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A GENERALIZED EXPONENTIAL RATIO-PRODUCT ESTIMATOR FOR POPULATION VARIANCE IN SIMPLE RANDOM SAMPLING

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ABSTRACT
This study focuses on proposing the generalized exponential ratio-product estimator for population variance in simple random sampling by using the combination of some existing members with scalars. Mean square error and bias expressions with optimality conditions has been derived and at the optimal conditions proposed generalized class is a suitable alternative to linear regression estimator. Both analytical and numerical comparison with some existing estimators shows better performances from the members of the proposed class.

KEYWORDS: Exponential ratio-product estimator, Bias, MSE, optimality conditions, Efficiency

MSC: 62D05

RESUMEN
Este studio está enfocado en proponer un esimdor generalizado exponential razón-producto para la varianza poblaionel en el muestreo simple aleaoria uando combination de laugno de lo smiembros xostentes con escalares. Expresiones para el Error Cuadrlatico Medio y el sesgo bajo conditoncnes de optimalidad han sido derivadas y estas conditones levan a proponer una vlase generalizada que es adecuada para el stimdor lineal de regression . Coapraciones analyticas y numericas se llvn a cabo rspecto a otros estimdores existentes mustan un mejro compratmaeito d elso miembos d ela propuesta class.

KEYWORDS: Exponential ratio-product estimator, Bias, MSE, optimality conditions, Efficiency

1. INTRODUCTION

Existence of finite population $Q=[q_1, q_2, \ldots, q_Z]$ is the basic assumption for survey sampling model, where units are perfectly identifiable and sample of size $z \le Z$ is selected from Q. However the supplementary information of the finite population under consideration is quite often available from previous experience, census or administrative databases and so while incorporating such ancillary information in survey sampling has also resulted an increase in precision, while estimating population parameters. From the literature on survey sampling, there are various methods by which this supplementary information is incorporated such as ratio, product, regression, difference method of estimation, but in this study we have incorporated the ancillary information of Gini's Mean Difference, Downton's Method and Probability weighted moment of auxiliary variate. Use of such type of ancillary information in this study is made, because these parameters are not affected by extreme values as they are robust to extreme values. While estimating the population variance with more precision various authors have tried their best to obtain more precision and the significant contribution in this area are due to Das and Tripathi (1978), Isaki (1983), Kadilar and Cingi (2006), Singh *et al.* (2013; 2014), Solanki *et al.* (2015) and Singh and Pal (2016) has also suggested the class of ratio estimators for population variance. Recently Bhat *et al.* (2018a, 2018b) and Maqbool *et al.* (2018) have also proposed different ratio estimators for estimating finite population variance by incorporating different parameters of auxiliary variable in order to get more precise results. So in this study we have proposed a generalized exponential ratio-product estimator for population variance, which is a suitable alternative to linear regression estimator and is more efficient that existing ones mentioned in this study in terms of efficiency both from Analytical comparison and Numerical illustration.

a) Notations

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Consider a finite population $Q=[q_1, q_2, \dots, q_Z]$ of Z units and let (u, v) be the (study, auxiliary) variable defined on Q taking values (u_i, v_i) respectively, on $Q_i = [1,2,\dots, Z]$. It is desired to estimate the population variance T_u^2 of the study variable u using information on an auxiliary variable v. Let a simple random sample of size z be drawn without replacement from the finite population Q. Several existing estimators have been mentioned in this study and is given in Table 1. We denote

$\bar{U} = Z^{-1}\sum_{i=1}^{Z} u_i$; The population mean of the study variable u,

$\bar{V} = Z^{-1}\sum_{i=1}^{Z} v_i$; The population mean of the auxiliary variable v,

$\bar{u} = z^{-1}\sum_{i=1}^{z} u_i$; The sample mean of the study variable u,

$\bar{v} = z^{-1}\sum_{i=1}^{z} v_i$; The sample mean of the auxiliary variable v,

$C_u = \frac{T_u}{\bar{U}}$; Coefficient of variation of the study variable u,

$C_v = \frac{T_v}{\bar{V}}$; Coefficient of variation of the auxiliary variable v,

$T_u^2 = (Z-1)^{-1}\sum_{i=1}^{Z}(u_i - \bar{U})^2$; The population variance of the study variable u,

