LEARNING EFFECT ON INVENTORY MODEL IN FUZZY ENVIRONMENT WITH TRADE- CREDIT FINANCING

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ABSTRACT

Now-a-days, learning's awareness is increasing in various disciplines because impact of learning has a direct impact on profit or loss, and it is a promotional deemed effective tool for inventory management. The supplier wants an appreciable coordination with the buyers and analyzes with full detail about the concerned cost and the demand parameters as to how much suitable the demand and the respective costs should be for the buyer. The fuzzy analysis is a good tool for examining the performance as well the output of imprecise parameters involved in the business dealing's procedure. The selling price, demand rate and ordering cost per unit has been assumed imprecise in nature. In addition, these entities are also called fuzzy triangular numbers. As per considerations, a mathematical model has been developed with learning concept and credit financing under fuzzy environment. After computation, minimum annual cost is obtained owing to the learning phenomenon in the inventory carrying cost and a suitable range of unit selling price, demand rate and ordering cost due to the fuzzy properties. The optimal cost in fuzzy environment is de-fuzzified with the help of the centroid process. Lastly, some numerical examples as well as sensitivity analysis have been illustrated to verify the present model.

KEYWORDS: Learning effect, Inventory, EOQ, Trade credit financing, Fuzzy environment, Centroid method.

MSC: 90B05

RESUMEN

Actualmente el interés en el proceso de aprendizaje está incrementándose en varias disciplinas por el impacto que el aprendizaje tiene directamente sobre ganancias y pérdidas, y que es profundamente promocionalmente efectivo como herramienta en el manejo de inventarios. El suministrador desea coordinar con compradores y analiza detalladamente lo concerniente a costos y a los parámetros de la demanda sobre la adecuacidad de la demanda y los respectivos costos para el comprador . El análisis fuzzy es una buena herramienta para examinar el comportamiento y los resultados de tener imprecisiones en los parámetros del procedimiento del manejo del negocio. El precio de venta, la tasa de demanda y los costos de orden por unidad han sido asumidos como de naturaleza imprecisa. Además, estas entidades son también llamadas "fuzzy triangular numbers". La consideración del matemático modelo ha sido desarrollado con el concepto de aprendizaje y del crédito financiero en un ambiente fuzzy . Después de los cómputos, los mínimos costos anuales son obtenidos en el fenómeno del aprendizaje, costo de inventario y un adecuado rango de los precios de venta por unidad, tasa de demanda y costo de orden debidos a las propiedades fuzzy. El óptimo costo en un ambiente fuzzy es des-fuzzificado con la ayuda del proceso del centroide. Finalmente , algunos ejemplos numéricos así como un análisis de sensibilidad son usados para verificar el presente modelo.

PALABRAS CLAVE: Efecto del Aprendizaje, , Inventario, EOQ, Financiamiento del Crédito de Comercio, Ambiente Fuzzy, Método del Centroide

1. INTRODUCTION

Typically, in the business market, mostly the seller provides a trade-credit financing plan to upgrade sales and profit and to become a producer for the new buyers. Consequently, impractical exercise, the supplier permits a designated pre-determined time period to arrange money for the buyer that the seller owes to the buyer for the things delivered. Prior to the conclusion of the specified trade credit financing time, the items to be sold by the buyer, earn interest on the same, and accumulate revenue. In case the cash could not be arranged within the stipulated time for trade- credit financing, a higher rate of interest is charged and imposed. Goyal (1985) initially, recommended the formulation of EOQ utilizing trade-credit financing. Shah (1993a, 1993b), Aggarwal and Jaggi (1995), proposed the EOQ model with trade credit financing for articles which are decaying in quality. Learning phenomenon is a mathematical tool which acts during exercising

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business for reduction of cost as ordering cost, holding cost as well as screening cost. Many researchers have evolved mathematical models employing the impact of learning.

Learning curve developed by Wright (1936), is a mathematical tool to show the behavior of a parameter at different stages. In his first attempt in 1936, he finds out the mathematical renewed formula which derives the relationship between learning changeable parameters in quantitative form and derived the result in the proposition of the LC (learning curve). Again, different to the excess of review on LC, there is scarcity of review on forgetting curves. This scarcity of study has been credited almost certainty to the sensible difficulties occupied in getting information regarding the period of forgetting which function of time by Levin is et al. (1989). Hammer (1957) debated on the logical revision of LC as a means of involving work values. Baloff (1966) discussed about the mathematical behavior of the learning theory (learning slope varied widely and also explained with viable justification, the outcomes of the practical aspects to arrange the LC factor by urbanized skill and hesitant study in gathering knowledge). Cunningham (1980) discussed LR(learning rate) by using dissimilar kind of data, i.e. composed learning rates reported in 15 diverse U.S. industries between the years (1860-1978) and Thomas, Dutton (1984) justified learning rates under distribution in 108 forms. Argote et al. (1990) discussed about the factors by which the rate of learning varied in different situations which is one of the major factors in their search field. Salameh et al. (1993) considered a limited manufactured stock form (Production inventory model) with the outcome of human knowledge and also discussed variable demand rate and learning in time to optimize the cost. Jaber et al. (1996) explained the theory of forgetting using manufacture breaks, learning curves and discussed optimal manufacturing amount and minimize the whole stock price. Jaber et al .(1996) assumed an optimal lot sizing problem using the conditions of bounded learning cases and focuses on economic order quantity and minimization of entries to reprises keeping the learning shape into consideration.

