SOME NEW FUNCTIONAL FORMS OF THE RATIO AND THE PRODUCT ESTIMATORS OF THE POPULATION MEAN

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ABSTRACT
In this paper, some new functional forms of ratio and product estimators of population mean, namely, logarithmic ratio and product estimators have been introduced. The expressions of biases and mean square errors (MSEs) of these estimators have been obtained up to order n^{-1}. Further, the proposed estimators have been compared with the mean per unit, usual ratio and product estimators, and it has been found that the former are more efficient than the latter under a certain set of conditions. Also, under some practical situations, biases of proposed estimators are less than the corresponding biases of exponential ratio and product type estimators. Moreover, to improve the efficiency of proposed estimators, the transformation have been considered by shifting the origin of auxiliary variable and the optimum transformations have also been found for the proposed estimators for which MSEs of these estimators become minimum.

KEYWORDS— Efficiency, Ratio Estimation, Simple Random Sampling, Transformation.

MSC: 62D05

RESUMEN
En este paper son introducidas algunas formas nuevas de estimadores de razón y producto de la media de la población, llamados razón y producton logarítmicos. Las expresiones del sesgo y Error Cuadrtáico Medio (MSE) de ellos fueron obtenidos hasta el orden n^{-1}. Además, los propuestos estimadores han sido comparados con la media por unidad , los usuales estimadores de razón y producto, y ha sido hallado que los propuestos son más eficientes bajo ciertas condiciones. También, bajo algunas condiciones prácticas, los sesgos de los propuestos estimadores son menores que la de los estimadores del razón exponencial y producto. Màs aún , el incremento de la eficiencia de la propuesta, ha sido comparada con una transformación que ajusta al origen la variable auxiliar obteniéndose además condiciones para que los MSE´s sean mínimos.

PALABRAS CLAVE— Eficiencia, Estimación de RAazón, Muestreo Simple Aleatorio, Transformación.

1. INTRODUCTION

One of the main objectives of sampling is to estimate population parameters with high accuracy in less time and cost. Justifying the objective of sampling, [3] introduced the ratio estimator for population mean using auxiliary information. Although ratio estimators are biased, they give better results as compared to mean per unit estimator under the condition that $\rho > \frac{C_y}{2C_x}$, where C_y and C_x are coefficients of variation of study variable (y) and auxiliary variable (x) respectively and ρ is the coefficient of correlation between y and x. Further, [4] proposed the product estimator of population mean using auxiliary variable which is beneficial when $\rho < -\frac{C_x}{2C_y}$. [1] suggested the exponential ratio and product type estimators of population mean using auxiliary information which is better than mean per unit estimator, usual ratio estimator and usual product estimator under some conditions. From the last few decades, a number of statisticians such as [7],[5], [6] and [8] etc. have suggested different ratio and product type estimators using the additional information of the auxiliary variable. [10], [2] also discussed the effect of change of origin and scale in ratio method of estimation under simple random sampling.
In the present paper, alternative methods have been suggested to estimate population mean using auxiliary

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information. Here we have proposed logarithmic ratio and product estimators for population mean and have shown that the performance of these estimators is better than the mean per unit estimator, usual ratio and product estimators under some conditions, whereas the expression for MSEs of logarithmic ratio and product estimators are same as in the case of exponential ratio and product type estimators but the corresponding expressions for biases are different. Further, we have considered the effect of change of origin of auxiliary variable on the proposed estimators and have also found the optimum transformations which minimize the MSEs of the corresponding estimators. Further, a simulation study has been done using software R to verify the theoretical results. In the last, conclusion of the paper has been drawn.

2. NOTATIONS

Consider a finite population of N distinguishable and identifiable units labelled as $U=(U_1,U_2,U_3,\dots\dots\dots,U_N)$. Let $Y_1,Y_2,\dots\dots,Y_N$ and $X_1,X_2,\dots\dots,X_N$ be the values of y and x for the corresponding units in the population. Let $(y_1,x_1),(y_2,x_2),(y_3,x_3),\dots\dots\dots,(y_n,x_n)$ be a sample of size n drawn from the population by using simple random sampling without replacement(SRSWOR). Here note that Y_i,s and X_i,s usually take positive values in actual survey otherwise can be adjusted to become positive. Following notation are used throughout the paper:

