MULTI-OBJECTIVE PRODUCTION PLANNING PROBLEM FOR A LOCK INDUSTRY: A CASE STUDY AND MATHEMATICAL ANALYSIS

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ABSTRACT

This case study develops a multi-objective production planning for a lock industry. The specific lock industry, i.e. Mezia lock industry produces various types of locks during the production run. The main objective of the firm is to minimize the production cost as well as profit maximization with consideration of the realistic constraints. This type of model formulated by considering the input information collected from the decision-maker. However, in many cases, it has been seen that the input information provided by the manufacturer is not precisely known. Formulation of the model has been solved by the Lingo 16.0 under a certain limitation, which can be further explored.

KEYWORDS: Production Planning Problem; Multi-objective Optimization; Case study.

MSC: 90B30, 90B90, 90C29

RESUMEN

Este es un estudio desarrollado sobre el planeamiento multi-objetivo de la producción en una industria de candados. Esta posee especificidades , i.e. el proceso en la industria de candado de Mezia la que produce varios tipos de candados durante el proceso de producción. El principal objetivo de la firma es minimizar el costo de producción, así como maximizar la ganancia considerando restricciones realísticas. Este tipo de modelo es formulado considerando la información de entrada obtenida del decisor. Sin embargo, en muchos casos, ha sido detectado que esta no es conocida con precisión. La formulación el modelo fue resuelta usando Lingo 16.0 bajo ciertas limitaciones, las que pueden ser exploradas posteriormente.

PALABRAS CLAVE: Problema del Planeamiento de la Producción; Optimización Multi-objetivo; Estudio de un Caso.

1. INTRODUCTION

Production planning sets the production targets and estimates the resources needed to achieve the goals. It creates a detailed plan for obtaining the production goals economically and efficiently within a certain time. It also predicts the problems, which may arise in the production process. It is considered as the transformation of raw material into finished goods by using different applications with the proper procedure under optimal costs and optimal consumption of raw materials. The primary goal of production planning is to successfully understand the market situations, which satisfy the customer's demands and brings profit to the manufacturer. For the production process, there are different touchstone which plays a decisive role in the production. It is necessary to evaluate the universality of the machines for a specific production, which can be done through the amount of the produced items in a specific time period, the number of different types of items, demands employees' qualification, cycle length for the production, etc. Therefore, it is required to pay more attention to the production plan to ensure the optimal profit and service level. Most of the manufacturing enterprises compelled to optimize the production process for the existing in the competitive market.

Nowadays, for the optimum profit or production, academicians and industry specialist are using the optimization techniques. Ghosh and Mondal (2017) developed a production-distribution planning model in the two-echelon supply chain environment with consideration of plants-warehouses and customer regions, including transhipment among the warehouses. A goal programming (GP) model is used to formulate the problem mathematically for the objective of the paper. Hossain and Hossain (2018) formulated a model for the production and distribution cost with the consideration of the fuzzy MOLPP.

One of the fundamental activity for manufacturing enterprises is production planning. Before starting the production plan for a financial year, most of the companies prepare a strategy for the production. The prepared production plan gives the idea about the demand for each period during the financial year to be

*Corresponding author: Sumit Maheshwari Email: sumitduor@gmail.com produced. For any firm, the production plan can be executed monthly, quarterly or even yearly depending on the products of the firm. The production plan is the allocation of available resources over the time to best meet the criteria such as quality, delivery time, demand and supply. A production problem includes production scheduling problem, machine capacity planning problem, storage and freight scheduling problems. From the last two decades, it is clear that the technological advancements and competitive market dynamics have brought a significant impact on the manufacturing industry. For the manufacturing industries, efficiency and productivity are the two important factors. Industries always demanded a high level of efficiency for the more flexible production. Saad (1982) described the nature of the production planning problems in various phases of the production. It is known as the pre and post existing function of the set. Nowadays, it is clear that production planning problems have the multi-objectives to sustain in the competitive environment. Academicians and practitioners used the e-constraint method for transforming the multi-objective problem into a single objective, where all other objectives are considered as constraints. In a practical scenario, almost every company produces multiple products. Therefore different types of fuzzy parameters existing nature.