$T_v^2 = (Z-1)^{-1}\sum_{i=1}^{Z}(v_i - \bar{V})^2$; The population variance of the auxiliary variable v,

$\rho_c = \frac{T_{uv}}{T_u T_v}$; Correlation coefficient between the auxiliary and study variables,

$\lambda = \frac{T_{uv}}{T_v^2}$; Regression Coefficient

$k = \rho_c \frac{C_u}{C_v}$; Population constant

$\delta = \frac{1-f}{z}$, while $f = \frac{z}{Z}$ is the sampling fraction

$G = (4/Z-1)\sum_{i=1}^{Z}\{(2i - Z - 1)/2Z\}V_{(i)}$ Gini's mean difference

$D = \left(2\sqrt{\lambda}/Z(Z-1)\right)\sum_{i=1}^{Z}\left\{i - \frac{Z+1}{2}\right\}V_{(i)}$ Downton's method

$S_{pw} = \left(\sqrt{\lambda}/Z^2\right)\sum_{i=1}^{Z}\{2i - Z - 1\}V_{(i)}$ Probability Weighted Moments

Table 1: Some existing members mentioned in this study

Estimator	MSE	Bias
$t^2_{u\,(clR)} = t_u^2\left(\frac{T_v^2}{t_v^2}\right)$ Cochran (1940)	$\frac{(1-f)}{z}T_u^4\left[C_u^2 + C_v^2(1-2k)\right]$	$\frac{(1-f)}{z}T_u^2 C_u^2(1-k)$
$t^2_{u\,(clP)} = t_u^2\left(\frac{t_v^2}{T_v^2}\right)$ [(Robson;1957), (Murthy;1964)]	$\frac{(1-f)}{z}T_u^4\left[C_u^2 + C_v^2(1+2k)\right]$	$\frac{(1-f)}{z}T_u^2 C_u^2 k$

$t^2_{u(\exp R)} = t^2_u \exp\left(\frac{T^2_v - t^2_v}{t^2_v + T^2_v}\right)$ Bahl and Tuteja; 1991	$\frac{(1-f)}{z} T^4_u \left[C^2_u + \frac{C^2_v}{4}(1-4k)\right]$	$\frac{(1-f)}{8z} T^2_u C^2_u (3-4k)$
$t^2_{u(\exp P)} = t^2_u \exp\left(\frac{t^2_v - T^2_v}{t^2_v + T^2_v}\right)$ Bahl and Tuteja; 1991	$\frac{(1-f)}{z} T^4_u \left[C^2_u + \frac{C^2_v}{4}(1+4k)\right]$	$\frac{(1-f)}{8z} T^2_u C^2_u (4k-1)$
$t^2_{u(chR)} = t^2_u \left(\frac{T^2_v}{t^2_v}\right)^2$ Kadilar and Cingi; 2003	$\frac{(1-f)}{z} T^4_u \left[C^2_u + 4C^2_v(1-k)\right]$	$\frac{(1-f)}{z} T^2_u C^2_u (1+2k)$
$t^2_{u(chP)} = t^2_u \left(\frac{t^2_v}{T^2_v}\right)^2$ Kadilar and Cingi; 2003	$\frac{(1-f)}{z} T^4_u \left[C^2_u + 4C^2_v(1+k)\right]$	$\frac{(1-f)}{z} T^2_u C^2_u (1+2k)$
$t^2_{u(LR)} = t^2_u + \lambda(T^2_u - t^2_u)$ Linear Regression estimator	$\frac{(1-f)}{z} T^4_u C^2_u (1-\rho^2_c)$	0

2. PROPOSED ESTIMATOR

We suggest the generalized exponential ratio-product estimator for population variance denoted as

$$t^2_{u(pr)} = t^2_u \left[\eta_1 \left(\frac{t^2_v}{T^2_v}\right) \exp\left\{\frac{\theta_1 (t^2_v - T^2_v)}{t^2_v + T^2_v}\right\} + \eta_2 \left(\frac{T^2_v}{t^2_v}\right) \exp\left\{\frac{\theta_2 (t^2_v - T^2_v)}{t^2_v + T^2_v}\right\}\right] \tag{2.1}$$

where $\eta_1 + \eta_2 = 1$ while θ_1 and θ_2 suitably chosen scalars.

a) Bias and Mean square of the proposed generalized exponential ratio-product estimator.

