OPTIMAL CARBON FOOTPRINTS FOR CHEMICAL INDUSTRIES INCORPORATING TRADE CREDIT UNDER TRENDED DEMAND

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ABSTRACT
Nowadays, world's focus is on the reduction of carbon footprints to mitigate the global warming. To overcome this, various countries have introduced the trend of offering carbon credits to the industries. Chemical industry is one of the industries which harm the environment significantly by its carbon emission. Also the demand in the chemical industries is very dynamic as new research and new development force them to produce different products at different time. In this article, our goal is to investigate the chemical industries' optimal ordering policies under trended demand. In such industries, cash on delivery is hardly followed. Most of the industries do their business on credit. This credit boosts the demand of the industries. Illustrations with numerical data are presented to validate results. It is seen that profit of inventory system is maximized with minimum carbon emission.

KEY WORDS: Trended Demand, Carbon Footprints, Cap and Trade System, Carbon Cap, Carbon Credits

MSC: 90B05

RESUMEN
Actualmente el mundo se enfoca en la reducción de las huellas del carbón para mitigar el calentamiento global. Para superar esto varios países han introducido la tendencia de ofertar créditos por el carbón a las industrias. La industria química es una de las dañan el medio ambiente significativamente por sus emisiones de carbón. También la demanda de estas industrias es muy dinámica ya que las nuevas investigaciones y el desarrollo de nuevas tendencias a producir diferentes productos en momentos diferentes. En este articulo, nuestro objetivo es investigar en las industrias químicas sus políticas de optimizar sus órdenes bajo las tendencias de la demanda. En esas industrias, la distribución en cash es raramente seguida. Muchas industrias hacen sus negociaciones en base a crédito. Estos créditos mayorean la demanda de ellas. Ilustraciones con datos numéricos son presentados para validar resultados. Se evidencia que la ganancia del sistema de inventario se maximiza con un mínimo de la emisión de carbón.

PALABRAS CLAVE: Demanda Tendenciosa, Huellas de Carbón, Sistema "Cap and Trade" , "Carbon Cap", Créditos del Carbón.

1. INTRODUCTION

For classical EOQ model, the common policy is retailer must pay after receiving the goods ordered. To propose credit period is an important tool of promotion for profit enhancement for all players. In this allowable time, retailer sell items and get the income and earn interest on it if he keeps in interest generating firms. Goyal (1985) built up model to work out optimum quantity when postponement is allowed in payment. Aggarwal and Jaggi (1995) studied a case in which deteriorating products are sold on certain time bound to pay. The literature review by Shah *et al.* (2010) reviewed a complete scenario for delayed payment. Sarker *et al.* (2000) generated inventory system with inflation for products which deteriorate over time and agreed for late payment also. Chang (2004) talked about an EOQ system for goods which gets deteriorated and system entertains inflation and where credits are associated to order quantity. To attain optimal cycle time along with delayed payment Chung and Huang (2003) discussed EPQ inventory system. Teng *et al.* (2005) discussed system with delayed payment to uncover best selling value and stock amount. Liao (2007) analyzed EPQ system adding factors as decaying goods and late payment. Chang *et al.* (2008) presented review for inventory order size with company of trade credits.
Huang (2003) proved that seller gets more advantage when time from late payment he got from supplier is carried towards his clients also. When supplier gifts time M for late payment to retailer and retailer again offers time N to his clients with $N<M$, order quantity is computed to get higher profit. Situation discussed

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here is recognized as two- stage trade credits. Pal *et al.* (2013) studied 3 stage trade credit policies for 3 layer system.

