Que son transportados desde varias fuentes a diferentes destinos. En este se formula un problema de transporte como uno de múltiple objetivo fraccional transportación, para productos rompibles. Un algoritmo es generado para determinar una solución inicial básica eficiente para resolver el problema fraccional lexicográfico minimal, para productos rompibles relacionados. El algoritmo es soportado por un ejemplo de la vida real de Ashi India Glass Limited, India para minimizar el múltiple costo para transportar láminas de vidrio protegidas.

PALABRAS CLAVE: Multiobjetivo, Programación Fraccional, productos rompibles, Transportación, Lexicográfica, productos rompibles.

1. INTRODUCTION

Transportation problems with fractional objective functions arise in many real life situations e.g., in transportation management situations and in analysis of financial aspects of transportation, where an individual, or a group, or a commodity is faced with the problem of maintaining good ratios between some very important crucial parameters (e.g. total actual to total standard transportation cost, total actual to total standard maintenance cost, total return on total investment etc.) concerned with the transportation of commodities from certain sources to various destinations. The transportation problems with $k > 1, (k = 1,2, \dots, K)$ linear fractional objective functions are very important from practical point of view because they take care of those real life planning problems from the economic world which have the mathematical structure of a transportation problem but are characterized by the existence of several fractional objective functions. Obviously, these objective functions have different units and are measured on different scales. Therefore, it is very difficult for a transportation system decision maker to combine these objective functions into one overall utility function. Multiple objective fractional transportation problems and its variants have been studied by various authors (Cetin and Tiryaki [1]; Doke and Jadhav [2]; Maruti [3]; Porchelvi and Sheela [4]; Sadia et al. [5]) and others.
This paper presents a multiple objective fractional transportation problem for breakable commodity. To determine an initial efficient basic solution for the problem, an algorithm is developed by solving the related lexicographic minimum fractional transportation problem for breakable commodity. The algorithm is illustrated by glass-wrap of flat glass transportation problem of Ashi India Glass Limited, India.

2. MATHEMATICAL FORMULATION

The multiple objective fractional transportation problem for breakable commodity is the problem of minimizing the k scalar-valued fractional objective functions considered except for conflicts among them. It may be stated as:

[1] email: madhuridayalbagh@gmail.com

$$\min w_k = \frac{\sum_i \sum_j p_{ij}^k x_{ij}}{\sum_i \sum_j q_{ij}^k x_{ij}} \tag{1}$$

subject to

$$\sum_j x_{ij} = s_i \tag{2}$$

$$\sum_i x_{ij} = d_j \tag{3}$$

$$\sum_i r_{ijl} x_{ij} \le \theta_{jl} \tag{4}$$

$$x_{ij} \ge 0 \qquad \forall\, i \text{ and } j \tag{5}$$

$$(i = 1,2,\dots,M; j = 1,2,\dots,N; l = 1,2,\dots,L; k = 1,2,\dots,K)$$

where

s_i = amount of the breakable commodity available at origin i

d_j = requirement of the breakable commodity at destination j

x_{ij} = amount of the breakable commodity transported from origin i to destination j

r_{ijl} = units of L breakages $(l = 1,2,\dots,L)$ in one unit of the breakable commodity when it is transported from origin i to destination j

θ_{jl} = units of highest amount of breakage l of the breakable commodity that can be received by destination j

$\frac{p_{ij}^k}{q_{ij}^k}$ = the proportional contribution to the value of k^{th} fractional objective function of shipping one unit of breakable commodity from origin i to destination j.

Here s_i and d_j are given non-negative numbers and

$$\sum_i s_i = \sum_j d_j \tag{6}$$

3. LEXICOGRAPHIC MINIMUM FRACTIONAL TRANSPORTATION PROBLEM FOR BREAKABLE COMMODITY

Let $\widetilde{M} = \{1,2,\dots,M\}$, $\widetilde{N} = \{1,2,\dots,N\}$, $\widetilde{L} = \{1,2,\dots,L\}$, $\tilde{J} = \{(i,j) \mid i \in \widetilde{M}, j \in \widetilde{N}\}$. The initial efficient basic solution for the problem (1) to (5) can be obtained by solving the following formulated lexicographic minimum fractional transportation problem for breakable commodity:

$$\operatorname{lexmin}\left[w = \frac{\sum_{(i,j)\in\tilde{J}} p_{ij} x_{ij}}{\sum_{(i,j)\in\tilde{J}} q_{ij} x_{ij}} \;\middle|\; \begin{array}{ll} \sum_{j\in\widetilde{N}} x_{ij} = s_i & \forall\, i \in \widetilde{M} \\ \sum_{i\in\widetilde{M}} x_{ij} = d_j & \forall\, j \in \widetilde{N} \\ \sum_{i\in\widetilde{M}} r_{ijl} x_{ij} \le \theta_{jl} & \\ x_{ij} \ge 0, & \forall\, (i,j) \in \tilde{J} \end{array} \right] \tag{7}$$

where $w \in \mathbb{R}^k$, $p_{ij} = (p_{ij}^1, \dots, p_{ij}^k)^T$ and $q_{ij} = (q_{ij}^1, \dots, q_{ij}^k)^T$. The additional breakage restrictions can be written as:

$$\sum_{i\in\widetilde{M}} r_{ijl} x_{ij} + x_{M+l,j} = \theta_{jl} \tag{8}$$

$$x_{M+l,j} \ge 0 \tag{9}$$

where $x_{M+l,j}$ are the slack variables and a feasible basic solution will consist of $NL + M + N - 1$ basic variables.

Assumptions:

i) $s_i > 0, i \in \widetilde{M}$; $d_j > 0,\ j \in \widetilde{N}$ and

$$\sum_{i\in\widetilde{M}} s_i = \sum_{j\in\widetilde{N}} d_j \tag{10}$$

i.e. total destination requirement equals the total origin capacity. This condition ensures the existence of a feasible solution to the problem (7)

ii) $\sum_{(i,j)\in\tilde{J}} q_{ij} x_{ij} > 0$ for all feasible solutions.