To obtain the bias and mean square error for the estimator $t^2_{u(pr)}$, expressing (2.1) in terms of (1.4) and simplifying the part first of the (2.1), we obtain

$$\eta_1 \left(\frac{t^2_v}{T^2_v}\right) \exp\left\{\frac{\theta_1 (t^2_v - T^2_v)}{t^2_v + T^2_v}\right\} = \eta_1 \frac{T^2_v (1+e_v)}{T^2_v} \exp\left\{\frac{\theta_1 (T^2_v (1+e_v) - T^2_v)}{T^2_v (1+e_v) + T^2_v}\right\}$$

$$= \eta_1 (1+e_v) \exp\left\{\frac{\theta_1}{2} e_v \left(1 + \frac{e_v}{2}\right)^{-1}\right\}$$

Using Taylor's series expansion up to second order approximation and assuming higher orders are negligible, we obtain

$$\eta_1 \left(\frac{t^2_v}{T^2_v}\right) \exp\left\{\frac{\theta_1 (t^2_v - T^2_v)}{t^2_v + T^2_v}\right\} = \eta_1 + \eta_1 \frac{\theta_1}{2} e_v - \eta_1 \frac{\theta_1}{4} e^2_v + \eta_1 \frac{\theta^2_1}{8} e^2_v + \eta_1 e_v + \eta_1 \frac{\theta_1}{2} e^2_v \tag{2.2}$$

Similarly, the second term in (2.1) $\eta_2 \left(\frac{T^2_v}{t^2_v}\right) \exp\left\{\frac{\theta_2 (t^2_v - T^2_v)}{t^2_v + T^2_v}\right\}$ is expandable as follows;

$$\eta_2 \left(\frac{T^2_v}{t^2_v}\right) \exp\left\{\frac{\theta_2 (t^2_v - T^2_v)}{t^2_v + T^2_v}\right\} = \eta_2 \frac{T^2_v}{T^2_v (1+e_v)} \exp\left\{\frac{\theta_2 (T^2_v (1+e_v) - T^2_v)}{T^2_v (1+e_v) + T^2_v}\right\}$$

Using Taylor series expansion up to second order approximation that is, neglecting terms with power greater than 2, we obtain

$$\eta_2 \left(\frac{T^2_v}{t^2_v}\right) \exp\left\{\frac{\theta_2 (t^2_v - T^2_v)}{t^2_v + T^2_v}\right\} = \eta_2 + \eta_2 \frac{\theta_2}{2} e_v - \eta_2 \frac{\theta_2}{4} e^2_v + \eta_2 \frac{\theta^2_2}{8} e^2_v - \eta_2 e_v + \eta_2 \frac{\theta_2}{2} + e^2_v + \eta_2 e^2_v \tag{2.3}$$

Adding (2.2) and (2.3) together, simplifying the result and substituting for t_u^2 the proposed generalized exponential ratio-product estimator $t_{u(pr)}^2$ becomes

$$t_{u(pr)}^2 = T_u^2 \left\{ \begin{array}{l} 1 + (\eta_1\theta_1 + 2\eta_1 + \eta_2\theta_2 - 2\eta_1)\dfrac{e_v}{2} - \left(\begin{array}{l} 2\eta_1\theta_1 - \eta_1\theta_1^2 - 4\eta_1\theta_1 + 2\eta_2\theta_2 - \eta_2\theta_2^2 \\ + 4\eta_2\theta_2 - 8\eta_2 \end{array} \right) \dfrac{e_v^2}{8} \\ + e_u + (\eta_1\theta_1 + 2\eta_1 + \eta_2\theta_2 - 2\eta_2)\dfrac{e_v e_u}{2} \end{array} \right\} \quad (2.4)$$

