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PRODUCTION INVENTORY MODELS FOR DETERIORATIVE ITEMS WITH CONTINUOUS COMPOUND DEMAND AND GROWTH OF DEMAND

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ABSTRACT
A Production Inventory model is an inventory control model that determines the amount of product to be produced on a single facility so as to meet a deterministic demand over an infinite planning horizon. In all inventory models a general assumption is that products generated have indefinitely long lives. In general, almost all items deteriorate over time. In production inventory model, the demand rate is constant and it does not change over the period. But in real life, the demand rate is fluctuating over the period. At the end of the particular period, the demand rate will get changed. So, the continuous compound demand rate is analyzed in the model 1 and integrated continuous compound demand with growth of demand is introduced in the model 2. To our knowledge, no researchers have considered both time dependent demands in a single model. The rate of Growth in the production period is $Y(1+i)^n$ and in the consumption period is $Y(1-i)^n$. This research considers inventory systems for production inventory models where the objective is to find the optimal cycle time, which minimize the total cost and optimal amount of shortage if it is allowed. The relevant model is built, solved and closed formulas are obtained. Necessary and sufficient conditions are derived. An illustrative example is provided and numerically verified. The validation of result in this model was coded in Microsoft Visual Basic 6.0

KEYWORDS: Economic Production Quantity (EPQ), Deteriorative items, Cycle time, Optimality, Integration, Growth of Demand**,** Continuous compound Demand (CCD) and Production**.**

MSC : 90B05

RESUMEN
Un modelo de inventario de producción es uno de control que determina la cantidad de productos a ser producida por una facilidad simple para alcanzar una demanda determinística en un horizonte infinito de planeamiento. En todos los modelos de inventario una general asunción es que los productos generados tienen una vida larga e indefinida. En general, casi todos los ítems se deterioran con el tiempo. En los modelos de inventario de producción, la tasa de demanda es constante y no cambia en el periodo. Pero en la vida real, la tasa de demanda fluctúa en el periodo. Al final del particular periodo, la tasa de demanda cambiará. Así que el componente continuo de la tasa de demanda es analizada en el modelo 1 y la demanda integrada continua compuesta con crecimiento se introduce en el modelo 2. A nuestro conocimiento, ningún investigador ha considerado al mismo tiempo demandas dependientes del tiempo en un mismo modelo. La tasa de crecimiento en el periodo de producción es $Y(1+i)^n$ y en el de consumo $Y(1-i)^n$. Esta investigación considera sistemas de inventario para modelos de inventarios de producción donde el objetivo es hallar el óptimo del ciclo de tiempo, que minimice el costo total y la cantidad optimal de las carencias, si esta es aceptada. El modelo relevante se construyó, resolvió y fórmulas analíticas son obtenidas. Se derivan condiciones necesarias y suficientes. Un ejemplo ilustrativo se presenta y se verifica numéricamente. La validación del modelo resultante se codificó en Microsoft Visual Basic 6.0

PALABRAS CLAVE: Cantidad Económica de Producción (EPQ), ítems Deteriorativos, Ciclo de tiempo, Optimalidad, Integración, Crecimiento de la Demanda**,** Demanda continua compuesta (CCD) y Producción**.**

1. INTRODUCTION

The fundamental goals and strategies of most of the manufacturing firms is to satisfy the customer's demand and to attain minimum cost. The company has to use their resources effectively to attain these goals. Several decades ago the first mathematical model was introduced to assist companies in minimizing the total inventory costs which balances cost of inventory and cost of setup per set with the derivation of optimal order quantity. The EOQ inventory model is in existence based on its easiness. In the manufacturing sector, when items are produced internally instead of being obtained from an outside supplier, the economic production quantity (EPQ) model is often employed to determine the optimal production lot size that minimizes overall production/inventory costs. The production inventory model considers the supreme case where the value of inventory items are unaffected by time and replenishment is done instantaneously. But the supreme case is not favorable for the real life situations. Inventories are often replenished periodically at a certain production rate

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which is seldom infinite. In the past decades, deteriorating inventory problems were typically approached by developing mathematical models that considered practical factors in a real world situation. Demand is the major factor in the inventory management. Therefore, decisions of inventory are to be made because of the present and future demands. As demand plays a key role in modeling of deteriorating inventory, researchers have recognized and studied the variations of demand from the viewpoint of real life situations. Demand may be constant, time-varying, stock-dependent and price-dependent, etc. Inventory modelers have so far considered only three types of time-dependent demands, linear, exponential and quadratic demand. Linear time-dependence of demand implies a uniform change in the demand rate of the product per unit time. This is rarely seen to occur in the real market. An exponentially time-varying demand also seems to be unrealistic because of exponential rate of changes is very high and it is uncertain whether the market demand of any product may undergo such a high rate of changes as exponential. In quadratic time-dependence of demand, it may represent all types of time-dependence depending on the signs of the parameters of the time-quadratic demand function (and perhaps more realistic). An alternative approach is considered to the rate of growth of demand. Growth of demand refers to an increase in demand over an extended period. Growth can best be described as a process of transformation. Deteriorating items are common in our daily life; however, academia has not reached a consensus on the definition of the deteriorating items. Ghare and Schrader (1963) developed a inventory model in which the decrease in inventory level is based upon not only on demand but also by deterioration. Heng, Labban and Linn (1991) developed production inventory model with finite production rate and the effect of decay for which the solution of optimal lot size and order level is obtained from the algorithm. Shib Sankar Sana (2004) considered production inventory model for deteriorating item with a linear time varying demand and finite production rate in which shortages are allowed. Alfares et al. [2005] introduced a solution algorithm for incorporating quality and maintenance aspects into a production inventory system for deteriorating items. Closed-form solutions include the quality considerations. Chiu et al. [2007] developed an EPQ model with imperfect production quality, imperfect inspection and rework. Srivastava and Gupta (2007) developed an inventory model for deteriorative items with constant demand for certain period and linear demand for the remaining period. Maity et al (2008) considered production inventory model in fuzzy and the demand is inversely depending on the selling price and the selling price is serviceable with stock dependent. The holding cost is fuzzy variables and the authors considered the optimal control approach is considered to optimize the production. Mahata[2011] considered a realistic inventory model with imprecise inventory cost components have been formulated for deteriorating items under trade credit policy within the EPQ framework. It is assumed that the retailer maintains a powerful position and can obtain full trade credit offered by supplier yet retailer just offers the partial trade credit to his/her customers under which the replenishment rate is finite. Feng, Yan and Viswanathan [2011] proposed mathematical models for general multi manufacturing and remanufacturing setup policies. Chung and Wee [2011] considered short life-cycle deteriorating product remanufacturing in a green supply chain inventory control system. Widyadanaetal. [2012] developed EPQ models for deteriorating items with preventive maintenance, random machine breakdown and immediate corrective action. Corrective and preventive maintenance times are assumed to be stochastic and the unfulfilled demands are lost sales. Widyadana etal. [2012] developed an economic production quantity model for deteriorating items with rework. In one cycle, production facility can produce items in m production setups and one rework setup, (m.1) policy. Sivashankari and Panayappan (2015) considered production inventory models for deteriorating items with growth of demand and allowed shortages. The rate of growth in the production period is $D(1 + i)^n$ and in the consumption period is $D(1 - i)^n$. Prasenjit Manna (2016) considered inventory model for deteriorative items with ramp type demand function. The unit of production cost is inversely proportional to the demand rate and shortages are not permitted. Tripathi, Sarla Pareek and Manjit Kaur (2017) considered a purchasing inventory model for time dependent exponential deterioration items with time dependent exponential demand and shortages are permitted. And also, unit production cost and demand rate are considered as proportional to each other. Sumit Saha and Nabendu Sen (2017) considered an inventory model for deteriorating items with negative exponential demand and shortages are permitted with partial back logging. In this model, three types of probabilistic deterioration function is studied to determine decision variables. Pervin and Roy (2018) developed an inventory model for deteriorative items with integrated vendor buyer inventory model along with time-dependent demand to attract more customers and time dependent holding cost. Shortages are permitted and partial backordering is followed. Anima Bag and Tripathy P.K. (2019) developed an inventory model for deteriorative items with time and selling price induced quadratic demand and also developed the production period consisting of many sub periods with different production rates. Tahirov et al (2019) developed a production inventory models for four-level closed-loop supply chain with remanufacturing. Customer demand is met from either newly manufactured item, remanufacturing used items collected from customer for recovery or from both. Sunil Tiwaria, Leopoldo Eduardo Cárdenas, Barrónb Ali, AkbarShaikh and Mark Gohad (2018) developed inventory model for deteriorating items to determined theoretical results with shortages and partial delay in payment. Shaikh, Cárdenas-Barrón and Tiwari, S. (2019) considered a two-warehouse inventory model for non-instantaneous deteriorating items with interval-valued inventory costs and stock-dependent demand under inflationary conditions. The proposed inventory model permits shortages, and the backlogging rate is variable and dependent on the waiting time for the next order, and inventory parameters are interval-valued. The main aim of this research is to obtain the retailer's optimal replenishment policy that minimizes the present worth of total cost per unit time. In this paper, the continuous compound demand is analyzed in the model 1 and growth of demand with continuous compound demand rate is considered in model 2 and shortages are not permitted. The rate of Growth in the production period is $Y(1+i)^n$ and in the consumption period is $Y(1-i)^n$. This research considers inventory systems for production inventory models where the objective is to find the optimal cycle time, which minimize the total cost and optimal amount of shortage if it is allowed. This paper is organized as follows. Section 2 is concerned with assumptions and notations, Section 3 presents mathematical model for finding the optimal solutions and numerical example. Section 4, consider comparative study between constant demand, continuous compound demand and Continuous compound demand with growth of demand. Finally, the paper summarizes and concludes in section 5.