Definition: The solution $x^* = (x_{11}^*, x_{12}^*, \dots\dots, x_{MN}^*)$ is said to be an efficient or non dominated solution for (7) if and only if there is no other feasible solution x^{**} for (7) such that

$$w^{**} = \frac{\sum_{(i,j)\in\tilde{J}} p_{ij} x_{ij}^{**}}{\sum_{(i,j)\in\tilde{J}} q_{ij} x_{ij}^{**}} \le \frac{\sum_{(i,j)\in\tilde{J}} p_{ij} x_{ij}^{*}}{\sum_{(i,j)\in\tilde{J}} q_{ij} x_{ij}^{*}} = w^* \tag{11}$$

Note that a feasible solution for the problem (1) to (5), which is a unique optimal solution with respect to the scalar-valued objective function w_1, is an optimal basic solution for (7), and therefore, an efficient basic solution for the problem (1) to (5). But, if there is no unique optimal basic solution with respect to the scalar-valued objective function w_1, some of these optimal basic solutions may not be efficient solution for the problem (1) to (5). By the lexicographic minimum fractional transportation problem for breakable commodity denoted by (7), one feasible basic solution, which is optimal with respect to w_1, is identified that is an efficient basic solution for the problem (1) to (5).

Remark: Let $\mathbb{R}$ denote the set of the real numbers, $\mathbb{R}^0$ the set of the non-negative real numbers. With regard to lexicographic vector inequalities, the following convention will be applied: For $a, b \in \mathbb{R}^h$, the strict lexicographic inequality $a \succ b$ holds, if and only if, $a_{\tilde{c}} \succ b_{\tilde{c}}$ holds for $\tilde{c} = \min\{c|\ c = 1, 2, \ldots\ldots, h;\ a_c \neq b_c\}$ and the weak lexicographic inequality $a \succsim b$ holds, if and only if, $a \succ b$ or $a = b$.

4. VECTOR-VALUED DUAL VARIABLES AND OPTIMALITY CONDITIONS

Consider the k-component vector-valued dual variables (simplex multipliers) u_i^{k1}, u_i^{k2} $(i \in \tilde{M})$; v_j^{k1}, v_j^{k2} $(j \in \tilde{N})$; $\lambda_{jl}^{k1}, \lambda_{jl}^{k2}$ $(j \in \tilde{N}, l \in \tilde{L})$ defined such that

$$p_{ij}^k - \left(u_i^{k1} + v_j^{k1} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k1} r_{ijl}\right) = 0 \tag{12}$$

$$q_{ij}^k - \left(u_i^{k2} + v_j^{k2} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k2} r_{ijl}\right) = 0 \tag{13}$$

(for those i, j for which x_{ij} is in the basis)

and

$$\lambda_{jl}^{k1} = 0 \tag{14}$$

$$\lambda_{jl}^{k2} = 0 \tag{15}$$

(for those j, l for which $x_{M+l,j}$ is in the basis)

Also let,

$$p_{ij}^{k\,\prime} = p_{ij}^k - \left(u_i^{k1} + v_j^{k1} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k1} r_{ijl}\right) \tag{16}$$

$$q_{ij}^{k\,\prime} = q_{ij}^k - \left(u_i^{k2} + v_j^{k2} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k2} r_{ijl}\right) \tag{17}$$

Let

$$w = \frac{\sum_{(i,j)\in J} p_{ij}^k x_{ij}}{\sum_{(i,j)\in J} q_{ij}^k x_{ij}} = \frac{A}{B} \tag{18}$$

Then

$$A = \sum_{i\in\tilde{M}} \sum_{j\in\tilde{N}} p_{ij}^k x_{ij} + \sum_{i\in\tilde{M}} u_i^{k1}\left(s_i - \sum_{j\in\tilde{N}} x_{ij}\right) + \sum_{j\in\tilde{N}} v_j^{k1}\left(d_j - \sum_{i\in\tilde{M}} x_{ij}\right) + \sum_{j\in\tilde{N}} \sum_{l\in\tilde{L}} \lambda_{jl}^{k1}\left(\theta_{jl} - \sum_{i\in\tilde{M}} r_{ijl} x_{ij} - x_{M+l,j}\right) \tag{19}$$

Since the quantities in parenthesis are zero from the supply and demand constraints of (7) and additional breakage restrictions (8); therefore, A of equation (18) is equal to equation (19) or

$$A = \left[\sum_{(i,j)\in E} p_{ij}^{k\,\prime} x_{ij} - \sum_{(j,l)\in E_1} \lambda_{jl}^{k1} x_{M+l,j} + V_N^k\right] \tag{20}$$

where $\sum_{(i,j)\in E}$ and $\sum_{(j,l)\in E_1}$ denote the summation extending over the set of non-basic variables x_{ij} and $x_{M+l,j}$ respectively.

And

$$V_N^k = \left[\sum_{i\in\tilde{M}} s_i u_i^{k1} + \sum_{j\in\tilde{N}} d_j v_j^{k1} + \sum_{j\in\tilde{N}} \sum_{l\in\tilde{L}} \lambda_{jl}^{k1} \theta_{jl}\right] \tag{21}$$

Similarly

$$B = \left[\sum_{(i,j)\in E} q_{ij}^{k\,\prime} x_{ij} - \sum_{(j,l)\in E_1} \lambda_{jl}^{k2} x_{M+l,j} + V_D^k\right] \tag{22}$$

where

$$V_D^k = \left[\sum_{i\in\tilde{M}} s_i u_i^{k2} + \sum_{j\in\tilde{N}} d_j v_j^{k2} + \sum_{j\in\tilde{N}} \sum_{l\in\tilde{L}} \lambda_{jl}^{k2} \theta_{jl}\right] \tag{23}$$

Therefore, the objective function (18) becomes

$$w = \frac{A}{B} = \frac{\left[\sum_{(i,j)\in E} p_{ij}^{k\,\prime} x_{ij} - \sum_{(j,l)\in E_1} \lambda_{jl}^{k1} x_{M+l,j} + V_N^k\right]}{\left[\sum_{(i,j)\in E} q_{ij}^{k\,\prime} x_{ij} - \sum_{(j,l)\in E_1} \lambda_{jl}^{k2} x_{M+l,j} + V_D^k\right]} \tag{24}$$

Now from (24), differentiating w with respect to the non-basic variables x_{ij} (i, j ranging over the set E), and let $\left[\frac{\partial w}{\partial x_{ij}}\right]_*$ denote the value of $\left[\frac{\partial w}{\partial x_{ij}}\right]$ at the feasible basic solution x^*, then

$$\left[\frac{\partial w}{\partial x_{ij}}\right]_* = \frac{V_D^k p_{ij}^{k\,\prime} - V_N^k q_{ij}^{k\,\prime}}{\left[V_D^k\right]^2} \tag{25}$$