Jaber et al. (1997) proposed and explored on a comparative study of learning and forgetting theory, focused on the comparison of different types of models such as VRVF, VRIF and LFCM. Jaber et al. (1995) discussed about optimal lot sizing with shortage and backordering considering them under learning. Jaber et al. (2008) proposed the EOQ model for defective articles with defective percentage per batch, decreases according to the LC (learning Curve). Jaber and Bonney (2003) considered the lot shape with theory of learning as well as forgetting in set-up and in manufactured goods excellence and focused on minimizing production time, reducing rework process and optimizing production quantity. Balkhi (2003) discussed on maximum manufacturing of lot volume for decaying items and shortage case material with time unreliable order and rates with the help of impact of learning. Jaber et al. (2004) presented that the LC for manner producing imperfects necessary reworks and generates rate defects as stable and improves by Wright on LC. Khan et al. (2010) assumed a mathematical formulation of economic order quantity for articles with defective features that use learning in screening and maximizing production and minimizing the cost of production. Jaber et al. (2010) discussed on, how to merge the average dispensation time process to give way with respect to the number of lot.

Anzanello and Fogliatto (2011) suggested a different kind of an application of learning curves model and authors focused on how the model could be used and implied in distinct mathematical form. Konstantaras et al. (2011) urbanized a mock-up to exploit construction with the situation of lacks for defective things with assessment as learning concept. Jaggi, Goyal and Mittal (2013) discussed on construction stock representation with financing policies of defective things under suitable backlogging case. Jaber et al. (2013) assumed a manufactured inventory mathematical model with LC and FC "learning and forgetting" theory in consideration and also discussed how much minimization of the number of length (order) of a lot from manufacture to the succeeding sequence was required or implied. Mostly researchers considered all the parameters of inventory model such as demand rate, selling price, holding cost as ordering cost etc, either as fixed, needy on time or probabilistic in environment for the improvement of EOQ. Generally, an author assumes various types of parameters, in the formulation of inventory design or models either as a fixed, reliant on period or uncertain or flexible in environment for the improvement of the EOQ. Though, in reality, such type of components may have little formulations from the certain values, and they do not necessarily follow any type of probability distribution as they are discontinuous in nature. Suppose, such components are treated as fuzzy components, then the other components will be considered as sensible. Teng et al. (2014) proposed the task by using the maximum trade financing strategies and batch length strategies in production models with LC affect manufacture costs. Givi et al. (2015) discussed the modeling of worker reliability with learning and fatigue. Sangal and Rani (2016) discussed the working policy of a fuzzy concept model with shortages under the learning effect. Sangal et al. (2017) proposed effect of learning with non-instant deteriorating model. Jaggi, Tiwari and Goel (2017) exposed the policy of trade credit financing in different inventory ordering strategies for non-instantaneous deteriorating things and concluded that demand is a function of selling fewer than two storage amenities. Rani et al (2018) used the concept of green supply chain with learning effect for non-instantaneous deteriorating inventory model. Tiwari et al. (2018) proposed a combined store and pricing model for decaying items with ending dates and partial backlogging under two level trade credit policies in the provided sequence. Patro et al. (2018) proposed a fuzzy EOQ model for decaying things with defective quality using balanced reduction under the impact of learning. Jayaswal et al. (2019) discussed the learning phenomenon on seller ordering strategy for defective quality articles with permissible delay in payment. Jayaswal, Sangal and Mittal (2019) found out the impact of learning on inventory -policies with defective quality and decaying things under the trade financing strategy. Yadav et al. (2019) analyzed the behavior of learning on best strategy of source chain followers for defective quality things: Game theory concepts. In practical implications, the components of a stoke model are indeterminate, inaccurate and the purpose to get maximum size length is difficult as it is a non-stochastic indistinguishable managerial process. Such types of problems have been tried to resolve by this model and the verification of the result has been illustrated by appropriate examples. In the business organization, demand parameters, ordering cost, purchasing cost and many more which is related to the inventory cost are not always fixed. In this situation seller wants to increase his sale by using of credit financing and it is very dangerous for seller because of undefined inventory parameters. Present paper tries to eliminate such type of risky problem for seller and some parameters like demand parameters, ordering and purchasing cost are in imprecise in nature treated as fuzzy variables. Credit financing policy is good co-ordination tool and it is considered that seller provides a fixed credit period to his buyer for more profit during business and buyer accepted this offer. The buyer wants to minimize the total cost termed as fuzzy whole cost with new strategy and eliminate non-productive activities through repetition termed as learning for this action it is assumed that holding cost follows the learning effect. The present model has the extended form of Shah et al. (2012) by incorporating the learning effect and the whole optimal cost in fuzzy environment is de-fuzzified with the help of the centroid process with respect to optimal cycle length termed as fuzzy cycle length. Sensitivity analysis as well as conclusion has been presented in end of the section. This concept is motivated for the seller and buyer during transaction of business where some inventory parameters imprecise in nature and seller provides credit financing period as well as buyer wants to reduce holding cost per shipments.

Table 1. Different author's contribution

Author(s)	Learning Effects	Trade credit Financing	Fuzzy environment
Write (1936)	✓		
Hammer (1957)	✓		
Baloff (1966)	✓		
Cunnigham (1980)	✓		
Dutton (1984)	✓		
Argote et al. (1990)	✓		
Salameh et al. (1993)	✓		
Jaber et al. (1996)	✓		
Jaber et al. (2008)	✓		
Khan et al. (2010)	✓		
Anazanello et al. (2011)	✓		
Shah et al. (2012)		✓	✓
Jaggi et al. (2013)		✓	
Sair et al. (2014)	✓	✓	
Khan et al. (2014)	✓		
Zhou et al. (2015)		✓	
Sangal et al. (2016)	✓		
Tiwari et al. (2016)		✓	

Sangal et al. (2017)	✓		
Jaggi et al. (2017)		✓	
Jayaswal et al. (2019)	✓	✓	
Patro et al. (2018)	✓		✓
Tiwari et al. (2018)		✓	
Present Paper	✓	✓	✓

2. ASSUMPTIONS AND NOTATONS

The mathematical model is derived using the following notations and assumptions.