$\bar{Y}=\frac{1}{N}\sum_{i=1}^{N} Y_i$, be the population mean of y

$\bar{X}=\frac{1}{N}\sum_{i=1}^{N} X_i$, be the population mean of x

$\bar{y}=\frac{1}{n}\sum_{i=1}^{n} y_i$, be the sample mean of y

$\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i$, be the sample mean of x

$S_y^2=\frac{1}{N-1}\sum_{i=1}^{N} (Y_i-\bar{Y})^2$, be the population mean square of y

$S_x^2=\frac{1}{N-1}\sum_{i=1}^{N} (X_i-\bar{X})^2$, be the population mean square of x

$S_{yx}=\frac{1}{N-1}\sum_{i=1}^{N} (Y_i-\bar{Y})(X_i-\bar{X})$, be the population covariance between y and x

$\rho=\frac{S_{yx}}{S_yS_x}$, be the correlation coefficient between y and x

$C_y=\frac{S_y}{\bar{Y}}$, $C_x=\frac{S_x}{\bar{X}}$ and $C=\rho\frac{C_y}{C_x}$.

3. EXISTING ESTIMATORS

For estimating the population mean $\bar{Y}$ of the study variable y, [3] suggested ratio estimator as

$$\bar{y}_R=\frac{\bar{y}}{\bar{x}}\bar{X} \tag{1}$$

Approximate expressions for bias and MSE of $\bar{y}_R$ are :

$$Bias(\bar{y}_R)=\frac{\bar{Y}}{n}(1-f)\left[C_x^2-\rho\, C_y\, C_x\right],$$

$$MSE(\bar{y}_R)=\frac{\bar{Y}^2}{n}(1-f)\ \left[C_y^2+C_x^2-2\rho\, C_y\, C_x\right] \tag{2}$$

Where, $f=\frac{n}{N}$ Throughout the paper, the approximate expressions for biases and MSEs have been obtained under the assumptions that

$$\left|\frac{\bar{x}-\bar{X}}{\bar{X}}\right|<1 \quad \& \quad \left|\frac{\bar{y}-\bar{Y}}{\bar{Y}}\right|<1.$$

[4] suggested the product estimator of $\bar{Y}$ as

$$\bar{y}_P=\frac{\bar{y}}{\bar{X}}\bar{x} \tag{3}$$

Approximate expressions for bias and MSE of $\bar{y}_P$ are :

$$Bias(\bar{y}_P)=\frac{\bar{Y}}{n}(1-f)\rho C_yC_x\ ,$$

$$MSE(\bar{y}_P)=\frac{\bar{Y}^2}{n}(1-f)\ \left[C_y^2+C_x^2+2\rho\, C_y\, C_x\right]. \tag{4}$$

For details of these expressions, see [11].

[1] suggested the ratio and product type exponential estimator of $\bar{Y}$ as

$$\bar{y}_{Re}=\bar{y}\ \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{5}$$

$$\bar{y}_{Pe} = \bar{y}\ \exp\ \left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right) \tag{6}$$

Approximate expressions for bias and MSE of $\bar{y}_{Re}$ and $\bar{y}_{Pe}$

$$Bias(\bar{y}_{Re}) = \frac{\bar{Y}}{n}(1-f)\ \left[\frac{3}{8}C_x^2 - \frac{1}{2}\rho\ C_y C_x\right], \tag{7}$$

$$Bias(\bar{y}_{Pe}) = \frac{\bar{Y}}{n}(1-f)\ \left[-\frac{C_x^2}{8} + \frac{1}{2}\rho\ C_y C_x\right], \tag{8}$$

$$MSE(\bar{y}_{Re}) = \frac{\bar{Y}^2}{n}(1-f)\ \left[C_y^2 + \frac{C_x^2}{4} - \rho\ C_y C_x\right], \tag{9}$$

$$MSE(\bar{y}_{Pe}) = \frac{\bar{Y}^2}{n}(1-f)\ \left[C_y^2 + \frac{C_x^2}{4} + \rho\ C_y C_x\right]. \tag{10}$$

4. PROPOSED ESTIMATORS

For estimating the mean $(\bar{Y})$ of a finite population, we define logarithmic ratio and product estimators as:

$$\bar{y}_{LR} = \bar{y}\ \frac{\bar{X}}{\bar{x}-\bar{X}}\ln\left(\frac{\bar{x}}{\bar{X}}\right) \tag{11}$$

$$\bar{y}_{LP} = \bar{y}\frac{\bar{X}}{\bar{X}-\bar{x}}\ln\left(\frac{2\bar{X}-\bar{x}}{\bar{X}}\right) \tag{12}$$

These proposed estimators converge to $\bar{Y}$ in probability since $\bar{x} \xrightarrow{P} \bar{X}$, $\bar{y} \xrightarrow{P} \bar{Y}$. Therefore, they are consistent estimators of $\bar{Y}$.