2. ASSUMPTIONS AND NOTATIONS

2.1. Assumptions

The following assumptions are used to formulate the problem:
(1) Initial inventory level is zero and planning horizon is infinite, (2) The demand rate is continuous compound demand in model 1 and CCD with Growth of Demand in Model 2., (3) Shortages are not allowed, (4) Cost of carrying inventory per unit per unit time is known, (5) The lead time is known and constant, (6) Items are produced and added to the inventory, (7) The production rate is always greater than or equal to the sum of the demand rate, (8) The rate of deteriorative is μ .

2.2. Notations.

The following notations are used in our analysis.
(1) X - Production rate in units per unit time, (2) Y - Ye^{Rt} Demand rate is continuous compound demand in model 1 but in model 2, Y - $Y(1+i)^n e^{Rt}$ is considered, (3) Q*-Optimal size of production run (decision variable), (4) P_C - Production cost per unit (5) μ - Rate of perishable product, (6) H_C - Cost of carrying inventory per unit per unit time, (7) S_C - Cost of setup per set , (8) T - optimal cycle time (decision variable), (9) T_1 - cost of production and the inventory is building up at a constant rate of X – Y units per unit time, (10) I_I - maximum inventory level at time T_1, (11) R - Rate of increase in demand, (12) i – rate of interest and (13) n – number of periods (years)

Computational Algorithm:
Step 1: Assign values to the parameters with proper units
Step 2: To find the two variables T and Q in model 1 and model 2. Here two variables T1 and T has to be calculated so the partial differential equation is used.
Step 3 : The partial differential equation for optimality is as follows

1. $$\frac{\partial TC(T)}{\partial T_2} = 0 \quad \text{and} \quad \frac{\partial^2 TC(T)}{\partial T_2^{\,2}} > 0$$

2. $$\frac{\partial TC(T)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial^2 TC(T)}{\partial T^2} > 0$$

Step 4: The cubic equation is solved by using the following algorithm.

1. Let the cubic equation be $ay^3 + by^2 + cy + d = 0$
2. Let us consider an example, $y^3 - 0.7660\,y^2 + 0.1345\,y - 0.0058 = 0$
3. where A =1, B = - 0.7660, C = 0.1345 and D = -0.0058
4. The cubic equations have to be solved in several steps:
5. Define a variable " f_2 ". Therefore, $f_2 = \frac{1}{3}\left[\frac{3C}{A} - \frac{B^2}{A^2}\right]$ = -0.06105
6. Define variable " g_2 ". Therefore, $g_2 = \frac{1}{27}\left[\frac{2B^3}{A^3} - \frac{9BC}{A^2} + \frac{27D}{A}\right]$ = - 0.00474
7. Define variable " h_2 ". Therefore, $h_2 = \frac{g^2}{4} + \frac{f^3}{27}$ = -0.0000028
8. Define variable " i " therefore, $i = \left[\frac{g_2{}^2}{4} - h_2\right]^{\frac{1}{2}}$ = 0.0029
9. Define variable "j ", therefore, $j = i^{\frac{1}{3}}$ = 0.14266
10. Define variable " k ", therefore, $k = arc.Cos\sin\left[-\frac{g}{2i}\right]$ = 0.6147
11. Define variable "L", therefore, L = -j = - 0.14266
12. Define variable "M", therefore, M= $\cos(k/3)$ = 0.9791
13. Define variable "N", therefore, N = $\sqrt{3}\sin(k/3)$ = 0.3524
14. Define variable "P", therefore, P = $\frac{-B}{3A}$ = 0.2553

Therefore, the roots of cubic equation are as follows:

$y_1 = 2j\cos(k/3) - B/3A$ = 0.5347;

$y_2 = L(M+N) + P$ = 0.0654;

$y_3 = L(M-N) + P$ = 0.1659.

From above, all roots are real.

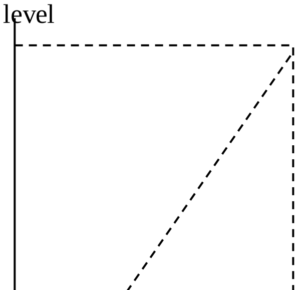
Step 5: All datas are programmed and generated from visual basic 6.0 software.

3. MATHEMATICAL MODELS

3.1. Model: 1 Production inventory model for Deteriorative items with Continuous Compound Demand (CCD)

The objective of the inventory models is to determine the optimal cycle time or the corresponding optimal production quantity in order to minimize the total relevant cost. Consequently, the production time and the maximum inventory level can easily be calculated. Figure 1 represents the economic production quantity (EPQ) model with continuous compound demand. The inventory on-hand increases with the rate X – Y during the production time, which is the production rate minus consumption rate. After that point, the inventory level decreases with the consumption rate Y, until it becomes zero at the end of the cycle T, when the production process is resumed again.

$I(t)$ - Inventory level



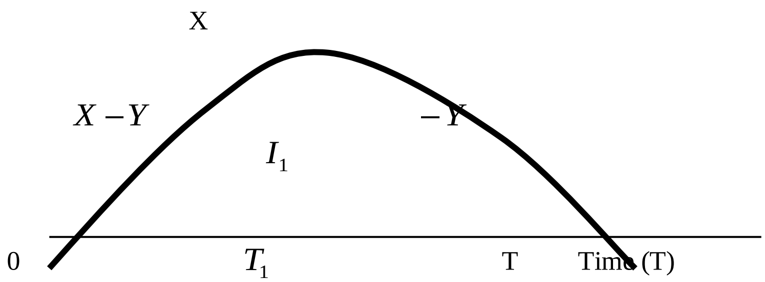


Figure -1 Production Inventory Cycle

During the production time or stage, the inventory increases due to production but decreases due to demand with the rate of deteriorative items. Thus, the inventory differential equation is

$$\frac{dI(t)}{dt} + \mu I(t) = X - Ye^{Rt};\ 0 \le t \le T_1 \tag{1}$$

During consumption period, no production is carried out but there is reduction in the inventory level due to rate of deteriorative items and sales. Thus, the inventory differential equation is

$$\frac{dI(t)}{dt} + \mu I(t) = -Ye^{Rt}\ ;\ T_1 \le t \le T \tag{2}$$

The differential equations boundary conditions are

$$I(0) = 0,\ I(T_1) = I_1,\ I(T) = 0 \tag{3}$$

During the first cycle, the inventory level $I(t)$, at time t is equal to
The solution of the differential equation (1),

$$I(t) = \frac{X}{\mu}\left(1 - e^{-\mu t}\right) + \frac{Y}{R+\mu}\left(e^{-\mu t} - e^{Rt}\right) \tag{4}$$

The solution of the differential equation (2),

$$I(t) = \frac{Y}{R+\mu}\left(e^{(R+\mu)T - \mu t} - e^{-RT}\right) \tag{5}$$

To find T_1 and I_1**:** From the equations (4), (5) and boundary conditions as per equation (3), the production time T_1 $\frac{X}{\mu}\left(1 - e^{-\mu T_1}\right) + \frac{Y}{R+\mu}\left(e^{-\mu T_1} - e^{RT_1}\right) = \frac{Y}{R+\mu}\left(e^{(R+\mu)T - \mu t} - e^{Rt}\right)$

