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EGGSHELL BREAKAGE RESTRICTED BI-CRITERIA EGGS TRANSPORTATION PROBLEM

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ABSTRACT
Eggs are most fundamental and important foodstuff in everyday life for all developed and developing countries. It is not sufficient only to produce and transport eggs at a reasonable cost but arrangements must be made to ensure that the eggs reach the consumers well in time.In between the two extremes of minimization of eggs shipping cost and minimization of eggs shipping time, there exist a number of situations where an eggs transportation system decision maker would like a partial trade-off on shipping cost to attain a certain degree of shipping time advantages.From laying to the final destination, more eggs are broken during shipping than in any other step; therefore, losses due to eggshell breakage are greatest during shipping. In this paper, an algorithm and its underlying theory is developed to solve eggshell breakage restricted bi-criteria eggs transportation problem. This paper discusses a more realistic and general assumption that the eggs shipping time of bi-criteria eggs transportation problem depends on the quantity of the eggs transported and is an increasing piecewise constant function. The algorithm is also supported by a real life eggs shipping problem of Tamil Nadu, India.

KEYWORDS: Eggs Transportation Problem, Bi-criteria, Trade-off, Eggshell Breakage.

MSC: 90B06, 90C29, 90C46.

RESUMEN
Los huevos son un alimento muy fundamental eimportanteen la vida diaria para todos los países en desarrollo. No es suficiente solo producir y transportar los huevos un razonablecosto, sino asegurar que su manejo garantice hacerlesllegar a losconsumidores en tiempo. Entre los extremos de minimizar el costo del traslado de los huevos y hacerlo con el tiempo, hay una serie de situaciones donde para el sistema de transporte el decisor querrá hacer acuerdo parcial entre el costo de transporte, para obtener un cierto grado de ventajarespecto al tiempo de arribos. Hasta la llegada a su final destino, más huevos se rompen durante el traslado que en ningúnotro paso, por tanto, las pérdidas debido a roturas son mayores durante el traslado. En este paper, un algoritmo y su teoría son desarrollados par resolver el problema de la rotura de los huevos restringido a un problema bi-criterial del problema de transporte de huevos. El paper usa la asunción másrealista y general: de que el traslado de huevos y el tiempo de un problema de transportebi-criterialdepende de la cantidad de huevostransportados y que es una función creciente a saltos constante. El algoritmo es tambiénilustrado por la solución de un problema del traslado de huevos de Tamil Nadu, India.

PALABRAS CLAVE: problema de transportación de huevos, bi-criterial, acuerdo, rotura de huevos.

1. INTRODUCTION

Eggs are most fundamental and important foodstuff in everyday life for all developed and developing countries because it contains equal quantity of animal protein as pork and poultry meat, about two-thirds that of cheese and roughly three-quarters that of beef. India is the fifth largest producer of eggs in the world and the rate of consumption is estimated to triple by the end of the year 2020. With increasing urbanization, eggs will need to be transported in good condition from egg producers and farm owners to distant cities and distributed through wholesalers, wholesalers-cum-retailers and retail outlets conveniently situated near consumers. The eggs are shipped in egg cartons or trays, generally by big lorries or cargo trucks. The study conducted by Omar et al. [15] identified some problems of layer farming and marketing of egg, and suggested measures for solving these problems. The authors also noticed that there was a wide seasonal price variation of egg in the selected markets due to change in demand and supply at different times of the year.
Most of the real world shipping problems appear with two objectives and are known as bi-criteria transportation problem. When a transportation system decision maker considers a bi-criteria transportation problem with two objectives, say minimization of total shipping cost and minimization of shipping time, one fails to get an optimum solution satisfying both the objectives. In such interesting situations, one considers for

a sequence of solutions termed as trade-off solutions. Algorithms for the trade-off transportation problem have been provided by (Prakash et al. [17]; Das et al. [3]; Khurana[10]; Chakraborty and Chakraborty[1]).

The weighted sum method and epsilon constraint method are used for multi objective optimization problems. Both these methods have certain disadvantages also and have been stated by different researchers. According to Pike-Burke [16] most methods [e.g. weighted sum method, epsilon-constraint method] produced solutions that were lexicographically optimal for one of the objective functions, only the game theoretic approach produced a compromise solution, but this came at the cost of solving a non-linear program. Therefore, it would be useful to produce methods for generating compromise solutions that are more computationally efficient. In multi-objective optimization, different methods are often used to generate a set of efficient solutions from which the decision maker can choose. Hence methods that are able to produce the entire set of efficient solutions [such as the two-phase method for multi-objective combinatorial optimization problems (Ulungu E.L. and Teghem [21])] are preferable and more of these methods should be investigated. Each of the methods discussed has advantages and disadvantages and a lot of them can be adapted for specific problems. However, there is still no general `best' method that can be used to solve multi-objective optimization problems. In the weighted sum method if the positivity requirement on w_i is weaken to w_i greater or equal to 0 there is a potential to get only weakly efficient solutions (Marler and Arora [12]). The weighted sum method is simple to implement but the results obtained are highly dependent on the weights used, which have to be specified before the optimization process begins. Additionally, the weighted sum method is not able to represent complex preferences and in some cases will only approximate the decision maker's preferences. The one issue with the Epsilon constraint approach is that it is necessary to preselect which objective to minimize and the epsilon j values. This is problematic as for many values of epsilon; there may not be a feasible solution (Pike-Burke [16]).