Again from (24), differentiating w with respect to the non-basic variables $x_{M+l,j}$ (j, l ranging over the set E_1), and let $\left[\frac{\partial w}{\partial x_{M+l,j}}\right]_*$ denote the value of $\left[\frac{\partial w}{\partial x_{M+l,j}}\right]$ at the feasible basic solution x^*, then

$$\left[\frac{\partial w}{\partial x_{M+l,j}}\right]_* = \frac{V_N^k\,\lambda_{jl}^{k2}\ -V_D^k\lambda_{jl}^{k1}}{\left[V_D^k\right]^2} \tag{26}$$

Due to arguments similar to those of Swarup [6], the optimality criteria comes out to be

$$\delta_{ij}^k = \left[V_D^k p_{ij}^{k\,\prime} - V_N^k q_{ij}^{k\,\prime}\right] \geq 0 \tag{27}$$

and

$$\delta_{M+l,j}^k = \left[V_N^k\,\lambda_{jl}^{k2}\ \ - V_D^k\lambda_{jl}^{k1}\right] \geq 0 \tag{28}$$

5. MULTIPLE OBJECTIVE FRACTIONAL DUAL AND OPTIMALITY CONDITIONS

The multiple objective fractional dual of the k-component multiple objective fractional transportation problem for breakable commodity is derived as:

$$\text{lexmax } W = \frac{\sum_{i\in\widetilde{M}} s_i u_i^{k1} + \sum_{j\in\widetilde{N}} d_j v_j^{k1} + \sum_{j\in\widetilde{N}}\sum_{l\in\widetilde{L}} \lambda_{jl}^{k1}\theta_{jl}}{\sum_{i\in\widetilde{M}} s_i u_i^{k2} + \sum_{j\in\widetilde{N}} d_j v_j^{k2} + \sum_{j\in\widetilde{N}}\sum_{l\in\widetilde{L}} \lambda_{jl}^{k2}\theta_{jl}} = \frac{V_N^k}{V_D^k}$$

subject to

$$\left[V_D^k\left\{\left(u_i^{k1} + v_j^{k1} + \sum_{l\in\widetilde{L}} \lambda_{jl}^{k1} r_{ijl}\right) - p_{ij}^k\right\} - V_N^k\left\{\left(u_i^{k2} + v_j^{k2} + \sum_{l\in\widetilde{L}} \lambda_{jl}^{k2} r_{ijl}\right) - q_{ij}^k\right\}\right] \leq 0$$

$$\left[\,V_D^k\lambda_{jl}^{k1} - V_N^k\,\lambda_{jl}^{k2}\right]\ \leq 0$$

$$V_D^k \geq 0$$

and $u_i^{k1}, u_i^{k2}; v_j^{k1}, v_j^{k2}; \lambda_{jl}^{k1}, \lambda_{jl}^{k2}$, are unrestricted in sign where $u_i^{k1}, u_i^{k2}; v_j^{k1}, v_j^{k2}; \lambda_{jl}^{k1}, \lambda_{jl}^{k2}$, are k-component vector-valued dual variables.

Now by the main duality theorem of fractional programming (Swarup [6]):

$$\frac{\sum_{(i,j)\in\widetilde{J}} p_{ij}^k x_{ij}}{\sum_{(i,j)\in\widetilde{J}} q_{ij}^k x_{ij}} = \frac{\sum_{i\in\widetilde{M}} s_i u_i^{k1} + \sum_{j\in\widetilde{N}} d_j v_j^{k1} + \sum_{j\in\widetilde{N}}\sum_{l\in\widetilde{L}} \lambda_{jl}^{k1}\theta_{jl}}{\sum_{i\in\widetilde{M}} s_i u_i^{k2} + \sum_{j\in\widetilde{N}} d_j v_j^{k2} + \sum_{j\in\widetilde{N}}\sum_{l\in\widetilde{L}} \lambda_{jl}^{k2}\theta_{jl}}$$

giving

$$\sum_{(i,j)\in\widetilde{J}} q_{ij}^k x_{ij}\left[\sum_{(i,j)\in\widetilde{J}}\left(u_i^{k1} + v_j^{k1} + \sum_{l\in\widetilde{L}} \lambda_{jl}^{k1} r_{ijl}\right) x_{ij} + \sum_{j\in\widetilde{N}}\sum_{l\in\widetilde{L}} \lambda_{jl}^{k1} x_{M+l,j}\right]$$

$$= \sum_{(i,j)\in\widetilde{J}} p_{ij}^k x_{ij}\left[\sum_{(i,j)\in\widetilde{J}}\left(u_i^{k2} + v_j^{k2} + \sum_{l\in\widetilde{L}} \lambda_{jl}^{k2} r_{ijl}\right) x_{ij} + \sum_{j\in\widetilde{N}}\sum_{l\in\widetilde{L}} \lambda_{jl}^{k2} x_{M+l,j}\right]$$

$$\Rightarrow\ \sum_{(i,j)\in\widetilde{J}}\left[V_D^k\left\{\left(u_i^{k1} + v_j^{k1} + \sum_{l\in\widetilde{L}} \lambda_{jl}^{k1} r_{ijl}\right) - p_{ij}^k\right\} - V_N^k\left\{\left(u_i^{k2} + v_j^{k2} + \sum_{l\in\widetilde{L}} \lambda_{jl}^{k2} r_{ijl}\right) - q_{ij}^k\right\}\right] x_{ij}$$

$$+ \sum_{(i,j)\in\widetilde{J}}\left[V_D^k\lambda_{jl}^{k1} - V_N^k\lambda_{jl}^{k2}\right] x_{M+l,j} = 0$$

From the dual constraints each term in above equation is less than or equal to zero.
Hence

$$\left[V_D^k p_{ij}^{k\,\prime} - V_N^k q_{ij}^{k\,\prime}\right] x_{ij} = 0$$

And

$$\left[V_N^k\lambda_{jl}^{k2} - V_D^k\lambda_{jl}^{k1}\right] x_{M+l,j} = 0$$

i.e. for $x_{ij} > 0$, $\left[V_D^k p_{ij}^{k\,\prime} - V_N^k q_{ij}^{k\,\prime}\right] = 0$

for $x_{M+l,j} > 0$, $\left[V_N^k\lambda_{jl}^{k2} - V_D^k\lambda_{jl}^{k1}\right] = 0$

Therefore, the optimality criteria are:

For basic variables

$$\delta_{ij}^k = \left[V_D^k p_{ij}^{k\,\prime} - V_N^k q_{ij}^{k\,\prime}\right] = 0$$

$$\delta_{M+l,j}^k = \left[V_N^k\lambda_{jl}^{k2} - V_D^k\lambda_{jl}^{k1}\right] = 0$$