Expanding the exponential functions and neglecting second and higher power of θ for small value of θ, the

$$T_1 = \frac{Y}{X}T \tag{6}$$

From the equations (4), (5) and boundary conditions as per equation (3), the maximum inventory I_1 is as follows: $I(T_1) = I_1 \Rightarrow I_1 = \frac{X}{\mu}\left(1 - e^{-\mu T_1}\right) + \frac{Y}{R+\mu}\left(e^{-\mu T_1} - e^{RT_1}\right)$ and $I(T_1) = I_1 \Rightarrow I_1 = \frac{Y}{R+\mu}\left(e^{(R+\mu)T - \mu t} - e^{Rt}\right)$

On simplification,

$$I_1 = (X - Y)T_1 \tag{7}$$

Total Cost: Total cost comprises of setup cost, holding cost, production cost, and deteriorative cost

(1) Setup cost = $\frac{S_C}{T}$ (8)

(2) Production Cost = YP_C (9)

(3) Holding Cost: Holding cost is applicable to both stages of the production cycle, as described by

$$\text{HC} = \frac{H_C}{T}\left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T} I(t)dt\right] = \frac{H_C}{T}\left[\int_0^{T_1}\left\{\frac{X}{\mu}\left(1-e^{-\mu t}\right)+\frac{Y}{R+\mu}\left(e^{-\mu t}-e^{Rt}\right)\right\}dt + \int_{T_1}^{T}\frac{Y}{R+\mu}\left(e^{(R+\mu)T-\mu t}-e^{Rt}\right)dt\right]$$

$$= \frac{H_C}{T}\left[\begin{array}{l}\frac{X}{\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)-\frac{Y}{R\mu(R+\mu)}\left(\text{Re}^{-\mu T_1}+\mu e^{RT_1}-R-\mu\right)\\ -\frac{Y}{R\mu(R+\mu)}\left(\text{Re}^{RT}+\mu e^{RT}-\text{Re}^{(R+\mu)T-\mu T_1}-\mu e^{RT_1}\right)\end{array}\right] = \frac{H_C}{T}\left[\frac{X}{\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)-\frac{Y}{R\mu(R+\mu)}\left(\text{Re}^{-\mu T_1}+(R+\mu)(e^{RT}-)-\text{Re}^{(R+\mu)T-\mu T_1}\right)\right]$$

$$= \frac{H_C}{T}\left[\frac{X}{\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)-\frac{Y}{R\mu(R+\mu)}\left((R+\mu)(e^{RT}-1)+\text{Re}^{-\mu T_1}-e^{(R+\mu)T-\mu T_1}\right)\right]$$

$$= \frac{H_C}{T}\left[\frac{X}{\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)-\frac{Y}{R\mu(R+\mu)}\left((R+\mu)(e^{RT}-1)+\text{Re}^{-\mu T_1}\left(1-e^{(R+\mu)T}\right)\right)\right] \tag{10}$$

(4) Cost of deteriorative = $\mu D_C\left[\int_0^{T_1} I_1(t)e^{-Rt}dt + \int_{T_1}^{T} I_2(t)e^{-Rt}dt\right]$

$$= \frac{\mu D_C}{T}\left[\frac{X}{\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)-\frac{Y}{R\mu(R+\mu)}\left((R+\mu)(e^{RT}-1)+\text{Re}^{-\mu T_1}\left(1-e^{(R+\mu)T}\right)\right)\right] \tag{11}$$

Theorem 1: The average system cost functions $TC(T_1)$ and $TC(T)$ are strictly convex.

Proof: The optimality conditions can be easily shown that $TC(T_1)$ and $TC(T)$ are convex function in T_1 and T. Hence, an optimum cycle time T_1 and T can be calculated from

(i) $\frac{\partial}{\partial T_1}\left[TC(T)\right]=0$ and $\frac{\partial^2}{\partial T_1^{\,2}}\left[TC(T)\right]>0$ and

(ii) $\frac{\partial}{\partial T}\left[TC(T)\right]=0$ and $\frac{\partial^2}{\partial T^2}\left[TC(T)\right]>0$

The total cost function comprises of setup cost, holding cost, production cost and perishable cost. Therefore, the total cost function is as follows:

$$\text{TC(T)} = \frac{S_c}{T}+YP_C+\left(H_C+\mu D_C\right)\left[\frac{X}{\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)-\frac{Y}{R\mu(R+\mu)}\begin{pmatrix}(R+\mu)(e^{RT}-1)\\ +\text{Re}^{-\mu T_1}\left(1-e^{(R+\mu)T}\right)\end{pmatrix}\right] \tag{12}$$

Partially differentiate the equation (12) with respect to T_1

$$\frac{\partial}{\partial T_1}(TC(T)) = \left[\frac{X}{\mu^2}\left(\mu-\mu e^{-\mu T_1}\right)-\frac{Y}{R\mu(R+\mu)}\left\{-R\mu e^{-\mu T_1}\left(1-e^{(R+\mu)T}\right)\right\}\right]=0$$

And $\frac{\partial^2}{\partial T_1^{\,2}}\left[TC(T)\right]>0$

On simplification of the above equation, then the production time $T_1=\frac{YT}{X}$ (13)

Therefore, $TC(T_1)$ is strictly convex.

Partially differentiate the equation (12) with respect to T, then

$$\frac{\partial}{\partial T}[TC(T)] = \frac{-S_C}{T^2}+\left(H_C+\mu D_C\right)\left[\begin{array}{l}\frac{-X}{T^2\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)\\ -\frac{Y}{R\mu(R+\mu)T^2}\begin{pmatrix}(R+\mu)T\,\text{Re}^{RT}-(R+\mu)(e^{rT}-1)\\ +\text{Re}^{-\mu T_1}\left(-(R+\mu)Te^{(R+\mu)T}-\left(1-e^{(R+\mu)T}\right)\right)\end{pmatrix}\end{array}\right]=0$$

And also $\frac{\partial^2}{\partial T^2}\left[TC(T)\right]>0$

Therefore, the average system cost functions $TC(T_1)$ and $TC(T)$ are strictly convex.

The reduced equation is as follows:

$$\left[\begin{array}{l}\dfrac{-X}{\mu^2}\left(\mu T_1 + e^{-\mu T_1} - 1\right) \\ -\dfrac{Y}{R\mu(R+\mu)}\left(\begin{array}{l} R(R+\mu)Te^{RT} - (R+\mu)(e^{RT}-1) \\ + \mathrm{Re}^{-\mu T_1}\left(-(R+\mu)Te^{(R+\mu)T} - \left(1 - e^{(R+\mu)T}\right)\right)\end{array}\right)\end{array}\right] = \frac{S_C}{H_C + \mu D_C}$$

To reduce the above equation in fourth order equation for the optimum solution of T ,the higher power of T^5 and above are eliminated on some simplification

$$\left[\begin{array}{l}\dfrac{-XT_1^2}{2} + \dfrac{X\mu T_1^3}{6} - \dfrac{X\mu^2 T_1^4}{24} + \dfrac{YT^2}{2} + \dfrac{Y\mu T^3}{3} + \dfrac{2YRT^3}{3} + \dfrac{YR^2T^4}{8} + \dfrac{YR\mu T^4}{8} \\ + \dfrac{Y\mu^2T^4}{24} - \dfrac{YRT^2T_1}{2} - \dfrac{Y\mu T^2T_1}{2} - \dfrac{YR^2T^3T_1}{3} - \dfrac{Y\mu^2T^3T_1}{3} - \dfrac{2YR\mu T^3T_1}{3} + \dfrac{YR\mu T^2T_1^2}{4}\end{array}\right] = \frac{S_C}{H_C + \mu D_C}$$

Substitute the value of T_1 and on some simplification, the equation is reduced to cubic equation which is the optimal solution for the cycle time T

$$\begin{aligned}&\left[3X^3YR^2 - Y^4\mu^2 + 3X^2YR\mu + X^3Y\mu^2 - 8X^2Y^2R^2 + 8X^2Y^2\mu^2 - 16X^2Y^2R\mu + 6X^3YR\mu\right]T^4 \\ &+ 4X\left[Y^3\mu + 2X^2Y\mu + 4X^2YR - 3XY^2R - 3XY^2\mu\right]T^3 + 12X^2Y(X-Y)T^2 = \frac{24X^3S_C}{H_C + \mu D_C}\end{aligned} \tag{14}$$

