

A DETERMINISTIC VENDOR MANAGED INVENTORY MODEL UNDER TWO PAYMENT POLICIES: CASH ON DELIVERY AND CASH BEFORE DELIVERY

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ABSTRACT

Vendor managed inventory (VMI) is a concept of supply chain in which a vendor (supplier or manufacturer) has the charge for optimizing the inventory held by a buyer (or retailer). In traditional VMI modelling, it is often assumed that the vendor offers cash on delivery payment terms to the buyer. But in real world the vendors usually offer different types of payment policies to promote their commodities and influence the buyers ordering policies. Some of the common payment policies are cash on delivery, cash before delivery (advance payment), delayed payment (trade credit) etc. In the present paper mathematical model is developed for VMI system by comparing two payment policies between vendor and buyer. In policy (I) buyer pays for an item as soon as he receives it from the vendor, i.e. **CASH ON DELIVERY** and in policy (II) buyer pays for an item at the time of placing an order, i.e. **CASH BEFORE DELIVERY**. Discounted cash flow (DCF) approach has been used for proper recognition of cash flows. Numerical illustrations and sensitivity analysis are performed between two payment policies and it is shown that VMI system is beneficial for both vendor and buyer under both payment policies and policy II is better than policy I for both VMI system and non VMI system.

KEYWORDS: Vendor managed inventory (VMI), Discounted cash flow (DCF) approach, payment policies, Cash on delivery (COD), Cash before delivery (CBD)

MSC:90B50

RESUMEN

El inventario administrado por el proveedor (VMI) es un concepto de cadena de suministro en el que un proveedor (proveedor o fabricante) tiene el cargo de optimizar el inventario que posee un comprador (o minorista). En el modelado tradicional de VMI, a menudo se supone que el vendedor ofrece condiciones de pago contra reembolso al comprador. Pero en el mundo real, los proveedores suelen ofrecer diferentes tipos de políticas de pago para promover sus productos e influir en las políticas de pedidos de los compradores. Algunas de las políticas de pago comunes son contra reembolso, efectivo antes de la entrega (anticipo), pago retrasado (crédito comercial), etc. En el presente documento, se desarrolla un modelo matemático para el sistema VMI comparando dos políticas de pago entre el proveedor y el comprador. En la política (I), el comprador paga los artículos tan pronto como los recibe del proveedor, es decir, EFECTIVO ENTREGA y en la política (II) el comprador paga un artículo en el momento de realizar un pedido, es decir, EFECTIVO ANTES DE LA ENTREGA. El enfoque de flujo de efectivo descontado (DCF) se ha utilizado para el reconocimiento adecuado de los flujos de efectivo. Las ilustraciones numéricas y el análisis de sensibilidad se realizan entre dos políticas de pago y se muestra que el sistema VMI es beneficioso tanto para el vendedor como para el comprador tanto en las políticas de pago como en la política II es mejor que la política I tanto para el sistema VMI como para el sistema no VMI.

PALABRAS CLAVE: inventario gestionado por el proveedor (VMI), enfoque de flujo de caja descontado (DCF), políticas de pago, pago contra reembolso (COD), efectivo antes de la entrega (CBD)

1. INTRODUCTION

Vendor Managed Inventory (VMI) is widely accepted tactics for inventory management in the retail sector. VMI is an approach to optimize supply chain performance in which vendor (or manufacturer) takes the charge of maintaining the buyers inventory levels and can acquire the buyer's inventory data to create the purchase orders. The vendor manages the buyer's inventory levels by supplying the inventory in smaller batches, more frequently based on the information accessed from the buyers.

VMI concept was initiated in the late 1980s, by a successful collaboration of P&G with Wal-Mart. After that VMI was adopted as one of the leading management strategy by several organizations like Barilla,

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Johnson and Johnson, what's more, Kodak Canada Inc., Whitbread Beer Company, have successfully implemented VMI initiatives [7].

Over the years many researchers have developed mathematical models to study the behaviour of VMI systems. In one of the earliest attempts to mathematically model a VMI systems, Lee et.al[2] developed ordering and delivery scheduling model for integrating inventory and transportation decisions. Some of the notable research in the area of VMI has been presented by Cachon and Fisher [1], Chaouch [3], Fry et.al. [12], Dong and Xu [11], Disney et.al. [10] etc.

In 2007, Yao et.al. [24] discussed how key parameters of inventory i.e. ordering cost and inventory carrying costs affects the profit gained from VMI and how it will be distributed among vendor and buyer in supply chain by developing mathematical model. Extending the work of Yao et.al.[24], Sadeghi et.al. [21] developed analytical model for single-vendor, multi-retailer and single warehouse under VMI system with respect to order and budgetary constraints.

In almost all of the traditional VMI models, it is presumed that payment for stock of items is made at the time of delivery. But it is a common knowledge that in real world, many different payment policies are prevalent between vendor and buyer like cash on delivery, cash before delivery (advance payment) and delayed payment (trade credit). Often these payment terms are negotiated between vendor and buyer based on mutual benefits.

To make VMI modelling more realistic, it is imperative to study the impact of various payment policies made available to the buyer by the vendor. One of the major concerns for the researchers to compare and model the impact of payment policies on the VMI decisions is the different timings of cash flows in the system. DCF analysis is one of the most fundamental and omnipresent theory in finance. It allows an unambiguous identification of timing of each cash flow in the system and considers the time value of money. It is broadly acknowledged that the net present worth or DCF approach prompts superior decision making (Kim and Feist [16]).

Some of the important investigations in the field of inventory management with the incorporation of VMI system, payment policies and DCF approach is presented in Table 1.

Table 1:Review of related literature

PAPER	DCF APPROACH	VMI SYSTEM	PAYMENT POLICY
Chung(1989)	✓	✗	Delayed payment
Chung and Lin (2001)	✓	✗	✗
Lee et.al (2000)	✗	✓	✗
Cachon and Fisher (2000)	✗	✓	✗
Chaouch (2001)	✗	✓	✗
Fry et al. (2001)	✗	✓	✗
Dong and Xu (2002)	✗	✓	✗
Disney et.al. (2003)	✗	✓	✗
Soni et.al.(2006)	✓	✗	Progressive payment scheme
Jaggi et.al.(2006)	✓	✗	✗
Yao et.al.(2007)	✗	✓	✗
Chung and Liao (2009)	✓	✗	Delayed payment
Darwish and Odah (2010)	✗	✓	✗
Daya et.al. (2012)	✗	✓	✗
Yu et.al.(2012)	✗	✓	✗
Hariga et.al.(2013)	✗	✓	✗
Zhang(2014)	✓	✗	Delayed payment, advance payment
Mateen and Chatterjee(2015)	✗	✓	✗
Taleizadeh et.al.(2015)	✗	✓	✗
Sajadieh et.al.(2017)	✗	✓	✗
Rabbani et.al. (2018)	✗	✓	✗
Parsa et.al. (2016)	✓(TVM)	✓	✗
Li et.al.(2017)	✓	✗	Advance cash credit payment
Present paper	✓	✓	Cash on delivery and cash before delivery

As it can be seen from the Table 1, none of the authors have considered the impact of payment policies in VMI system. Parsa et.al [19] were the first authors who introduces time value of money (TVM) into VMI system.