2.1 Assumptions

The subsequent assumptions have been incorporated to expand the present model

- Demand rate for an item is imprecise in nature
- Demand of this model is fulfilled and there are no shortages in this model
- Selling price of items is a major factor of any inventory model during business and it is assumed imprecise in nature
- The ordering cost of an item is imprecise in nature
- The rate of refilling is instantaneous
- Lead time is zero and insignificant
- The seller provides a predetermined credit financing period to clear up the accounts to the buyer which is suggested by Jaggi et al. (2013)
- Carrying cost is partially fixed and partially reduces in each shipment (n) owing to the learning power of employees, suggested by Aggarwal et al. (2017))

$$h(n) = ho + \frac{h_1}{n^{\lambda}}, ho, h_1 > 0 \, and \;\; 0 < \lambda < 1$$

λ is learning factor, ho and h_1 are a fixed holding cost,

2.2 Notations

D	fixed demand /year
$\tilde{D}$	fuzzy demand /year
A_c	ordering cost / order (in \$)
$\tilde{A}_c$	fuzzy ordering cost/order (in \$)
P_c	selling price/unit (in \$)
$\tilde{P}_c$	fuzzy unit selling price /unit (in \$)
C_c	unit purchase cost /unit (in \$)
$h(n)$	unit inventory variable carrying cost defined in assumption.
Q_o	lot size (unit)
M_s	The provided trade financing policy by the seller to the purchaser to arrange the account /year
IHC_1	holding cost / cycle/unit (in\$)

IC_1	interest paid/charged under the condition, $M_s \leq T_c$ /year
IE_1	interest gained under the condition, $M_s \leq T_c$ /year
IC_2	interest paid under the condition, $M_s \geq T_c$ /year
IE_2	interest paid under the condition, $M_s \geq T_c$ /year
I_c	interest paid /$ /year in store by the retailer
I_e	interest gained / $ /year in store by the buyer
T_c	cycle time in year
$\Psi_1(T_c)$	the whole concerned cost /unit time with case $M_s \leq T_c$
$\Psi_2(T_c)$	the whole concerned cost / unit time with case $M_s \geq T_c$
$\tilde{\Psi}_1(T_c)$	fuzzy whole concerned cost /unit time with case $M_s \leq T_c$
$\tilde{\Psi}_2(T_c)$	fuzzy whole concerned cost per unit time with case $M_s \geq T_c$
$\tilde{\Psi}_3(T_c)$	de- fuzzy whole concerned cost / unit time with case $M_s \leq T_c$
$\tilde{\Psi}_4(T_c)$	de-fuzzy whole concerned cost / unit time with case $M_s \geq T_c$
n	number of shipments
λ	learning factor
T_{c1}	the whole cycle time (in year) under the condition, $M_s \leq T_c$
T_{c2}	the whole cycle time (in year) under the condition, $M_s \geq T_c$
$\tilde{T}_{c1}$	de-fuzzy the whole cycle time (in year) under the condition, $M_s \leq T_c$
$\tilde{T}_{c2}$	de-fuzzy whole cycle time (in year) under the condition, $M_s \geq T_c$

3. CRISP FORMULATION MODEL

Assume that $q(t)$ is the stock at $t\ (0 \leq t \leq T_c)$. At initial position the inventory level is Q_0. The cost parameters below are related to the inventory stock, suggested by Naddor (1966).
The ordering cost,

$$OC_1 = \frac{A_c}{T_c} \tag{3.1}$$

The carrying cost per cycle,

$$IHC_1 = \frac{1}{T_c} h(n) \int_0^{T_c} Dt\, dt = \frac{h(n) DT_c}{2} \tag{3.2}$$

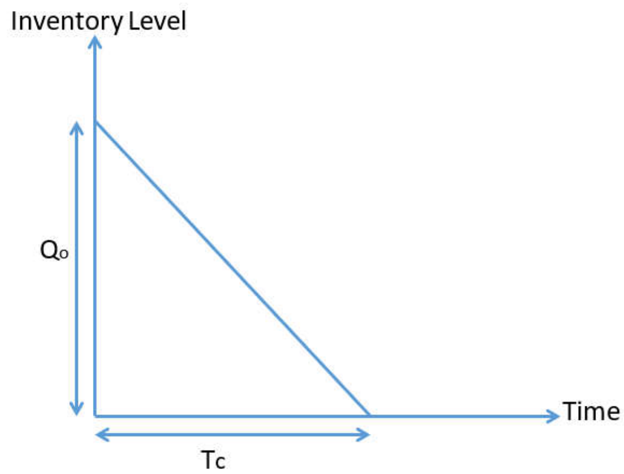

Figure 3.1 Inventory system with order quantity and time

The interest paid and interest gained, are the two cases that arise depending on the lengths of T_c and M_s .These cases are presented in the graphed form in figure 3.2 and figure 3.3.

Case-1: $M_s \le T_c$

The purchaser gains extra money in the form of interest I_e at suitable rate on the usual sales income created up to 0 to M_s. Additional, the purchaser has to organize the explanation at credit time M_s and must settle for assets to pay the seller for the lasting store at the precise rate of interest, I_c, from M_s to T_c .