For non-basic variables

$$\delta_{ij}^k = \left[V_D^k p_{ij}^{k\,\prime} - V_N^k q_{ij}^{k\,\prime}\right] \geq 0 \tag{29}$$

$$\delta_{M+l,j}^k = \left[V_N^k\lambda_{jl}^{k2} - V_D^k\lambda_{jl}^{k1}\right] \geq 0 \tag{30}$$

where

$$p_{ij}^{k\,\prime} = p_{ij}^k - \left(u_i^{k1} + v_j^{k1} + \sum_{l\in\widetilde{L}} \lambda_{jl}^{k1} r_{ijl}\right) \tag{31}$$

$$q_{ij}^{k}{}' = q_{ij}^{k} - \left(u_i^{k2} + v_j^{k2} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k2} r_{ijl}\right) \quad (32)$$
$$V_N^k = \left[\sum_{i\in\tilde{M}} s_i u_i^{k1} + \sum_{j\in\tilde{N}} d_j v_j^{k1} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{k1}\,\theta_{jl}\right] \quad (33)$$

$$V_D^k = \left[\sum_{i\in\tilde{M}} s_i u_i^{k2} + \sum_{j\in\tilde{N}} d_j v_j^{k2} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{k2}\,\theta_{jl}\right] \quad (34)$$

Lemma. If $\boldsymbol{X^{\alpha}} = \left(\boldsymbol{x_{ij}^{\alpha}}, \boldsymbol{x_{M+l,j}^{\alpha}}\right)$, $(\boldsymbol{i},\boldsymbol{j}) \in \tilde{\boldsymbol{J}}; \boldsymbol{l} \in \tilde{\boldsymbol{L}}$, is any feasible solution to $\boldsymbol{k}$-component fractional cost objective function in (7) and $\boldsymbol{k}$-component vector valued variables $\boldsymbol{u_i^{\alpha^{k1}}}$, $\boldsymbol{u_i^{\alpha^{k2}}}$, $\boldsymbol{v_j^{\alpha^{k1}}}$, $\boldsymbol{v_j^{\alpha^{k2}}}$, $\boldsymbol{\lambda_{jl}^{\alpha^{k1}}}$, $\boldsymbol{\lambda_{jl}^{\alpha^{k2}}}$ be any feasible solution to $\boldsymbol{k}$-component fractional cost objective function $\boldsymbol{W}$ defined as:

$$lexmax\ W = \frac{\sum_{i\in\tilde{M}} s_i u_i^{k1} + \sum_{j\in\tilde{N}} d_j v_j^{k1} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{k1}\theta_{jl}}{\sum_{i\in\tilde{M}} s_i u_i^{k2} + \sum_{j\in\tilde{N}} d_j v_j^{k2} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{k2}\theta_{jl}} = \frac{V_N^k}{V_D^k} \quad (35)$$

subject to

$$\left[V_D^k\left\{\left(u_i^{k1} + v_j^{k1} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k1} r_{ijl}\right) - p_{ij}^k\right\} - V_N^k\left\{\left(u_i^{k2} + v_j^{k2} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k2} r_{ijl}\right) - q_{ij}^k\right\}\right] y_{ij} \geq 0 \quad (36)$$
$$\left[V_D^k \lambda_{jl}^{k1} - V_N^k\, \lambda_{jl}^{k2}\right] y_{M+l,j} \geq 0 \quad (37)$$
$$V_D^k \geq 0 \quad (38)$$

and u_i^{k1}, u_i^{k2}, v_j^{k1}, v_j^{k2}, $\lambda_{jl}^{k1}, \lambda_{jl}^{k2}$, are unrestricted in sign where $\left\{Y = \left(y_{ij}; y_{M+l,j}\right) \in \mathbb{R}_0 \,|\mathbb{R}_0 \text{ is the set of non} - \text{negative numbers}\right\}$. Let E^{α} denote the set of all feasible solutions for k-component fractional cost objective function in (7) and E^{β} denote the set of all efficient solutions for k-component fractional cost objective function W in (35). Then

$$\frac{\sum_{(i,j)\in\tilde{J}} p_{ij}^k x_{ij}^{\alpha}}{\sum_{(i,j)\in\tilde{J}} q_{ij}^k x_{ij}^{\alpha}} \leq \frac{\sum_{i\in\tilde{M}} s_i u_i^{\alpha^{k1}} + \sum_{j\in\tilde{N}} d_j v_j^{\alpha^{k1}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k1}}\theta_{jl}}{\sum_{i\in\tilde{M}} s_i u_i^{\alpha^{k2}} + \sum_{j\in\tilde{N}} d_j v_j^{\alpha^{k2}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k2}}\theta_{jl}}$$

does not hold.

Proof: Since $x_{ij}^{\alpha} \geq 0$and $x_{M+l,j}^{\alpha} \geq 0$, therefore from (7) and (9):

$$V_D^k\left\{\left(\sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} u_i^{\alpha^{k1}} x_{ij}^{\alpha} + \sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} v_j^{\alpha^{k1}} x_{ij}^{\alpha} + \sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k1}} r_{ijl}\, x_{ij}^{\alpha}\right) - \sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} p_{ij}^k x_{ij}^{\alpha}\right\}$$
$$- V_N^k\left\{\left(\sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} u_i^{\alpha^{k2}} x_{ij}^{\alpha} + \sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} v_j^{\alpha^{k2}} x_{ij}^{\alpha} + \sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k2}} r_{ijl}\, x_{ij}^{\alpha}\right) - \sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} q_{ij}^k x_{ij}^{\alpha}\right\}$$
$$+V_D^k\left\{\sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k1}}\, x_{M+l,j}^{\alpha}\right\} - V_N^k\left\{\sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k2}}\, x_{M+l,j}^{\alpha}\right\}$$

for no $X^{\alpha} \in E^{\alpha}$ replacing the values

$$V_D^k\left(\sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} \boldsymbol{u_i^{\alpha^{k1}}}\, x_{ij}^{\alpha} + \sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} \boldsymbol{v_j^{\alpha^{k1}}} x_{ij}^{\alpha} + \sum_{l\in\tilde{L}} \boldsymbol{\lambda_{jl}^{\alpha^{k1}}}\theta_{jl}\right)$$
$$- V_N^k\left(\sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} \boldsymbol{u_i^{\alpha^{k2}}}\, x_{ij}^{\alpha} + \sum_{i\in\tilde{M}}\sum_{j\in\tilde{N}} \boldsymbol{v_j^{\alpha^{k2}}} x_{ij}^{\alpha} + \sum_{l\in\tilde{L}} \boldsymbol{\lambda_{jl}^{\alpha^{k2}}}\theta_{jl}\right)$$
$$+\mathrm{V_D^k}\left\{\sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k1}}\, \mathrm{x}_{M+l,j}^{\alpha}\right\} - \mathrm{V_N^k}\left\{\sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k2}}\, \mathrm{x}_{M+l,j}^{\alpha}\right\}$$