In this study, we are proposing VMI model for single buyer single vendor by considering DCF approach to understand the impact of payment policies. We have discussed two payment policies between vendor and buyer:

Policy I: Buyer pays for items as he receives it from the vendor, i.e. *cash on delivery*. In this policy order received from the buyer will be split and delivered in multiple small batches during the inventory cycle. Buyer will make payment to the vendor for the items in an order in multiple batches after receiving the consignment.

Policy II: Buyer pays for all the items included in purchase order (for all the batches) at the time of placing the order, i.e. *cash before delivery*. Here, in policy II buyer will pay in the beginning of the cycle.

As the timing of cash flow is different in both the policies, DCF approach is used to evaluate the policies. Mathematical model is developed for VMI system considering total profit as an objective function for both the policies.

In the paper section 2 includes model notations and section 3 describes assumptions applied to construct the mathematical model which is further elaborated in section 4, mathematical model. Then, in section 5 and 6, optimality and solution method of the mathematical model is discussed respectively. Section 7 presents the numerical examples which is used to illustrate the model and then in section 8 sensitivity analysis is done on key parameters then results are discussed. Section 9 describes the applicability of the model along with managerial implications. Finally, section 10 presents conclusion of the study.

2. NOTATIONS

A_B	Buyer's ordering cost per order
A_v	Vendor's ordering cost per order
C_B	Buyer's unit purchase cost
C_v	Vendor's unit purchase cost
h_B	Buyer's unit holding cost per unit per unit time
h_v	Vendor's unit holding cost per unit per unit time
r	The discount rate representing the time value of money
n	Number of replenishment frequency
q_b	Buyer's order quantity
Q	Vendor's order quantity
T	Buyer's replenishment cycle
T'	Vendor's replenishment cycle which is equal to VMI system cycle length
λ^o	Demand rate in units per unit time
P	Buyer's selling price per unit of item
PVOC	Present worth of ordering cost
PVPC	Present worth of purchasing cost
PC_{v1}	PVPC of vendor in policy I
PC_{v2}	PVPC of vendor in policy II
PC_{B1}	PVPC of buyer in policy I
PC_{B2}	PVPC of buyer in policy II
PVHC	Present worth of inventory holding cost
$I_{v1}(t)$	PVHC of vendor in policy I
$I_{v2}(t)$	PVHC of vendor in policy II
$I_{B1}(t)$	PVHC of buyer in policy I
$I_{B2}(t)$	PVHC of buyer in policy II
PVGR	Present worth of gross revenue
GR_{v1}	PVGR of vendor in policy I
GR_{v2}	PVGR of vendor in policy II
GR_{B1}	PVGR of buyer in policy I

GR_{B2}	PVGR of buyer in policy II
PVTC	Present worth of total cost
$TC_{v1}(n, q_b)$	PVTC of vendor in policy I
$TC_{v2}(n, q_b)$	PVTC of vendor in policy II
$TC_{B1}(n, q_b)$	PVTC of buyer in policy I
$TC_{B2}(n, q_b)$	PVTC of buyer in policy II
$TC_1(n, q_b)$	PVTC in policy I
$TC_2(n, q_b)$	PVTC in policy II
PVTP	Present worth of total profit
$TP_{v1}(n, q_b)$	PVTP of vendor in policy I
$TP_{v2}(n, q_b)$	PVTP of vendor in policy II
$TP_{B1}(n, q_b)$	PVTP of buyer in policy I
$TP_{B2}(n, q_b)$	PVTP of buyer in policy II
$TP_1(n, q_b)$	PVTP in policy I
$TP_2(n, q_b)$	PVTP in policy II

3. ASSUMPTIONS

1. Demand rate of the buyer is uniform, deterministic and constant
2. Lead time is negligible.
3. Replenishment rate of buyer is infinite
4. Stockouts are not permitted.
5. DCF approach is applied to properly account the cost components at different times, r is discount rate. Continuous compounding of money is assumed.
6. Planning horizon is infinite.
7. The vendor splits and delivers the order, received from the buyer in a cycle into n small shipments of size q_b such that $(Q = nq_b)$ after every time interval of T .
8. The cycle length T' has been separated into n replenishment equivalent cycles of length T , such that $T' = nT$.

4. MATHEMATICAL MODEL

In the proposed VMI systems, at the start of each cycle vendor will receive an order of the size Q from the buyer. The vendor divides and delivers the each ordered received from the buyer in a cycle into n small batches of size q_b , (so that $Q = nq_b$) after every time interval of T . The cycle length T' has been split into n replenishment equal cycles of length T , such that $T = \frac{T'}{n}$. Consider the j^{th} replenishment cycle of buyer, i.e. $T_{j-1} \leq T \leq T_j$, where $T_0 = 0, T_n = T', T_j - T_{j-1} = T$ and $T_j = jT$ ($j = 1, 2, 3, \dots, n$).

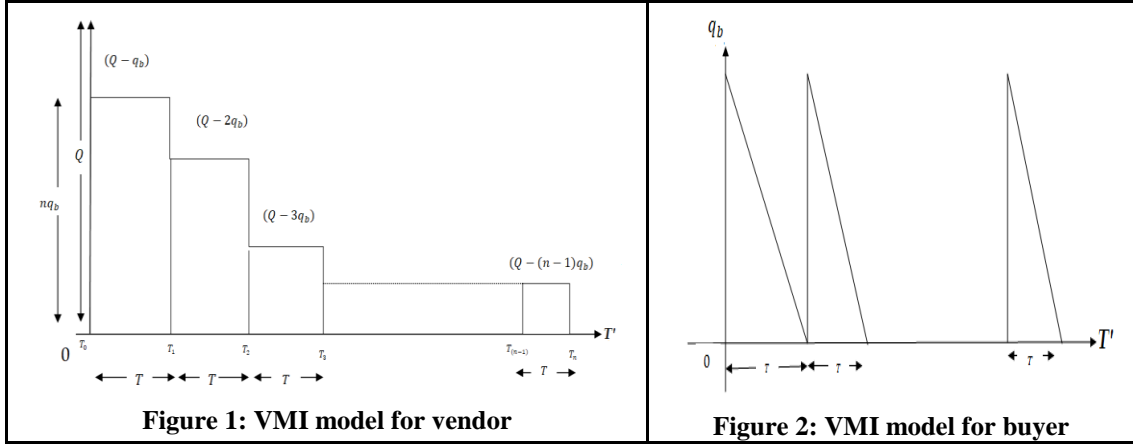
Figure 1 and Figure 2 depicts vendor's and buyer's inventory levels in a cycle respectively. As appeared in Figure 1 at the start of the cycle (T'), vendor has inventory level of Q ($Q = nq_b$) units, which is delivered to the buyer in batch size of q_b units after replenishment cycle T . Thus after every T period, the inventory level of vendor decreases by q_b units until it becomes zero. After that vendor replenishes its inventory level upto Q units again. Figure 2 represents the buyer's inventory levels. At the start of each cycle (T') the buyer has requirement of Q items, which he receives in n batches of size q_b each. As soon as buyer's inventory levels reaches zero, vendor supplies q units immediately to the buyer, same pattern will be followed in the entire planning horizon. The aim of the profit maximizing model is to determine the impact of payment terms between vendor and buyer on the replenishment policies.