For the more efficient solution of the production planning problems, the fuzzy multi-objective linear programming model has been considered because of the different industries have more than single objective function to establish the response and flexibility of the system. Several studies are available based on the application of the fuzzy optimization, Zimmermann (1978) used the concept of the fuzzy set which was firstly given by Zadeh (1965) and studied fuzzy programming and LP with multiple objectives. Kaveh and Ayda (2014) developed a fuzzy GP approach to formulate the multi-objective mixed-integer production planning model. This model considered simultaneously the three objectives, which are minimizing total cost, maximizing customer service level, and maximizing the quality of the end product. Okada et al. (1993) suggested a fuzzy production planning model with the help of a crisp model. Navid et al. (2013), Najmeh and Kuan (2014) developed the model for production planning with consideration of the performance and availability of the production lines. The significance of these two factors is shown by the comparison of the results in developing a real and practical production plan. Rinks (1982) evolved the production and workforce algorithm with the consideration of the relational assignment rules and explored the aggregate production by using the fuzzy logic.

In real-life situations, every problem has more than one objective function. Generally, in such types of problem, GP plays a vital role for the decision-maker, which is the extension of the LP. The academicians, practitioners and researchers are more aware of the existence of the multi-criteria in the real-life problem of management science. This case study aims to build a multi-objective production planning model for a private hardware firm. The proposed model is trying to achieve the following goals, first is to minimize the production cost, and second is to maximize the net profit. GP approach has been used for solving the formulated MOPPP.

2. MODEL BUILDING

The prime objective of the production company is to maintain the profit for the sustain in the competitive market. Some factors must be considered to achieve this goal. These include meeting customer demands, timely delivery of goods and services and much more. To achieve these objectives, companies must have technical experts and competent managers to respond to current management skills and trends in the needed technology. It is recommended that the company develop a production plan based on scientific methods and clarify the direction of execution of the production process. This case study aims to set up an effective production plan that minimizes the total production cost and maximize the profit of the firm. This work shows the demand for products from the market and the capacity of the company to meet this demand. The following input information is important before formulating the problem:

- 1. Information about the resources and the existing facilities including production equipment, manufacturing time for machines in hours, number of machines available for the production
- 2. Production cost through various production alternatives, such as available raw material cost.

The following assumptions and limitations are essential for the production planning of the model, considered as;

- The present model is multi-objective, where it optimizes the profit of the manufacturing company under certain conditions.
- The multi-item production model is to be considered.

- At a time, one machine cannot perform more than a job.
- Time horizon is finite.
- Shortages are not allowed for materials during the production time.
- Demand should be only for the finished products.
- In any case, the storage capacity of the warehouse and machine cannot exceed the maximum value.
- The manufacturing cost depends on the raw material and labour charges.
- The raw material cost is different for each product depending on its weight.

The following notations which are used during the formulation of the problem: **Nomenclature**

Indices

k - Index for the objective function, k=1, 2, ..., K**j** - Index for the manufactured item, j=1, 2, ..., J**i** - Index for machine timing, i=1,2,...,I**Decision Variable:**

 X_i -Manufactured items

Parameters:

 R_i -Profit of j^{th} unit (in Rs.)

 P_i -The production cost of the j^{th} unit (in Rs.)

B -Total Budget (in *Rs*.)

 M_{ii} -Working time (in hours) of $i^{\rm th}$ machine on $j^{\rm th}$ unit

 b_i -Total Working time (in hours) for i^{th} machine

 D_{1i} -Mean of demand

 D_{2i} -Upper (3 σ) limit of demand

Objective Function:

- Z_1 minimize the production cost
- Z_2 maximize the profit

Based on certain assumptions and notations, the mathematical model is solved for the problem as follows:

Objectives

1. Minimize the total cost of the finished products which includes the various types of cost (raw material cost, labour cost, transportation cost etc.)

$$Min \ Z_1 = \sum_{j=1}^J P_j x_j \tag{1}$$

2. Each company has the main objective for maximizing the profits by minimizing the total production cost.

$$Max \ Z_2 = \sum_{j=1}^{J} R_j x_j \tag{2}$$

Constraints

1. Production cost and fixed capital per month so that the total production cost will be less than or equal to the fixed capital of the firm. Constraints 1 related to balancing the equation for limitation production cost for each item does not exceed the fixed capital.

$$\sum_{j=1}^{J} P_j x_j \le B \tag{3}$$

2. Sum of the machine timing on each product of the month is less than or equal to available machine timing of that machine per month. Constraints 2 related to the balancing the equation for a limitation on working time (in hours) of the i^{th} machine on the j^{th} unit does not exceed to the total working time (in hours) for the i^{th} machine.

$$\sum_{j=1}^{J} M_{ij} x_j \le b_i, \qquad i = 1, 2, \dots, I$$
(4)

3. This Constraint puts the restrictions on the aggregate demand for the different types of products. $D_{1j} \le x_j \le D_{2j}, \qquad j = 1, 2, \dots, J$ (5)

