AN EXTENSION OF THE PARABOLIC DEA FRONTIER TO DEAL WITH INTEGER VARIABLES: A CASE STUDY IN BRAZILIAN RAILWAY STATIONS

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ABSTRACT
In this paper, we extend the Parabolic DEA frontier to deal with integer values. Thus, we propose a hybrid approach that combines the Parabolic DEA frontier with the Sequential Algorithm to Resource Allocation in DEA. This is a two-phase approach, in which the non-integer reallocation obtained by the parabolic DEA frontier is transformed into an integer reallocation by the use of the Sequential Algorithm, while maintaining the highest DMUs' efficiencies. We use this hybrid approach to redistribute employees among 79 railway stations in Rio de Janeiro. By using this approach, all the employees were redistributed respecting the integer values, and the system's average efficiency has increased from 52% to 96% through input redistribution.

KEYWORDS: Parabolic DEA; Parametric DEA; Resources Redistribution; Integer Resources, Fixed-sum Inputs

MSC: 90B06

RESUMEN

En este documento, ampliamos la frontera parabólica DEA para tratar con valores enteros. Por lo tanto, proponemos un enfoque híbrido que combina la frontera parabólica DEA con el algoritmo secuencial para la distribuición de recursos en DEA. Este es un enfoque de dos fases, en el que la redistribución no entera obtenida por la frontera parabólica DEA se transforma en una redistribución entera mediante el uso del algoritmo secuencial, mientras se mantienen las eficiencias más altas de las DMUs. Utilizamos este enfoque híbrido para redistribuir a los empleados entre 79 estaciones de ferrocarril en Río de Janeiro. Al utilizar este enfoque, todos los empleados fueron redistribuidos respetando los valores enteros, y la eficiencia promedio del sistema aumentó del 52% al 96% a través de la redistribución de inputs.

PALABRAS CLAVE:
DEA parabólica; DEA paramétrica; Redistribución de recursos; Recursos enterros, Inputs de Suma Fija

1. INTRODUCTION

Classic Data Envelopment Analysis (DEA) models (Charnes *et al.* 1978) (Banker *et al.* 1984) are tools used for efficiency analysis of a set of units, the so-called Decision Making Units (DMUs), through a non-parametric approach based on linear programming models comparing produced outputs with consumed inputs.

The models build a piecewise linear frontier in which all DMUs located there are efficient. Classic DEA, since it is not parametric, does not presume any functional form for the efficiency frontier. The piecewise linear efficiency frontier incorporates some peculiarities about efficient DMUs such as not obtaining a single set of optimal weights and indeterminacy about scale variation (increasing, decreasing or invariable) in Variable Returns to Scale models (VRS) (Benicio & Soares de Mello, 2015)(Benicio & Soares de Mello, 2019):

In classical models, an inefficient DMU would have freedom of production, that is, it could reach the efficient frontier by modifying its inputs or outputs independently, that is, without affecting other DMUs. However, this situation is not feasible in realities of competition or cooperation among DMUs and in limited resources environment, where it is undesirable or impossible to change the total sum of some input or output. Thus, for receiving more resources would result in losses for some other DMUs.

Lins *et al*. (2003) have introduced Zero Sum Gains DEA model (ZSG – DEA): Many researchers have been using it to distribute variables without changing its total sum. One example is Cai and Ye (2019) that have transformed the Chinese national goal for reducing CO2 emissions in individualized targets for each of the 30 provinces (DMUs): Another example is Bernardo *et al*. (2020) that have resized staff at Brazilian university's libraries to improve their performances. Fonseca *et al.* (2010) have extended the ZSG – DEA model for non-radial projections.

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In the literature, researchers have been using strategies for redistributing among DMUs to make them efficient regarding scenarios of limited resources or products. Parametric DEA is one of the methods for redistribution, whose name derives from the specific form the efficiency frontier takes after the reallocation of constant output or input. This method requires from the decision maker a priory definition about the functional form.

In 2007, Avellar *et al.* developed the spherical parametric frontier for distribution of a new input among DMUs. After that, Avellar *et al.* (2010) have extended this model for redistribution of already existing input. They have pointed that, in this approach, some DMUs should increase their input to become efficient, which is not a reasonable concept on classic DEA models. In addition, they have presented another approach in which DMUs would not have to increase their inputs to become efficient. This specific case does not keep the total sum constant.