In the following part expressions for present worth of buyer's, vendor's and system profit are obtained under two payment policies.

Policy I: Cash on delivery (Buyer pays for an item as soon as they receives items in a batch)

Here, in this policy buyer will make payment at the time of receiving each batch of size q_b units. From Figure 1 and Figure 2, the buyer will place an order of size Q to the vendor and will receive the first

batch at T_0 and make the payment immediately for q_b units. Subsequent payments will be made at times $T_j = jT$ ($j = 1, 2, 3, \dots, n$).



Vendor's profit

The PVPC of vendor for the first cycle is,

$$PC_{v1} = C_v Q \tag{1}$$

The PVOC of vendor for the first cycle is,

$$A_{v1} = A_v \tag{2}$$

The PVHC of vendor for the first cycle is,

$$I_{v1}(t) = h_v C_v \left[\int_0^T (Q - q_b) e^{-rt} dt + e^{-rT} \int_0^T (Q - q_b) e^{-rt} dt + e^{-2rT} \int_0^T (Q - q_b) e^{-rt} dt + \dots \dots \dots + e^{-nrT} \int_0^T (Q - (n-1)q_b) e^{-rt} dt \right] \tag{3}$$

After solving the equation (3), we get

$$I_{v1}(t) = h_v C_v \left[\left(\frac{(1 - e^{-rT})(Q - q_b)}{r} \right) \left(\frac{1 - e^{-nrT}}{1 - e^{-rT}} \right) \right] \tag{4}$$

The PVTC of vendor for the first cycle is,

$$TC_{v1} = PC_{v1} + A_{v1} + I_{v1}(t) \tag{5}$$

The PVTC of vendor for the i^{th} cycle is,

$$TC_{v1i} = e^{-irT'} TC_{v1} = e^{-irnT} TC_{v1}$$

The PVTC of vendor for entire planning horizon is,

$$TC_{v1} = \sum_{i=0}^{\infty} TC_{v1i} = TC_{v1} \left[\frac{1}{(1 - e^{-nrT})} \right] \tag{6}$$

Substituting the values of equation (1), (2), (4) in (6) and after simplification, we get

$$TC_{v1}(n, q_b) = \left\{ C_v n q_b + A_v + h_v C_v \left[\left(\frac{(1 - e^{-mq_b/\lambda^o})(nq_b - q_b)}{r} \right) \left(\frac{1 - e^{-rnmq_b/\lambda^o}}{1 - e^{-mq_b/\lambda^o}} \right) \right] \right\} \left[\frac{1}{(1 - e^{-rnmq_b/\lambda^o})} \right] \tag{7}$$

The PVGR of vendor from the sales for first cycle is,

$$GR_{v1} = C_B q_b + C_B q_b e^{-rT} + C_B q_b e^{-2rT} + \dots + C_B q_b e^{-(n-1)rT}$$

The PVGR of vendor for the i^{th} cycle is,

$$GR_{v1i} = e^{-irT'} GR_{v1} = e^{-irnT} GR_{v1}$$

The PVGR of vendor for the entire planning horizon is,

$$= \sum_{i=0}^{\infty} GR_{v1i} = C_B q_b \frac{1}{[1-e^{-rT}]} = C_B q_b \left[\frac{1}{1-e^{-r q_b / \lambda^o}} \right] \quad (8)$$

Now, the PVTP of vendor for entire planning horizon is given by,

$$\text{Total Profit} = \text{Gross revenue} - \text{Total cost of the vendor for the } j^{th} \text{ cycle}$$

The PVTP of vendor is,

$$TP_{v1}(n, q_b) = C_B q_b \left[\frac{1}{1-e^{-r q_b / \lambda^o}} \right] - \left\{ C_v n q_b + A_v + h_v C_v \left[\frac{(1-e^{-m q_b / \lambda^o})(n q_b - q_b)}{r} \right] \times \left[\frac{1-e^{-m n q_b / \lambda^o}}{1-e^{-m q_b / \lambda^o}} \right] \right\} \left[\frac{1}{1-e^{-m n q_b / \lambda^o}} \right] \quad (9)$$

Buyer's profit

The PVPC of buyer for the first cycle is,

$$PC_{B1} = \sum_{j=1}^n C_B q e^{-rT_j} = C_B \left(\frac{1-e^{-mT}}{1-e^{-rT}} \right) \quad (10)$$

The PVOC of buyer for the first cycle is,

$$A_{B1} = \sum_{j=1}^n A_B q_b e^{-rT_j} = A_B \left(\frac{1-e^{-mT}}{1-e^{-rT}} \right) \quad (11)$$

The PVHC of buyer for the first cycle is,

$$I_{B1}(t) = h_B C_B \left[\int_0^T (q_b - \lambda^o t) e^{-rt} dt + e^{-rT} \int_0^T (q_b - \lambda^o t) e^{-rt} dt + e^{-2rT} \int_0^T (q_b - \lambda^o t) e^{-rt} dt + \dots + e^{-nrT} \int_0^T (q_b - \lambda^o t) e^{-rt} dt \right] \quad (12)$$

After solving the equation (12), we get

$$I_{B1}(t) = \left\{ \frac{h_B C_B}{r} \left[Q(1-e^{-rT}) + T e^{-rT} + \frac{\lambda^o}{r} (1-e^{-rT}) \right] \right\} \left[\frac{1-e^{-nrT}}{1-e^{-rT}} \right] \quad (13)$$

The PVTC of buyer for the first cycle is,

$$TC_{B1} = PC_{B1} + A_{B1} + I_{B1}(t) \quad (14)$$

The PVTC of buyer for the i^{th} cycle is,

$$TC_{B1i} = e^{-irT'} TC_{B1} = e^{-irnT} TC_{B1}$$

The PVTC of buyer for entire planning horizon is,

$$TC_{B1} = \sum_{i=0}^{\infty} (TC_{B1})_i = TC_{B1} \left[\frac{1}{1-e^{-mT}} \right] \quad (15)$$

Substituting the values of equation (10), (11) and (13) in (15) and after simplification, we get

$$TC_{B1}(n, q_b) = \left\{ \begin{aligned} & C_B q_b \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda}} \right) + A_B \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda}} \right) \\ & + \left(\frac{h_B C_B}{r} \left[nq_b (1 - e^{-rq_b/\lambda^o}) + \frac{q_b}{\lambda^o} e^{-rq_b/\lambda^o} + \frac{\lambda^o}{r} (1 - e^{-rq_b/\lambda^o}) \right] \right) \end{aligned} \right\} \times \left[\frac{1}{1 - e^{-mq_b/\lambda^o}} \right] \\ \times \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right) \quad (16)$$