The above-formulated equations are summarized in a model as follows: **Model with exact information**

$$\begin{aligned} Min \ & Z_1 = \sum_{j=1}^J P_j x_j \quad , Max \ & Z_2 = \sum_{j=1}^J R_j x_j, \\ \text{Subject to the constraint} \\ & \sum_{j=1}^J P_j x_j \leq B \\ & \sum_{j=1}^J M_{ij} x_j \leq b_i, \quad i = 1, 2, \dots, I \\ & D_{1j} \leq x_j \leq D_{2j}, \ j = 1, 2, \dots, J \end{aligned}$$

3. PROCEDURE FOR SOLVING MULTI-OBJECTIVE PRODUCTION PLANNING

GP is a flexible and effective technique which can be incorporated a variety of decision-making problems with consideration of multiple objectives. The following steps which have been used are as follows:

Step 1: Formulate the production planning problem as MOPPP with certainty. Firstly, the model is solved for the single objective problem with consideration of both the objective functions and only one objective can be solved at a time and ignoring the other objective functions. Thus, the obtained solutions of these models can be ideal and anti-ideal solutions. For the setting of the aspiration level of these objectives, the obtained solution serves a lot to the decision-maker.

Step 2: Set the aspiration level to each of the objective function, which is also to be considered as the goal value $(g_k, k = 1, 2)$.

Find $X = (x_1, x_2, ..., x_n)$ to optimize the following goals

$$Z_1(X) \preceq g_1, \quad Z_2(X) \succeq g_2,$$

Subject to the constraint of Model Where, $(g_1) = Min(Z_1(x))$ and $(g_2) = Max(Z_2(x))$. From the objective functions, the first objective is for minimization, i.e. $Z_1(X)$ which should be less than or equal to the aspiration level g_1 up to a certain tolerance limit. The second objective function, i.e. $Z_2(X)$ which should be

greater than or equal to the aspiration level g_2 up to a certain tolerance limit. **Step 3**: Construct the linear membership function for the Model. Hence the membership function of the goal of $Z_1(X) \preceq g_1$ (*i.e.*, min) can be defined as:

$$\mu_1(Z_1(X)) = \begin{cases} 1, & \text{if } Z_1(X) \le g_1 \\ \frac{U_1 - Z_1(X)}{U_1 - g_1}, & \text{if } g_1 \le Z_1(X) \le U_1 \\ 0, & \text{if } Z_1(X) \ge U_1 \end{cases}$$

For the goal $Z_1(X)$, U_1 is the upper tolerance limit.

Moreover, for the goal $Z_2(X) \succeq g_2$ (*i.e.*, max), the membership function can be defined as:

$$\mu_{2}(Z_{2}(X)) = \begin{cases} 1, & \text{if } Z_{2}(X) \ge g_{2} \\ \\ \frac{Z_{2}(X) - L_{2}}{g_{2} - L_{2}}, & \text{if } L_{2} \le Z_{2}(X) \le g_{2} \\ \\ 0, & \text{if } Z_{2}(X) \le L_{2} \end{cases}$$

For the goal $Z_2(X)$, L_2 is the lower tolerance limit.

Step 4: Finally, by following all the above-given steps, the formulated MOPPP takes the following form:

$$Max \ D(\mu) = \mu_1(Z_1(X)) + \mu_2(Z_2(X))$$

subject to constraint of Model with exact information

$$\mu_1(Z_1(X)) = \frac{U_1 - Z_1(X)}{U_1 - g_1},$$

$$\mu_2(Z_2(X)) = \frac{Z_2(X) - L_2}{g_2 - L_2},$$

$$0 \le \mu_1(Z_1(X)) \le 1$$

$$0 \le \mu_2(Z_2(X)) \le 1$$

 $D(\mu)$ is called the achievement function. This is a single objective optimization problem which can be solved by using a suitable classical optimization technique. The model has been solved by LINGO 16.0.