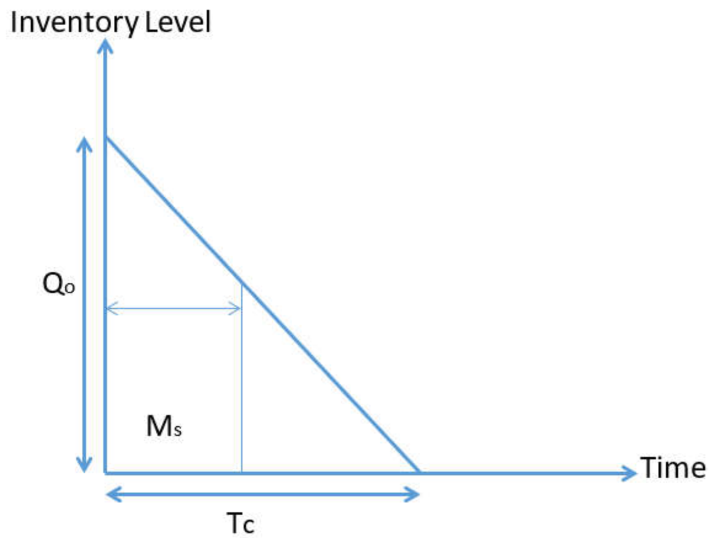

Figure 3.2 Inventory process of trade credit financing for case-1

Therefore, the buyers gain an extra money in form of interest charge for the usual store during period of time 0 to M_s i.e. $I_e P_c D\left(M_s\right)^2 / 2T_c$ and buyers pay an interest for the unsold items after M_s which is equal to $C_c I_c D\left(T_c - M_s\right)^2 / 2T_c$.

Hence, the whole concerned cost per unit time is,

$$\Psi_1(T_c) = OC_1 + IHC_1 + IC_1 - IE_1$$

$$\Psi_1(T_c) = \frac{A_c}{T_c} + \frac{h(n)DT_c}{2} + \frac{C_c I_c D(T_c - M_c)^2}{2T_c} - \frac{P_c I_e D(M_s)^2}{2T_c} \qquad \textbf{(3.3)}$$

Case-2: $M_s \geq T_c$

In this case, no interest is payable by the buyer, who only gains an interest on the income generated from 0 to M_s and equal to $P_c I_e D\left(M_s - \frac{T_c}{2}\right)$.

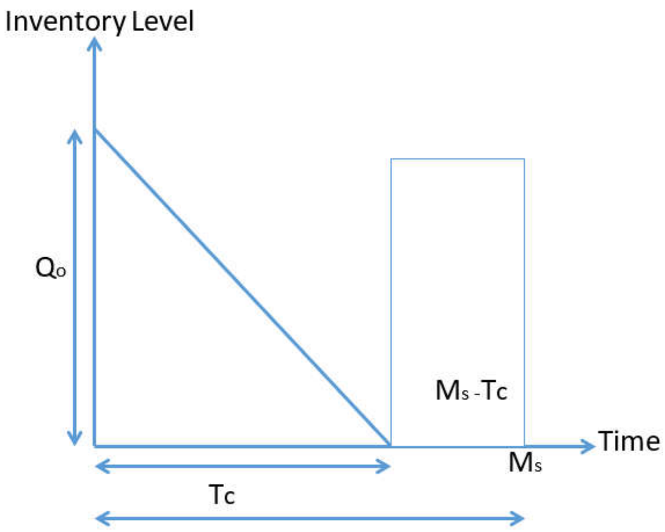

Figure 3.3 Inventory process of trade credit financing for case-2

Hence, the whole concerned cost per unit time is,

$$\Psi_2(T_c) = OC_1 + IHC_1 + IC_2 - IE_2$$

$$\Psi_2(T_c) = \frac{A_c}{T_c} + \frac{h(n)DT_c}{2} - P_c I_e D\left(M_s - \frac{T_c}{2}\right) \qquad (3.4)$$

Now, the whole related cost $\Psi(T_c)$ per unit time is,

$$\Psi(T_c) = \begin{cases} \Psi_1(T_c), & M_s \leq T_c \\ \Psi_2(T_c), & M_s \geq T_c \end{cases} \qquad (3.5)$$

where,

$$\Psi_1(T_c) = \frac{A_c}{T_c} + \frac{h(n)DT_c}{2} + \frac{C_c I_c D(T_c - M_s)^2}{2T_c} - \frac{P_c I_e D(M_s)^2}{2T_c}$$

and

$$\Psi_2(T_c) = \frac{A_c}{T_c} + \frac{h(n)DT_c}{2} - P_c I_e D\left(M_s - \frac{T_c}{2}\right).$$

It can be easily checked or verified that $\Psi_1(M_s)=\Psi_2(M_s)$, so $\Psi(T_c)$ is a continuous function of T_c.

The essential and enough conditions for $\Psi_1(T_c)$ to be optimum are as follows,

$$\frac{d\Psi_1(T_c)}{dT_c}=-\frac{A_c}{T_c^{\,2}}+\frac{h(n)D}{2}+\frac{C_cI_cD}{2}-\frac{C_cI_cD(M_s)^2}{2T_c^{\,2}}+\frac{P_sI_eD(M_s)^2}{2T_c^{\,2}}$$

and

$$\frac{d^2\Psi_1(T_c)}{dT_c^{\,2}}=\frac{2A_c}{T_c^3}+\frac{C_cI_cD(M_s)^2}{T_c^3}-\frac{P_cI_eD(M_s)^2}{T_c^3}>0$$

Respectively. For the maximum cycle length of time T_{c1}, set $\frac{d\Psi_1(T_c)}{dT_c}=0$ which gives

$$T_c=T_{c1}(\text{say})=\sqrt{\frac{2A_c+D(M_s)^2(C_cI_c-P_cI_e)}{D(h(n)+C_cI_c)}} \tag{3.6}$$