According to Yu and Solvang [22] weighted sum is an a priori method, which means the weight of each objective function must be pre-determined and the optimal result obtained is significantly affected by the given weights. Therefore, the weighted sum is not an effective method when the relative importance of each objective is unclear or cannot be pre-determined by decision makers, and which is frequently encountered in the system planning of hazardous waste management. For posteriori decision-making, the weighted sum is neither able to generate evenly distributed pareto solutions nor a complete set of points at the pareto frontier. The authors may point out that more details on the weaknesses of the weighted sum method are given by (Das and Dennis [4]). According to Chirop and Zammit-Mangion [2], in most cases, the weighted sum method is unable to capture the middle ground of the pareto set, rendering it fairly useless as a means of studying the trade-off between conflicting objectives.

According to Mavrots[13], the value selected for epsilon is determined by payoff matrix, and it has great influence on the pareto frontier generated. The payoff matrix calculated by conventional epsilon-constraint method may lead to dominated or weakly efficient solutions which result in an unevenly distributed pareto optimal curve.

Bi-criteria eggs transportation problems are very important from practical point of view because they take care of those real life eggs transport planning and control problems from the economic world which have the mathematical structure of eggs transportation problems but are characterized by the existence of two objective functions: minimization of total eggs shipping cost and minimization of eggs shipping time. This paper presents minimization of shipping cost and minimization of shipping time in the objective function of a bi-criteria eggs transportation problem. The majority of the widely quoted authors (Glickman and Berger [6], Srinivasan and Thompson [18], Derigs [5], Srinivasan and Thompson [19], Gupta [8]) have worked on shipping cost and shipping time objective functions in bi-criteria transportation problem. Gupta and Arora [7] developed an algorithm to find optimum cost-time trade-off pairs in a fractional plus fractional capacitated transportation problem. Khurana and Arora [11] developed an algorithm to find an efficient cost-time trade off pairs in a fixed charge bi-criterion quadratic transportation problem.

From the viewpoint of a developing economy like India, cost considerations are important in eggs transportation; hence the traditional transportation cost view is incorporated along with eggs shipping time. Services for the transportation of eggs by road are in particular demand, as road transport guarantees the fastest and cheapest delivery. Eggs being a perishable commodity, shipping time and shipping cost considerations are very relevant and shipping time of eggs should be minimized to avoid substantial quality deterioration, contamination and price losses. Jacobs et al. [9] presented a study and investigated the effects of transportation duration and parental flock age on chick welfare, productivity and quality. The main aim of the research presented by Mertens et al. [14] was to monitor the percentage of eggshell breakage in four different production and logistic chains, from laying to final destination, to reveal critical points at which breakage occurs. Thompson and Hamilton [20] stated that most eggs are broken during transportation, rather than any other step during processing and distribution.

After the eggshell breakage or partial breakage, the shells of damaged eggs present in egg cartons or trays are a perfect foil for bacterial infections and the total value of such eggs is zero and it is a loss. Hence it is necessary to restrict the eggshell breakage to a known specified level. In this paper, an algorithm and its underlying theory is developed to solve eggshell breakage restricted bi-criteria eggs transportation problem. The paper also discusses a more realistic and general assumption that the eggs shipping time $t_{ij}(x_{ij})$ of bi-criteria eggs transportation problem depends on the quantity x_{ij} of eggs transported and is an increasing piecewise constant function. The algorithm generates all eggs shipping cost-time solution pairs that are pareto optimal with respect to the eggs shipping time and eggs shipping cost and for demonstration of the efficacy of the algorithm, a real life eggs shipping problem of Suguna Poultry Farm, Tamil Nadu (a state in India) is taken.