Global warming is a great threat to our earth nowadays. Main causes for world wide high temperature are growing carbon emissions. Carbon emissions from industries can be reduced by implementing newer technologies as well as by using efficient operating systems. Caro and Corbett's(2013) work aimed to start for a structure to combine the economics and life cycle assessment dependent views on supply - chain carbon foot printing. Cachon (2014) studied that if retailer wants to reduce the emission then how factors affecting such as networking stores are densed, bigger in size and geometric position. Hoen *et al.* (2014) studied group which is carbon conscious that reconsiders the transport mode selection to reduce carbon emission. Chen *et al.* (2013) studied how emission can be reduced by changing the order size without significantly increasing cost. Benjaafar *et al.* (2013) demonstrated how decisions can be made for order size, production and management of inventory with taking care of carbon emission. Chen *et al.* (2017) investigated how retailer changes the order quantity in presence of cap-and-trade structure and late payment to achieve better total costs and carbon footprint. Sarkar *et al.*(2018)studied carbon release for production model with ecological supply chain system.

In this paper, we have decided optimal order quantity for the industries especially chemical industries with trended demand under trade credit with existence of cap and trade structure to reduce carbon emissions. For a retailer, decision polices are changed by adjusting time for new order and size of new order to minimize carbon emissions.

In the process of manufacturing, transportation and storing of chemical in chemical industries, carbon emission is seen. In that sense this article is supportive to optimal carbon footprints for chemical industries

2. NOTATIONS & ASSUMPTIONS

2.1 Notations

A	Order cost / order
C	Regular Procurement cost / unit
s	Sale Price / Unit ; where $s > C$
h	Cost of Holding items per single unit per unit time
I_e	Interest earned / $ / Yr
I_c	Interest charged / $ for unsold item /year by the supplier
M	Period of Credit presented by supplier to retailer
$I(t)$	Level of Inventory at t , $0 \le t \le T$
T	Time for single cycle (decision variable)
Q	Size of order
e	Carbon Emission Amount / order
g	Carbon Emission Amount for each part retailer seize stock / unit t

k	Carbon Emission quantity / unit purchased
α	Carbon limit / yr
X	Quantity of Carbon which retailer sold or bought in market / yr
p	Value of Carbon / part
$\pi(T)$	Retailer's Gross Profit / unit time

2.2. Assumptions

1. The Demand rate is, $R(t) = a(bt+1)a$ is function of t where $a > 0$ is scale demand and $0 < b < 1$ denotes the rate of change of demand.
2. Planning possibility is endless.
3. We have not considered any Shortages. Lead-time is 0 or insignificant.
4. $I_c \geq I_e$.
5. Total amount of carbon emissions is consisting of emission with ordering, holding and purchasing inventory.
6. Ordering cost includes the transportation cost.

3. MATHEMATICAL REPRESENTATION

Retailer's starting stock of Q diminishes to 0 when $t = T$ because of customer need.. So, the alteration of stock level t is administrated by the D. E.

$$\frac{dI(t)}{dt} = -a(1+bt), \quad 0 \leq t \leq T \qquad (1)$$

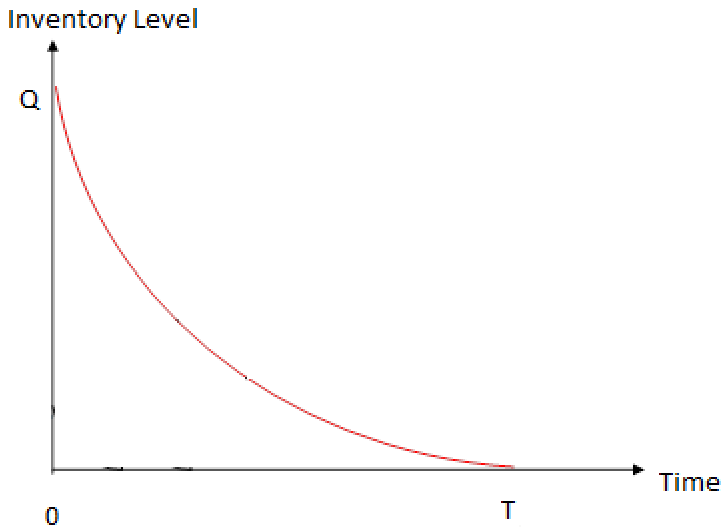

Figure (1) Stock Inventory Level with respect to time

Figure 1 exhibits Stock intensity at any instantaneous time .