The PVGR of buyer from the sales for first cycle is,

$$GR_{B1} = p \int_0^T \lambda^o e^{-rt} dt + e^{-rT} p \int_0^T \lambda^o e^{-rt} dt + \dots + e^{-(n-1)rT} p \int_0^T \lambda^o e^{-rt} dt$$

The PVGR of buyer for the i^{th} cycle is,

$$GR_{B1i} = e^{-irT} GR_{B1} = e^{-irnT} GR_{B1}$$

The PVGR of buyer for entire planning horizon is,

$$= \sum_{i=0}^{\infty} GR_{B1i} = \frac{p\lambda^o}{r}$$

Now, the PVTP of buyer's the j^{th} replenishment cycle is given by,

$$\text{Total Profit} = \text{Gross revenue} - \text{Total cost of the buyer for the } j^{th} \text{ cycle}$$

The PVTP of the buyer is,

$$TP_{B1}(n, q_b) = \frac{p\lambda^o}{r} - \left\{ \begin{aligned} & C_B q_b \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right) + A_B \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right) \\ & + \left(\frac{h_B C_B}{r} \left[nq_b (1 - e^{-rq_b/\lambda^o}) + \frac{q_b}{\lambda^o} e^{-rq_b/\lambda^o} + \frac{\lambda^o}{r} (1 - e^{-rq_b/\lambda^o}) \right] \right) \end{aligned} \right\} \times \left[\frac{1}{1 - e^{-mq_b/\lambda^o}} \right] \quad (17) \\ \times \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right)$$

Now, the PVTC of VMI system for policy I is

$$TC_1(n, q_b) = \left\{ \begin{aligned} & \left[C_v nq_b + A_v + h_v C_v \times \left[\left(\frac{(1 - e^{-mq_b/\lambda^o})(nq_b - q_b)}{r} \right) \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right) \right] \right] \left[\frac{1}{1 - e^{-mq_b/\lambda^o}} \right] \right\} \\ & + \left\{ \begin{aligned} & C_B q_b \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right) + A_B \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right) \\ & + \left(\frac{h_B C_B}{r} \left[nq_b (1 - e^{-rq_b/\lambda^o}) + \frac{q_b}{\lambda^o} e^{-rq_b/\lambda^o} + \frac{\lambda^o}{r} (1 - e^{-rq_b/\lambda^o}) \right] \right) \left(\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right) \end{aligned} \right\} \left[\frac{1}{1 - e^{-mq_b/\lambda^o}} \right] \end{aligned} \right\} \quad (18)$$

The PVTP for policy I is,

$$TP_1(n, q_b) = \left\{ \left[\left[\frac{qC_B}{1 - e^{-r q_b / \lambda^o}} \right] - \left[C_v n q_b + A_v + h_v C_v \times \left[\left(\frac{(1 - e^{-m q_b / \lambda^o})(n q_b - q_b)}{r} \right) \left(\frac{1 - e^{-n r q_b / \lambda^o}}{1 - e^{-r q_b / \lambda^o}} \right) \right] \right] \left[\frac{1}{1 - e^{-r m n q_b / \lambda^o}} \right] \right\} + \left\{ \left[\frac{p \lambda^o}{r} \right] - \left[C_B q_b \left(\frac{1 - e^{-n r q_b / \lambda^o}}{1 - e^{-r q_b / \lambda^o}} \right) + A_B \left(\frac{1 - e^{-n r q_b / \lambda^o}}{1 - e^{-r q_b / \lambda^o}} \right) \right] \left[\frac{1}{1 - e^{-r q_b / \lambda^o}} \right] \right\} + \left\{ \left[\frac{h_B C_B}{r} \left[n q_b (1 - e^{-r q_b / \lambda^o}) + \frac{q_b}{\lambda^o} e^{-r q_b / \lambda^o} + \frac{\lambda^o}{r} (1 - e^{-r q_b / \lambda^o}) \right] \right] \left(\frac{1 - e^{-n r q_b / \lambda^o}}{1 - e^{-r q_b / \lambda^o}} \right) \right\} \left[\frac{1}{1 - e^{-r q_b / \lambda^o}} \right] \right\} \quad (19)$$

The optimization problem in this VMI system is,

$$\text{Max}(TP_1(n, q_b))$$

subject to $q_b \geq 0$

Policy II: Cash before delivery (buyer will pay at the time of placing an order)

In this policy, buyer will make payment of lot size Q at the time of placing an order i.e. at the beginning of the cycle. From Figure 1 and Figure 2, the buyer will place an order of size Q to the vendor at T_0 and will receive the first batch of q_b units at T_0 .

Vendor's profit

The PVPC of vendor for the first cycle is,

$$PC_{v2} = C_v Q \quad (20)$$

The PVOC of vendor for the first cycle is,

$$A_{v2} = A_v \quad (21)$$

The PVHC of vendor for the first cycle is,

$$I_{v2}(t) = h_v C_v \left[\int_0^T (Q - q_b) e^{-rt} dt + e^{-rT} \int_0^T (Q - q_b) e^{-rt} dt + e^{-2rT} \int_0^T (Q - q_b) e^{-rt} dt + \dots \dots \dots + e^{-nrT} \int_0^T (Q - (n-1)q_b) e^{-rt} dt \right] \quad (22)$$

After solving the equation (22), we get

$$I_{v2}(t) = h_v C_v \left[\left(\frac{(1 - e^{-rT})(Q - q_b)}{r} \right) \left(\frac{1 - e^{-nrT}}{1 - e^{-rT}} \right) \right] \quad (23)$$

The PVTC of vendor for the first cycle is,

$$TC_{v2} = PC_{v2} + A_{v2} + I_{v2}(t) \quad (24)$$

The PVTC of vendor for the i^{th} cycle is,

$$TC_{v2i} = e^{-irT} TC_{v2} = e^{-irnT} TC_{v2}$$

The PVTC of vendor of the VMI system, for the entire planning horizon is given by,

$$TC_{v2} = \sum_{i=0}^{\infty} TC_{v2i} = TC_{v2} \left[\frac{1}{(1 - e^{-nrT})} \right] \quad (25)$$

Substituting the values of equation (20), (21) and (23) in (25) and after simplification, we get

$$TC_{v2}(n, q_b) = \left\{ C_v n q_b + A_v + h_v C_v \left[\left(\frac{(1 - e^{-m q_b / \lambda^o})(n q_b - q_b)}{r} \right) \left(\frac{1 - e^{-m n q_b / \lambda^o}}{1 - e^{-m q_b / \lambda^o}} \right) \right] \right\} \left[\frac{1}{(1 - e^{-m n q_b / \lambda^o})} \right] \quad (26)$$