Now,

$$\frac{d\psi_2(T_c)}{dT_c}=-\frac{A_c}{T_c^2}+\frac{h(n)D}{2}+\frac{P_cI_eD}{2}$$

and

$$\frac{d^2\Psi_2(T_c)}{dT_c^{\,2}}=\frac{2A_c}{T_c^{\,3}}>0$$

For the optimal cycle time T_{c2}, set $\frac{d\Psi_2(T_c)}{dT_c}=0$ which gives

$$T_c=T_{c2}(\text{say})=\sqrt{\frac{2A_c}{D(h(n)+P_cI_e)}} \tag{3.7}$$

4. FUZZY METHODOLOGY

Here A_c, D and P_c are assumed imprecise in nature and let A_c, D and P_c be defined as triangular fuzzy numbers such that $\widetilde{A}_c=[a_1,a_2,a_3]$, $\widetilde{D}=[d_1,d_2,d_3]$ and $\widetilde{P}_c=[p_1,p_2,p_3]$

Here $(d_1<d_2<d_3),(p_1<p_2<p_3)$ and $(a_1<a_2<a_3)$ are founded on individual decisions.

The relationship functions for $\widetilde{A}_c,\widetilde{D}$ and $\widetilde{P}_c$ are clear as follows

$$\mu_{\widetilde{A}_c}(\widetilde{A}_c)=\begin{cases}0, & if\ A_c<a_1\\ \dfrac{A_c-a_1}{-a_1+a_2}, & if\ A_c\in[a_1,a_2)\\ \dfrac{a_3-A_c}{a_3-a_2}, & if\ A_c\in[a_2,a_3)\\ 0, & if\ A_c\geq a_3\end{cases}$$

$$\mu_{\widetilde{D}}\left(\widetilde{D}\right)=\begin{cases}0, & if\ d_1 > D\\ \dfrac{-d_1+D}{-d_1+d_2}, & if\ D\in\left[d_1,d_2\right)\\ \dfrac{-D+d_3}{-d_2+d_3}, & ifD\in\left[d_2,d_3\right)\\ 0, & if\ d_3\geq D\end{cases}$$

$$\mu_{\widetilde{P}_c}\left(\widetilde{P}_c\right)=\begin{cases}0, & if\ P_c < p_1\\ \dfrac{-P_1+P_c}{p_2-p_1}, & ifP_c\in\left[p_1,\ p_2\right)\\ \dfrac{p_3-P_c}{\widetilde{p}_3-\widetilde{p}_2}, & if\ P_c\in\left[p_2,p_3\right)\\ 0, & if\ P_c\geq p_3\end{cases}$$

The de-fuzzified $\widetilde{\Psi}_1\left(T_c\right)$ and $\widetilde{\Psi}_2\left(T_c\right)$ by centroid method are defined as illustrated below,

$$\widetilde{\Psi}_3\left(T_c\right)=\frac{\widetilde{\Psi}_{11}\left(T_c\right)+\widetilde{\Psi}_{12}\left(T_c\right)+\widetilde{\Psi}_{13}}{3T_c}.$$

$$\widetilde{\Psi}_4\left(T_c\right)=\frac{\widetilde{\Psi}_{21}\left(T_c\right)+\widetilde{\Psi}_{22}\left(T_c\right)+\widetilde{\Psi}_{23}\left(T_c\right)}{3T_c}.$$

4.1 Fuzzy Inventory Model

During the formulation of fuzzy process, we consider that the call rate, set up cost and vending price are all imprecise in nature and are termed as fuzzy numbers and are denoted by $\widetilde{D},\ \widetilde{A}_c$ and $\widetilde{P}_c$ correspondingly. Here, we consider that $\widetilde{D}=\left(d_1,d_2,d_3\right),\ \widetilde{A}=\left(a_1,a_2,a_3\right)$ and $\widetilde{P}_c=\left(p_1,p_2,p_3\right)$ are positive triangular fuzzy numbers.

4.2 Derivation of $\widetilde{\Psi}_1\left(T_c\right)$ And $\widetilde{\Psi}_2\left(T_c\right)$

The fuzzy yearly whole concerned price can be formulated as,

$$\widetilde{\Psi}\left(T_c\right)=\begin{cases}\widetilde{\Psi}_1\left(T_c\right), & M_s\leq T_c\\ \widetilde{\Psi}_2\left(T_c\right), & M_s\geq T_c\end{cases}\tag{4.8}$$

Where,

$$\widetilde{\Psi}_1\left(T_c\right)=Y_{11}\widetilde{A}_c+Y_{12}\widetilde{D}+Y_{13}\widetilde{P}_c\widetilde{D}$$

$$\widetilde{\Psi}_{11}\left(T_c\right)=Y_{11}a_1+Y_{12}d_1+Y_{13}p_1d_1\tag{4.9}$$

$$\widetilde{\Psi}_{12}\left(T_c\right)=Y_{11}a_2+Y_{12}d_2+Y_{13}p_2d_2\tag{4.10}$$

$$\widetilde{\Psi}_{13}\left(T_c\right)=Y_{11}a_3+Y_{12}d_3+Y_{13}p_3d_3\tag{4.11}$$

and

$$\widetilde{\Psi}_2\left(T_c\right)=Y_{21}\widetilde{A}_c+Y_{22}\widetilde{D}+Y_{23}\widetilde{P}_c\widetilde{D}$$

$$\widetilde{\Psi}_{21}\left(T_c\right)=Y_{21}a_1+Y_{22}d_1+Y_{23}p_1d_1\tag{4.12}$$

$$\tilde{\Psi}_{22}(T_c) = Y_{21}a_2 + Y_{22}d_2 + Y_{23}p_2d_2 \tag{4.13}$$

$$\tilde{\Psi}_{23}(T_c) = Y_{21}a_3 + Y_{22}d_3 + Y_{23}p_3d_3 \tag{4.14}$$

where, $Y_{11} = Y_{21} = \frac{1}{T_c}$, $Y_{12} = \frac{h(n).T_c}{2} + \frac{C_cI_c}{2}\left[T_c + \frac{(M_s)^2}{T_c} - 2M_s\right]$, $Y_{13} = -\frac{I_e(M_s)^2}{2T_c}$

$Y_{22} = \frac{h(n).T_c}{2}$, $Y_{23} = I_e\left[\frac{T_c}{2} - M_s\right]$.