Using $I(t)=0,\ at\ t=T$,

$$I(t)=\frac{(2+(T+t)b)\ a\ (T-t)}{2}$$

(2)

Therefore

$$Q=I(0)=\frac{a}{2}T(2+Tb)$$

(3)

Hence

$$SR=s\int_0^T (bt+1)\,a\,dt$$

$$=\frac{s\,a\,T\,(2+Tb)}{2}$$

(4)

Therefore

$$OC=A \tag{5}$$

Hence

$$PC=CQ$$

$$=\frac{C\,T\,a\,(2+Tb)}{2}$$

(6)

Cost of holding is;

$$HC=h\int_0^T I(t)\,dt$$

$$=\frac{haT^2(3+2Tb)}{6}$$

(7)

Under cap & trade system, retailer's stock strategy is influenced by *M* and quantity of carbon release, carbon limit and value price of carbon. Assuming carbon emission (Chen *et al* . (2013) & Hua *et al.* (2011)) are $E(T)=g\int_0^T I(t)\,dt+k\int_0^T I(t)\,dt+\frac{e}{T}$ where $\frac{e}{T}$, $g\int_0^T I(t)\,dt$ & kR are the release with ordering, holding and buying inventory respectively.

Under both the *M*, t & cap and trade structure one can find the optimal decision of the retailer. Under cap & trade structure, if retailer exceeds carbon cap then he must purchase the required credit from marketplace. Conversely if retailer is underneath carbon cap, he may put up for sale the extra carbon credit and make income. So we have carbon balance equation as follows

$$\frac{e}{T}+g\int_0^T I(t)dt+k\int_0^T I(t)dt+X=\alpha$$

where X is the total quantity of carbon credit that retailer buy or put up for sale in marketplace per annum. This might be –ve, +ve or 0 depending on whether the retailer exceeds carbon cap.

Emission cost is;

$$EC = -pX$$

$$= \frac{1}{6T} p\left(T^5 abg + 2a\left(bk + \frac{3}{4}g\right)T^4 + 3kT^3 a - 6\alpha T + 6e\right)$$

(A) Case – I $M \leq T$

Here, the retailer earns interest till credit period M on items sold is

$$IE_1 = s\, I_e \int_0^M R(t)\, dt$$

$$= \frac{1}{2} s I_e a M \left(Mb + 2\right)$$

and the interest charged is for all stuff is expressed by

$$IC_1 = \left(\int_M^T I(t) dt \right) I_C\, C$$

$$= \frac{\left(3 + \left(M + 2T\right) b\right)\left(M - T\right)^2 C a I_c}{6}$$

Hence,

$$\pi_1(T) = \frac{SR - PC - OC - HC - IC_1 + IE_1 + EC}{T}$$

$$= \frac{1}{12T^2} \begin{pmatrix} 2T^5 abgp + 4a\left(\left(-CI_c + kp - h\right)b + \frac{3}{4} pg\right)T^4 + 6\left(\left(C I_c M - C + s\right)b + kp - I_c C - h\right)aT^3 \\ +12a\left(CI_c M - C + s\right)T^2 + \left(\begin{pmatrix} \left(-2CI_c M^3 + 6I_e M^2 s\right)b - 6I_c M^2 \\ +12sI_e M \end{pmatrix} a - 12p\alpha - 12A\right)T + 12ep \end{pmatrix}$$

(8)

(B) Case – II $M \geq T$

The retailer trade all stock prior than the M, so IC_2 is zero. Retailer generates income from initiation of the T and settle the dues at $M \geq T$.

Here, retailer's earned interest is

$$IE_2 = \left[\int_0^T R(t) + (M - T) \int_0^T R(t) \right] s\, I_e$$

$$= \frac{1}{2} s I_e a T \left(Tb + 2\right)\left(M - T + 1\right)$$

So,

$$\pi_2(T) = \frac{SR - PC - OC - HC - IC_2 + IE_2 + EC}{T}$$

$$= \frac{1}{12T^2}\left(3a\left(pg - 2sI_e - \frac{4}{3}h \right)bT^4 + 6a\left(\begin{array}{l} \left(\left(1+(M+1)I_e\right)s + kp - C\right)b \\ + pg - 2sI_e - h \end{array} \right)T^3 + 12\left(\left(1+(M+1)I_e\right)s + kp - C\right)aT^2 + (-12\alpha p - 12A)T + 12ep \right) \quad (9)$$

4. NUMERICAL EXAMPLES

Needed provision to optimize income fun is to put

$$\frac{\partial \pi_i(T)}{\partial T} = 0$$

and track the following method to get the optimum solution.