The PVGR of vendor from the sales is,

$$GR_{v_2} = nq_b C_B + nq_b C_B e^{-rT} + nq_b C_B e^{-2rT} + \dots$$

$$= \frac{nq_b C_B}{(1 - e^{-rT})} = \frac{nq_b C_B}{(1 - e^{-mq_b/\lambda^o})} \quad (27)$$

Now, the PVTP of the vendor's for j^{th} replenishment cycle is given by,

Total Profit = Gross revenue – Total cost of the vendor for the j^{th} cycle

The PVTP of the vendor is,

$$TP_{v_2}(n, q_b) = \frac{nq_b C_B}{(1 - e^{-mq_b/\lambda^o})} - \left\{ C_v nq_b + A_v + h_v C_v \left[\frac{(1 - e^{-mq_b/\lambda^o})(nq_b - q_b)}{r} \right] \times \left[\frac{1 - e^{-nmq_b/\lambda^o}}{1 - e^{-mq_b/\lambda^o}} \right] \right\} \left[\frac{1}{(1 - e^{-nmq_b/\lambda^o})} \right] \quad (28)$$

Buyer's profit

The PVPC of buyer for the first cycle is,

$$PC_{B2} = C_B Q \quad (29)$$

The PVOC of buyer for the first cycle is,

$$A_{B2} = \sum_{j=1}^n A_B e^{-rT_j} = A_B \quad (30)$$

The PVHC of buyer for the first cycle is,

$$I_{B2}(t) = I_{B1}(t) = h_B C_B \left[\int_0^T (q_b - \lambda^o t) e^{-rt} dt + e^{-rT} \int_0^T (q_b - \lambda^o t) e^{-rt} dt + e^{-2rT} \int_0^T (q_b - \lambda^o t) e^{-rt} dt + \dots + e^{-nrT} \int_0^T (q_b - \lambda^o t) e^{-rt} dt \right] \quad (31)$$

After solving the above equation (31), we get

$$I_{B2}(n, q_b) = \left\{ \frac{h_B C_B}{r} \left[nq_b (1 - e^{-rq_b/\lambda^o}) + \frac{q_b}{\lambda^o} e^{-rq_b/\lambda^o} + \frac{\lambda^o}{r} (1 - e^{-rq_b/\lambda^o}) \right] \right\} \left[\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right] \quad (32)$$

The PVTC of buyer for the first cycle is,

$$TC_{B2} = PC_{B2} + A_{B2} + I_{B2}(n, q_b) \quad (33)$$

The PVTC of buyer for the i^{th} cycle is,

$$TC_{B2i} = e^{-irT} TC_{B2} = e^{-irnT} TC_{B2}$$

The PVTC of buyer for entire planning horizon is given by,

$$TC_{B2} = \sum_{i=0}^{\infty} (TC_{B2})_i = TC_{B2} \left[\frac{1}{(1 - e^{-nrT})} \right] \quad (34)$$

Substituting the values of equation (29), (30) and (32) in (34) and after simplification, we get

$$TC_{B2}(n, q_b) = \left\{ C_B nq_b + A_B + \left[\frac{h_B C_B}{r} \left[nq_b (1 - e^{-rq_b/\lambda^o}) + \frac{q_b}{\lambda^o} e^{-rq_b/\lambda^o} + \frac{\lambda^o}{r} (1 - e^{-rq_b/\lambda^o}) \right] \right] \right\} \left[\frac{1 - e^{-nrq_b/\lambda^o}}{1 - e^{-rq_b/\lambda^o}} \right] \left[\frac{1}{1 - e^{-nrq_b/\lambda^o}} \right] \quad (35)$$

The PVGR of buyer from the sales for first cycle is,

$$GR_{B2} = p \int_0^T \lambda^o e^{-rt} dt + e^{-rT} p \int_0^T \lambda^o e^{-rt} dt + \dots + e^{-(n-1)rT} p \int_0^T \lambda^o e^{-rt} dt$$

The PVGR of buyer for the i^{th} cycle is,

$$GR_{B2i} = e^{-irT'} GR_{B2} = e^{-irnT} GR_{B2}$$

The PVGR of buyer for entire planning horizon is,

$$= \sum_{i=0}^{\infty} GR_{B2i} = \frac{p\lambda^o}{r}$$

Now, the PVTP of buyer for the j^{th} replenishment cycle is given by,

Total Profit = Gross revenue – Total cost of the buyer for the j^{th} cycle

The PVTC of the buyer is,

$$TP_{B2}(n, q_b) = \frac{p\lambda^o}{r} \left\{ C_B n q_b + A_B + \left[\frac{h_B C_B}{r} \left[n q_b (1 - e^{-r q_b / \lambda^o}) + \frac{q_b}{\lambda^o} e^{-r q_b / \lambda^o} + \frac{\lambda^o}{r} (1 - e^{-r q_b / \lambda^o}) \right] \right] \left(\frac{1 - e^{-nr q_b / \lambda^o}}{1 - e^{-r q_b / \lambda^o}} \right) \right\} \left[\frac{1}{1 - e^{-rn q_b / \lambda^o}} \right] \quad (36)$$

Now, the PVTC for policy II is

$$TC_2(n, q_b) = \left\{ \left[C_v n q_b + A_v + h_v C_v \left[\frac{(1 - e^{-m q_b / \lambda^o})(n q_b - q_b)}{r} \right] \right] \left[\frac{1}{1 - e^{-rn q_b / \lambda^o}} \right] \right. \\ \left. + \left[C_B n q_b + A_B + \frac{h_B C_B}{r} \left[n q_b (1 - e^{-r q_b / \lambda^o}) + \frac{q_b}{\lambda^o} e^{-r q_b / \lambda^o} + \frac{\lambda^o}{r} (1 - e^{-r q_b / \lambda^o}) \right] \right] \left(\frac{1 - e^{-nr q_b / \lambda^o}}{1 - e^{-r q_b / \lambda^o}} \right) \right\} \left[\frac{1}{1 - e^{-rn q_b / \lambda^o}} \right] \quad (37)$$

The PVTP for policy II is,

$$TP_2(n, q_b) = \left\{ \left[\frac{n q_b C_B}{1 - e^{-nr q_b / \lambda^o}} \right] - \left[C_v n q_b + A_v + h_v C_v \left[\frac{(1 - e^{-m q_b / \lambda^o})(n q_b - q_b)}{r} \right] \right] \left[\frac{1}{1 - e^{-rn q_b / \lambda^o}} \right] \right. \\ \left. + \frac{p\lambda^o}{r} \left[C_B n q_b + A_B + \frac{h_B C_B}{r} \left[n q_b (1 - e^{-r q_b / \lambda^o}) + \frac{q_b}{\lambda^o} e^{-r q_b / \lambda^o} + \frac{\lambda^o}{r} (1 - e^{-r q_b / \lambda^o}) \right] \right] \left(\frac{1 - e^{-nr q_b / \lambda^o}}{1 - e^{-r q_b / \lambda^o}} \right) \right\} \left[\frac{1}{1 - e^{-rn q_b / \lambda^o}} \right] \quad (38)$$