Guedes *et al.* (2012) have presented the adjusted spherical frontier model that achieves results more adherent to the coherence property than other redistribution models such as the original SFM and Beasley (Beasley, 2003): Bianca Alves *et al.* (2014) have introduced the parameterized boundary in elliptical geometry. Milioni *et al.* (2011a) and Silva *et al.* (2018) have published about hyperbolic DEA frontier.

Milioni and Bianca Alves (2013) have made a brief overview about related studies on DEA Parametric. In addition, Milioni *et al.* (2011b) have compared parametric DEA, ZSG-DEA and Beasley (Beasley, 2003) for calculating the quota allocation of CO2 among countries that signed the Kyoto Protocol.

It is noteworthy that all parametric DEA models previously mentioned only deal with CRS scenarios. Silveira *et al.* (2019) have developed the parabolic model that respects VRS conditions by generating a paraboloid curve after resource reallocation.

Although with different objectives, parametric DEA provides smoothed frontiers as in Nacif *et al.* (2009), Brandão and Soares de Mello (2017) and Brandão *et al.* (2020):

On this paper, we will introduce a hybrid approach using the parabolic DEA model (Silveira *et al.*, 2019) along with the Sequential Algorithm to Resource Allocation in DEA (Soares de Mello *et al.*, 2006)(Gomes *et al.*, 2008): This combination enables the parabolic DEA to deal with integer variables. This approach will be used to reallocate employees among Rio de Janeiro’s railway stations and make the system efficient.

2. PARABOLIC DEA FRONTIER

Reallocating constant inputs or outputs is the main use of Parametric DEA models. After redistribution, the frontier takes the smooth form of the function previously defined by the decision maker.

The parabolic DEA model, developed by Silveira *et al.* (2019), is a particular case of parametric models to perform input reallocation in VRS scenarios. After applied, it generates a parabolic frontier in which all DMUs become arranged following the new resource configuration determined by the model without changing the total sum of the redistributed input. Thus, all DMUs should become extremely efficient without changing their other variables values.

In Silveira *et al.* (2019), the objective function is to minimize the difference between the original input of a DMU and the input obtained after redistribution. Thus, the new frontier ensures that the optimal solution calculated is as similar as possible to the original scenario. The parabolic model also ensures some properties of a DEA frontier such as convexity, increasing monotonicity and non-negative values for inputs. In addition, unlike classical DEA, the redistribution by parabolic classic DEA requires a solution of a single linear program.

The linear program (1) describes the parabolic frontier for cases using one input and multiple outputs. In a set of *k* DMUs, the new input of a DMU_i will be represented by x_i and the outputs by y_{ij}, in which *j* varies from 1 to *s*. The original input is represented by x_{oi} and M_i is an auxiliary variable used to linearize the objective function’s absolute value (or modulus):

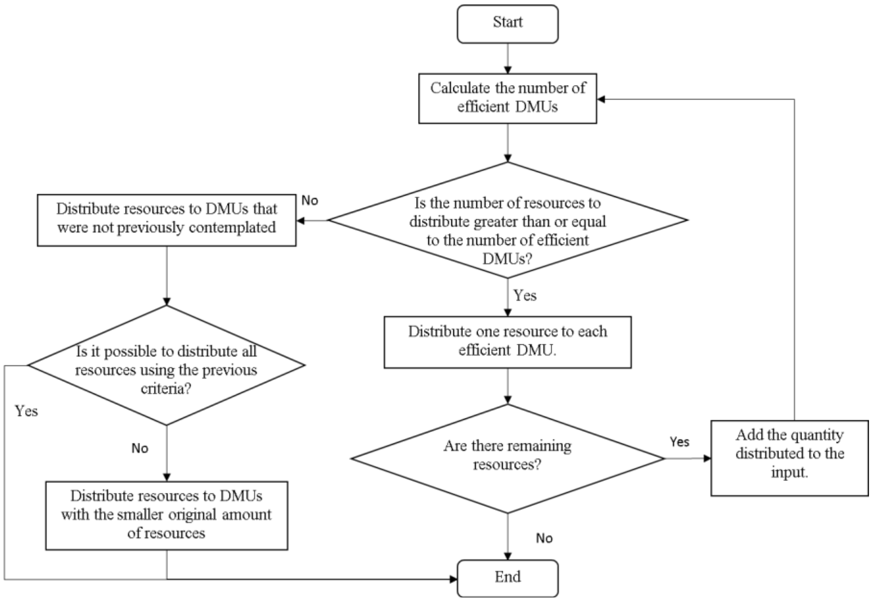
$$\begin{gathered} Min \sum_{i=1}^{k} M_i \\ \text{Subject to} \\ M_i \geq x_{oi} - x_i \ ; \ \forall i \\ M_i \geq -x_{oi} + x_i \ ; \ \forall i \\ \sum_{i=1}^{k} (x_{oi} - x_i) = 0 \\ x_i = ay_{i1}^2 + by_{i2} + cy_{i2}^2 + dy_{i2} + \cdots + my_{is}^2 + ny_{is} + e ; \forall i \\ \frac{d^2x}{dy_{ij}^2} \geq 0, \forall i \ and \ \forall j \\ \frac{dx}{dy_{ij}} \geq 0, \forall i \ and \ \forall j \end{gathered} \tag{1}$$