1 : Fix values to all Variables.

2 : If $M \leq T$, work out $\frac{\partial \pi_1}{\partial T} = 0$ or $\frac{\partial \pi_2}{\partial T} = 0$.

3 : Check second order (sufficiency) conditions analytically or graphically.

4 : Determine profit $\pi_i(T)$ from (8) or (9) and ordering quantity Q from (3).

Example 1: Take $A = \$1500$ / order, C is $\$6$ / unit, $h = \$0.5$ / unit /yr, $a = 1000$ units, $b = 0.1$, $\alpha = 1$, $e = 0.02$, $g = 0.1$, $k = 0.1$, $I_e = 10\%/\$/$ year, $I_c = 15\%/\$/$ year, $s = \$15$, $p = \$3$ and $M = \frac{30}{365}$ year, then best possible *T* is 2.29 yrs and matching profit is \$8378.81. Visibly, M less than or equal to T is observe. Retailer's procure 2564 units. Concavity of $\pi_i(T)$ is verified in fig. 2.

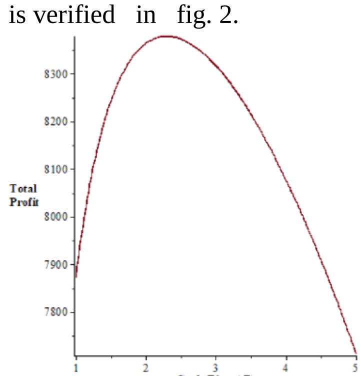

Figure. 2 Concavity of $\pi_i(T)$ with respect to *T* for M less than or equal to T

Example 2 : Consider $A = \$1500$ / order, $C = \$6$ / unit, $h = \$0.5$ / unit / yr, $a = 1000$ units, $b = 0.21$, $\alpha = 1$, $e = 0.02$, $g = 0.1$, $k = 0.1$, $I_e = 10\%/\$/$ year,

$I_c = 15\% / \$ /$ year, $s = \$15$, $P = \$3$ and $M = \frac{600}{365}$ year then best T is 1.45 yrs & corresponding profit is \$ 11557.33. Here M greater than or equal to T is observed. Retailer's get 1666 units. Concavity of $\pi_i(T)$ is established in fig.3.

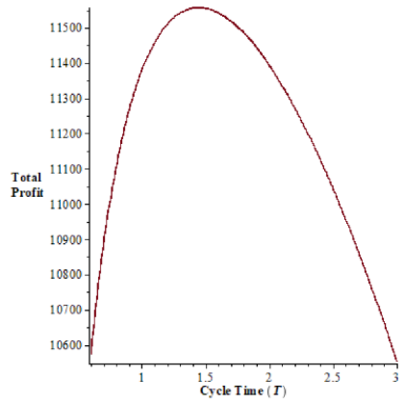

Figure. (3) Concavity of $\pi_i(T)$ with respect to T for M greater than or equal to T

Now, example 2, we examine the variations in cycle time (fig.2) and profit realization (fig.3) by changing variables as -40 %, -20 %, 20 % & 40 % with help of numerical data given in table I.