To obtain the approximate expressions of biases and MSEs of these estimators, we first expand these estimators by using Taylor's series as:

$$\bar{y}_{LR} = \bar{Y}(1+\varepsilon_1)\left(1 - \frac{\varepsilon_2}{2} + \frac{\varepsilon_2^2}{3} + \dots\right)$$

$$\bar{y}_{LR} - \bar{Y} = \bar{Y}\left(\varepsilon_1 - \frac{\varepsilon_2}{2} + \frac{\varepsilon_2^2}{3} - \frac{\varepsilon_1\varepsilon_2}{2} + \dots\right).$$

Similarly

$$\bar{y}_{LP} - \bar{Y} = \bar{Y}\left(\varepsilon_1 + \frac{\varepsilon_2}{2} + \frac{\varepsilon_2^2}{3} + \frac{\varepsilon_1\varepsilon_2}{2} + \dots\right),$$

where $\varepsilon_1 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $\varepsilon_2 = \frac{\bar{x}-\bar{X}}{\bar{X}}$. Assume that $|\varepsilon_1|, |\varepsilon_2| < 1$, so as to ignore the terms of ε's with power more than two.

From the above expressions, one can easily obtain the approximate expressions for the respective biases and MSEs of $\bar{y}_{LR}$, $\bar{y}_{LP}$ as follow:

$$Bias(\bar{y}_{LR}) = \frac{\bar{Y}}{n}(1-f)\left[\frac{C_x^2}{3} - \rho\frac{C_y C_x}{2}\right], \tag{13}$$

$$MSE(\bar{y}_{LR}) = \frac{\bar{Y}^2}{n}(1-f)\ \left[C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x\right]. \tag{14}$$

Similarly,

$$Bias(\bar{y}_{LP}) = \frac{\bar{Y}}{n}(1-f)\left[\frac{C_x^2}{3} + \rho\frac{C_y C_x}{2}\right], \tag{15}$$

$$MSE(\bar{y}_{LP}) = \frac{\bar{Y}^2}{n}(1-f)\ \left[C_y^2 + \frac{C_x^2}{4} + \rho C_y C_x\right]. \tag{16}$$

4.1. Relative performances of the estimators

From (14) and $V(\bar{y}) = \left(\frac{1}{n} - \frac{1}{N}\right)S_y^2$, we conclude that the proposed estimator $\bar{y}_{LR}$ is better than the mean per unit estimator $\bar{y}$ iff,

$$\rho > \frac{C_x}{4C_y} \qquad \text{or} \qquad C > \frac{1}{4}. \tag{17}$$

From (2) and (14), we conclude that $\bar{y}_{LR}$ is efficient than usual ratio estimator $\bar{y}_R$ when

$$\rho < \frac{3C_x}{4C_y} \quad \text{or} \quad C < \frac{3}{4}. \tag{18}$$

From equations (17) and (18), we can conclude that proposed estimator $\bar{y}_{LR}$ is more efficient than the usual ratio estimator $\bar{y}_R$ and than the mean per unit estimator $\bar{y}$ provided,

$$\frac{1}{4} < C < \frac{3}{4}.$$

Proposed estimator $\bar{y}_{LP}$ is better than the mean per unit estimator $\bar{y}$ iff,

$$\rho < -\frac{C_x}{4C_y} \quad \text{or} \quad C < -\frac{1}{4}. \tag{19}$$

Proposed estimator $\bar{y}_{LP}$ is efficient than usual product estimator $\bar{y}_P$ iff,

$$\rho > -\frac{3C_x}{4C_y} \quad \text{or} \quad C > -\frac{3}{4}, \tag{20}$$

The above equations (19) and (20) show that the logarithmic product estimator is more efficient than the mean per unit estimator and the usual product estimator provided

$$-\frac{3}{4} < C < -\frac{1}{4}.$$

4.2. Comparison of Biases

We have compared various existing estimators with proposed estimators and have found the condition under which these estimators performed better than the existing ones. But the proposed logarithmic ratio and product type estimators are equally efficient corresponding to the exponential ratio and product type estimators because their corresponding MSEs are same. So it is necessary to compare the biases of proposed estimators with corresponding exponential ratio and product type estimators.