for no $X^{\alpha} \in E^{\alpha}$ and using (38)

$$\frac{\sum_{(i,j)\in\tilde{J}} p_{ij}^k x_{ij}^{\alpha}}{\sum_{(i,j)\in\tilde{J}} q_{ij}^k x_{ij}^{\alpha}} \leq \frac{\sum_{i\in\tilde{M}} s_i u_i^{\alpha^{k1}} + \sum_{j\in\tilde{N}} d_j v_j^{\alpha^{k1}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k1}}\theta_{jl}}{\sum_{i\in\tilde{M}} s_i u_i^{\alpha^{k2}} + \sum_{j\in\tilde{N}} d_j v_j^{\alpha^{k2}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\alpha^{k2}}\theta_{jl}}$$

for no $X^{\alpha} \in E^{\alpha}$

Hence the result.

Theorem 1. If $\boldsymbol{X^{\beta}} = \left(\boldsymbol{x_{ij}^{\beta}}, \boldsymbol{x_{M+l,j}^{\beta}}\right)$, $(\boldsymbol{i},\boldsymbol{j}) \in \tilde{\boldsymbol{J}}; \boldsymbol{l} \in \tilde{\boldsymbol{L}}$, is any feasible solution to $\boldsymbol{k}$-component fractional cost objective function in (7). The solution $\boldsymbol{X^{\beta}}$ is an efficient solution for $\boldsymbol{k}$-component fractional cost objective function of (7) iff there exist a feasible solution for $\boldsymbol{k}$-component fractional cost objective function of (7) such that

$$\frac{\sum_{(i,j)\in\tilde{J}} p_{ij}^k x_{ij}^{\beta}}{\sum_{(i,j)\in\tilde{J}} q_{ij}^k x_{ij}^{\beta}} = \frac{\sum_{i\in\tilde{M}} s_i u_i^{\beta^{k1}} + \sum_{j\in\tilde{N}} d_j v_j^{\beta^{k1}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k1}}\theta_{jl}}{\sum_{i\in\tilde{M}} s_i u_i^{\beta^{k2}} + \sum_{j\in\tilde{N}} d_j v_j^{\beta^{k2}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k2}}\theta_{jl}}$$

then the solution is itself an efficient solution for k-component fractional cost objective function W in (35).

Proof: By Assumption

$$\frac{\sum_{(i,j)\in\tilde{J}} p_{ij}^{k} x_{ij}^{\beta}}{\sum_{(i,j)\in\tilde{J}} q_{ij}^{k} x_{ij}^{\beta}} = \frac{\sum_{i\in\tilde{M}} s_i u_i^{\beta^{k1}} + \sum_{j\in\tilde{N}} d_j v_j^{\beta^{k1}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k1}} \theta_{jl}}{\sum_{i\in\tilde{M}} s_i u_i^{\beta^{k2}} + \sum_{j\in\tilde{N}} d_j v_j^{\beta^{k2}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k2}} \theta_{jl}}$$

Applying Lemma for any feasible solution X^{α} to k-component fractional cost objective function w in (7),

$$\frac{\sum_{(i,j)\in\tilde{J}} p_{ij}^{k} x_{ij}^{\alpha}}{\sum_{(i,j)\in\tilde{J}} q_{ij}^{k} x_{ij}^{\alpha}} \leq \frac{\sum_{i\in\tilde{M}} s_i u_i^{\beta^{k1}} + \sum_{j\in\tilde{N}} d_j v_j^{\beta^{k1}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k1}} \theta_{jl}}{\sum_{i\in\tilde{M}} s_i u_i^{\beta^{k2}} + \sum_{j\in\tilde{N}} d_j v_j^{\beta^{k2}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k2}} \theta_{jl}} = \frac{\sum_{(i,j)\in\tilde{J}} p_{ij}^{k} x_{ij}^{\beta}}{\sum_{(i,j)\in\tilde{J}} q_{ij}^{k} x_{ij}^{\beta}}$$

for no $X^{\alpha} \in E^{\alpha}$

Hence $X^{\beta} \in E^{\beta}$. Similar arguments hold for k-component fractional cost objective function W in (35).

Theorem 2. Let $\boldsymbol{X^{\beta} = \left(x_{ij}^{\beta}, x_{M+l,j}^{\beta}\right)^{T}}$, $\boldsymbol{(i,j) \in \tilde{J}; l \in \tilde{L}}$, be a feasible solution to $\boldsymbol{k}$-component fractional cost objective function $\boldsymbol{w}$ in (7) if

$$\sum_{(i,j)\in\tilde{J}} \delta_{ij}^{\beta} y_{ij} \leq 0, \qquad y_{ij} \in \mathbb{R}_0 \; for \; (i,j) \in \tilde{J} \tag{39}$$

$$\sum_{(i,j)\in\tilde{J}} \delta_{M+l,j}^{\beta} y_{M+l,j} \leq 0, \qquad y_{M+l,j} \in \mathbb{R}_0 \; for \; j \in \tilde{N}, l \in \tilde{L} \tag{40}$$

has no solution $Y = \left(y_{ij}, y_{M+l,j}\right)$, then X^{β} is an efficient solution for k-component fractional cost objective function w in (7).