But $\eta_2 = 1 - \eta_1$

$$t_{u(pr)}^2 = T_u^2 \left\{ \begin{array}{l} 1 + (\eta_1\theta_1 + 4\eta_1 + \theta_2 - \eta_1\theta_2 - 2)\dfrac{e_v}{2} - \\ \left(2\eta_1\theta_1 - \eta_1\theta_1^2 + 2\theta_2 + 2\eta_2\theta_2 - 2\eta_1\theta_2 - \theta_2^2 + \eta_1\theta_2^2 - 4\eta_1\theta_2 - 8 + 8\eta_1\right)\dfrac{e_v^2}{8} \\ + e_u + (\eta_1\theta_1 + 4\eta_1 + \theta_2 - \eta_1\theta_2 - 2)\dfrac{e_v e_u}{2} \end{array} \right\} \quad (2.5)$$

The bias of the proposed generalized exponential ratio-product estimator to its first order approximation is obtained from (2.5) as follows

$$B\left(t_{u(pr)}^2\right) = E\left[t_{u(pr)}^2 - T_u^2\right]$$

$$B\left(t_{u(pr)}^2\right) = T_u^2 \left\{ \begin{array}{l} [\eta_1\theta_1 + 4\eta_1 + \theta_2 - \eta_1\theta_2 - 2]\delta\rho_c C_u C_v - \\ [\eta_1\theta_1^2 - 2\eta_1\theta_1 - \eta_1\theta_1^2 + 2\theta_2 - 2\eta_1\theta_2 - \theta_1^2 + 4\theta_2 - 4\eta_1\theta_2 - 8 - 8\eta_1]\dfrac{\delta C_v^2}{8} \end{array} \right\} \quad (2.6)$$

The expression for the MSE of the proposed generalized exponential ratio-product estimator is also obtained from (2.5) as follows;

$$MSE\left(t_{u(pr)}^2\right) = E\left[t_{u(pr)}^2 - T_u^2\right]^2$$

$$= \left(T_u^2\right)^2 \left\{ e_u^2 + (\eta_1\theta_1 + 4\eta_1 + \theta_2 - \eta_1\theta_2 - 2)e_u e_v + (\eta_1\theta_1 + 4\eta_1 - \eta_1\theta_2 + \theta_2 - 2)^2 \frac{e_v^2}{4} \right\}$$

$$= \left(T_u^2\right)^2 \delta \left\{ C_u^2 + (\eta_1\theta_1 + 4\eta_1 + \theta_2 - \eta_1\theta_2 - 2)\rho_c C_u C_v + (\eta_1\theta_1 + 4\eta_1 - \eta_1\theta_2 + \theta_2 - 2)^2 \frac{C_v^2}{4} \right\} \quad (2.7)$$

b) Optimality Conditions for proposed generalized exponential ratio-product estimator for population variance in simple random sampling

To get the optimal value of η_1 that will minimize the equation (2.7) while substituting that value in that equation. So we have take partial derivative of (2.7) with respective to η_1, equated to zero and obtain that optimal value.

$$\frac{\partial MSE\left(t_{u(pr)}^2\right)}{\eta_1} = (\theta_1 + 4 - \theta_2)\rho_c C_u C_v + 2(\eta_1\theta_1 + 4\eta_1 - \eta_1\theta_2 + \theta_2 - 2)(\theta_1 + 4 - \theta_2)\frac{C_v^2}{4} = 0$$

$$\eta_1 = \frac{2 - \theta_2 - 2k}{\theta_1 + 4 - \theta_2} \quad (2.8)$$

Substituting the value of (2.8) in (2.7) we obtain the optimal mean square error of $t_{u(pr)}^2$ as

$$MSE\left(t_{u(pr)}^2\right)_{opt} = \delta\left(T_u^2\right)^2 \left\{ \begin{array}{l} C_u^2 + (2 - \theta_2 - 2k)\rho_c C_u C_v + (\theta_2 - 2)\rho_c C_u C_v \\ + \left[(2 - \theta_2 - 2k)^2 + 2(2 - \theta_2 - 2k)(\theta_2 - 2) + (\theta_2 - 2)^2\right]\dfrac{C_v^2}{4} \end{array} \right\}$$

$$MSE\left(t_{u(pr)}^2\right)_{opt} = \delta\left(T_u^2\right)^2 C_u^2\left(1-\rho_c^2\right) \tag{2.9}$$

From (2.9), it is observed that the optimum MSE of the proposed generalized exponential ratio-product estimator is the same as the MSE of the linear regression estimator.