From (7) and (13), we have

$$\frac{|Bias(\bar{y}_{LR})|}{|Bias(\bar{y}_{Re})|} = \left|\frac{\frac{1}{3}-\frac{C}{2}}{\frac{3}{8}-\frac{C}{2}}\right|.$$

That is, bias of $\bar{y}_{LR}$ is less than the bias of $\bar{y}_{Re}$ if $C \leq 0.708$.

Similarly, from (8) and (15), we have

$$\frac{|Bias(\bar{y}_{LP})|}{|Bias(\bar{y}_{Pe})|} = \left|\frac{\frac{1}{3}+\frac{C}{2}}{-\frac{1}{8}+\frac{C}{2}}\right|.$$

Clearly, the bias of $\bar{y}_{LP}$ is less than that of $\bar{y}_{Pe}$ if $C \leq -0.210$.

5. EFFECT OF CHANGE OF ORIGIN

Now, we consider the transformation by shifting the origin of the auxiliary variable as $x \rightarrow x + k$. Then the modified logarithmic ratio and product estimators of $\bar{Y}$ can be defined as

$$\bar{y}_{LR}\ (k) = \bar{y}\ \frac{\bar{X}+k}{\bar{x}-\bar{X}} \ln\left(\frac{\bar{x}+k}{\bar{X}+k}\right) \tag{21}$$

$$\bar{y}_{LP}(k) = \bar{y}\ \frac{\bar{X}+k}{\bar{X}-\bar{x}} \ln\left(\frac{2\bar{X}-\bar{x}+k}{\bar{X}+k}\right) \tag{22}$$

where the optimum value of k can be determined by minimizing the MSEs of these modified proposed estimators.

$$Bias\big(\bar{y}_{LR}\ (\theta)\big) = \frac{\bar{Y}}{n}(1-f)\ \left[\frac{\theta^2\, C_x^2}{3} - \frac{\theta\, \rho\, C_y\, C_x}{2}\right],$$

where $\theta = \frac{\bar{X}}{\bar{X}+k}$.

$$Bias\big(\bar{y}_{LP}\ (\theta)\big) = \frac{\bar{Y}}{n}(1-f)\ \left[\frac{\theta^2\, C_x^2}{3} + \frac{\theta\, \rho\, C_y C_x}{2}\right],$$

$$MSE\big(\bar{y}_{LR}\ (\theta)\big) = \frac{\bar{Y}^2}{n}(1-f)\ \left[C_y^2 + \frac{\theta^2\, C_x^2}{4} - \theta\, \rho\, C_y C_x\right], \tag{23}$$

$$MSE\big(\bar{y}_{LP}\ (\theta)\big) = \frac{\bar{Y}^2}{n}(1-f)\ \left[C_y^2 + \frac{\theta^2\, C_x^2}{4} + \theta\, \rho\, C_y C_x\right]. \tag{24}$$

Optimum value of θ can be obtained by solving the following equations:

$$\frac{d\ MSE(\bar{y}_{LR}\ (\theta))}{d\theta} = 0 \tag{25}$$

and $$\frac{d\ MSE(\bar{y}_{LP}\ (\theta))}{d\theta} = 0. \tag{26}$$
From equation (25), we have
$$\theta = 2\rho\ \frac{C_y}{C_x} = 2C = \theta_{opt1}(\text{say}) \tag{27}$$
$$\text{or} \qquad k = \bar{X}\left(\frac{1-2C}{2C}\right) = k_{opt1}(\text{say}). \tag{28}$$
Similarly, from equation (26) we have
$$\theta = -\ 2\rho\ \frac{C_y}{C_x} = -2C = \theta_{opt2}(\text{say}) \tag{29}$$
$$\text{or} \qquad k = -\bar{X}\left(\frac{1+2C}{2C}\right) = k_{opt2}(\text{say}). \tag{30}$$
After putting $\theta = \theta_{opt1}$ in (23) or $\theta = \theta_{opt2}$ in (24), we get
$$MSE_{min}\big(\bar{y}_{LR}(k)\big) = MSE_{min}\big(\bar{y}_{LP}(k)\big) = \ \frac{\bar{Y}^{`2}}{n}(1-f)\ C_y^2(1-\rho^2). \tag{31}$$
The expression of minimum MSE of $\bar{y}_{LR}(k)$ or $\bar{y}_{LP}(k)$ is same as that of the usual regression estimator. Since the values of θ_{opt1} and θ_{opt2} depend upon some unknown parameters, therefore in practical situations one can use their estimated values by replacing parameters by their consistent estimators. Further, [9] proved that if we replace the parameters by their respective consistent estimators, then upto order n^{-1}, the minimum MSE remains same.
One can easily prove that the modified logarithmic ratio and product estimators of population mean are more efficient than mean per unit estimator, usual ratio estimator, usual product estimator, logarithmic ratio and product estimators.