Proof: Let (39) and (40) have no solution. Since from the duality theorem

$$\sum_{(i,j)\in\tilde{J}} \delta_{ij}^{\beta} x_{ij} \leq 0$$

$$\sum_{(i,j)\in\tilde{J}} \delta_{M+l,j}^{\beta} x_{M+l,j} \leq 0 \qquad \text{holds for no } X \in E^{\alpha}$$

$$\Rightarrow \sum_{(i,j)\in\tilde{J}} \left[V_D^k \left\{\left(u_i^{\beta^{k1}} + v_j^{\beta^{k1}} + \sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k1}} r_{ijl}\right) - p_{ij}^k\right\} - V_N^k \left\{\left(u_i^{\beta^{k2}} + v_j^{\beta^{k2}} + \sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k2}} r_{ijl}\right) - q_{ij}^k\right\}\right] x_{ij}$$

$$+ \sum_{(i,j)\in\tilde{J}} \left[V_D^k \lambda_{jl}^{\beta^{k1}} - V_N^k \lambda_{jl}^{\beta^{k2}}\right] x_{M+l,j} \leq 0 \text{ holds for no } X \in E^{\alpha}$$

$$\Rightarrow \qquad \frac{\sum_{(i,j)\in\tilde{J}} p_{ij}^{k} x_{ij}}{\sum_{(i,j)\in\tilde{J}} q_{ij}^{k} x_{ij}} \leq \frac{\sum_{i\in\tilde{M}} s_i u_i^{\beta^{k1}} + \sum_{j\in\tilde{N}} d_j v_j^{\beta^{k1}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k1}} \theta_{jl}}{\sum_{i\in\tilde{M}} s_i u_i^{\beta^{k2}} + \sum_{j\in\tilde{N}} d_j v_j^{\beta^{k2}} + \sum_{j\in\tilde{N}}\sum_{l\in\tilde{L}} \lambda_{jl}^{\beta^{k2}} \theta_{jl}} \text{ holds for no } X \in E^{\alpha}$$

$$\Rightarrow \qquad \frac{V_N^k}{V_D^k} \leq \frac{V_N^{\beta^k}}{V_D^{\beta^k}} \text{ holds for no } X \in E^{\alpha}$$

where last inequality follows from the fact that V_D^k, $V_D^{\beta^k} > 0$. Hence $X^{\beta} \in E^{\beta}$.

6. THE ALGORITHM

The complete solution procedure to enumerate optimal and all efficient solutions in a finite number of iterations is explored in the steps given below:

Step 1: Determine an initial feasible basic solution X^1 to the multiple objective fractional transportation problem for breakable commodity by inspection method.

Step 2: Determine recursively the k-dimensional vector-valued dual variables u_i^{k1}, u_i^{k2} $(i \in \tilde{M})$; $v_j^{k1}, v_j^{k2}, (j \in \tilde{N})$; $\lambda_{jl}^{k1}, \lambda_{jl}^{k2}$ $(j \in \tilde{N}, l \in \tilde{L})$ defined such that

$$p_{ij}^k - \left(u_i^{k1} + v_j^{k1} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k1} r_{ijl}\right) = 0 \tag{41}$$

$$q_{ij}^k - \left(u_i^{k2} + v_j^{k2} + \sum_{l\in\tilde{L}} \lambda_{jl}^{k2} r_{ijl}\right) = 0 \tag{42}$$

(for those i, j for which x_{ij} is in the basis), and

$$\lambda_{jl}^{k1} = 0 \tag{43}$$

$$\lambda_{jl}^{k2} = 0 \tag{44}$$

(for those j, l for which $x_{M+l,j}$ is in the basis)

Step 3: Designate the set of pairs of indices (i,j) of the basic variables by H. Evaluate the relative criterion vectors:

$$\delta_{ij}^k = \left[V_D^k p_{ij}^{k'} - V_N^k q_{ij}^{k'}\right]$$

and

$$\delta_{M+l,j}^k = \left[V_N^k \lambda_{jl}^{k2} - V_D^k \lambda_{jl}^{k1}\right], \text{ for all } (i,j) \in \tilde{J}\backslash H$$

Here $p_{ij}^{k'}, q_{ij}^{k'}$ and V_N^k, V_D^k are given by equations (31)-(34).

Step 4: If δ_{ij}^k, $\delta_{M+l,j}^k$ are lexicographically greater than or equal to the zero vector for all $(i,j) \in \tilde{J}\backslash H$, then the current feasible basic solution is optimal which implies going to Step 7, otherwise go to Step 5.

Step 5: Select

$$\begin{bmatrix} \delta_{i_* j_*}^k \\ \delta_{M+l_* j_*}^k \end{bmatrix} = lexmin \begin{bmatrix} \delta_{ij}^k & | & \delta_{ij}^k \lesssim 0 \\ \delta_{M+l,j}^k & | & \delta_{M+l,j}^k \lesssim 0 \end{bmatrix}$$

Now $x_{i_* j_*}$ or $x_{M+l_* j_*}$ becomes a basic variable of the new feasible basic solution.

Step 6: Change the current solution to the new feasible basic solution using equations:

$$\sum_a \gamma_{ab} = 0 \tag{45}$$

$$\sum_b \gamma_{ab} = 0 \quad (46)$$
$$\sum_y r_{aby}\gamma_{ab} + \gamma_{M+y,b} = 0 \quad (47)$$

and

$$\psi = \min_{\substack{\gamma_{ab}<0 \\ \gamma_{M+y,b}<0}} \left[-\frac{x_{ab}}{\gamma_{ab}}; -\frac{x_{M+y,b}}{\gamma_{M+y,b}} \right] \quad (48)$$

$(a = 1,2, \dots, M; b = 1,2, \dots, N; y = 1,2, \dots, L)$

Go to Step 2.

Step 7: Designate the current feasible basic solution by X^0. The solution $X^0 = (x_{ij}^0, x_{M+l,j}^0)$ are optimal solution for lexicographic minimum fractional transportation problem for breakable commodity denoted by (7) and hence, the initial efficient basic solution for multiple objective fractional transportation problem for breakable commodity denoted by (1) to (5). The optimal value of objective function is

$$w_k^0 = \frac{\sum_{(i,j)\in J} p_{ij}^k x_{ij}^0}{\sum_{(i,j)\in J} q_{ij}^k x_{ij}^0}$$

The following are the salient features of the proposed algorithm:

1. The developed algorithm allows the optimization of multiple fractional conflicting objectives while permitting an explicit consideration of the existing decision environment.
2. The developed algorithm allows the transportation system decision maker to review critically the priority structure for the objectives in view of the efficient/non dominated/ pareto optimal solution derived by the algorithm.
3. The most important property of the developed algorithm is its great flexibility which allows model experimentation with numerous variations of constraints and priority structure of objectives.
4. The construction of a sequence of solutions having different objective values as well as quality helps in cases not only where one objective is an equally crucial factor besides other objectives but also when analyzing the practicability and sensitivity of an existing transportation situation.
5. The algorithm takes into account the special structure of the problem and will prove to be useful in making the multiple objective fractional transportation problem formulated more realistic in logic and other application areas.