Taking demand Y overall common and take to RHS

$$\begin{aligned}&\left[3X^3R^2 - Y^3\mu^2 + 3X^2R\mu + X^3\mu^2 - 8X^2YR^2 + 8X^2Y\mu^2 - 16X^2YR\mu + 6X^3R\mu\right]T^4 \\ &+ 4X\left[Y^2\mu + 2X^2\mu + 4X^2R - 3XYR - 3XY\mu\right]T^3 + 12X^2(X-Y)T^2 = \frac{24X^3S_C}{Y(H_C + \mu D_C)}\end{aligned}$$

Again, value of T is approximately equal both in forth and third order higher equation. Therefore, the fourth order equation is reduced to third order equation

$$\left[Y^2\mu + 2X^2\mu + 4X^2R - 3XYR - 3XY\mu\right]T^3 + 3X(X-Y)T^2 = \frac{6X^2S_C}{Y(H_C + \mu D_C)} \tag{15}$$

Numerical Example 1, Let us consider the cost parameters

X = 12000 units, Y = 11000 units, H_C =15, P_C = 120, S_C = 500, μ = 0.01, *R*=0.01, D_C= 120

Optimum solution

Optimum Cycle time T = 0.2594, Optimum Quantity *Q** = 2834.97; Production time T_1 = 0.2362, Maximum Inventory I_1= 236.25, Production cost = 1320000, Setup cost = 1940.05, Holding cost = 1726.19, deteriorative items = 138.09, and Total cost = 1323804.34

Numerical example 2, If R = 0.1 (10%), then Optimum cycle time T= 0.2448, Optimum Quantity *Q** = 2693.27; T_1 = 0.2244, maximum inventory I_1= 224.43, Production cost = 1320000, setup cost = 2042.12, holding cost = 1530.80, cost of deteriorative = 122.46, total cost = 1323695.39

Table 1 Variation of Rate of Deteriorating Items with inventory and total Cost

θ	T	Q	T_1	I_1	Setup Cost	Holding Cost	Perishable Cost	Total Cost
0.01	0.2577	2834.97	0.2362	236.25	1940.05	1726.19	138.09	1323804.34
0.02	0.2486	2734.92	0.2279	227.91	2011.02	1637.08	261.93	1323910.03
0.03	0.2404	2644.70	0.2203	220.39	2079.62	1557.56	373.81	1324011.00
0.04	0.2330	2562.82	0.2135	213.56	2146.07	1486.07	475.54	1324107.69
0.05	0.2262	2488.05	0.2073	207.34	2210.56	1421.37	568.55	1324200.49
0.06	0.2199	2419.43	0.2016	201.62	2273.25	1362.48	653.99	1324289.72
0.07	0.2142	2356.16	0.1963	196.34	2334.30	1308.58	732.80	1324375.68
0.08	0.2088	2297.57	0.1914	191.46	2393.82	1259.02	805.77	1324458.62
0.09	0.2039	2243.12	0.1869	186.92	2451.93	1213.26	873.55	1324538.75
0.10	0.1993	2192.35	0.1826	182.69	2505.72	1170.86	936.69	1324616.27

Note : Production cost constant = 450,000

From the table 1, a study in the rate of deteriorative items with optimum quantity, cycle time, production time (T_1) , maximum inventory (I_1), setup cost, holding cost, deteriorative cost and total cost. There is a positive relationship between the increase in the rate of deteriorative items with cost of setup, perishable cost and total cost and there is negative relationship between increase in the rate of deteriorative with optimum quantity, cycle time, production time, maximum inventory.

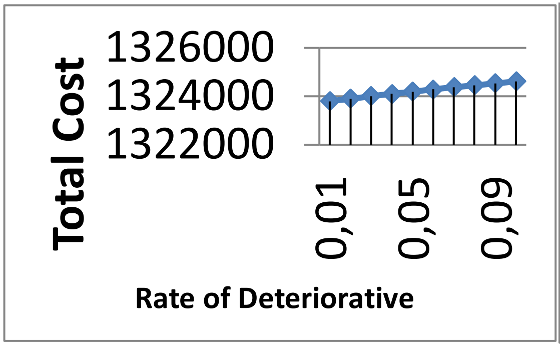


Figure 2 Relationship between Rate of Deteriorative item with Total cost.

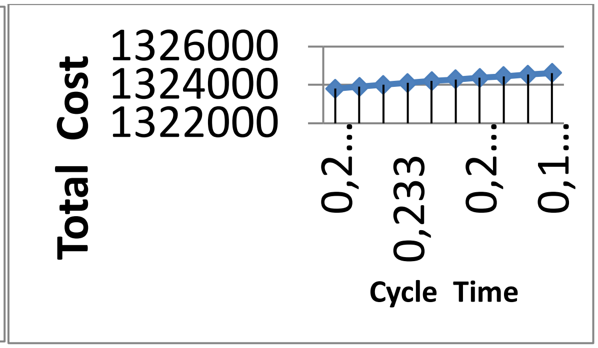


Figure 3 Relationship between Cycle Time with Total cost

Sensitivity Analysis:

The total cost functions are the real solutions in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity i.e. the effect of making chances in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for the models of this research are required to observe whether the current solutions remain unchanged, the current solutions become not feasible, etc.

Table 2, Effect of Demand and cost parameters on optimal values

Parameters		Optimum Values							
		T	Q	T_1	I_1	Setup Cost	Holding Cost	Deteriorative Cost	Total Cost
μ	0.01	0.2577	2834.97	0.2362	236.25	1940.05	1726.19	138.09	1323804.34
	0.02	0.2486	2734.92	0.2279	227.91	2011.02	1637.08	261.93	1323910.03
	0.03	0.2404	2644.70	0.2203	220.39	2079.62	1557.56	373.81	1324011.00
	0.04	0.2330	2562.82	0.2135	213.56	2146.07	1486.07	475.54	1324107.69
	0.05	0.2262	2488.05	0.2073	207.34	2210.56	1421.37	568.55	1324200.49
R	0.01	0.2577	2834.97	0.2362	236.25	1940.05	1726.19	138.09	1323804.34
	0.02	0.2561	2817.35	0.2347	234.78	1952.18	1702.22	136.17	1323790.58
	0.03	0.2545	2800.26	0.2333	233.35	1964.09	1678.88	134.31	1323777.28
	0.04	0.2530	2783.68	0.2319	231.97	1975.80	1656.14	132.49	1323764.44
	0.05	0.2515	2767.56	0.2306	230.63	1987.31	1633.98	130.72	1323752.01
S_C	400	0.2306	2537.49	0.2114	211.45	1733.99	1549.35	123.94	1323407.29
	450	0.2445	2690.43	0.2242	224.20	1839.85	1640.39	131.23	1323611.47
	500	0.2577	2834.97	0.2362	236.25	1940.05	1726.19	138.09	1323804.34
	550	0.2702	2972.36	0.2476	247.69	2035.41	1807.53	144.60	1323987.54
	600	0.2821	3103.56	0.2586	258.42	2126.59	1884.99	150.80	1324162.39
H_C	13	0.2751	3026.65	0.2522	252.22	1817.18	1594.32	147.16	1323558.68
	14	0.2660	2926.10	0.2438	243.84	1879.63	1661.49	142.41	1323683.53
	15	0.2577	2834.97	0.2362	236.25	1940.05	1726.19	138.09	1323804.34
	16	0.2501	2751.88	0.2293	229.32	1998.63	1788.69	134.15	1323921.47
	17	0.2432	2675.70	0.2229	222.97	2055.53	1849.18	130.53	1324035.25
D_C	100	0.2593	2852.52	0.2377	237.71	1928.11	1736.59	115.79	1323780.48
	110	0.2585	2843.70	0.2369	236.97	1934.69	1731.37	126.96	1323792.43
	120	0.2577	2834.97	0.2362	236.25	1940.05	1726.19	138.09	1323804.34
	130	0.2569	2826.32	0.2355	235.52	1945.98	1721.06	149.15	1323816.21
	140	0.2561	2817.75	0.2348	234.81	1951.91	1715.98	160.15	1323828.05
P_C	100	0.2593	2852.52	0.2377	237.71	1928.11	1736.59	115.79	1100000.00 1103804.34
	110	0.2585	2843.70	0.2369	236.97	1934.69	1731.37	126.96	1210000.00 1213804.34
	120	0.2577	2834.97	0.2362	236.25	1940.05	1726.19	138.09	1320000.00

									1323804.34
	130	0.2569	2826.32	0.2355	235.52	1945.98	1721.06	149.15	1430000.00 1433804.34
	140	0.2561	2817.75	0.2348	234.81	1951.91	1715.98	160.15	1540000.00 1543804.34

Managerial observations: A sensitivity analysis is performed to study the effects of change in the system parameters, rate of deteriorative items (μ), setup cost per set (S_C), holding cost per unit/ unit time (H_C), rate of increase in demand (R), production cost per unit ((C_P) , cost of deteriorative item per unit (D_C) on optimum cycle time (T), maximum inventory (I_1), Production time (T_1) , setup cost, production cost, holding cost, deteriorative cost and total cost. The sensitivity analysis is performed by changing (increasing or decreasing) the parameter taking at a time, keeping the remaining parameters at their original values. The following influences can be obtained from sensitivity analysis based on table 2.