6. SIMULATION STUDY

To validate theoretical results of suggested estimators, a simulation study has been performed by using statistical software R. For this purpose, 10,000 samples have been drawn from each bivariate normal population $BN(\bar{Y} = 17, \bar{X} = 11, \sigma_y^2 = 25, \sigma_x^2 = 9, \rho),\ \rho = -0.9, -0.7, -0.5, -0.3, 0.3, 0.5, 0.7, 0.9$ for different sample sizes $n = 5, 9, 25$. The values of biases and MSEs of various estimators have been shown in Table-1 and 2 respectively. Further, the percentage efficiencies of these estimators with respect to mean per unit estimator have been obtained by using the formula as
$$Eff(\hat{\theta}) = \frac{V(\bar{y})}{MSE(\hat{\theta})} \times 100$$
and the values of these efficiencies have been shown in Table-3.

6.1. Some Results Based on Table:-1, 2 and 3

1. Biases of all the estimators decrease as sample size increases. Moreover, biases of all ratio type estimators decrease as positive correlation coefficient increases and biases of all product type estimators decrease as negative correlation coefficient increases.
2. Proposed estimator $\bar{y}_{LR}(\theta)$ is an almost unbiased estimator for population mean.
3. Table-1 proves that the $Bias(\bar{y}_{LR})$ is less than the $Bias(\bar{y}_{Re})$ when $C \leq 0.708$ and the $Bias(\bar{y}_{LP})$ is less than the $Bias(\bar{y}_{Pe})$ when $C \leq -0.210$.
4. Table-2 shows that MSEs of all the estimators decrease as sample size increases.
5. Also, MSEs of ratio type estimators decrease as positive correlation increases, whereas MSEs of product type estimators decrease as negative correlation increases.
6. Table-3 verifies that $\bar{y}_{LR}$ is more efficient than $\bar{y}$ and $\bar{y}_R$ when conditions (17) and (18) are satisfied respectively.
7. $\bar{y}_{LP}$ is more efficient than $\bar{y}$ and $\bar{y}_P$ when conditions (19) and (20) are satisfied respectively.
8. Efficiency of proposed estimator $\bar{y}_{LR}(\theta)$ is approximately same that of the most efficient estimator $\bar{y}_D$ for $-0.75 < \rho < 0.75$.