4. NUMERICAL CASE STUDY

To validate the model building, we have collected data from a hardware firm, which produces different types of hardware locks in Aligarh (UP), India. Following are the 17 types of locks manufactured by a firm, (i) International (Hardend), (ii) PRESTIGE, (iii) SINGLE LOCKING, (iv) MAX, (v) 2In1, (vi) X5, (vii) SECURE, (viii) MAGIC GOLD, (ix) MAGIC BRIGHT, (x) HEAVY, (xi) BULLET, (xii) LONG SHACKLE (xiii) ATOOT, (xiv) HARDY, (xv) PREMIUM, (xvi) SHUTTER, (xvii) HALF ROUND. Data for machine availability, production cost, expected per unit profit, an average time consumed by a machine during the production run, the demand for each item (in dozen) were obtained from a hardware firm. The primary purpose of adding these hypothetical data is to make production planning model more efficient. Table 1 the number of available machines during the production time

TABLE 1. Machine Availability										
Machines	No. of Availability									
Power press	4									
Tool grinding machine	3									
Caps ton machine	2									
Surface grinder	3									
Polish machine	4									
Milling machine	2									

 TABLE 1: Machine Availability

Drill machine	2
Hand press machine	3
Numbering machine	2



Figure 1. Mezia Lock Industry

 Table 2 summarizes production cost and expected profit an item manufactured by the firm

 TABLE 2: Production Cost and Profit per Dozen Units

Items	Production costs	Expected Profit
International (Hardend) (MLIH)	2295	405
PRESTIGE (MLP)	410.40	159.6
SINGLE LOCKING (MLSL)	443.16	90.84
MAX (MLMX)	318	72
2In1 (MLMTO)	558	82
X5 (MLX)	342	108
SECURE (MLSEC)	504	121
MAGIC GOLD (MLMG)	894	174
MAGIC BRIGHT (MLMB)	1080	276
HEAVY (MLHVY)	510	156
BULLET (MLBLT)	534	140
LONG SHACKLE (MLLS)	474	138
ATOOT (MLAT)	726	210
HARDY (MLHDY)	744	216
PREMIUM (MLPRM)	942	198
SHUTTER (MLSTR)	912	288
HALF ROUND (MLHR)	198	54

Table 3 summarizes the average time consumed by each machine during production hours.

					IA	BLE	J: A	vera	ge 11	me C	onsun	iea					
Machine/ Item	X ₁	<i>X</i> ₂	<i>X</i> ₃	X ₄	<i>X</i> ₅	<i>X</i> ₆	<i>X</i> ₇	<i>X</i> ₈	<i>X</i> 9	<i>X</i> ₁₀	<i>X</i> ₁₁	<i>X</i> ₁₂	<i>X</i> ₁₃	<i>X</i> ₁₄	<i>X</i> ₁₅	X ₁₆	X ₁₇
M ₁	9	7	6	8	14	7	6	6	9	14	15	14	15	14	12	17	14

TABLE 3: Average Time Consumed

<i>M</i> ₂	5	8	5	7	12	6	9	10	14	8	9	9	7	8	12	15	11
<i>M</i> ₃	6	7	6	4	10	8	6	5	8	6	5	5	6	8	5	9	7
M ₄	6	7	7	4	16	4	19	8	11	6	6	7	5	6	8	19	5
M ₅	4	7	7	7	10	6	14	12	8	10	6	6	7	10	10	22	20
M ₆	4	6	6	6	9	4	7	6	6	6	7	4	6	7	4	8	6
M ₇	4	4	4	5	8	5	8	6	7	10	9	4	5	6	5	7	5
<i>M</i> ₈	4	7	5	4	14	8	16	9	9	6	6	6	8	7	6	19	5
M ₉	6	5	6	6	8	4	7	6	9	6	6	6	5	6	6	11	6

Table 4 summarizes monthly demand of each item (in Dozen)

 TABLE 4: Demand of each Item (in Dozen)

Month/Items	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	<i>X</i> ₉	X ₁₀	<i>X</i> ₁₁	<i>X</i> ₁₂	<i>X</i> ₁₃	<i>X</i> ₁₄	<i>X</i> ₁₅	<i>X</i> ₁₆	<i>X</i> ₁₇
JAN	36	1174	64	135	141	121	117	86	230	38	35	76	228	41	41	302	514
FEB	39	1282	66	142	146	127	124	85	230	37	41	78	230	42	39	298	529
MAR	42	1334	66	145	152	132	125	88	234	41	44	86	235	45	42	317	550
APR	41	1308	64	144	151	126	123	85	233	40	42	82	234	42	38	313	536
MAY	40	1280	65	139	146	125	126	86	231	37	41	79	228	42	37	304	524
JUNE	38	1160	63	134	142	122	117	85	229	36	37	75	223	40	36	297	511
JULY	32	1040	60	127	133	114	112	83	228	33	38	72	217	35	33	289	470
AUG	29	1006	62	118	127	110	109	81	226	32	30	64	218	34	32	266	445
SEP	31	1040	61	123	124	112	110	80	224	35	34	68	219	39	37	279	460
ОСТ	34	1067	62	128	134	115	113	82	227	33	34	67	218	35	31	283	480
NOV	35	1109	61	132	136	117	111	84	227	34	36	70	221	37	34	286	487
DEC	35	1120	62	129	136	119	117	83	229	36	32	71	217	36	32	294	494