1) There is a positive relationship between in increase in rate of deteriorative items μ with the setup cost, deteriorative cost and total cost and there is a negative relationship between increase in rate of perishable items with optimum quantity, cycle time, production time, maximum inventory.

2) there is a positive relationship between increase in the cost of set per unit (S_C) with optimum cycle time (T), optimum quantity (Q), production time (T_1), production time (T_1), maximum inventory (I_1), setup cost, holding cost, deteriorative cost and total cost.

3) there is a positive relationship between increase in the cost of carrying inventory per unit per per unit time (H_C), with setup cost, holding cost and total cost but there is a negative relationship between increase in the cost of carrying inventory per unit per unit time with cycle time (T) and optimal lot size (Q), production time (T_1) , maximum inventory (I_1), deteriorative cost .

4) Similarly, other parameters deteriorative cost per unit (D_C), production cost per unit (P_C), can also be observed from the table 2.

3.2. Model 2 Production Inventory Model for Deteriorative items integrated with Continuous Compound Demand and Growth of Demand

In this model, an production inventory model for deteriorative items with continuous compound demand and integrated with the growth of demand is considered. The growth rate of demand in the production time T_1 is $Y(1+i)^n$ and the growth of demand in the decline period is $Y(1-i)^n$. The inventory level at time T_1 is $\left(X - Y(1+i)^n\right)T_1$ and the remaining are same as given in the model 1 of this paper. The differential equation for the production period is

$$\frac{d}{dt}I(t) + \mu I(t) = X - Y(1+i)^n e^{Rt},\ 0 \le t \le T_1 \tag{16}$$

The differential equation for the decline period

$$\frac{d}{dt}I(t) + \mu I(t) = -Y(1-i)^n e^{Rt},\ T_1 \le t \le T \tag{17}$$

With the conditions of boundary

$$I(t) = 0, I(T_1) = I_1, I(T) = 0 \tag{18}$$

The derivation of the equation (15) is

$$I(t) = \frac{X}{\mu}\left(1 - e^{-\mu t}\right) + \frac{Y(1+i)^n}{R+\mu}\left(e^{-\mu t} - e^{Rt}\right) \tag{19}$$

Note: When n = i = 0, then the above equation becomes CCD as per model 1

$$I(t) = \frac{X}{\mu}\left(1 - e^{-\mu t}\right) + \frac{Y}{R+\mu}\left(e^{-\mu t} - e^{Rt}\right)$$

The derivation of the equation (16) is

$$I(t) = \frac{Y(1-i)^n}{R+\mu}\left(e^{(R+\mu)T-\mu t} - e^{Rt}\right) \tag{20}$$

Note: When n = i = 0, then the above equation becomes CCD as per model 1

$$I(t) = \frac{Y}{R+\mu}\left(e^{(R+\mu)T-\mu t} - e^{Rt}\right)$$

To find T_1 and I_1 :

From the diagram, I(t) for production period and I(t) for the decline period are same. Therefore, substitute T_1 in the equations (19) and (20) for calculating T_1 and Q_1

$$\frac{X}{\mu}\left(1-e^{-\mu T_1}\right)+\frac{Y(1+i)^n}{R+\mu}\left(e^{-\mu T_1}-e^{RT_1}\right)= \frac{Y(1-i)^n}{R+\mu}\left(e^{(R+\mu)T-\mu T_1}-e^{RT_1}\right)$$

On simplification

$$T_1 = \frac{Y(1-i)^n T}{X - Y(1+i)^n + Y(1-i)^n} \tag{21}$$

Note: When n = i = 0 then, $T_1 = \frac{YT}{X}$ which is model 1 and

$$I_1 = \left(X - Y(1+i)^n\right)T_1 \text{ (or) } I_1 = Y(1-i)^n (T - T_1) \tag{22}$$

Theorem 2: The average system cost functions $TC(T_1)$ and $TC(T)$ are strictly convex.

Proof: The optimality conditions can be easily shown that $TC(T_1)$ and $TC(T)$ are convex function in T_1 and T. Hence, an optimum cycle time T_1 and T can be calculated from

(i) $\frac{\partial}{\partial T_1}\left[TC(T)\right]=0$ and $\frac{\partial^2}{\partial T_1^{\,2}}\left[TC(T)\right]>0$ and

(ii) $\frac{\partial}{\partial T}\left[TC(T)\right]=0$ and $\frac{\partial^2}{\partial T^2}\left[TC(T)\right]>0$

The total cost function comprises of setup cost, holding cost, production cost and perishable cost. Therefore, the total cost function is as follows:

1. Setup cost = $\frac{S_C}{T}$ (23)

2. Production cost = YP_C (24)

3. Holding cost (HC) $= \frac{H_C}{T}\left[\int_0^{T_1} I(t)\,dt + \int_{T_1}^{T} I(t)dt\right]$

$$= \frac{H_C}{T}\left[\int_0^{T_1}\left\{\frac{X}{\mu}\left(1-e^{-\mu t}\right)+\frac{Y(1+i)^n}{R+\mu}\left(e^{-\mu t}-e^{Rt}\right)\right\}dt+\int_{T_1}^{T}\frac{Y(1-i)^n}{R+\mu}\left(e^{(R+\mu)T-\mu t}-e^{Rt}\right)dt\right]= \frac{H_C}{T}\left[\begin{array}{l}\frac{X}{\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)-\frac{Y(1+i)^n}{R\mu(R+\mu)}\left(\mathrm{Re}^{-\mu T_1}+\mu e^{RT_1}-R-\mu\right)\\ -\frac{Y(1-i)^n}{R\mu(R+\mu)}\left(\mathrm{Re}^{RT}+\mu e^{RT}-\mathrm{Re}^{(R+\mu)T-\theta T_1}-\mu e^{RT_1}\right)\end{array}\right]$$

$$= \frac{H_c}{T}\left[\begin{array}{l}\frac{X}{\mu^2}\left(\mu T_1+e^{-\mu T_1}-1\right)-\frac{Y(1+i)^n}{R\mu(R+\mu)}\left(R(e^{-\mu T_1}-1)+\mu(e^{RT_1}-1\right)\\ -\frac{Y(1-i)^n}{R\mu(R+\mu)}\left((R+\mu)e^{RT}-\mu e^{RT_1}-\mathrm{Re}^{(R+\mu)T-\mu T_1}\right)\end{array}\right] \tag{25}$$

Note: When n = i = 0, then HC = $\frac{H_C}{T}\left[\begin{array}{l}\frac{X}{\mu^2}\left(\mu T_1 + e^{-\mu T_1} - 1\right) - \frac{Y}{R\mu(R+\mu)}\left(R(e^{-\mu T_1} - 1) + \mu(e^{RT_1} - 1)\right) \\ - \frac{1}{R\mu(R+\varpi)}\left((R+\mu)e^{RT} - \mu e^{RT_1} - \mathrm{Re}^{(R+\mu)T-\mu T_1}\right)\end{array}\right]$ which is like model 1

4. Deteriorative cost = $\frac{\mu D_C}{T}\left[\begin{array}{l}\frac{X}{\mu^2}\left(\mu T_1 + e^{-\mu T_1} - 1\right) - \frac{Y(1+i)^n}{RY(R+\mu)}\left(R(e^{-\mu T_1} - 1) + \mu(e^{RT_1} - 1\right) \\ - \frac{Y(1-i)^n}{R\mu(R+\mu)}\left((R+\mu)e^{RT} - \mu e^{RT_1} - \mathrm{Re}^{(R+\mu)T-\mu T_1}\right)\end{array}\right]$ (26)

Theorem 2: The average system cost functions $TC(T_1)$ and $TC(T)$ are strictly convex.