Table -1: Biases of various estimators

ρ	C	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}_{Re}$	$\bar{y}_{Pe}$	$\bar{y}_{LR}$	$\bar{y}_{LP}$	$\bar{y}_{LR}(\theta)$	$\bar{y}_D$
$n = 5$										
-0.9	-0.97059	-0.00672	0.504637	-0.24246	0.210942	-0.15634	0.20039	-0.04081	0.098794	0.003484
-0.7	-0.7549	-0.00672	0.444747	-0.1854	0.181548	-0.12767	0.171079	-0.01192	0.055013	0.002641
-0.5	-0.53922	-0.00672	0.386847	-0.12991	0.153133	-0.09986	0.142717	0.016428	0.02574	0.000217
-0.3	-0.32353	-0.00672	0.329751	-0.07522	0.125179	-0.07249	0.1148	0.044525	0.006448	-0.00264
-0.1	-0.10784	-0.00672	0.273191	-0.02112	0.097549	-0.04543	0.087196	0.072415	-0.00435	-0.00544
0.1	0.107843	-0.00672	0.217148	0.032479	0.070205	-0.01865	0.059867	0.100095	-0.00702	-0.00786
0.3	0.323529	-0.00672	0.161714	0.085563	0.043149	0.007858	0.032825	0.127533	-0.0011	-0.00958
0.5	0.539216	-0.00672	0.107043	0.138064	0.016423	0.034079	0.006113	0.154657	0.014609	-0.01029
0.7	0.754902	-0.00672	0.05338	0.189767	-0.00988	0.059912	-0.02017	0.181298	0.042466	-0.00949
0.9	0.970588	-0.00672	0.001387	0.239896	-0.03542	0.085	-0.04568	0.206902	0.0882	-0.00604
$n = 9$										
-0.9	-0.97059	0.000721	0.276494	-0.12603	0.118909	-0.08045	0.112975	-0.01549	0.060262	0.011608
-0.7	-0.7549	0.000721	0.236706	-0.08812	0.099357	-0.06136	0.093478	0.003395	0.042447	0.014985
-0.5	-0.53922	0.000721	0.200234	-0.05349	0.081454	-0.04392	0.07563	0.020623	0.026261	0.013202
-0.3	-0.32353	0.000721	0.165585	-0.02052	0.06444	-0.02732	0.058665	0.037069	0.013364	0.009024
-0.1	-0.10784	0.000721	0.132389	0.011226	0.048119	-0.01136	0.042384	0.052978	0.004007	0.003623
0.1	0.107843	0.000721	0.100544	0.04191	0.032432	0.004043	0.026727	0.068437	-0.00158	-0.0022
0.3	0.323529	0.000721	0.07009	0.071528	0.017395	0.018871	0.011707	0.083454	-0.00287	-0.00771
0.5	0.539216	0.000721	0.041215	0.099928	0.003101	0.033049	-0.00258	0.097962	0.001152	-0.01212
0.7	0.754902	0.000721	0.014427	0.126666	-0.0102	0.046344	-0.0159	0.11175	0.012485	-0.01425
0.9	0.970588	0.000721	-0.00845	0.150121	-0.02163	0.057926	-0.02737	0.12404	0.035482	-0.01133
$n = 25$										
-0.9	-0.97059	0.009656	0.115616	-0.04463	0.056024	-0.02382	0.053909	-0.00068	0.020721	0.004639
-0.7	-0.7549	0.009656	0.100538	-0.02996	0.048553	-0.01645	0.046451	0.0066	0.018069	0.008499
-0.5	-0.53922	0.009656	0.086486	-0.01617	0.041574	-0.00954	0.039481	0.013486	0.015212	0.010338
-0.3	-0.32353	0.009656	0.072986	-0.00283	0.034857	-0.00286	0.032767	0.020177	0.012622	0.010833
-0.1	-0.10784	0.009656	0.059898	0.010195	0.028333	0.003647	0.026243	0.026725	0.010493	0.010271
0.1	0.107843	0.009656	0.047162	0.022931	0.021975	0.010005	0.019883	0.033149	0.009005	0.008848
0.3	0.323529	0.009656	0.034756	0.035382	0.015776	0.016215	0.013681	0.039446	0.008384	0.006741
0.5	0.539216	0.009656	0.022698	0.047508	0.00975	0.022259	0.007649	0.045595	0.008936	0.004146
0.7	0.754902	0.009656	0.011075	0.05919	0.003944	0.028079	0.001838	0.051533	0.011078	0.00135
0.9	0.970588	0.009656	0.00025	0.069995	-0.00145	0.033466	-0.00356	0.057033	0.015529	-0.00097

Note:

1. Biases are actual biases so they may differ a bit from the approximate biases.
2. $\bar{y}_D$ is the difference estimator of population mean.