$$c \geq 0$$
$$x_i \geq 0, \forall i$$

3. SEQUENTIAL ALGORITHM TO RESOURCE ALLOCATION IN DEA

Soares de Mello *et al*. (2006) have developed the Sequential Algorithm for DEA for integer Resource Allocation. Later, Gomes *et al*. (2008) have combined this algorithm with the ZSG-DEA model. This algorithm does not require any judgements from decision makers. Figure 1 describes the algorithm.

Figure 1 – Steps of the sequential algorithm to resource allocation in DEA



The authors have stated that the first step is the calculous of DMU's efficiencies using classical DEA. If the total number of resources is equal or greater than the number of efficient DMUs, each of them receives one resource unit. After that, the decision maker must recalculate the efficiencies using the new resources configuration and repeat the previously described steps until all resources have been distributed.

Otherwise, if the total number of resources is lower than the number of efficient DMUs at some stage of the process, the decision maker must allocate resources only to DMUs that have not been awarded in previous steps. This rule contributes to an equal distribution. However, if the tie persists, it prioritizes distribution to efficient DMUs with fewer original resources. This rule is based on the principle that the input is most useful for DMUs that have fewer resources.

4. PROPOSED HYBRID MODEL: PARABOLIC DEA AND SEQUENTIAL ALGORITHM TO RESOURCES ALLOCATION IN DEA.

As previously mentioned, the parabolic DEA model is suitable only for non-integer resources redistribution. One way to deal with integer values is rounding them off. However, it may lead to a total sum different from the original.

On the other hand, the Sequential Algorithm to Resource Allocation properly deals with integer resources. However, this method needs a previously definition of the number of resources to be redistributed. In addition, it may become very slow if the number of resources is greater than the number of efficient DMUs (Soares de Mello *et al*., 2006):

Similar as developed by Gomes *et al.* (2008), we propose herein the joint use of parabolic DEA, instead of ZSG-DEA, and the Sequential Algorithm combining the advantages and potentialities of both models. That is, the parabolic DEA will be used as first phase to speeds up the resource redistribution and determines the number of resources that will be distribute by the Sequential Algorithm in the second phase.

In the first phase, the parabolic DEA model allocates resources assuming that the variable is non-integer. However, as it is a discrete variable, each DMU receives only the integer part, resulting in a resource's total sum smaller than the original. In the second phase, the Sequential Algorithm distributes the difference between the resource's original total sum and the total sum allocated in the first phase, that is, the remaining resources.

Therefore, the following steps describe the proposed hybrid model:

1) Resources redistribution using DEA parabolic frontier. (Phase 1)
2) Truncate resources to convert them all in integer numbers. (Phase 1)
3) Allocate only the resource's integer part for each DMU. (Phase 1)
4) Distribute the remaining resources using the sequential algorithm to resource allocation in DEA in Figure 1. (Phase 2)

The hybrid model redistributes integer resources so that the DMUs can get as close as feasible to the efficiency frontier without decision maker's judgements. Unlike in parabolic DEA, the proposed hybrid model does not ensure that all DMUs will become extremely efficient at the end of the process since the Sequential Algorithm may modify the parabolic shape of the DEA's efficiency frontier achieved in the first phase, when assuming the resources as a non-integer variable.

5. CASE STUDY

Railway stations are the main interface between passengers and the railroad because it is through them that users can access the rail system, board or disembark and change lines. As stated by Sameni *et al.* (2016), stations usually are the railway's bottlenecks. However, few studies focus on evaluating and ranking their performances.