Table 2 shows the different members of the family of the proposed generalized exponential ratio-product $t_{u(pr)}^2$ estimator obtained from suitably choosing θ_1, θ_2, and η_1. By substituting various members were formed with their remained that of the linear regression estimator.

Table 2: Special members of the generalized exponential ratio-product $t_{u(pr)}^2$ estimator

η_1	η_2	θ_1	θ_2	Estimator	Bias
0	1	$1-G$	$1-G$	$t_{u(pr1)}^2 = t_y^2\left[\frac{T_v^2}{t_v^2}\exp\left\{\frac{(1-G)(t_v^2-T_v^2)}{t_v^2+T_v^2}\right\}\right]$	$\frac{T_u^2\delta}{8}\left\{3C_v^2-3\rho_c^2C_u^2\right\}$
$\frac{1-D}{2}$	$\frac{1+D}{2}$	0	0	$t_{u(pr2)}^2 = t_y^2\left[\left(\frac{1-D}{2}\right)\frac{t_v^2}{T_v^2}+\left(\frac{1+D}{2}\right)\frac{T_v^2}{t_v^2}\right]$	$\frac{T_u^2\delta}{2}\left\{C_v^2+\rho_c C_u C_v-2\rho_c^2C_u^2\right\}$
$\frac{2-3S_{pw}}{4}$	$\frac{2+3S_{pw}}{4}$	S_{pw}	S_{pw}	$t_{u(pr3)}^2 = t_y^2\left[\left(\frac{2-3S_{pw}}{4}\right)\frac{t_v^2}{T_v^2}\exp\left\{\frac{S_{pw}(t_u^2-T_u^2)}{t_u^2+T_u^2}\right\}+\left(\frac{2+3S_{pw}}{4}\right)\frac{T_v^2}{t_v^2}\exp\left\{\frac{S_{pw}(t_u^2-T_u^2)}{t_u^2+T_u^2}\right\}\right]$	$\frac{T_u^2\delta}{4}\left\{2C_v^2+\rho_c C_u C_v-5\rho_c^2C_u^2\right\}$

3. EFFICIENCY COMPARISON

The efficiency comparison in this study are done using the mean square error of the proposed generalized exponential ratio-product estimator for population variance and the other existing estimators mentioned in this study.

a) Efficiency Comparison with the Classical ratio estimator

The mean square error of the classical ratio estimator is given as

$$MSE\left(t_{u(CLR)}^2\right) = \delta T_u^4\left[C_u^2 + C_v^2(1-2k)\right]$$

The condition for the proposed estimators to be more efficient than the classical ratio estimator is defined by

$$\delta T_u^4\left[C_u^2 + C_v^2(1-2k)\right] - \delta\left(T_u^2\right)^2 C_u^2\left(1-\rho_c^2\right) > 0$$

$\Rightarrow (1-k)^2 > 0$, which Is always true

b) Efficiency Comparison with the Classical product estimator

The mean square error of the classical product estimator is given as

$$MSE\left(t_{u(CLR)}^2\right) = \delta T_u^4\left[C_u^2 + C_v^2(1-2k)\right]$$

The condition for the proposed estimators to be more efficient than the classical product estimator is defined by

$$\delta T_u^4\left[C_u^2 + C_v^2(1+2k)\right] - \delta\left(T_u^2\right)^2 C_u^2\left(1-\rho_c^2\right) > 0$$

$\Rightarrow (1+k)^2 > 0$, which Is always true

c) Efficiency Comparison with the Exponential ratio estimator

The mean square error of the exponential ratio estimator is given as

$$MSE\left(t_{u(\exp R}^2\right) = \delta T_u^4\left[C_u^2 + \frac{C_v^2}{4}(1-4k)\right]$$

The condition for the proposed estimators to be more efficient than the exponential ratio estimator is defined by

$$\delta T_u^4\left[C_u^2+\frac{C_v^2}{4}(1-4k)\right]-\delta\left(T_u^2\right)^2C_u^2\left(1-\rho_c^2\right)>0$$

$\Rightarrow (1-2k)^2 > 0$, which Is always true

d) Efficiency Comparison with the Exponential product estimator

The mean square error of the exponential product estimator is given as

$$MSE\left(t_{u(\exp P}^2\right)=\delta T_u^4\left[C_u^2+\frac{C_v^2}{4}(1+4k)\right]$$