When parameters of the problem are precisely known

Before formulating the problem, we have to find out the 3σ limit of the demand (given in Table 4) for the items to be produced by the manufacturer. TABLE 5: 3σ Limit for the Demand

					IAB	LE 3:	3 σ L	Imit	ior th	le De	man	la					
Items	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Mean	36	1160	63	133	139	120	117	84	229	36	37	74	224	39	36	294	500
S.D.	4	116	2	8	9	7	6	2	3	3	4	6	7	3	4	14	33
\overline{X} +3 σ	48	1508	69	157	166	141	135	90	238	45	49	92	245	48	48	336	599

Table 5 helps the DM to set out the upper and lower limit of the items to be manufactured during the production time. Using the stepwise procedure as defined in Section 4, the bounds for the two objective functions are determined as: $1794757 \le Z_1 \le 1966311$, and $526622 \le Z_2 \le 576048.5$. Using these

bounds, the corresponding linear membership functions for the two objective functions are constructed as follows:

$$\mu_1(Z_1(X)) = \begin{cases} 1, & \text{if } Z_1(X) \leq 1794757 \\ \frac{1966311 - Z_1(X)}{1966311 - 1794757}, & \text{if } 1794757 \leq Z_1(X) \leq 1966311 \\ 0, & \text{if } Z_1(X) \geq 1966311 \\ \end{array} \\ \mu_2(Z_2(X)) = \begin{cases} 1, & \text{if } Z_2(X) \geq 576048.5 \\ \frac{Z_2(X) - 526622}{576048.5 - 526622}, & \text{if } 526622 \leq Z_2(X) \leq 576048.5 \\ 0, & \text{if } Z_2(X) \leq 526622 \end{cases}$$

Now, we calculate the compromise solution for the model, which was introduced in section 4. The formulated model is solved with the help of optimizing software LINGO 16.0. Table 6 is the evidence of the optimum solution of the considered problem with the optimal quantities as follows:

	Table 6: Optimal Compromise Solution
Objective Values	The optimal number of quantities to be produced by the firm.
Z ₁ =1892085	$x_1=36, x_2=1396, x_3=63, x_4=133, x_5=139, x_6=120, x_7=117, x_8=84, x_9=229,$
$Z_2 = 564425.5$	x_{10} =36, x_{11} =37, x_{12} =75, x_{13} =224, x_{14} =39, x_{15} =36, x_{16} =294, x_{17} =500
1	

Let us suppose that DM accepts this solution and considers it the preferred compromise solution with the acceptance rate of **0.5987545**.

5. MANAGERIAL INSIGHTS, CONTRIBUTIONS AND LIMITATIONS

5.1. Managerial Insights

Following are the insight drawn from our proposed production planning model:

- The problem that has been formulated in this paper is influenced by a real case study considering the situations which are now regularly faced by the industries managers.
- This model will also be helpful to the manager in optimizing the production cost, which is directly related to the profit.

5.2. Contributions

The proposed production planning model makes the following contribution:

- We found out the limit for which the solution remains idle.
- MOPPP model provides an optimal product mix of the finished products by minimizing the production cost and also by maximizing the net profit.

5.3. Limitations

In general, every model has certain limitation for the existence of the problem, thus for the present study, certain limitations are given below:

- The role of ordering cost, carrying a cost, labour cost, and replenishment cost in this model is yet to be explored, i.e. the inventory circumstances.
- Transportation cost and supplier selection in the model formulation are also not considered, which is also an integral part of production planning.

6. CONCLUSION

In the proposed model, an attempt has been made for the production planning problem with multi-products, multi-periods and multi-machines under a certain environment that takes into account to minimize the production cost and maximize the net profit subject to some realistic set of constraints. In a multi-objective optimization problem, objective functions are usually conflicting with each other, and any improvement in one of the objective functions can be achieved only compromising with another objective function. To deal with such situations in MOPPP, the GP approach has been used to obtain the optimal solution of the formulated problem, and this optimal solution can only be obtained by achieving the highest degree of each of the membership goals. It will give an insight view to the manager of the firm about what type of production policy should he adapt to optimize the realistic situations. In future, heuristics methods or genetic algorithm can be used for the optimal solution minimum computational time.

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ADDITIONAL INFORMATIONS:

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