The rail system costs high for operating and maintenance. Thus, the best use of the available resources, such as materials, energy and employees, is mandatory to make the operation financially viable. Rio de Janeiro's metropolitan trains system has 102 stations distributed among 220km divided in 7 branches. Two of them run by diesel locomotive and the 165km remainder run by electric traction.

Besides the type of traction, the size of the gauge (space between the inner sides of the rails) differentiates the two branches operated by locomotives from the others. These have 1 meter gauge (narrow gauge) while those powered by electricity have 1,60 meter gauge (wide gauge): Thus, there is segregation between the trains that run on electrified and diesel traction branches. Regarding the conditions presented, this work only deals with stations located on wide gauge extensions.

Employees located at stations are responsible for ensuring that the passengers will get the best services while staying at the station. Ticket's sellers are responsible for selling tickets, used to access station's paid areas through turnstiles. Security agents are in charge of monitoring the site and ensuring the safety of customers, employees and the company's material property. Attendants are responsible for transmitting travel information, informing about changes in the schedule and answering passenger's questions. Cleaning assistants are responsible for the site's hygiene.

Studies related to DEA's application on transports can be found at (Sameni *et al*., 2016), (Moreira *et al*., 2019) and (Pereira *et al*., 2019):

5.1 DEA modeling

We have selected variables based on the following statement: the greater the flow of passengers and the station's area, the greater the need for employees. In addition, stations that receive a large flow of people during peak hour, even if the average day is not significant, need more staff to avoid operational troubles such as waiting lines at the box office and clutter situations. Thus, we have selected the following variables:

- Input: Total employees per station (Employees) - ticket agents, security officers, public attendants and cleaning professionals.
- Output 1: Average of passengers during working days (AWD Pax) per station;
- Output 2: Train movement per station (Movement) – how many times the trains stop at the station for boarding and disembarking of passengers, regardless direction.
- Output 3: Number of lines per station (Lines) - railways passing through the station. Usually, there are at least two lines with opposite ways of movement.
- Output 4: Average of the maximum arrivals during peak hour in business days (AWD Max) - highest number of passengers within 60 minutes per day corresponding to the maximum passenger served during peak hours.

AWD Max differs from AWD Pax because while the first one is the maximum passengers during peak hour, the latter is the average passenger over a business day. Therefore, a station that receives many

passengers at the same time needs more workers than another one that receives the same number of clients spread out during the day.
Station's operational characteristics, such as the number of lines and train movements, interfere in the number of passengers. The chances of more simultaneous boarding and disembarking on different trains at the same station increase with the number of lines, which directly increases passenger flow. In addition, the number of lines correlates with the number of station's boarding platforms and total area that requires surveillance. Thus, we have inferred that the higher the passenger flow and the number of lines at the station, the greater the need of staff.
We have used real inputs and outputs values dated from May 2017 multiplied by a correction factor to preserve the information confidentiality. This change has no influence on the results since DEA is invariant to scale, that is, although the numerical data is disguised, the efficiencies are reliable and can be used to support management decision-making.
We have decided to use BBC model due to the non-proportionality between variables, since modifications in the number of lines or passengers do not cause proportional changes in the number of employees. Moreover, the relationship between inputs and outputs does not go through origin 0, i.e., the absence of passengers at any given time does not imply the unavailability of staff at the station.
We also have seen efficient DMUs by default as an advantage in this study case, because the model respects the minimum number of employees at stations defined by the company, not allowing results less than the minimum established.
We have selected 79 DMUs corresponding to the broad gauge railway but Saracuruna. This station has been excluded from the study because is the terminal station of three branches – Saracuruna, Vila Inhomirim and Guapimirim (the last two are narrow gauge branches, which were not used on this study case): In terminal stations, the passengers flow requires more staff to assist the public once people arrive at the station to change lines or leave the system in batches.

5.2 Results

First, using classical DEA BCC input-oriented model for calculate efficiencies, we have found that Central do Brasil, Maracanã, Deodoro and Silva Freire are efficient in the original scenario. After that, we have used the hybrid proposed model's steps, which results are described in the following sections.