7. GLASS-WRAP TRANSPORTATION PROBLEM OF ASHI INDIA GLASS LIMITED

The Algorithm is illustrated by the following real life example:

In Ashi India Glass Limited, the basic ingredients lime, silica and soda etc. are first blended with recycled broken glass known as cullet and then heated at a very high temperature around 1600 centigrade in a furnace to form molten glass which is then fed onto the top of a molten tin bath. A flat glass ribbon of almost uniform thickness is produced by flowing molten glass on the tin bath under controlled heating. At the end of the tin bath, the flat glass is then slowly cooled down, and is fed into the annealing lehr for further controlled gradual cooling down. Ashi India Glass Limited has different types of glass-wrap of flat glass in each of the four plants j, located at Taloja-Maharashtra, Roorkee-Uttarakhand, Bawal-Haryana and Chennai-Tamil Nadu. The plants j are receiving a fixed quantity of glass-wrap of flat glass i which has four different grades. The glass-wrap of flat glass transportation means one thing above all for Ashi India Glass Limited: high costs for packaging, transportation and transportation damage. After the breakage or partial breakage, the total value of such glass-wrap of flat glass is zero and it is a loss. Hence it is necessary to restrict the breakage to a known specified level. The basic goal is to determine a feasible transportation schedule which minimizes the total actual/total standard shipping cost, total actual/total standard loading/unloading cost, total actual/total standard overtime cost of transporting glass-wrap of flat glass, while satisfying the extra requirement that the quantity of breakage present in glass-wrap of flat glass is less than a certain level.

In Table 1, the total actual transportation cost, total actual loading/unloading cost, total actual overtime cost, p_{ij}^1, p_{ij}^2, p_{ij}^3, are written in left bracket while total standard transportation cost, total standard loading/unloading cost, total standard overtime cost q_{ij}^1, q_{ij}^2, q_{ij}^3 are written in the right bracket. Availabilities of glass-wrap of flat glass s_i and the quantities of breakage τ_i are listed in the last column while requirements of glass-wrap of flat glass d_j and maximum quantity of breakages in glass-wrap of flat glass σ_j are shown in the last row. Let x_{ij} be the tonnage of glass-wrap of flat glass sent from i to j , then it is required to

$$\min w_k = \frac{\sum_i \sum_j p_{ij}^k x_{ij}}{\sum_i \sum_j q_{ij}^k x_{ij}}$$

subject to

$$\sum_j x_{ij} = s_i; \sum_i x_{ij} = d_j; \ \sum_i \tau_i x_{ij} \le \sigma_j d_j; x_{ij} \ge 0 \ , \forall\, i \text{ and } j$$
$$(i = 1,2, \dots ,4; j = 1,2, \dots ,4; k = 1,2,3)$$

Table 1: Data for Glass-wrap Transportation Problem

		Plants j					
		1	2	3	4	s_i	τ_i
Glass-wrap of flat glass	1	$\begin{bmatrix}5\\10\\10\end{bmatrix}\begin{bmatrix}4\\6\\12\end{bmatrix}$	$\begin{bmatrix}6\\7\\3\end{bmatrix}\begin{bmatrix}5\\9\\7\end{bmatrix}$	$\begin{bmatrix}2\\2\\4\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}$	$\begin{bmatrix}1\\3\\6\end{bmatrix}\begin{bmatrix}2\\2\\2\end{bmatrix}$	8	0.4

i	2	$\begin{bmatrix}11\\1\\1\end{bmatrix}\begin{bmatrix}7\\1\\7\end{bmatrix}$	$\begin{bmatrix}3\\5\\16\end{bmatrix}\begin{bmatrix}8\\10\\14\end{bmatrix}$	$\begin{bmatrix}12\\8\\1\end{bmatrix}\begin{bmatrix}6\\3\\1\end{bmatrix}$	$\begin{bmatrix}5\\6\\3\end{bmatrix}\begin{bmatrix}10\\7\\4\end{bmatrix}$	11	0.8
	3	$\begin{bmatrix}4\\4\\3\end{bmatrix}\begin{bmatrix}1\\5\\6\end{bmatrix}$	$\begin{bmatrix}10\\13\\4\end{bmatrix}\begin{bmatrix}3\\12\\8\end{bmatrix}$	$\begin{bmatrix}2\\3\\3\end{bmatrix}\begin{bmatrix}12\\3\\2\end{bmatrix}$	$\begin{bmatrix}7\\2\\5\end{bmatrix}\begin{bmatrix}8\\5\\1\end{bmatrix}$	7	0.6
	4	$\begin{bmatrix}8\\7\\2\end{bmatrix}\begin{bmatrix}2\\2\\3\end{bmatrix}$	$\begin{bmatrix}4\\12\\3\end{bmatrix}\begin{bmatrix}4\\10\\4\end{bmatrix}$	$\begin{bmatrix}14\\1\\1\end{bmatrix}\begin{bmatrix}11\\2\\3\end{bmatrix}$	$\begin{bmatrix}10\\4\\2\end{bmatrix}\begin{bmatrix}9\\5\\5\end{bmatrix}$	3	0.7
Tons Reqd. d_j		6	10	9	4		
Max Breakage σ_j		0.7	0.7	0.7	0.7		

Using the initial feasible basic solution, the 3-dimensional vector-valued dual variables are found using equations (41), (42), (43), (44) and then relative criterion vectors δ_{ij}^k and $\delta_{M+l,j}^k$ are calculated. Table 2 shows the transportation tableau with the initial feasible basic solution $X^1 = (x_{ij}, x_{M+l,j}), \delta_{ij}^k$ and $\delta_{M+l,j}^k$. The marginal two columns contain the values of u_i^{k1} and u_i^{k2}, while lower first two marginal row contain the values of v_j^{k1},v_j^{k2} and lower next two marginal row contain the values of λ_{jl}^{k1} and λ_{jl}^{k2}. The values of σ_j and d_j are displayed in the top two rows of the table while τ_i and s_i are shown in first and second left columns respectively. For X^1, the total actual/total standard shipping cost, total actual/total standard loading/unloading cost and total actual/ total standard overtime cost of transporting the glass-wrap of flat glass are 0.827, 0.991 and 0.996 respectively.

As X^1 is not optimal, therefore applying the selection rule of Step 5, the variable x_{53} becomes an entering basic variable and so γ_{53} is added to this variable and $\gamma_{ab}, \gamma_{M+y,b}$is added to all the basic variables $x_{ab}, x_{M+y,b}$. The $\gamma's$ satisfy the equations (45), (46) and (47).