Proof: The optimality conditions can be easily shown that $TC(T_1)$ and $TC(T)$ are convex function in T_1 and T. Hence, an optimum cycle time T_1 and T can be calculated from

(i) $\frac{\partial}{\partial T_1}\left[TC(T)\right] = 0$ and $\frac{\partial^2}{\partial T_1{}^2}\left[TC(T)\right] > 0$ and

(ii) $\frac{\partial}{\partial T}\left[TC(T)\right] = 0$ and $\frac{\partial^2}{\partial T^2}\left[TC(T)\right] > 0$

The total cost function comprises of setup cost, holding cost, production cost and perishable cost. Therefore, the total cost function is as follows:

TC(T)= $\frac{H_C}{T} + YP_C + \frac{H_C + \mu D_C}{T}\left[\begin{array}{l}\frac{X}{\mu^2}\left(\mu T_1 + e^{-\mu T_1} - 1\right) - \frac{Y(1+i)^n}{R\mu(R+\mu)}\left(R(e^{-\mu T_1} - 1) + \mu(e^{RT_1} - 1)\right) \\ - \frac{Y(1-i)^n}{R\mu(R+\mu)}\left((R+\mu)e^{RT} - \mu e^{RT_1} - \mathrm{Re}^{(R+\mu)T-\mu T_1}\right)\end{array}\right]$ (27)

The equation (27) is partiallydifferentiatew.r.t. T_1

$$\frac{\partial}{\partial T_1}TC(T) = \left[\begin{array}{l}\frac{X}{\mu^2}\left(\mu - \mu e^{-\mu T_1}\right) - \frac{Y(1+i)^n}{R\mu(R+\mu)}\left(R\mu e^{RT_1} - R\mu e^{-\mu T_1}\right) \\ - \frac{Y(1-i)^n}{R\mu(T+\mu)}\left(R\mu e^{(R+\mu)T}e^{-\mu T_1} - R\mu e^{RT_1}\right)\end{array}\right] = 0,\ \frac{\partial^2}{\partial T_1^2}TC(T) > 0$$

Therefore, the average system cost functions $TC(T_1)$ is strictly convex.

On simplification, $T_1 = \frac{Y(1-i)^n T}{X - Y(1+i)^n + Y(1-i)^n}$ (28)

Therefore, the average system cost functions $TC(T_1)$ and $TC(T)$ are strictly convex.

Compare with the equation (21) and (27) with the above value, then both values are same.

The equation (26) is differentiate partially w.r.t. T,

$$\frac{\partial}{\partial T}TC(T) = -S_C + (H_C + \mu D_C)\left[\begin{array}{l}\frac{-X}{\mu^2}\left(\mu T_1 + e^{-\varpi T_1} - 1\right) + \frac{Y(1+i)^n}{R\mu(R+\mu)}\left(R(e^{-\mu T_1} - 1) + \mu(e^{RT_1} - 1)\right) \\ - \frac{Y(1-i)^n}{R\mu(R+\mu)}\left(\begin{array}{l}(R+\mu)Te^{RT} - (R+\mu)e^{RT} + \mu e^{RT_1} \\ - R(R+\mu)T e^{(R+\mu)T}e^{-\mu T_1} + \mathrm{Re}^{(R+\mu)T}e^{-\mu T_1}\end{array}\right)\end{array}\right] = 0$$

$$\frac{\partial^2}{\partial T^2}TC(T) > 0$$

Therefore, the average system cost functions $TC(T)$ is strictly convex. Therefore, TC(T) and $TC(T_1)$ both are strictly convex.

On simplification

$$\begin{bmatrix} \frac{-X}{\mu^2}\left(\mu T_1 + e^{-\mu T_1} - 1\right) + \frac{Y(1+i)^n}{R\mu(R+\mu)}\left(R(e^{-\mu T_1} - 1) + \mu(e^{RT_1} - 1)\right) \\ -\frac{Y(1-i)^n}{R\mu(R+\mu)}\begin{pmatrix} T(R+\mu)Te^{RT} - (R+\mu)e^{RT} + \mu e^{RT_1} \\ - R(R+\mu)T\, e^{(R+\mu)T} e^{-\mu T_1} + \mathrm{Re}^{(R+\mu)T}\, e^{-\mu T_1} \end{pmatrix} \end{bmatrix} = \frac{S_C}{H_C + \mu D_C}$$

Making higher order equation with some simplifications,

$$\begin{bmatrix} -\frac{XT_1^2}{2} + \frac{X\mu T_1^3}{6} - \frac{X\mu^2 T_1^4}{24} + Y(1+i)^n\left[\frac{T_1^2}{2} + (R-\mu)\frac{T_1^3}{6} + (R^2 - R\mu + \mu^2)\frac{T_1^4}{24}\right] \\ -Y(1-i)^n\begin{bmatrix} \frac{T_1^2}{2} + (R-\mu)\frac{T_1^3}{6} + (R^2 - R\mu + \mu^2)\frac{T_1^4}{24} - \frac{T^2}{2} + \frac{(R+\mu)T^2T_1}{2} \\ -\frac{\mu(R+\mu)T^2T_1^2}{4} - \frac{2RT^3}{3} - \frac{\mu T^3}{3} + \frac{(R+\mu)^2T^3T_1}{3} - \frac{3R^2T^4}{8} - \frac{3R\mu T^4}{8} - \frac{\mu^2T^4}{8} \end{bmatrix} \end{bmatrix} = \frac{S_C}{H_C + \mu D_C}$$

$$\begin{bmatrix} -\left(X - Y(1+i)^n + Y(1-i)^n\right)\frac{T_1^2}{2} + \frac{Y(1-i)^n T^2}{2} \\ \frac{1}{6}\left(X\mu + RY(1+i)^n - \mu Y(1+i)^n - RY(1-i)^n + \mu Y(1-i)^n\right)T_1^3 \\ +\frac{1}{3}\left(2RY(1-i)^n + \mu Y(1-i)^n\right)T^3 - \frac{1}{2}\left(RY(1-i)^n + \mu Y(1-i)^n\right)T^2T_1 \\ \frac{1}{24}\left(-X\mu^2 + (Y(1+i)^n - Y(1-i)^n)(R^2 - R\mu + \mu^2)\right)T_1^4 + \frac{\mu DY(1-i)^n}{4}(R+\mu)T^2T_1^2 \\ +\frac{Y(1-i)^n}{8}(3R^2 + 3R\mu + \mu^2)T^4 - \frac{Y(1-i)^n}{3}(R^2 + 2R\mu + \mu^2) \end{bmatrix} = \frac{S_C}{H_C + \mu D_C}$$

Substitute the value of T_1 and simplify

$$\frac{1}{24}\begin{bmatrix} \frac{Y(1-i)^n\left(Y(1+i)^n - Y(1-i)^n\right)(R^2 - R\mu + \mu^2) - X\mu^2}{\left(X - Y(1+i)^n + Y(1-i)^n\right)^4} \\ +3Y(1-i)^n(3R^2 + 3R\mu + \mu^2) + \frac{6\left(Y(1-i)^n\right)^3\mu(R+\mu)}{\left(X - Y(1+i)^n + Y(1-i)^n\right)^2} \\ -\frac{8\left(Y(1-i)^n\right)^2(R^2 + 2R\mu + \mu^2)}{(X - Y(1+i)^n + Y(1-i)^n)} \end{bmatrix}T^4$$
$$+\frac{1}{6}\begin{bmatrix} \frac{\left(Y(1-i)^n\right)^3\left(X\mu + (R-\mu)Y(1+i)^n - (R-\mu)Y(1-i)^n\right)}{\left(X - Y(1+i)^n + Y(1-i)^n\right)^3} \\ +2(2R+\mu)D(1-i)^n - \frac{3(R+\mu)\left(Y(1-i)^n\right)^2}{X - Y(1+i)^n + Y(1-i)^n} \end{bmatrix}T^3$$
$$+\frac{Y(1-i)^n\left(X - Y(1+i)^n\right)}{2\left(X - Y(1+i)^n + Y(1-i)^n\right)}T^2 = \frac{S_c}{H_C + \mu D_C}$$

The fourth order equation is reduced into third order equation as

$$\frac{1}{6}\begin{bmatrix} \frac{\left(Y(1-i)^n\right)^3\left(X\mu + (R-\mu)\left(Y(1+i)^n - Y(1-i)^n\right)\right)}{(X - Y(1+i)^n + Y(1-i)^n)^3} \\ +2(2R+\mu)Y(1-i)^n - \frac{3(R+\mu)\left(Y(1-i)^n\right)^2}{X - Y(1+i)^n + Y(1-i)^n} \end{bmatrix}T^3 \tag{29}$$
$$+\frac{Y(1-i)^n\left(X - Y(1+i)^n\right)}{2\left(X - Y(1+i)^n + Y(1-i)^n\right)}T^2 = \frac{S_C}{H_C + \mu D_C}$$

which is optimum solution in third order equation.