5.3 Phase 1: using the parabolic DEA model

The first phase uses the Parabolic DEA model for reallocating employees between DMUs by making all of them efficient and keeping the total input's sum. Table 1 shows the results in which "Eff" corresponds to the classical input-oriented BCC efficiency.
The model also determines the coefficients for the paraboloid frontier equation, described in (2): As can be noticed, the model has not used "Movement" (Output 2) and "AWD Max" (Output 4) for shaping the new efficiency frontier, as shown by their null coefficient.

$$x_i = 0y_1^2 + 95{,}40y_1 + 0y_2^2 + 0y_2 + 0y_3^2 - 19{,}24y_3 + 0y_4^2 - 0y_4 + 3.74 \quad (2)$$

Table 1 – Parabolic DEA redistribution results

Stations	Original data						Parabolic DEA	
	Input	Output 1	Output 2	Output 3	Output 4	Eff.	Input	Eff.
Central do Brasil	40	138182	783	14	23627	1	118,4	1
Maracanã	19	13705	783	6	1979	1	21,5	1
Deodoro	31	7499	523	12	1230	1	25,4	1
Silva Freire	2	674	77	2	161	1	7	1
Marechal Hermes	9	6010	208	4	1043	0,918	13,4	1
Oswaldo Cruz	9	3273	208	4	654	0,915	11,5	1
Triagem	10	4558	261	4	1166	0,881	12,4	1
São Cristovão	26	30111	783	6	5398	0,858	32,8	1
Cascadura	10	4114	208	4	557	0,824	12,1	1
Realengo	10	5502	181	4	923	0,799	13	1
Riachuelo	7	4599	208	2	876	0,789	9,7	1
Piedade	7	3430	208	2	484	0,744	8,9	1

Stations	Original data						Parabolic DEA	
	Input	Output 1	Output 2	Output 3	Output 4	Eff.	Input	Eff.
Quintino	7	2903	208	2	554	0,744	8,5	1
Engenho de Dentro	19	11636	522	5	1579	0,733	18,6	1
Olaria	6	3974	164	2	469	0,73	9,2	1
São Francisco Xavier	12	2777	208	4	568	0,686	11,2	1
Engenho Novo	8	4320	208	2	711	0,67	9,5	1
Penha Circular	7	3204	164	2	769	0,638	8,7	1
Praça da Bandeira	9	4683	208	2	1074	0,635	9,7	1
Agostinho Porto	4	923	96	2	266	0,628	7,1	1
Tomás Coelho	4	702	96	2	140	0,614	7	1
Rocha Miranda	4	547	96	2	138	0,614	6,9	1
Cavalcante	4	518	96	2	118	0,614	6,8	1
Vila Militar	9	1115	181	3	246	0,612	8,6	1
Santíssimo	8	3415	181	2	644	0,589	8,8	1
Benjamim do Monte	9	1645	158	3	253	0,586	9	1
Cordovil	7	1687	164	2	290	0,585	7,7	1
Brás de Pina	7	1501	164	2	231	0,585	7,5	1
Guilherme da Silveira	8	3424	181	2	579	0,581	8,9	1
Bento Ribeiro	10	3546	208	3	638	0,579	10,3	1
Sampaio	9	1997	208	2	239	0,573	7,9	1
Comendador Soares	14	8514	167	4	1863	0,572	15,1	1
Penha	8	4458	164	2	502	0,558	9,6	1
Anchieta	10	3061	172	3	558	0,543	10	1
Ramos	8	3312	164	2	480	0,534	8,8	1
Padre Miguel	9	3834	181	2	647	0,524	9,1	1
Magalhães Bastos	9	3634	181	2	497	0,52	9	1
Parada de Lucas	8	2251	164	2	286	0,512	8	1
Augusto Vasconcelos	9	2349	181	2	380	0,5	8,1	1
Vila Rosali	4	184	70	2	48	0,5	6,6	1
Tancredo Neves	8	853	158	2	142	0,494	7,1	1
Pilares	5	914	96	2	167	0,492	7,1	1
Honório Gurgel	5	711	96	2	119	0,492	7	1
Paciência	11	5790	158	3	926	0,483	11,9	1
Madureira	36	34994	522	5	3533	0,476	34,8	1
Vigário Geral	9	1444	164	2	248	0,455	7,5	1
Coelho da Rocha	6	2056	96	2	486	0,454	7,9	1
Mesquita	12	5194	172	3	901	0,454	11,5	1
Senador Camará	11	4132	181	2	874	0,449	9,3	1
Méier	16	14821	208	2	1943	0,446	16,7	1
Ricardo de Albuquerque	11	4677	172	2	944	0,437	9,7	1
Cosmos	10	3731	158	2	744	0,431	9,1	1
Austin	19	8654	167	4	1756	0,421	15,2	1
Del Castilho	5	653	70	2	129	0,4	6,9	1
Olinda	11	2697	172	2	480	0,396	8,4	1
Presidente Juscelino	11	2813	172	2	460	0,396	8,4	1