From equations (9), (10) and (11) we have de-fuzzified $\tilde{\Psi}_1(T_c)$ by centroid method which is equal to

$$\tilde{\Psi}_3(T_c) = \frac{\tilde{\Psi}_{11}(T_c) + \tilde{\Psi}_{12}(T_c) + \tilde{\Psi}_{13}(T_c)}{3T_c}.$$

$$\tilde{\Psi}_3(T_c) = \frac{Y_{11}(a_1 + a_2 + a_3) + Y_{12}(d_1 + d_2 + d_2) + Y_{13}(p_1d_1 + p_2d_2 + p_3d_3)}{3T_c}. \tag{4.15}$$

From equations (12), (13) and (14) we have de-fuzzified $\tilde{\Psi}_1(T_c)$ by centroid method which is equal

$$\tilde{\Psi}_4(T_c) = \frac{\tilde{\Psi}_{21}(T_c) + \tilde{\Psi}_{22}(T_c) + \tilde{\Psi}_{23}(T_c)}{3T_c}.$$

$$\tilde{\Psi}_4(T_c) = \frac{Y_{21}(a_1 + a_2 + a_3) + Y_{22}(d_1 + d_2 + d_2) + Y_{23}(p_1d_1 + p_2d_2 + p_3d_3)}{3T_c}. \tag{4.16}$$

4.3 SOLUTION PROCEDURE

The essential and necessary conditions for $\tilde{\Psi}_3(T_c)$ to be optimum and for the optimal cycle time $\tilde{T}_{c1}$, set $\frac{d\tilde{\Psi}_3(T_c)}{dT_c} = 0$, from equation (15) which gives

$$\tilde{T}_{c1} = \sqrt{\frac{2(a_1+a_2+a_3)+(d_1+d_2+d_3)\,(M_s)^2\,\left(C_cI_c-(p_1+p_2+p_3)I_e\right)}{(d_1+d_2+d_3)\,\left(h(n)+C_c.I_c\right)}} \tag{4.17}$$

and for the optimal cycle time T_{c2}, set $\frac{d\tilde{\Psi}_4(T_c)}{dT_c} = 0$, from equation (16) which gives

$$T_c = \tilde{T}_{c2}(\text{say}) = \sqrt{\frac{2(a_1+a_2+a_3)}{(d_1+d_2+d_3)\,\left(h(n)+(p_1+p_2+p_3)I_e\right)}} \tag{4.18}$$

4.4. Algorithm Process

To find out the maximum cycle length of time (between $\tilde{T}_{C_1}$ and $\tilde{T}_{C_2}$) and whole maximum average cost (between $\tilde{\Psi}_3(\tilde{T}_{c1})$

and $\widetilde{\Psi}_4\left(\widetilde{T}_{c2}\right)$) for various values of M_s ,the subsequent algorithms employed in the form different sequential steps

Stage-1: Find out $\widetilde{T}_{c1}$ and $\widetilde{T}_{c2}$ by resolving equations (17) and (18).

Stage-2: If $\widetilde{T}_{c1} \le M_s$, then calculate $\widetilde{T}c_2$ and $\widetilde{\Psi}_4\left(\widetilde{T}_{c2}\right)$, then move to stage-3.

Stage-3: If $\widetilde{T}_{c1} > M_s$, then calculate $\widetilde{\Psi}_3\left(\widetilde{T}_{c1}\right)$.

Stage-4: Find out the concerning round length of time and whole maximum cost.

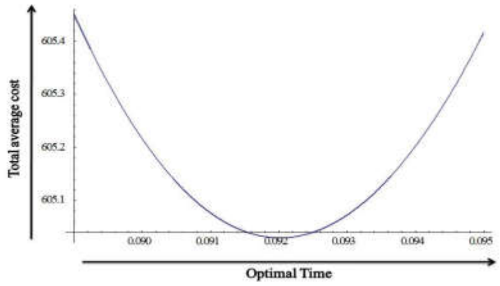

Figure 4.4 Optimized cycle length of time and optimized average cost for case-1

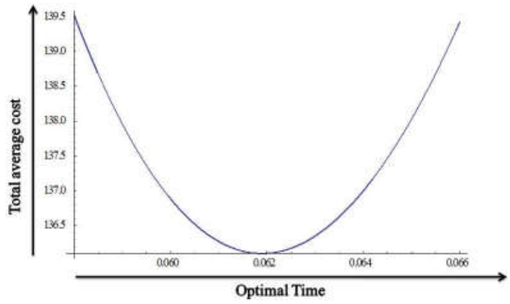

Figure 4.5 Optimized cycle length of time and optimized average cost for case-2

5. MODEL ILLUSTRATED EXAMPLES

Most of the input parameters in this example are obtained from Shah et al. (2012).