Note: When $T^3 = 0$ and n =0 then the equation (28) reduces to basic inventory model

$$T = \sqrt{\frac{2XS_C}{Y(X-Y)(H_C + \mu D_C)}}$$

Numerical Example Let us consider the cost parameters

X = 12000units, Y = 11000units, H_C =15, P_C = 120, S_C =500, D_C = 120, μ = 0.01, R=0.01 , n = 2, i = 0.01

Optimum solution

Optimum Quantity Q*= 3047.01; T_1 = 0.2583, T = 0.2770 , I_1 = 201.22,

Production cost = 1320000, Setup cost = 1805.04, Holding cost = 1428.71, ,

Deteriorating cost = 114.29, and Total cost = 1323348.05., Production cost 1320000

Table 3. Variation of Rate of Deteriorating Items with inventory and total Cost

μ	T	Q	T_1	I_1	Setup cost	Holding cost	Deteriorating cost	Total cost
0.01	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1323348.05
0.02	0.2670	2937.62	0.2490	193.99	1872.26	1379.09	220.65	1323471.99
0.03	0.2581	2838.93	0.2406	187.48	1937.35	1334.16	320.20	1323591.71
0.04	0.2499	2749.30	0.2330	181.55	2000.50	1293.26	413.84	1323707.61
0.05	0.2424	2667.42	0.2261	176.15	2061.91	1255.79	502.32	1323820.03
0.06	0.2356	2592.24	0.2197	171.18	2121.71	1221.32	586.23	1323929.27
0.07	0.2293	2522.89	0.2139	166.60	2180.03	1189.45	666.09	1324035.57
0.08	0.2235	2458.65	0.2084	162.36	2236.99	1159.87	742.32	1324139.18
0.09	0.2180	2398.94	0.2033	158.42	2292.67	1132.32	815.27	1324240.28
0.10	0.2130	2343.23	0.1986	154.74	2347.18	1106.58	885.26	1324339.03

The above table is a study of rate of the deteriorative items with cycle time, optimum quantity, Production time T_1 , the maximum inventory I_1 , setup cost, production cost, holding cost, deteriorating cost and total cost. When the rate of deteriorative items increases, the setup cost, deteriorating cost and total cost increases, as a result there is positive relation between them. When the rate of deteriorative items increases, the cycle time, optimum quantity, production time T_1 , the maximum inventory I_1 and holding cost decreases that result in a negative relationship between them.

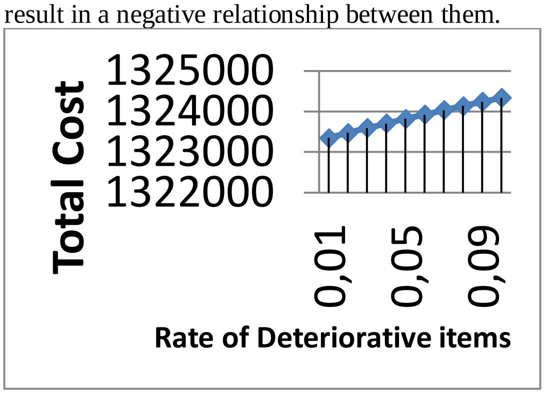


Figure 4 Relationship between Rate Deteriorative items with Total Cost

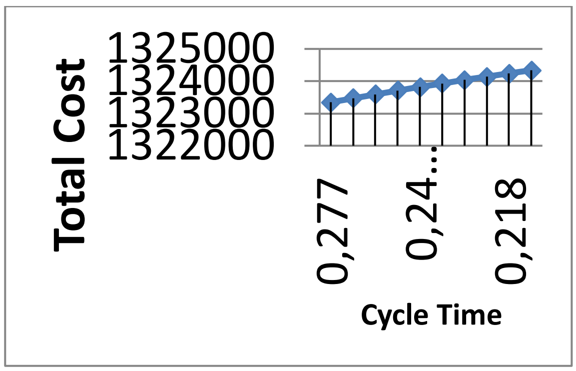


Figure 5 Relationship between Rate of Cycle Time with Total Cost Sensitivity Analysis

Table 4 Effect of Demand and cost parameters on optimal values

Parameters		Optimum Values							
		T	T_1	Q	I_1	Setup cost	Holding cost	Deteriorating cost	Total Cost
μ	0.01	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1323348.05
	0.02	0.2670	2937.62	0.2490	193.99	1872.26	1379.09	220.65	1323471.99
	0.03	0.2581	2838.93	0.2406	187.48	1937.35	1334.16	320.20	1325591.71
	0.04	0.2499	2749.30	0.2330	181.55	2000.50	1293.26	413.84	1323707.61
	0.05	0.2424	2667.42	0.2261	176.15	2061.91	1255.79	502.32	1323820.03
S_C	400	0.2489	2738.80	0.2322	180.86	1606.53	1291.50	103.32	1323001.36
	450	0.2634	2897.55	0.2456	191.34	1708.33	1362.38	108.99	1323179.70
	500	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1323348.05
	550	0.2898	3188.55	0.2703	210.56	1897.41	1491.16	119.29	1323507.86
	600	0.3021	3323.24	0.2817	219.46	1986.01	1550.27	124.02	1323660.31
H_C	13	0.2949	3244.34	0.2751	214.25	1695.25	1313.60	121.25	1323130.11
	14	0.2855	3140.95	0.2663	207.42	1751.06	1372.18	117.61	1323240.86
	15	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1323348.05
	16	0.2691	2961.16	0.2510	195.54	1857.37	1483.36	111.25	1323451.99
	17	0.2620	2882.29	0.2443	190.34	1908.28	1536.33	108.44	1323552.98
	100	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1100000.00 1103348.05

P_C	110	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1210000.00 1213348.05
	120	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1320000.00 1323348.05
	130	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1430000.00 1433348.05
	140	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1540000.00 1543348.05
D_C	100	0.2786	3065.11	0.2598	202.41	1794.38	1436.71	95.78	1323326.88
	110	0.2778	3056.02	0.2591	201.81	1799.72	1432.63	105.06	1323337.48
	120	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1323348.05
	130	0.2761	3038.08	0.2575	200.62	1810.35	1424.75	123.47	1323358.58
	140	02756	3029.23	0.2568	200.04	1815.64	1420.83	132.61	1323369.09
R	0.0025	0.2868	3155.82	0.2675	208.40	1742.87	1540.49	123.23	1323406.54
	0.005	0.2834	3117.43	0.2643	205.87	1764.27	1501.29	120.10	1323385.66
	0.01	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1323348.05
	0.02	0.2660	2926.26	0.2481	193.24	1879.52	1302.08	104.16	1323285.77
	0.03	0.2568	2825.41	0.2395	186.58	1946.62	1193.97	95.52	1323236.12
i	0.0025	0.2556	2812.29	0.2353	222.38	1955.69	1599.27	127.94	1323682.91
	0.005	0.2621	2883.82	0.2423	215.64	1907.18	1545.17	123.61	1323575.98
	0.01	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1323348.05
	0.02	0.3172	3489.99	0.3014	167.48	1575.93	1150.56	92.04	1322818.54
	0.03	0.3840	4224.49	0.3721	122.85	1301.93	766.76	61.34	1322130.03
n	1	0.2621	2883.46	0.2423	215.67	1907.42	1545.45	123.63	1323576.52
	2	0.2770	3047.01	0.2583	201.22	1805.04	1428.71	114.29	1323348.05
	3	0.2951	3246.25	0.2777	185.18	1694.26	1297.60	103.81	1323095.67
	4	0.3177	3495.46	0.3019	167.08	1573.46	1147.35	91.78	1322812.60
	5	0.3470	3817.95	0.3331	146.19	1440.56	970.19	77.61	1322488.35

Managerial insights: A sensitivity analysis is performed to study the effects of change in the system parameters, rate of deteriorative items (μ), ordering cost (S_C) , holding cost (C_h), production rate per unit (P_C),on optimal values that is optimal cycle time (T), optimal quantity (Q), production time (T_1), consumption time (T_2), maximum inventory (I_1), setup cost, holding cost, defective cost, reworking cost and total cost. The sensitivity analysis is performed by changing (increasing or decreasing) the parameter taking at a time, keeping the remaining parameters at their original values. The following influences can be obtained from sensitivity analysis based on table 1.