Stations	Original data Input	Output 1	Output 2	Output 3	Output 4	Eff.	Parabolic DEA Input	Eff.
Campos Elíseos	7	1527	106	2	268	0,393	7,5	1
Inhoaíba	11	3654	158	2	625	0,382	9	1
Edson Passos	13	4898	172	2	1009	0,375	9,9	1
Bangu	22	14585	181	4	1733	0,369	19,3	1
Japeri	23	4846	221	4	828	0,364	12,6	1
Nilópolis	16	11030	172	3	1774	0,359	15,5	1
Costa Barros	7	385	96	2	86	0,351	6,8	1
Jardim Primavera	9	2849	106	1	626	0,342	7,1	1
Barros Filho	6	495	70	2	78	0,333	6,8	1
Santa Cruz	25	15002	158	4	1993	0,325	19,6	1
Corte Oito	16	3971	165	2	1236	0,309	9,2	1
Campo Grande	26	22214	181	3	2890	0,306	23,2	1
Bonsucesso	17	7453	165	2	1022	0,293	11,6	1
Engenheiro Pedreira	20	11469	160	2	1860	0,278	14,4	1
Nova Iguaçu	30	23607	172	4	2534	0,278	25,5	1
Duque de Caxias	23	15689	165	2	2101	0,277	17,3	1
Gramacho	35	15511	271	3	3426	0,267	18,6	1
Belford Roxo	22	9846	96	2	2213	0,242	13,3	1
Manguinhos	19	2558	164	2	494	0,221	8,3	1
Queimados	30	16639	167	3	2598	0,219	19,4	1
Jacarezinho	11	220	70	2	33	0,182	6,6	1
Mercadão de Madureira	17	3398	96	2	624	0,168	8,8	1
Pavuna	18	2106	96	2	317	0,147	7,9	1
Total	1019						1019	

As expected, the values of resources redistributed by the model for each DMU are not integer. Therefore, each DMU has received only the integer part of its resource since number of employees is a discrete variable, once it is not possible to allocate parts of a person. However, this procedure results in a smaller number of employees than the original number, remaining 40 inputs to be redistributed. The sequential algorithm will distribute these resources in the second phase.

5.4 Phase 2: resource allocation algorithm

In the second phase, we follow the sequential algorithm steps to redistribute the 40 remaining resources. First, we determine the classic BCC efficiencies using the truncated integer input results obtained in the first phase. At this point, there are 36 DMUs at the efficiency frontier. According to the algorithm, each of them receives one resource.

We still have 4 employees left to be redistributed. Following the algorithm, we determine the efficiencies after this first redistribution. There are 28 efficient DMUs, which is greater than the number of resources left to be redistributed. Therefore, we use the first tiebreaker criterion, that is, we disregard DMUs that have already been awarded with resources in the previous round. This left use with 12 DMUs. However, this number is still greater than the 4 remaining employees to be redistributed. Then, we use the second tiebreaker criterion, which is to assign the reaming resources to 4 DMUs with the lowest values of inputs. As a result, Coelho da Rocha, Jardim Primavera, Piedade and Vila Militar have received one input each. Table 2 depicts the final results.

The relationship between station's employees losses and earnings can be better observed through Graph 1. It orders the stations by staff increasing after reallocation and its columns shows the original distribution of employees and the results. Thus, Central do Brasil, the first station presented, is the one that have received the most resources. On the other hand, Gramacho, the last station presented, is the one that have lost the most employees