Table I: Sensitivity study w. r. to key variables

	Value	T (Years)	Profit (\$)	Q (units)
	900	1.196947	12010.27	1347.378
	1200	1.332025	11773.19	1518.325
A	1500	1.446391	11557.33	1666.056
	1800	1.54654	11356.92	1797.677
	2100	1.636213	11168.44	1917.318
	3.6	1.663473	14347.85	1954.023
	4.8	1.547029	12945.77	1798.326
C	6	1.446391	11557.33	1666.056
	7.2	1.359164	10180.71	1553.133
	8.4	1.283214	8814.351	1456.111
	0.3	1.562417	11739.18	1818.738
	0.4	1.501317	11646.2	1737.982
h	0.5	1.446391	11557.33	1666.056
	0.6	1.396725	11472.14	1601.564
	0.7	1.351575	11390.23	1543.384
	600	1.743969	6558.4	1237.991
	800	1.570054	9046.653	1463.109
$-a$	1000	1.446391	11557.33	1666.056
	1200	1.352104	14083.56	1852.877
	1400	1.276842	16621.34	2027.237

Ie	0.06	2.273281	10945.01	2815.9
	0.08	1.740654	11185.82	2058.792
	0.1	1.446391	11557.33	1666.056
	0.12	1.258983	12001.46	1425.412
	0.14	1.127904	12492.08	1261.482
M	0.049315	1.373815	10425.07	1571.988
	0.065753	1.409049	10990.26	1617.519
	0.082192	1.446391	11557.33	1666.056
	0.09863	1.485995	12126.39	1717.854
	0.115068	1.528022	12697.57	1773.182
s	9	1.43529	3818.565	1651.589
	12	1.440841	7687.949	1658.823
	15	1.446391	11557.33	1666.056
	18	1.451487	15426.76	1672.703
	21	1.456178	19296.21	1678.826
b	0.06	1.173625	10876.05	1201.316
	0.08	1.310008	11216.69	1433.686
	0.1	1.446391	11557.33	1666.056
	0.12	1.582774	11897.98	1898.426
	0.14	1.726337	12262.02	2164.432
α	0.6	1.445965	11558.16	1665.501
	0.8	1.446178	11557.75	1665.779
	1	1.446391	11557.33	1666.056
	1.2	1.446604	11556.92	1666.334
	1.4	1.446817	11556.5	1666.611
e	0.012	1.446403	11557.32	1666.072
	0.016	1.446397	11557.33	1666.064
	0.02	1.446391	11557.33	1666.056
	0.024	1.446385	11557.34	1666.049
	0.028	1.446379	11557.34	1666.041
g	0.06	1.391274	11459.56	1594.517
	0.08	1.418111	11507.91	1629.271
	0.1	1.446391	11557.33	1666.056
	0.12	1.476234	11607.89	1705.057
	0.14	1.507773	11659.66	1746.477
k	0.06	1.437103	11419.17	1653.955
	0.08	1.44173	11488.24	1659.982
	0.1	1.446391	11557.33	1666.056
	0.12	1.451086	11626.46	1672.179
	0.14	1.455814	11695.62	1678.35
p	1.8	1.382426	11322.94	1583.092

2.4	1.413466	11439.41	1623.244
3	1.446391	11557.33	1666.056
3.6	1.481377	11676.81	1711.797
4.2	1.518619	11797.94	1760.77

5. MANAGERIAL INSIGHTS

Above table 1 shows the outcome of changes in variable on decision variables. From that one can derive some managerial implications as follows.

(1) (Fig. 4) Retailer's *T* is extremely alert to the *a* and *A*. By increasing *M*, we observe significant effect on *T*. *T* is negatively sensitive to *C*. Further decisive parameter is *Ie*. *Ie* decreases *T* significantly. While *T* decreases, retailer has to order repeatedly and therefore *A* raises. Increase in purchase cost and scale demand inversely effect cycle time. Increase in carbon price increases the *T*. Remaining inventory parameters have insignificant result on *T*.