The optimization problem in this VMI system is,

$Max(TP_2(n, q_b))$

subject to $q \geq 0$

5. OPTIMALITY

The total profit ($TP_1(n, q_b)$) and ($TP_2(n, q_b)$) contain discrete variable (n) as well as continuous variable (q_b) thus making them non-differentiable. Hence, it is not possible to prove the optimality through Hessian Matrix. In such situations the usual optimality criterion (Jaggi et al.[15]) is to first drop

the discrete conditions, and recursively check for the optimality conditions w.r.t. continuous variable for different values of discrete variable. After dropping the discrete conditions, the optimal value of total profit ($TP_1(n, q_b)$) and ($TP_2(n, q_b)$) can be found by using the necessary and sufficient condition of optimality i.e.

$$\frac{\partial TP_1(n, q_b)}{\partial q_b} = 0, \frac{\partial TP_2(n, q_b)}{\partial q_b} = 0 \text{ and } \frac{\partial TP_1(n, q_b)}{\partial q_b} < 0, \frac{\partial TP_2(n, q_b)}{\partial q_b} < 0 \text{ respectively.}$$

This VMI model aims to ascertain the optimal values of q_b and n that maximize the total profit function. Due to highly non-linear nature of equation (19) and (38), it is not possible to prove optimality condition of total profit function analytically even after dropping the discrete conditions. Hence, the optimality of profit function has been checked graphically for different values of discrete variable with the help of MS-EXCEL (Figure 3 and Figure 4).

6. SOLUTION METHOD

The procedure recommended by Jaggi et.al.,2006 is adopted to ascertain the optimal values of q_b and n which maximize the total profit $TP(n, q_b)$,

Step I: To obtain the value of q_b , solve equations (19) and (38) by putting $n = n_k$ and $n = n_{k+1}$, the consequent values of q_b will be $q_{b_{n_k}}$ and $q_{b_{n_{k+1}}}$, respectively. ($n_k = 1, 2, 3 \dots \dots$).

Step II: Calculate $(n_k, q_{b_{n_k}})$, $TP(q_{b_{n_{k+1}}}, n_k + 1)$.

Step III: If $TP(n_k, q_{b_{n_k}}) \geq TP(q_{b_{n_{k+1}}}, n_k + 1)$, the optimal values of q_b and n will be $n = n_k$ and $q_b = q_{b_{n_k}}$. We can ascertain optimal value of T , by using $= \frac{T'}{n}$, while by replacing q_b and n in equation (19) and (38) we can ascertain the optimal value of $TP(n, q_b)$ and optimal lot size (Q_i) for $i = 1, 2, 3, \dots \dots n$ can be achieved by using equation (19) and (38). Else, move to Step IV.

Step IV: Substitute n_k by n_{k+1} and move to Step I.

7. NUMERICAL ILLUSTRATION

Assume the inventory system with following data:

$$\lambda^o = 1100, h_B = 0.08, h_v = 0.06, C_B = 50, C_v = 30, A_v = 300, A_B = 200, P = 450 \text{ in significant units.}$$

Solution: The results attained for the above data are solved through LINGO-17 and MS- Excel software and by using aforementioned solution method. The results for policy I and policy II are depicted in Table 2 and Table 3.

Table 2: Optimal total profit for policy I (COD)

n	q_b	$TP_1(n, q_b)$
1	324.97	5044333
2	147.02	5241433
3	98.74	5296596
4	75.52	5316246
5	61.53	5321723
6	52.07	5320128
7	45.19	5314501
8	39.96	5306358
9	35.83	5296540
10	32.48	5285551

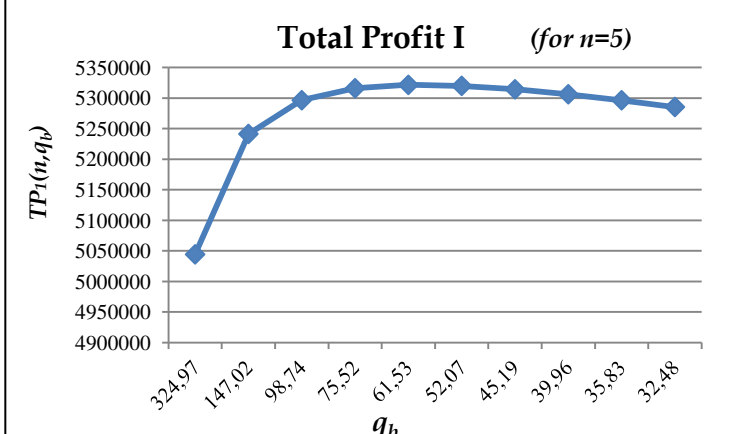


Figure 3

From Table 2, we can observe that PVTP is maximized at $n = 5$. Hence, the optimal solution is given as:
 $n = 5, q_b = 61.53, Q = 307. = 102.08 \text{ days} (.27967 \text{ years})$

Total profit for policy I=5321723

Total profit of vendor in policy I = 4750161

Total profit of buyer in policy I= 571561.9

Table 3: Optimal total profit for policy II (CBD)

n	Q	$TP_2(n, q_b)$
10	10.89	5399759
11	9.45	5401474
12	8.30	5402621
13	7.37	5403343
14	6.60	5403737
15	5.95	5403877
16	5.41	5403817
17	4.94	5403596
18	4.54	5403246
19	4.18	5403246

Figure 4

From table 3, we can observe that PVTP is maximized at $n = 15$. Hence, the optimal solution is given as:
 $n = 15, q_b = 5.95, Q = 89.25, T = 29.61 \text{ days} (.08112 \text{ years})$

Total profit for policy II=5403877

Total profit of vendor for policy II = 4774267

Total profit of buyer for policy II= 629610.8

From table 2 and table 3, we conclude that policy II (CBD) is better than policy I (COD). As total profit is more in policy II as compared to policy I.

8. SENSITIVITY ANALYSIS

$$\lambda^0 = 1100, h_B = 0.08, h_v = 0.06, C_B = 50, C_v = 30, A_v = 300, A_B = 200, P = 450 \text{ in significant units.}$$

In this section we have compared both the policies under VMI system and non VMI system. To examine the differences of inventory cost following the execution of VMI, we calculate the change in total profit in terms of percentage (V) (Yao et.al, [24]) :

The larger the V , the larger the profit achieved from VMI.

$$V = \frac{TP_{no\ VMI}^* - TP_{VMI}^*}{TP_{no\ VMI}^*}$$

Table 4: Effect of discount rate on the total profit

		r	0.02	0.04	0.06	0.08	0.1
POLICY I	No VMI	$q_{b\ noVMI}^*$	358	346	335	325	316
		TP_{noVMI}^*	11935990	8717724	6421689	5044333	4144547
	VMI	$q_{b\ VMI}^*$	64	63	62	61	60
		n	5	5	5	5	5
		Q_{VMI}^*	318	315	311	307	304
		TP_{VMI}^*	13043810	9272001	7857038	5321723	4366519

		V_1	-0.092	-0.063	-0.223	-0.054	-0.053
POLICY II	No VMI	$q_{b_{noVMI}}^*$	358	346	335	325	316
		TP_{noVMI}^*	11935990	8717724	6421689	5044333	4144547
	VMI	$q_{b_{VMI}}^*$	6.04	6.01	5.98	5.95	5.93
		n	15	15	15	15	15
		Q_{VMI}^*	90.6	90.1	89.7	89.2	88.9
		TP_{VMI}^*	13362640	9433053	6899854	5403877	4432890
		V_2	-0.11	-0.08	-0.07	-0.07	-0.06

From Table 4, we can observe that with increase in discount rate, total profit depletes in both the policies. We can observe that VMI system is more beneficial as compared to traditional system as profit in VMI system is more. Policy II (CBD) is better than policy I (COD), as total profit is more in policy II as compared to policy I.