Table 2 – Second phase results

Station	Phase 1 results		Phase 2 results		Station	Phase 1 results		Phase 2 results	
	Input	Eff.	Input	Eff.		Input	Eff.	Input	Eff.
Central do Brasil	118,4	1,0	119,0	1,0	Tancredo Neves	7,1	1,0	8,0	0,9
Maracanã	21,5	1,0	22,0	1,0	Pilares	7,1	1,0	8,0	0,9
Deodoro	25,4	1,0	26,0	1,0	Honório Gurgel	7,0	1,0	7,0	1,0
Silva Freire	7,0	1,0	7,0	1,0	Paciência	11,9	1,0	12,0	0,9
Marechal Hermes	13,4	1,0	13,0	1,0	Madureira	34,8	1,0	34,0	1,0
Oswaldo Cruz	11,5	1,0	12,0	0,9	Vigário Geral	7,5	1,0	8,0	1,0
Triagem	12,4	1,0	12,0	1,0	Coelho da Rocha	7,9	1,0	8,0	1,0
São Cristovão	32,8	1,0	33,0	1,0	Mesquita	11,5	1,0	11,0	1,0
Cascadura	12,1	1,0	12,0	0,9	Senador Camará	9,3	1,0	9,0	0,9
Realengo	13,0	1,0	13,0	0,9	Méier	16,7	1,0	16,0	1,0
Riachuelo	9,7	1,0	10,0	0,9	Ricardo de Albuquerque	9,7	1,0	10,0	0,9
Piedade	8,9	1,0	9,0	1,0	Cosmos	9,1	1,0	9,0	0,9
Quintino	8,5	1,0	9,0	0,9	Austin	15,2	1,0	15,0	1,0
Engenho de Dentro	18,6	1,0	19,0	1,0	Del Castilho	6,9	1,0	7,0	1,0
Olaria	9,2	1,0	9,0	0,9	Olinda	8,4	1,0	8,0	1,0
São Francisco Xavier	11,2	1,0	11,0	1,0	Presidente Juscelino	8,4	1,0	8,0	1,0
Engenho Novo	9,5	1,0	9,0	1,0	Campos Elíseos	7,5	1,0	8,0	0,9
Penha Circular	8,7	1,0	9,0	0,9	Inhoaíba	9,0	1,0	9,0	0,9
Praça da Bandeira	9,7	1,0	9,0	1,0	Edson Passos	9,9	1,0	9,0	1,0
Agostinho Porto	7,1	1,0	8,0	0,9	Bangu	19,3	1,0	19,0	1,0
Tomás Coelho	7,0	1,0	7,0	1,0	Japeri	12,6	1,0	12,0	1,0
Rocha Miranda	6,9	1,0	7,0	1,0	Nilópolis	15,5	1,0	15,0	1,0
Cavalcante	6,8	1,0	7,0	1,0	Costa Barros	6,8	1,0	7,0	1,0
Vila Militar	8,6	1,0	9,0	1,0	Jardim Primavera	7,1	1,0	8,0	1,0
Santíssimo	8,8	1,0	9,0	0,9	Barros Filho	6,8	1,0	7,0	1,0
Benjamim do Monte	9,0	1,0	9,0	0,9	Santa Cruz	19,6	1,0	19,0	1,0
Cordovil	7,7	1,0	8,0	1,0	Corte Oito	9,2	1,0	9,0	1,0
Brás de Pina	7,5	1,0	8,0	1,0	Campo Grande	23,2	1,0	23,0	1,0
Guilherme da Silveira	8,9	1,0	9,0	0,9	Bonsucesso	11,6	1,0	11,0	1,0
Bento Ribeiro	10,3	1,0	10,0	0,9	Engenheiro Pedreira	14,4	1,0	15,0	0,9
Sampaio	7,9	1,0	8,0	1,0	Nova Iguaçu	25,5	1,0	25,0	1,0
Comendador Soares	15,1	1,0	16,0	0,9	Duque de Caxias	17,3	1,0	18,0	0,9
Penha	9,6	1,0	9,0	1,0	Gramacho	18,6	1,0	19,0	1,0
Anchieta	10,0	1,0	9,0	1,0	Belford Roxo	13,3	1,0	13,0	1,0
Ramos	8,8	1,0	8,0	1,0	Manguinhos	8,3	1,0	8,0	1,0
Padre Miguel	9,1	1,0	9,0	0,9	Queimados	19,4	1,0	19,0	1,0
Magalhães Bastos	9,0	1,0	9,0	0,9	Jacarezinho	6,6	1,0	7,0	1,0
Parada de Lucas	8,0	1,0	8,0	1,0	Mercadão de Madureira	8,8	1,0	8,0	1,0
Augusto Vasconcelos	8,1	1,0	8,0	1,0	Pavuna	7,9	1,0	8,0	0,9

Station	Phase 1 results		Phase 2 results		Station	Phase 1 results		Phase 2 results	
	Input	Eff.	Input	Eff.		Input	Eff.	Input	Eff.
Vila Rosali	6,6	1,0	7,0	1,0					