Case-1: $M_s \le T_c$

$$\widetilde{A}_c = (48, 50, 52),\ \widetilde{P}_c = (118, 120, 122),\ \tilde{D} = (480, 500, 520),$$

$$M_S = 0.041 year, n = 1,\quad \lambda = 0.10,\ h_1 = \$5/unit/year,\ h_0 = \$4/unit/year, C_c = \$50/unit$$

$I_e = 0.12/year,\ I_p = 0.15/year,\ C_c = \$50/unit,$ After using algorithm, we got fuzzy optimal cycle time, $\widetilde{T}_{c1} = 0.092\ \ year$ and fuzzy whole cost corresponding fuzzy optimal time period is

$$\widetilde{\Psi}_3\left(\widetilde{T}_{c1}\right) = 605.03\,\$$$

Case-2: $M_s > T_c$

$$\widetilde{A}_c = (48, 50, 52),\ \widetilde{P}_c = (118, 120, 122),\ \tilde{D} = (480, 500, 520),$$

$$I_c = 0.15/year,\ I_e = 0.12/year,$$

$$M_S = 0.068 year, n = 1,\quad \lambda = 0.10,\ h_1 = \$5/unit/year,\ h_0 = \$4/unit/year, C_c = \$50/unit$$

After using algorithm, we got fuzzy optimal cycle time, $\widetilde{T}_{c2} = 0.061\ \ year$ and fuzzy total cost corresponding fuzzy optimal cycle time is $\widetilde{\Psi}_4\left(\widetilde{T}_{c2}\right) = 136.097\,\$$. After simplification of Eq. (16) with the help of Mathematica Software 0.8, we got, $\dfrac{d^2\Psi_4\left(\widetilde{T}_{c2}\right)}{dT_{c2}^{\ 2}} = 29 > 0$, where $\widetilde{T}_{c2} = 0.061\ \ year$ .The convexity of the cost function for the retailer's and shown in figure 4.4 and 4.5.

6. SENSITIVITY ANALYSIS

To examine the robustness of both of the models, we performed a sensitivity analysis on the key parameters.

Table2.Variation of maximum cycle length of time and whole average cost for various values of trade-credit period

$\tilde{P}_c$		$M_s = 0.041$	$M_s = 0.049$	$M_s = 0.054$
$(118,120,122)$	$\tilde{T}_{c1}$	**0.0920**	**0.0828**	**0.07500**
	$\tilde{T}_{c2}$	0.0618	0.0618	0.0618
	$\tilde{\Psi}_3(\tilde{T}_{c1})$	**605.03**	**498.34**	**413.27**
	$\tilde{\Psi}_4(\tilde{T}_{c2})$	727.87	550.34	431.98
$(158,160,162)$	$\tilde{T}_{c1}$	**0.0836**	**0.0688**	**0.0548**
	$\tilde{T}_{c2}$	0.0547	0.0547	0.0547
	$\tilde{\Psi}_3(\tilde{T}_{c1})$	**535.79**	**382.87**	**246.74**
	$\tilde{\Psi}_4(\tilde{T}_{c2})$	641.26	404.55	246.75
$(185,210,125)$	$\tilde{T}_{c1}$	**0.0821**	**0.0662**	0.0506
	$\tilde{T}_{c2}$	0.05383	0.05383	**0.0538**
	$\tilde{\Psi}_3(\tilde{T}_{c1})$	**523.57**	**361.26**	212.59
	$\tilde{\Psi}_4(\tilde{T}_{c2})$	624.54	377.96	**213.96**

Table 3. Variation of fuzzy average cost and optimal cycle length with respect to number of shipments under learning effect

Number of shipments, n	Optimal cycle $\tilde{T}_{c1}$	Optimal Cycle $\tilde{T}_{c2}$	Average Cost for the case-1 $\tilde{\Psi}_3(\tilde{T}_{c1})$	Average Cost for the Case-2 $\tilde{\Psi}_4(\tilde{T}_{c2})$
1	0.0920169	0.0618984	605.03	727.87
2	0.0920773	0.0620579	598.84	723.72
3	0.0931997	0.0621467	595.395	721.42
4	0.0934956	0.0622077	593.03	719.84

Table 4. Variation of fuzzy average cost and optimal cycle length with respect to interest gained for case-1 and case-2 under learning effect

Interest earned I_e	Optimal cycle $\widetilde{T}_{c1}$	Optimal Cycle $\widetilde{T}_{c2}$	Average Cost for the case-1 $\widetilde{\Psi}_3\left(\widetilde{T}_{c1}\right)$	Average Cost for the Case2 $\widetilde{\Psi}_4\left(\widetilde{T}_{c2}\right)$
0.12	0.0920169	0.0618984	605.03	727.87
0.14	0.0879212	0.0580259	571.24	687.75
0.16	0.0836252	0.0547997	535.79	641.26
0.18	0.0790963	0.0520579	498.43	589.43

Table 5. Impact of learning factor with variable shipment on cycle length and average cost

Learning factor γ	Optimal Cycle $\widetilde{T}_{c2}$	Average Cost for the Case- $\widetilde{\Psi}_4\left(\widetilde{T}_{c2}\right)$
0.10	0.061	136
0.20	0.062	128
0.30	0.062	118
0.40	0.062	109