Table 5: Effect of discount rate on the profit of vendor and buyer in policy I

POLICY I	No VMI	r	0.02	0.04	0.06	0.08	0.1
		$q_{b_{noVMI}}^*$	358	346	335	325	316
		TP_{noVMI}^*	11935990	8717724	6421689	5044333	4144547
		TP_{Vendor}^*	10878690	8188271	6068237	4778915	3931972
	VMI	TP_{Buyer}^*	1057300	529453.3	353452	265418.4	212574.9
		$q_{b_{VMI}}^*$	64	63	62	62	61
		n	5	5	5	5	5
		Q_{VMI}^*	318	315	311	307	304
		TP_{VMI}^*	13043810	9272001	7857038	5321723	4366519
		TP_{Vendor}^*	10752030	8127034	6028698	4750161	3909639
	V_1	TP_{Buyer}^*	2291785	1144966	762699.2	571561.9	456880.2
		V_{vendor}	0.011	0.007	0.006	0.006	0.005
		V_{Buyer}	-1.17	-1.16	-1.16	-1.15	-1.15

From Table 5, we can observe that with increase in discount rate, total profit of vendor in VMI system is more. Consequently, VMI system is more favourable to the vendor than it is to the buyer.

Table 6: Effect of discount rate on the profit of vendor and buyer in policy II

POLICY II	No VMI	r	0.02	0.04	0.06	0.08	0.1
		$q_{b_{noVMI}}^*$	358	346	335	325	316
		TP_{noVMI}^*	11935990	8717724	6421689	5044333	4144547
		TP_{Vendor}^*	10878690	8188271	6068237	4778915	3931972
	VMI	TP_{Buyer}^*	1057300	529453.3	353452	265418.4	212574.9
		$q_{b_{VMI}}^*$	6.04	6.01	5.98	5.95	5.93
		n	15	15	15	15	15
		Q_{VMI}^*	90.6	90.15	89.7	89.25	88.95
		TP_{VMI}^*	13362640	9433053	6899854	5403877	4432890
		TP_{Vendor}^*	1084810	8175121	4774387	4774267	3928982
	V_2	TP_{Buyer}^*	5027910	1257932	83905	629610	503908
		V_{vendor}	0.9	0	0.21	0	0

		V_{Buyer}	-3.76	-1.38	-1.37	-1.37	-1.37
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From Table 6, we can observe that with increase in discount rate, total profit of vendor in VMI system is more. Consequently, VMI system is more favourable to the vendor than it is to the buyer.

Table 7: Effect of inventory carrying charge of vendor on total profit

		h_v	0.02	0.04	0.06	0.08	0.1
POLICY I	No VMI	$q_{b_{noVMI}}^*$	325	325	325	325	325
		TP_{noVMI}^*	504433	504433	504433	504433	504433
	VMI	$q_{b_{VMI}}^*$	61	59	62	70	52
		n	7	6	5	4	5
		Q_{VMI}^*	425	351	308	279	261
		TP_{VMI}^*	5346783	5332409	5321723	5309765	5304900
		V_1	-0.06	-0.057	-0.055	-0.053	-0.052
POLICY II	No VMI	$q_{b_{noVMI}}^*$	325	325	325	325	325
		TP_{noVMI}^*	504433	504433	504433	504433	504433
	VMI	$q_{b_{VMI}}^*$	4.91	5.36	5.95	5.85	5.94
		n	23	18	15	14	13
		Q_{VMI}^*	113	96.5	89.3	81.9	77.2
		TP_{VMI}^*	5429292	5414504	5403877	5395290	5387978
	V_2	-0.076	-0.073	-0.071	-0.07	-0.068	

As inventory carrying charge of vendor increases, cycle length, order quantity depletes consequently the total profit depletes. We can observe that policy II is better than policy I as profit is more in policy II.

Table 8: Effect of inventory carrying charge of vendor on total profit in policy I

		h_v	0.02	0.04	0.06	0.08	0.1	0.12
POLICY I	No VMI	$q_{b_{noVMI}}^*$	325	325	325	325	325	325
		TP_{noVMI}^*	504433	504433	504433	504433	504433	504433
		TP_{Vendor}^*	4778915	4778915	4778915	4778915	4778915	4778915
		TP_{Buyer}^*	265418	265418	265418	265418	265418	265418
	VMI	$q_{b_{VMI}}^*$	60.69	58.56	61.53	69.86	52.29	61.53
		n	7	6	5	4	5	4
		Q_{VMI}^*	424.83	351.36	307.65	279.44	261.45	246.12
		TP_{VMI}^*	5346783	5332409	5321723	5309765	5304900	5298070
		TP_{Vendor}^*	4743704	4745783	4750161	4756692	4744804	4753237
		TP_{Buyer}^*	603079	586626	571562	553073	560096	544833
	V_1	V_{Vendor}	0.0074	0.0069	0.006	0.0047	0.0071	0.0054
		V_{Buyer}	-1.272	-1.21	-1.153	-1.084	-1.11	-1.053

As inventory carrying charge of vendor increases, cycle length, order quantity depletes consequently the total profit depletes and we can observe that vendor's profit is more in VMI system than it is to the buyer. Hence, VMI system is more favourable to the vendor.

Table 9: Effect of inventory carrying charge of vendor on total profit in policy II

POLICY II	h_v	0.02	0.04	0.06	0.08	0.1	0.12
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	No VMI	$q_{b_{noVMI}}^*$	325	325	325	325	325	325
		TP_{noVMI}^*	504433	504433	504433	504433	504433	504433
		TP_{Vendor}^*	4778915	4778915	4778915	4778915	4778915	4778915
		TP_{Buyer}^*	265418	265418	265418	265418	265418	265418
	VMI	$q_{b_{VMI}}^*$	4.91	5.36	5.95	5.85	5.94	6.2
		n	23	18	15	14	13	12
		Q_{VMI}^*	113	96.5	89.3	81.9	77.2	74.4
		TP_{VMI}^*	5429292	5414504	5403877	5395290	5387978	5381581
		TP_{Vendor}^*	4778950	4776033	4774266	4772053	4770369	4769231
		TP_{Buyer}^*	650342	638471	629610	623237	617609	612350
	V_2	V_{Vendor}	-7.37E-06	0.0006	0.001	0.0014	0.0018	0.002
		V_{Buyer}	-1.45	-1.406	-1.372	-1.348	-1.327	1.307

With increase in inventory carrying charge of the vendor, cycle length, order quantity depletes consequently the total profit depletes and we can observe that vendor's profit is more in VMI system than it is to the buyer. Hence, VMI system is more favourable to the vendor.