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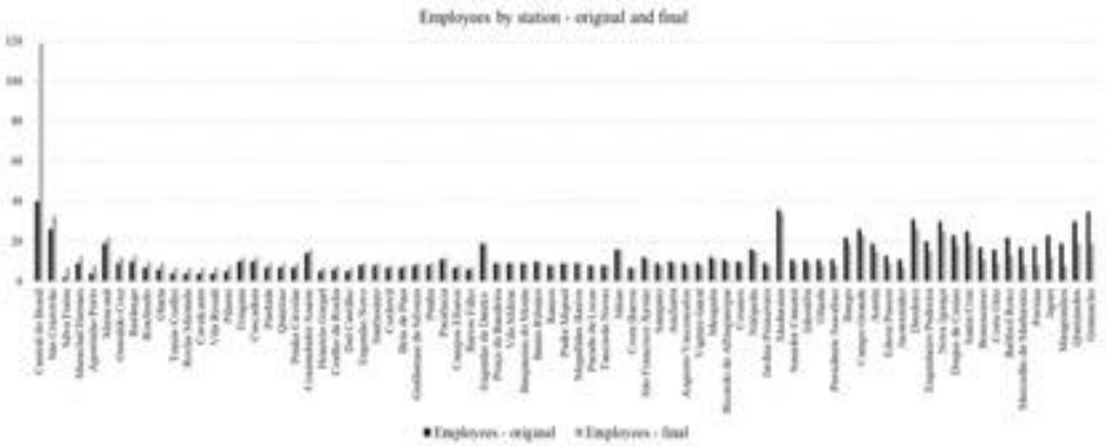
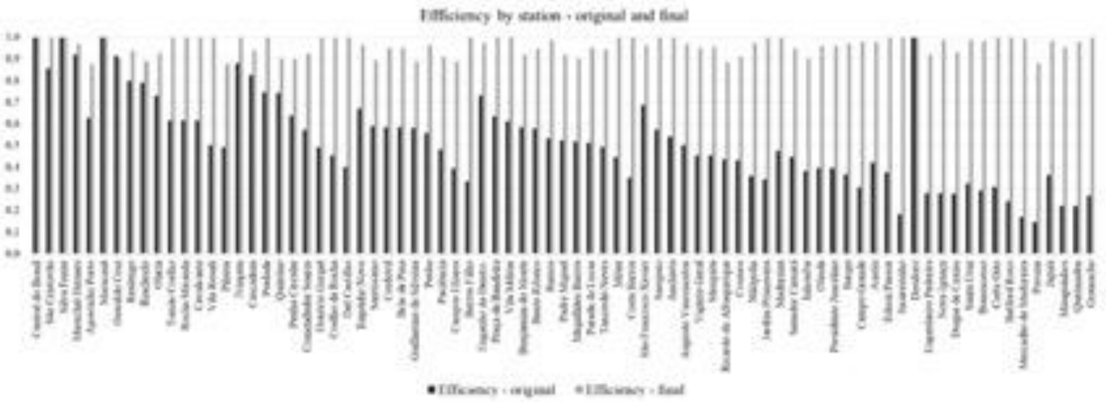


Graph 1 – Employees by station – original and final



Graph 2 – Efficiency by station – original and final

Graph 2 shows the relationship between original and after redistribution efficiencies of each station. It orders them using the same assumption as in Graph 1.

6. CONCLUSION

In this paper we have introduced a hybrid combination of Parabolic DEA and the Sequential Algorithm to Resource Allocation in DEA to redistribute an integer variable among all DMUs. Therefore, using this approach on the study case presented, we have increased the average efficiency of the set of stations from 52% to 96%.

As previously described, the hybrid model proposed to resource redistribution does not ensure 100% average efficiency. This is because the Sequential Algorithm may change the parabolic shape of the DEA’s efficiency frontier achieved in the first phase, when assuming the resources as a non-integer variable. The hybrid model redistributes integer resources without the decision maker’s judgements, setting the DMUs as close as feasible to the final efficiency frontier.

In the presented study case, we have noticed that the model has rewarded stations that have been using better their employees– higher efficiencies – and has punished with fewer inputs those that had them in excess – lower efficiencies.

We have inferred that using DEA methodology for calculating railway station’s efficiencies indicates that its performance can be improved by analysing and replicating management techniques that are adopted by efficient stations, if operationally feasible. In addition, the redistribution of employees between stations may contribute to improve and level the service offered to passengers among all DMUs regarding its individual characteristics.

As future studies, we intend to extend de parabolic model to redistribute more than one input and to redistribute outputs as well.

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