The condition for the proposed estimators to be more efficient than the exponential product estimator is defined by

$$\delta T_u^4\left[C_u^2+\frac{C_v^2}{4}(1+4k)\right]-\delta\left(T_u^2\right)^2C_u^2\left(1-\rho_c^2\right)>0$$

$\Rightarrow (1+2k)^2 > 0$, which Is always true

e) Efficiency Comparison with the Classical Chain ratio estimator

The mean square error of the Classical Chain ratio estimator is given as

$$MSE\left(t_{u(ChR}^2\right)=\delta T_u^4\left[C_u^2+4C_v^2(1-k)\right]$$

The condition for the proposed estimators to be more efficient than the Classical Chain ratio estimator is defined by

$$\delta T_u^4\left[C_u^2+4C_v^2(1-k)\right]-\delta\left(T_u^2\right)^2C_u^2\left(1-\rho_c^2\right)>0$$

$\Rightarrow k^2-4k+4>0$, which Is always true

f) Efficiency Comparison with the Classical Chain product estimator

The mean square error of the Classical Chain product estimator is given as

$$MSE\left(t_{u(ChP}^2\right)=\delta T_u^4\left[C_u^2+4C_v^2(1+k)\right]$$

The condition for the proposed estimators to be more efficient than the Classical Chain product estimator is defined by

$$\delta T_u^4\left[C_u^2+4C_v^2(1+k)\right]-\delta\left(T_u^2\right)^2C_u^2\left(1-\rho_c^2\right)>0$$

$\Rightarrow k^2+4k+4>0$, which Is always true, except $k=-2$.

4. NUMERICAL ILLUSTRATION

The performance of the proposed estimators $t_{u(pri)}^2, i=1,2,3$, which are members of suggested generalized exponential ratio-product estimator for population variance $t_{u(pr)}^2$ are evaluated against the classical ratio, classical product, exponential ratio, exponential product, classical chain ratio, classical chain product. For this we have used the different data sets; data set 1 is taken from Murthy (1967) page 228 in which fixed capital is denoted by V (auxiliary variable) and output of 80 factories are denoted by U (Study variable), data set 2 is taken from the book "Theory and Analysis of Sample Survey Designs" by Singh, D and Chaudhary, F. S. (1986) page 177, in which the data under wheat in 1971 and 1973 is given and in which area under wheat in the region was to be estimated during 1974 is denoted by U (study variable) by using the data of cultivated area under wheat in 1971 is denoted by V (auxiliary variable) and the data set 3 is taken from the Division of Agricultural Statistics Faculty of Horticulture Shalimar in which the data of apple production amount (as an interest of variate) and the number of apple trees (as an auxiliary variate) in 499 villages of District Baramulla of Jammu and Kashmir from 2010-2011. (Source: RCM project, pilot survey for estimation of cultivation and production of apple in District Baramulla, RCM approved project). First, we have stratified the data by area wise and from each stratum (region) and the samples (villages) have been selected randomly. Here, we have taken the sample size to 170. We joined two areas, then chose four strata where each one contains three blocks (as 1: Zaniger, Boniyar, Tangmarg; 2: Wagoora, Sopore, Baramulla; 3: Uri, Pattan, Rohama; 4: Rafiabad, Kunzer, Singpora) for this data. However, in the present study, we have used only the data of Uri, Pattan, Rohama of district Baramulla of Jammu and Kashmir, due to the interest in simple random sampling. We have applied our proposed ratio estimators on the data of apple production amount and number of apple trees in 117 villages of Uri, Pattan, Rohama of district Baramulla of Jammu and Kashmir, in which the apple production (in tons) is denoted by U (study variable), and the number of apple trees is denoted by V (auxiliary variable, 1 unit = 100 trees) and data statistics is given in Table 3. We have computed the percent relative efficiencies (PREs) of the suggested class of estimators which is suitable alternative to the linear regression estimator with the mentioned estimators in this study by using the formulae given in (4.1) and the statistical analysis of data set 1, data set 2 and data set 3 are given in Table 4, 5 and 6 respectively.

$$PRE\left(t^2_{u\,(i)}, t^2_{u(pri)}\right) = \frac{MSE\left(t^2_{u\,(i)}\right)}{t^2_{u(pri)}} \times 100 \tag{4.1}$$