Table 10: Effect of inventory carrying charge of buyer on total profit

		h_b	0.02	0.04	0.06	0.08	0.1	0.12
POLICY I	No VMI	$q_{b_{noVMI}}^*$	498.38	413.84	361.46	324.97	297.68	276.27
		TP_{noVMI}^*	5575175	5397493	5220641	5044333	4868420	4692808
	VMI	$q_{b_{VMI}}^*$	70.79	67.25	64.2	61.53	59.17	57.06
		n	5	5	5	5	5	5
		Q_{VMI}^*	354	336	321	308	296	285
		TP_{VMI}^*	5850160	5673818	5497681	5321723	5145921	4970259
		V_1	-0.049	-0.051	-0.053	-0.055	-0.057	-0.059
POLICY II	No VMI	$q_{b_{noVMI}}^*$	498.38	413.84	361.46	324.97	297.68	276.27
		TP_{noVMI}^*	5575175	5397493	5220641	5044333	4868420	4692808
	VMI	$q_{b_{VMI}}^*$	6.27	6.16	6.06	5.95	5.86	5.25
		n	15	15	15	15	15	16
		Q_{VMI}^*	94	92	91	89	87	84
		TP_{VMI}^*	5923411	5750213	5577035	5403877	5230738	5057622
	V_2	-0.062	-0.065	-0.068	-0.071	-0.074	-0.078	

As inventory carrying charge of buyer increases, cycle length is same, order quantity depletes consequently the total profit depletes. We can observe that policy II is better than policy I as total profit is more in policy II.

Table 11: Effect of inventory carrying charge of buyer on total profit in policy I

		h_b	0.02	0.04	0.06	0.08	0.1	0.12
POLICY I	No VMI	$q_{b_{noVMI}}^*$	498.4	413.8	361.5	325	297.7	276.3
		TP_{noVMI}^*	5575175	5397493	5220641	5044333	4868420	4692808
		TP_{Vendor}^*	5303589	5128453	4953573	4778915	4604440	4430117
		TP_{Buyer}^*	271586	269040	267068	265418	263980	262691
	VMI	$q_{b_{VMI}}^*$	70.79	67.25	64.2	61.53	59.17	57.06

		n	5	5	5	5	5	5
		Q_{VMI}^*	353.95	336.25	321	307.65	295.85	285.3
		TP_{VMI}^*	5850160	5673818	5497681	5321723	5145921	4970259
		TP_{Vendor}^*	5282826	5104857	4927329	4750161	4573298	4396695
		TP_{Buyer}^*	567334.1	568961	570352	571562	572623	573564
	V₁	V_{vendor}	0.0039	0.0046	0.0053	0.006	0.0068	0.0075
		V_{Buyer}	-1.089	-1.115	-1.136	-1.153	-1.169	-1.183

As inventory carrying charge of buyer increases, cycle length remains same, order quantity depletes consequently the total profit depletes and we can observe that vendor's profit is more in VMI system than it is to the buyer. Hence, VMI system is more favourable to the vendor.

Table 12: Effect of inventory carrying charge of buyer on total cost in policy II

		h_b	0.02	0.04	0.06	0.08	0.1	0.12
POLICY II	No VMI	q_{noVMI}^*	498.4	413.8	361.5	325	297.7	276.3
		TP_{noVMI}^*	5575175	5397493	5220641	5044333	4868420	4692808
		TP_{Vendor}^*	5303589	5128453	4953573	4778915	4604440	4430117
		TP_{Buyer}^*	271586	269040	267068	265418	263980	262691
	VMI	q_{bVMI}^*	6.27	6.16	6.06	5.95	5.86	5.25
		n	15	15	15	15	15	16
		Q_{VMI}^*	94.05	92.4	90.9	89.25	87.9	84
		TP_{VMI}^*	5923411	5750213	5577035	5403877	5230738	5057622
		TP_{Vendor}^*	5295099	5121453	4947869	4774267	4600765	4426572
		TP_{Buyer}^*	628312	628760	629166	629610	629973	631050
	V₂	V_{vendor}	0.0016	0.0014	0.0012	0.001	0.0008	0.0008
		V_{Buyer}	-1.31	-1.34	-1.36	-1.37	-1.39	-1.4

As inventory carrying charge of buyer increases, cycle length increases, order quantity depletes consequently the total profit depletes and we can observe that vendor's profit is more in VMI system than it is to the buyer. Hence, VMI system is more favourable to the vendor.

9. MANAGERIAL IMPLICATIONS

In supply chain management, integration of vendor and buyer is essential. Several companies are adopting VMI system for cost reduction and to achieve maximum profit. Payment policies offered by the vendor to the buyer have significant impact on inventory decisions, therefore it is important for the companies to negotiate the appropriate payment policies. In this paper, inventory model for single buyer single vendor has been introduced to compare the two payment policies between vendor and buyer i.e. cash on delivery (COD) and cash before delivery (CBD) in VMI system under DCF approach. Here DCF approach has been used to make the judgment more apparent. First it is established that VMI system is better than non-VMI system for both parties involved (i.e. Vendor and Buyer) and later detailed sensitivity analysis is conducted to compare both the policies under VMI system and non VMI system and some important observations are highlighted which are crucial for decision makers for the negotiations of payment policies.

10. CONCLUSION

DCF approach is an application of the time value of money principle which helps to find the present worth for the future cash flow. In the present study, VMI model is developed using DCF approach. Two

payment policies has been compared between vendor and buyer i.e. (i) cash on delivery and (ii) cash before delivery. Mathematical modelling has been applied to derive the net present worth of total profit per unit time by optimizing the order quantity and inventory cycle length jointly. Solution method is discussed to ascertain the optimal values which maximizes the total profit. Numerical illustrations, along with sensitivity analysis has been discussed to validate the mathematical model. From numerical results, we conclude that VMI system is beneficial for both vendor and buyer under both the payment policies and policy II i.e. cash before delivery (CBD) is better than policy I i.e. cash on delivery (COD) for both VMI system and non VMI system.

DISCLAIMER

The software used in this paper for numerical analysis like LINGO-17 and MS-Excel is used only for educational purpose. We do not have any direct financial gain to mentioned these commercial identities in the paper.

ACKNOWLEDGEMENT: The authors would like to thank the anonymous reviewers and the editor for their valuable comments and suggestions.

RECEIVED: JANUARY 2020.

REVISED: JUNE, 2020

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