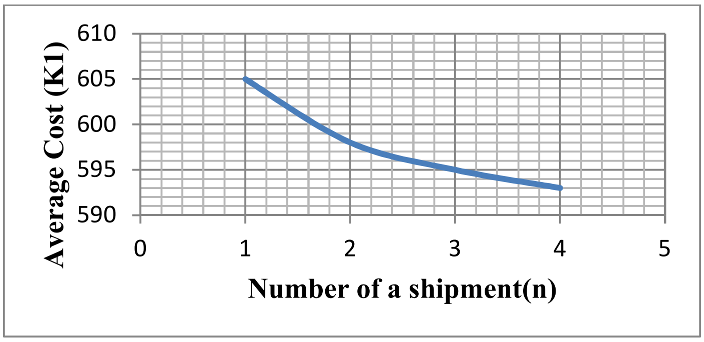

Figure 6.6 Variation of cost with the number of shipments under case-1

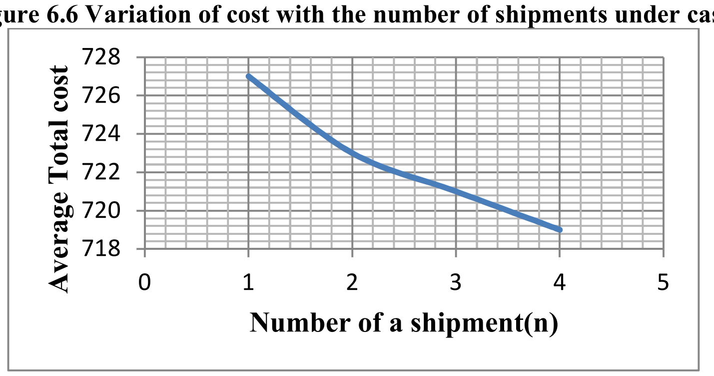

Figure 6.7 Variation of cost with the number of shipments under case-2

Figure 6.8 Variation of cost with interest earned for case-1

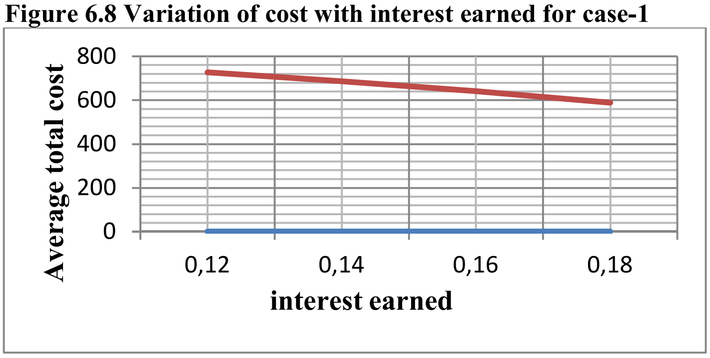

Figure 6.9 Variation of cost with interest earned for case-2

Observations

- From table 2, it can have observed that, when the values of M_s was increasing, the maximum cycle length of period and average cost for the seller will not increase owing to the learning phenomenon. Also, when P_c was increasing, the maximum cycle length of time and maximum value of cost was reducing. It can be analyzed that the seller will not instruction additional quantity to receive the gain of the interruption cash more often
- From table 3, it is analyzed that whenever the number of shipments was increasing, the optimal cycle time was increasing but average cost was decreasing in each shipment due to the learning effect
- From table 4, it can be observed that when the values of interest earned were increasing, the maximum cycle length of time and average cost was decreasing in each shipment owing to the process of learning
- From table 4, it can be observed that when the values of learning rate was increasing the maximum cycle length of time increased and the average cost decreased in each shipment owing to the process of learning
- From figures 6.6 and 6.7, it can be analyzed that whenever the number of shipments was increasing, the average cost was decreasing in each shipment due to the phenomenon of learning.
- From figures 6.8 and 6.9, it can be observed that when the values of interest earned were increasing, the average cost was decreasing per shipment due to the process of learning.

As per observation, the value of average cost with various parameters reduces owing to the effect of learning. It is also analyzed from the observations, which impact of learning as a direct impact over the calculation of values of average cost and minimizing the cost of inventory system.

7. MANAGERIAL IMPLICATION AND APPLICABILITY OF THE MODEL

The concept of this paper has a broad area of research covering operations research especially which is based on the human element, or man's ability to learn and improve and the elimination of non-productive activities through repetition termed as learning with co-ordination policy trade credit financing between buyer and seller. Now, it emerges its area with collaboration other business-related concepts like autorotation, flexible inventory cost, imprecise nature in inventory parameters and many more. Present paper covered as well as reported both the mathematical and management areas and designed preventive inventory cost maintenance employing and applying effect of learning. This combination of work has been done in this article for the industrial sector which is the contribution by the authors. In general, some industries have some constraints to apply machinery system instead of human labor. This area needs more research to find out the solution with profit maximization or cost minimization.

8. CONCLUSION

In the present paper, an inventory model has been improved with the effect of learning in fuzzy environment under trade financing policy. As per consideration holding cost obeys the impact of learning with the number of shipments termed as learning and demand, ordering cost as well as selling price are imprecise in nature which acts as major role in the inventory system. For any business organization some inventory parameters should be in the suitable form for future selling as well as gets more profit. In business field, seller and buyer want to get more profit and do more exercise. Seller wants to increase his/her sale and for this action, they used some policy like credit financing and to know about demand, how much cost should be applicable for buyer to balance the situation because considered parameters are imprecise in nature which controlled by fuzzy concept and it is beneficial for both players that have shown in this paper to get more profit. Cycle length, holding and purchase cost are the major factor for buyer and to minimize by such type of factor for more profit to eliminate non-productive activities through learning as have shown with the help of sensitive analysis. From the sensitive analysis, the situation exposed that if the number of shipment increases then total cost decreases and cycle length increases because holding cost is decrease with the number of shipment due to learning effect under the fuzzy environment with fixed credit period. Finally, form the numerical example some more information's like, how much should be fuzzy cost, what should be the suitable range of inventory cost as well as credit period provided by seller for his buyer for the good coordination and number of shipments for buyer. This paper allows scope for extension for further truthful circumstances such as decaying things, store addictions and stochastic strains with permissible delay in payments.

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