Where $t^2_{u\,(i)}, i = (CL_R),(CL_P),(CL_R),(Exp_R),(Exp_P),(CL_{CR}),(CL_{CP})$ and $t^2_{u(pri)}, i = 1,2,3$

Table 3: Data Statistics

Parameters	Data set 1	Data set 2	Data set 3
Z	80	34	177
z	20	20	40
$\bar{U}$	51.8264	856.4117	1263
$\bar{V}$	11.2646	208.8823	560
T_u	18.3569	733.1407	862
C_u	0.3542	0.8561	0.9728
T_v	8.4563	150.5059	235.5
C_v	0.7507	0.7205	0.7395
ρ_c	0.9413	0.4491	0.987
G	9.0408	155.446	205.142
D	8.0138	140.891	150.600
S_{pw}	7.9136	199.961	98.67

Table 4: Statistical Analysis of Estimators mentioned in this study for Data Set 1

Estimators	**Bias**	**MSE**	**PRE**
$t^2_{u\,(clR)}$	3.959	534.290	877.648
$t^2_{u\,(clP)}$	3.163	535.291	879.293
$t^2_{u\,(\exp R)}$	1.089	534.118	877.365
$t^2_{u\,(\exp P)}$	0.099	534.618	878.187
$t^2_{u\,(chR)}$	0.796	534.479	877.958
$t^2_{u\,(chP)}$	13.447	538.483	884.537
$t^2_{u\,(LR)}$	0.000	60.877	100.000
$t^2_{u(pr1)}$	2.144	60.877	100.000
$t^2_{u(pr2)}$	3.737	60.877	100.000
$t^2_{u(pr3)}$	2.596	60.877	100.000

Table 4: Statistical Analysis of Estimators mentioned in this study for Data Set 2

Estimators	Bias	MSE	PRE
$t^2_{u\,(clR)}$	2679.17	4359306815	125.26
$t^2_{u\,(clP)}$	3065.46	4359306817	125.26
$t^2_{u\,(\exp R)}$	621.51	4359306815	125.26
$t^2_{u\,(\exp P)}$	-48.29	4359306816	125.26
$t^2_{u\,(chR)}$	-386.29	4359306815	125.26
$t^2_{u\,(chP)}$	11875.54	4359306820	125.26
$t^2_{u\,(LR)}$	0.00	3480074693	100.00
$t^2_{u(pr1)}$	1540.81	3480074693	100
$t^2_{u(pr2)}$	2769.25	3480074693	100
$t^2_{u(pr3)}$	1593.93	3480074693	100

Table 4: Statistical Analysis of Estimators mentioned in this study for Data Set 3

Estimators	Bias	MSE	PRE
$t^2_{u\,(clR)}$	-1994.84	8596487959	3871.32
$t^2_{u\,(clP)}$	8680.37	8596487962	3871.32
$t^2_{u\,(\exp R)}$	-1833.11	8596487959	3871.32
$t^2_{u\,(\exp P)}$	-1334.40	8596487961	3871.32
$t^2_{u\,(chR)}$	-10675.21	8596487957	3871.32
$t^2_{u\,(chP)}$	24046.27	8596487968	3871.32
$t^2_{u\,(LR)}$	0.00	222055880	100.00
$t^2_{u(pr1)}$	-1719.34	222055880	100.00
$t^2_{u(pr2)}$	-3587.49	222055880	100.00
$t^2_{u(pr3)}$	-8575.19	222055880	100.00

5. CONCLUSION

From both analytical and empirical comparison it has been observed that our proposed generalized exponential ratio-product estimator at its optimal condition is a suitable alternative to the linear regression estimator which is clear from equation (2.9) and from the empirical study given in table 4, 5 and 6 reveals that mean square error of the proposed estimators of the generalized class has same as linear regression estimator does have. Thus, provides suitable alternative to linear regression estimator for practical applications while estimating population variance. Among the proposed estimators the suggested estimator 1 is more efficient than the remaining two suggested estimators in the present study. The suggested estimators in this paper can be modified to other sampling methods like stratified sampling, non response, rank set sampling etc. See for illustration; Subzar et al (2018), Al-Omari and Haq (2019), Bouza and Subzar (2019).

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