Table -2: MSEs of various estimators

ρ	C	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}_{Re}$	$\bar{y}_{Pe}$	$\bar{y}_{LR}$	$\bar{y}_{LP}$	$\bar{y}_{LR}(\theta)$	$\bar{y}_D$
					$n = 5$					
-0.9	-0.97059	5.020837	20.41145	0.847352	10.9698	1.786246	10.91698	1.906961	1.156459	0.955417
-0.7	-0.7549	5.020837	18.12767	2.748721	9.9116	2.750355	9.866721	2.879695	2.723295	2.567287
-0.5	-0.53922	5.020837	15.88788	4.658476	8.87106	3.710867	8.833077	3.862677	3.877363	3.777263
-0.3	-0.32353	5.020837	13.66833	6.575425	7.842141	4.668716	7.810213	4.855725	4.622895	4.580495
-0.1	-0.10784	5.020837	11.46754	8.498438	6.823714	5.624128	6.797008	5.858331	4.981833	4.975791
0.1	0.107843	5.020837	9.292138	10.42635	5.815971	6.57716	5.793644	6.870067	4.967801	4.964615
0.3	0.323529	5.020837	7.151249	12.35777	4.819572	7.527748	4.800763	7.890446	4.586505	4.550387
0.5	0.539216	5.020837	5.052425	14.29076	3.835266	8.47563	3.819067	8.918368	3.830563	3.73787
0.7	0.754902	5.020837	2.999742	16.22193	2.863733	9.420076	2.849159	9.950898	2.678532	2.53248
0.9	0.970588	5.020837	0.993565	18.1418	1.905833	10.35839	1.891842	10.97932	1.114032	0.939268
					$n = 9$					
-0.9	-0.97059	2.801431	10.86453	0.435062	6.021458	0.982599	6.001798	1.050112	0.602074	0.530636
-0.7	-0.7549	2.801431	9.655956	1.48258	5.451167	1.51395	5.43434	1.582786	1.472158	1.416476
-0.5	-0.53922	2.801431	8.460496	2.532377	4.885105	2.044669	4.870709	2.119913	2.123081	2.088004
-0.3	-0.32353	2.801431	7.278475	3.586778	4.323407	2.575444	4.311075	2.661627	2.556777	2.541483
-0.1	-0.10784	2.801431	6.11151	4.646797	3.766308	3.106607	3.755684	3.207874	2.773997	2.771573
0.1	0.107843	2.801431	4.960911	5.713147	3.214017	3.638411	3.204748	3.75854	2.774555	2.773975
0.3	0.323529	2.801431	3.827308	6.786482	2.666677	4.171097	2.658419	4.31348	2.556816	2.546481
0.5	0.539216	2.801431	2.710784	7.867548	2.124372	4.704941	2.11679	4.872537	2.118884	2.089783
0.7	0.754902	2.801431	1.611408	8.957459	1.587166	5.240357	1.579945	5.435566	1.461039	1.408518
0.9	0.970588	2.801431	0.530828	10.05891	1.055235	5.77833	1.048098	6.002454	0.584186	0.513914
					$n = 25$					
-0.9	-0.97059	1.012997	3.751964	0.151509	2.134556	0.358219	2.130324	0.382632	0.213129	0.194645
-0.7	-0.7549	1.012997	3.339171	0.536	1.933586	0.552223	1.929807	0.576432	0.532787	0.520694
-0.5	-0.53922	1.012997	2.93213	0.920771	1.734633	0.745701	1.731259	0.770078	0.770735	0.764721
-0.3	-0.32353	1.012997	2.529811	1.306262	1.537217	0.938988	1.534181	0.963975	0.927927	0.926033
-0.1	-0.10784	1.012997	2.131709	1.692514	1.341164	1.132141	1.338398	1.158259	1.004689	1.004597
0.1	0.107843	1.012997	1.737421	2.079461	1.146371	1.325143	1.143809	1.353005	1.001348	1.000948
0.3	0.323529	1.012997	1.346561	2.466965	0.952769	1.517934	0.950349	1.548259	0.918663	0.916056
0.5	0.539216	1.012997	0.958733	2.854777	0.760318	1.710397	0.757983	1.744039	0.757911	0.75119
0.7	0.754902	1.012997	0.573568	3.242403	0.569042	1.9023	0.566739	1.940292	0.520507	0.507785
0.9	0.970588	1.012997	0.19093	3.628333	0.379233	2.09294	0.376917	2.13666	0.206781	0.187351

Note: These MSEs of the estimators are actual MSEs. So their values are more than approximate MSEs.