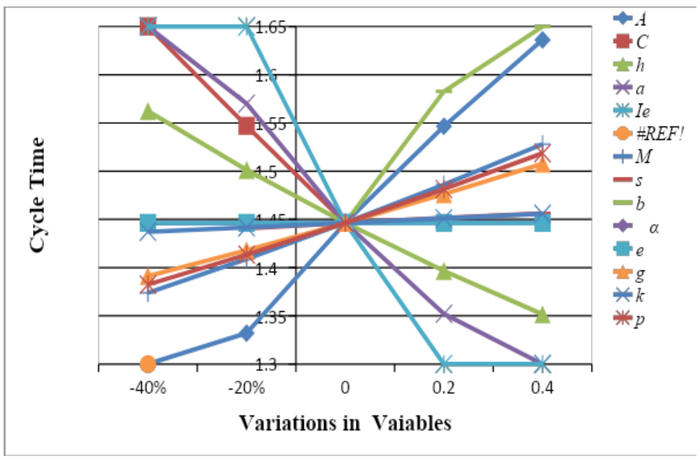

Figure. 4 Sensitivity Study for *T*

(2) (Fig. 5) It is very obvious and evident fact from the graph that if retailer has to pay later than the cycle time, then the retailer's profit is greatly increased. It is observed from figure that if selling price is higher, then retailer's profit is significantly increased. If demand is large, then profit is higher which is clearly observed from the figure. We observed a remarkable decrease in profit if purchase cost is increased.

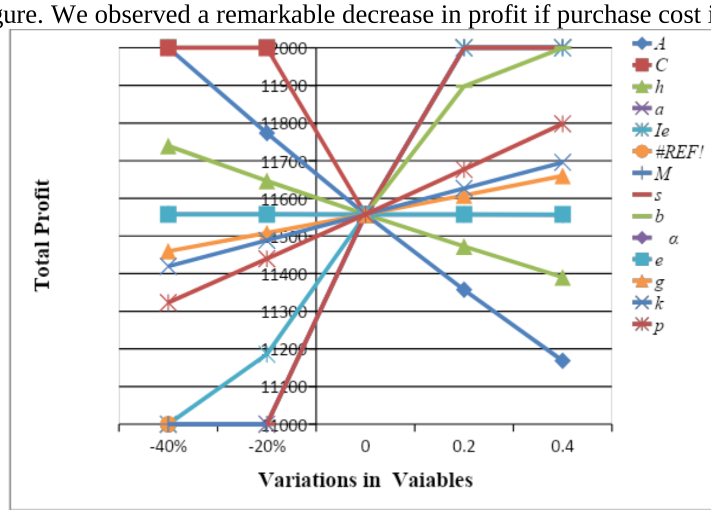

Figure. 5 Sensitivity analysis for total profit

6. CONCLUSIONS

To control the increase in overall temperature of earth, optimal carbon emission can be the key factor. All over the world, countries restrict the companies, factories, industrial units, logistics, plants and production houses either by limiting the carbon emission or imposing different types of taxes on them. As a result, they try to reduce carbon emission by improving quality of machines using new technologies, servicing etc. and using better processes then routine. In this article, we have derived optimal carbon footprints incorporating trade credit when demand is trended. This research is applicable to various industries which emit huge amount of carbon in the surroundings like chemical industries and others. Under cap and trade structure, retailer's ordering decision and entire cost is highly affected. If retailer emits more carbon than prescribed limit, he has to purchase from outside market required carbon credit. Also observation is that retailer's gross profit enhances when supplier offers higher time to make payment.

In this paper our attempt is to put some restriction on carbon emission by introducing emission cost. To optimized total profit, one should reduce the emission cost which could be done by reducing carbon emission which serves our purpose.

This paper is applicable where there are more chances of carbon emission. Especially chemical industries where more carbon is getting emitted which increases carbon emission for purchasing due to production of that item. Chemical industries involve manufacturing, storing, transportation of chemical, some of these chemicals are very sensitive with respect to different environmental condition, and byproduct is also some times to be taken care. Using proper method or adding some processes at the end the waste of chemical industries can be made less hazardous to environment.

Limitation and Future Scope:

The preset article is more useful where carbon emission is higher and our goal is to protect environment by reducing carbon emission. Government policies plays key role to control carbon emission, as developing countries require more limit of carbon emission because those countries needs to be developed in comparison of developed country. This work can be extended by adding more constraint related to control carbon emission. We can incorporate perishable items, more than single product, multi buyers, shortages, different types of demand, etc.

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