Using equation (48),

$$\psi = \min\left[5 \times 1,\ \frac{7}{2} \times 2, 2 \times 1, 15 \times \frac{1}{3}\right] = 2$$

Using this value of ψ, the new feasible basic solution can be obtained as:

$$x_{11} = 5 - 2 \times 1 = 3, x_{12} = 5/2 + 0 = 5/2$$

Similarly $x_{14} = 5/2, x_{22} = 15/2, x_{23} = 7/2, x_{33} = 11/2, x_{34} = 3/2, x_{41} = 3, x_{43} = 0,$ $x_{51} = 9, x_{53} = 2, x_{54} = 9.$

Proceeding in the manner described above, the subsequent values for various iterations are:

Initially,

$$X^1 = \begin{bmatrix} 5 & 5/2 & 0 & 1/2 \\ 0 & 15/2 & 7/2 & 0 \\ 0 & 0 & 7/2 & 7/2 \\ 1 & 0 & 2 & 0 \\ 15 & 0 & 0 & 5 \end{bmatrix} \qquad \text{Costs} = \begin{bmatrix} 0.827 \\ 0.991 \\ 0.996 \end{bmatrix}$$

First iteration,

$$X^2 = \begin{bmatrix} 3 & 5/2 & 0 & 5/2 \\ 0 & 15/2 & 7/2 & 0 \\ 0 & 0 & 11/2 & 3/2 \\ 3 & 0 & 0 & 0 \\ 9 & 0 & 2 & 9 \end{bmatrix} \qquad \text{Costs} = \begin{bmatrix} 0.733 \\ 0.964 \\ 1.113 \end{bmatrix}$$

Second iteration,

$$X^3 = \begin{bmatrix} 3 & 5/2 & 0 & 5/2 \\ 0 & 15/2 & 2 & 3/2 \\ 0 & 0 & 7 & 0 \\ 3 & 0 & 0 & 0 \\ 9 & 0 & 5 & 6 \end{bmatrix} \qquad \text{Costs} = \begin{bmatrix} 0.603 \\ 0.972 \\ 1.336 \end{bmatrix}$$

Third iteration,

$$X^4 = \begin{bmatrix} 3 & 5/2 & 3/2 & 1 \\ 0 & 15/2 & 1/2 & 3 \\ 0 & 0 & 7 & 0 \\ 3 & 0 & 0 & 0 \\ 9 & 0 & 11 & 0 \end{bmatrix} \qquad \text{Costs} = \begin{bmatrix} 0.539 \\ 0.896 \\ 1.042 \end{bmatrix}$$

Fourth iteration,

$$X^5 = \begin{bmatrix} 5/2 & 5/2 & 2 & 1 \\ 1/2 & 15/2 & 0 & 3 \\ 0 & 0 & 7 & 0 \\ 3 & 0 & 0 & 0 \\ 7 & 0 & 13 & 0 \end{bmatrix} \qquad \text{Costs} = \begin{bmatrix} 0.530 \\ 0.868 \\ 1.031 \end{bmatrix}$$

The solution X^5 is optimal solution for lexicographic minimum fractional transportation problem for breakable commodity and hence the initial efficient basic solution for multiple objective fractional transportation problem for breakable commodity. The optimal values are 0.530, 0.868 and 1.031 respectively.

Table 2: Glass-wrap Transportation Problem with X^1

$\tau_i \downarrow$	$\sigma_j \rightarrow$	0.7	0.7	0.7	0.7	u_i^{k1}	u_i^{k2}
$d_j \rightarrow$	s_i	6	10	9	4		

	↓						
0.4	8	x_{11}= 5	x_{12}= 5/2	$\delta_{13} = \begin{bmatrix} 5608.5 \\ -2901 \\ 6070 \end{bmatrix}$	x_{14}=1/2	(5,10,10)	(4,6,12)
0.8	11	$\delta_{21} = \begin{bmatrix} 1237.5 \\ 4005.5 \\ 3735 \end{bmatrix}$	x_{22} = 15/2	x_{23}=7/2	$\delta_{24} = \begin{bmatrix} -42 \\ 4345.5 \\ -3962.5 \end{bmatrix}$	(-9,14,0)	(-10,39,-10)
0.6	7	$\delta_{31} = \begin{bmatrix} 93 \\ -168.5 \\ 2100.5 \end{bmatrix}$	$\delta_{32} = \begin{bmatrix} 1282.5 \\ -951 \\ -3729.5 \end{bmatrix}$	x_{33}=7/2	x_{34}=7/2	(11,9,10)	(10,9,16)
0.7	3	x_{41}= 1	$\delta_{42} = \begin{bmatrix} -735.7 \\ -2637.7 \\ 1742 \end{bmatrix}$	x_{43}= 2	$\delta_{44} = \begin{bmatrix} -301.5 \\ -477 \\ -1864 \end{bmatrix}$	(8,7,2)	(2,2,3)
		x_{51}= 9	$\delta_{52} = \begin{bmatrix} 159.7 \\ 400.2 \\ 344.5 \end{bmatrix}$	$\delta_{53} = \begin{bmatrix} -1920 \\ 966 \\ 2099.5 \end{bmatrix}$	x_{54}= 7		
v_j^{k1}		(0,0,0)	(-10,-3,-30)	(-99,-6,-15)	(-4,-7,-4)		
v_j^{k2}		(0,0,0)	(-16,35,-34)	(-40,-42,-77)	(-2,-4,-10)		
λ_{jl}^{k1}		(0,0,0)	(11/4,-3/2,23/4)	(15,0,2)	(0,0,0)		
λ_{jl}^{k2}		(0,0,0)	(17/4,-8,29/4)	(7,6,11)	(0,0,0)		

8. CONCLUSION

In this paper, a totally new multiple objective fractional transportation problem for breakable commodity is formulated. A multiple objective fractional dual of the k-component multiple objective fractional transportation problem for breakable commodity is also developed. An innovative algorithm and its supporting mathematics are also presented to determine the initial efficient basic solution for the multiple objective fractional transportation problem for breakable commodity by solving the related lexicographic minimum fractional transportation problem. The algorithm developed in this paper for solving multiple objective transportation problem with respect to the fractional objectives offers a more universal apparatus for a wider class of real life decision priority problems than the single objective transportation problems. The multiple objective fractional transportation problems result in a subset of feasible solutions from which a transportation system decision maker is sure of a most preferred solution. This paper also gives an interesting real life application of Ashi India Glass Limited of developed algorithm and multiple objective fractional dual.

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