1) with the increase in the rate of deteriorative items, the setup cost, deteriorating cost and total cost increases, as a result there is positive relation between them and with the increase in the rate of deteriorative items increases, the cycle time, optimum quantity, Production time T_1 , the maximum inventory I_1 and holding cost decreases that results in a negative relationship between them.

2) with the increase in setup cost per unit (S_C) , optimum quantity (Q*), cycle time (T), production time T_1, consumption time (T_2) maximum inventory I_1 , setup cost, holding cost, deteriorative cost and total cost increases then there is positive relationship between them.

3) with the increase in holding cost per unit per unit time (H_C), the setup cost, holding cost, deteriorative cost and total cost increases then there is positive relationship between them but optimal cycle time (T) and optimal lot size (Q), production time (T_1), consumption time (T_2), maximum inventory (I_1) decreases then there is negative relationship between,

4) Similarly, other parameters, production cost per unit P_C , D_C, R, i and n number of years can also be observed from the table 4.

4. RELATIONSHIP BETWEEN CONSTANT DEMAND. CONTINUOUS COMPOUND DEMAND (CCD) AND INTEGRATED CCD WITH GROWTH OF DEMAND

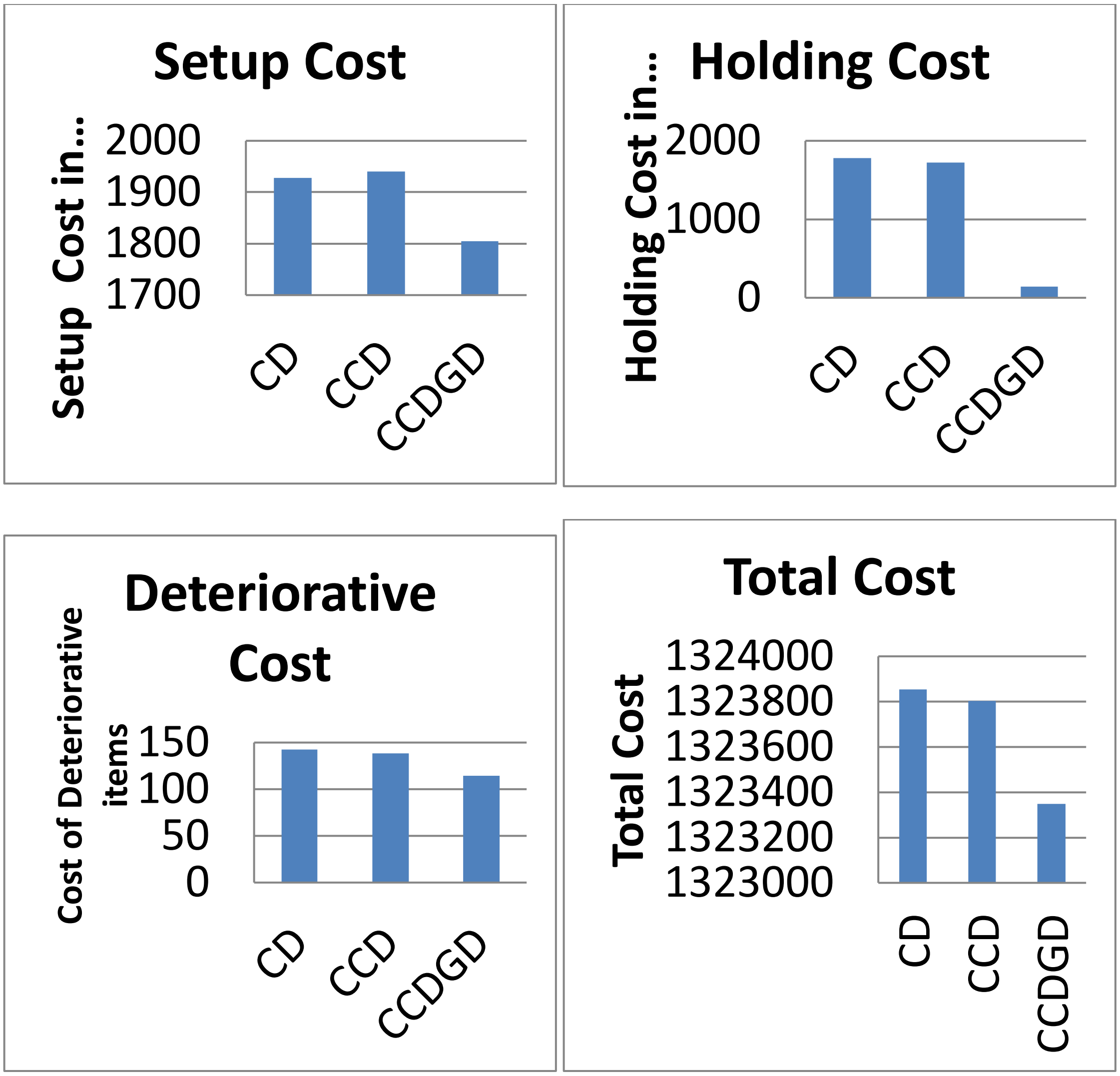


Figure – 6Relationship between constant demand, CCD and CCD with Growth of Demand

The following table and diagrams show the relationship between the constant demand, CCD and CCD with Growth of Demand. From the table, it is observed that the setup cost is less in constant demand with compared to continuous compound demand, the holding cost, perishable cost and total cost are is more in constant demand with compared to continuous compound demand**.** And also, the setup cost, holding cost, deteriorative cost and total cost are less in CCD with Growth of Demand compare with Constant and CCD.

The basic formula in production inventory model with constant demand rat is given below

$$T = \sqrt{\frac{2XS_C}{Y(X-Y)(H_C + \mu D_C)}}$$

For example, Let us consider the cost parameters for constant demand is given below

X = 12000 units, Y = 11000 units, H_C =15, P_C = 120, S_C = 500, μ = 0.01, R=0.01, D_C = 120

Optimum solution

T = 0.2594, Q = 2853.40, T_1 = 0.2377, I_1 = 237.70, Setup cost = 1927.52,

Production cost = 1320000, Holding cost = 1783.37, Cost of Deteriorative = 142.67,

Total cost = 1323853.56

Table 5. Relationship between constant demand CCD and CCD with Growth of Demand

Item of cost	Constant Demand	Continuous Compound Demand (CCD)	Integrated CCD with Growth of Demand
Setup cost	1927.52	1940.05	1805.04
Holding cost	1783.37	1726.19	1428.71
Perishable cost	142.67	138.09	114.29

Total cost	1323853.56	1323804.34	1323348.05

5. CONCLUSION

In this paper, a production inventory model for deteriorating items with continuous compound demand (CCD) and growth of demand are considered for developing the mathematical models. In model 1, continuous compound demand (CCD) and in the model2, continuous compound demand integrated with growth of demand is considered. The following points are observed during our study. All inventory cost (setup cost, holding cost, deteriorative cost and total cost) are less in model 2 compared with model 1 and constant demand rate. With the increase in the rate of deteriorative items, the setup cost, deteriorating cost and total cost increases, as a result there is positive relation between them and with the increase in the rate of deteriorative items increases, the cycle time, optimum quantity, Production time T_1, the maximum inventory I_1 and holding cost decreases that results in a negative relationship between them. With the increase in setup cost per unit (S_C), optimum quantity (Q*), cycle time (T), production time T_1, consumption time (T_2) maximum inventory I_1, setup cost, holding cost, deteriorative cost and total cost increases then there is positive relationship between them. With the increase in holding cost per unit per unit time (H_C), the setup cost, holding cost, deteriorative cost and total cost increases then there is positive relationship between them but optimal cycle time (T) and optimal lot size (Q), production time (T_1), consumption time (T_2), maximum inventory (I_1) decreases then there is negative relationship between,

Several extensions can be made to this research:

1. The production rates in two models were time dependent demand and the demand rate was increasing over growth rate of demand. Other extension to this research could be to consider probabilistic demand or production rate.
2. The models developed in this research were considered for a single time. One may relax this assumption and consider models with multiple items.
3. Another extension to this research could be to attempt to prove the convexity of the total cost function where interest rate is included in the total cost function.
4. In developing the models, only one concept was introduced at a time. One may want to investigate models with combination of several concepts and determine the optimal policies for these cases.

The proposed model can assist the manufacturer and retailer in accurately determining the optimal quantity, cycle time and inventory total cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as food items, fashionable commodities, stationary stores and others.

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