Table -3: Efficiencies of various estimators

ρ	C	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}_{Re}$	$\bar{y}_{Pe}$	$\bar{y}_{LR}$	$\bar{y}_{LP}$	$\bar{y}_{LR}(\theta)$	$\bar{y}_D$
				$n = 5$					

-0.9	-0.97059	24.59814	592.5327	45.76965	281.0832	45.99109	263.29	434.1561	525.5125
-0.7	-0.7549	27.69709	182.6609	50.65617	182.5523	50.88658	174.3531	184.3663	195.5698
-0.5	-0.53922	31.60167	107.7785	56.59794	135.3009	56.84131	129.9834	129.491	132.9226
-0.3	-0.32353	36.73336	76.35761	64.0238	107.5421	64.28553	103.4004	108.6081	109.6134
-0.1	-0.10784	43.78304	59.07953	73.57924	89.27316	73.86834	85.70422	100.7829	100.9053
0.1	0.107843	54.03317	48.15527	86.32844	76.33746	86.66112	73.0828	101.0676	101.1325
0.3	0.323529	70.20923	40.62899	104.176	66.69773	104.5841	63.63185	109.4698	110.3387
0.5	0.539216	99.3748	35.13345	130.9124	59.23851	131.4676	56.29771	131.0731	134.3235
0.7	0.754902	167.3756	30.95093	175.3249	53.29933	176.2217	50.45612	187.4473	198.2578
0.9	0.970588	505.3355	27.67552	263.4458	48.47123	265.3941	45.72993	450.6906	534.548
$n = 9$									
-0.9	-0.97059	25.7851	643.9148	46.52413	285.1042	46.67653	266.7745	465.2971	527.9385
-0.7	-0.7549	29.01247	188.9565	51.3914	185.0412	51.55053	176.9937	190.2942	197.7747
-0.5	-0.53922	33.1119	110.6246	57.34638	137.0115	57.51588	132.1484	131.9512	134.1679
-0.3	-0.32353	38.48926	78.10439	64.79684	108.7747	64.98219	105.2526	109.5689	110.2282
-0.1	-0.10784	45.83861	60.28735	74.38136	90.17655	74.59177	87.32983	100.989	101.0773
0.1	0.107843	56.4701	49.03481	87.16292	76.996	87.41502	74.53509	100.9686	100.9898
0.3	0.323529	73.19586	41.27958	105.0533	67.16294	105.3796	64.94596	109.5672	110.0119
0.5	0.539216	103.3439	35.60742	131.871	59.54232	132.3434	57.4943	132.2126	134.0537
0.7	0.754902	173.8499	31.27484	176.5052	53.45878	177.3119	51.5389	191.7424	198.8922
0.9	0.970588	527.7473	27.85025	265.4793	48.48167	267.2871	46.67143	479.5446	545.117
$n = 25$									
-0.9	-0.97059	26.99911	668.6074	47.45704	282.7873	47.55132	264.7442	475.2971	520.4323
-0.7	-0.7549	30.33678	188.9921	52.38956	183.4398	52.49213	175.7359	190.1318	194.5474
-0.5	-0.53922	34.54817	110.0162	58.39836	135.8449	58.51217	131.5447	131.4326	132.4663
-0.3	-0.32353	40.0424	77.54927	65.89813	107.8817	66.02853	105.0854	109.1677	109.391
-0.1	-0.10784	47.52041	59.85161	75.53121	89.47622	75.68728	87.45858	100.8269	100.8362
0.1	0.107843	58.30462	48.71439	88.36555	76.44435	88.56344	74.87018	101.1633	101.2038
0.3	0.323529	75.22848	41.06249	106.3213	66.73526	106.5921	65.42814	110.2686	110.5825
0.5	0.539216	105.6599	35.48428	133.2333	59.22585	133.6438	58.0834	133.6565	134.8523
0.7	0.754902	176.6132	31.24217	178.0181	53.25117	178.7415	52.20847	194.6173	199.4934
0.9	0.970588	530.5602	27.91907	267.117	48.40067	268.7584	47.4103	489.8888	540.6942

7. CONCLUSION

Proposed logarithmic estimator $\bar{y}_{LR}$ is more efficient than mean per unit estimator and usual ratio estimator when $\frac{1}{4} < C < \frac{3}{4}$. Similarly, the proposed product estimator $\bar{y}_{LP}$ is more efficient than mean per unit estimator and usual product estimator when $-\frac{3}{4} < C < -\frac{1}{4}$. Further, mean square error of proposed estimators is same as that of corresponding ratio and product type exponential estimators but biases of proposed estimators are less than that of corresponding exponential estimator under some practical situations. So that the proposed estimators are more beneficial than the existing estimators under some conditions.

Using the linear transformation by shifting origin of auxiliary variable, modified logarithmic ratio and product estimators have been suggested which have been found to be more efficient than the existing estimators. These modified estimators are also more efficient than logarithmic ratio and product estimators.

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