2. MATHEMATICAL FORMULATION

The mathematical formulation of eggshell breakage restricted bi-criteria eggs transportation problem [P1] is as follows:

[P1] Minimize $$Z(X) = \sum_{i=1}^{M'} \sum_{j=1}^{N'} c_{ij} x_{ij} \tag{1}$$

Minimize $$T(X) = \max_{i,j} \{t_{ij}(x_{ij})\} \tag{2}$$

subject to $$\sum_{j=1}^{N'} x_{ij} = a_i \qquad a_i > 0 \tag{3}$$

$$\sum_{i=1}^{M'} x_{ij} = b_j \qquad b_j > 0 \tag{4}$$

and $$\sum_{i=1}^{M'} f_{ijk} \cdot x_{ij} \le q_{jk}, \qquad \text{for all } j \text{ and } k \tag{5}$$

$$x_{ij} \ge 0, \qquad \text{for all } i \text{ and } j \tag{6}$$

also $$x_{ij} \le V_{ij}, \qquad \text{for } (i,j) \in J \tag{7}$$

$$(i = 1,2,\ldots\ldots, M';\ j = 1,2,\ldots\ldots, N';\ k = 1,2,\ldots\ldots, P')$$

where a_i is the quantity of eggs available at the i^{th} poultry farm and b_j is the quantity of eggs required at the j^{th} market. One unit of the egg's cartons or tray contains f_{ijk} units of P' breakages of eggshell $(k = 1,2,\ldots\ldots, P')$ when it is transported from poultry farm i to market j. q_{jk} is the units of highest amount of eggshell breakage that can be received by the market j, and x_{ij} is the amount of the eggs commodity transported from poultry farm i to market j. c_{ij} is unit shipping cost of eggs from poultry farm i to market j. Here $t_{ij}(x_{ij})$ is the shipping time of quantity x_{ij} of eggs commodity transported from poultry farm i to market j and is dependent on x_{ij}. It is an increasing piecewise constant function and that the eggs commodity interval $[0, V_{ij}]$ of possible values of x_{ij} is divided into a number of eggs commodity subintervals such that

$t_{ij}(x_{ij})$ is constant in each subinterval. For each poultry farm i to market j, the eggs commodity interval $[0, V_{ij}]$ is divided into eggs commodity subintervals as follows:

$$0 = \lambda_{ij}^{o} < \lambda_{ij}^{1} < \ldots < \lambda_{ij}^{e} = V_{ij} \tag{8}$$

and

$$t_{ij}^{1} < t_{ij}^{2} < \ldots < t_{ij}^{e} \tag{9}$$

Then

$$t_{ij}(x_{ij}) = \begin{cases} 0, & if \;\; x_{ij} = 0 \\ t_{ij}^{d}, & if \;\; \lambda_{ij}^{d-1} < x_{ij} \le \lambda_{ij}^{d}, \quad 1 \le d \le e \end{cases} \tag{10}$$

Here V_{ij} is the eggs capacity limitation from each poultry farm i to market j. Also J is the set of all cells (i, j) in the eggs transportation array for which there is a capacity limitation on the eggs commodity that can be shipped from poultry farm i to market j. J' is the set of all the remaining cells in the eggs transportation array. It is also not necessary that all the eggs commodity intervals $[0, V_{ij}]$ are subdivided into the same number of subintervals and it is assumed without loss of generality that

$$V_{ij} \le \min[a_i, b_j] \qquad \text{for all } (i, j) \tag{11}$$

In the following, the problem [P1] defined with the help of (1) through (10) will be denoted by problem R. Here a_i and b_j are given non-negative numbers and

$$\sum_{i=1}^{M'} a_i = \sum_{j=1}^{N'} b_j \tag{12}$$

In this context, it may be noted that eggshell breakage restrictions equation (5) can be written as:

$$\sum_{i=1}^{M'} f_{ijk} \cdot x_{ij} + x_{M'+k,j} = q_{jk} \tag{13}$$

$$x_{M'+k,j} \ge 0 \tag{14}$$

where $x_{M'+k,j}$ are the slack variables. There are total $N'P' + M'N'$ variables including slacks and $N'P' + M' + N'$ equations. Also a basic feasible solution will consist of $N'P' + M' + N' - 1$ basic variables.

3. FEASIBILITY OF EGGS SHIPPING TIME

T is said to be a feasible eggs shipping time for the problem R, if there exists a feasible solution X for the problem R, with $T(X) \le T$. Z is said to be optimal total eggs shipping cost and feasible solution X for problem R be cost optimal. Let $Z(X) = Z$ and $T(X) = T$. The eggs shipping cost-time pair (Z, T) is called a solution pair. This solution pair is said to be eggs trade-off solution pair, if there exists no other (Z^*, T^*) such that:

(a) $Z^* \le Z$ and $T^* < T$

(b) $Z^* < Z$ and $T^* \le T$

To check the feasibility of eggs shipping time T, consider the following commodity dependent shipping time based Eggs Shipping Cost Minimizing Transportation Problem (ESCMTP):

(ESCMTP) Minimize $$Z^*(X) = \sum_{i=1}^{M'} \sum_{j=1}^{N'} c_{ij}^{*} x_{ij} \tag{15}$$

subject to (3), (4), (5) and (6).

where
$$c_{ij}^* = \begin{cases} M, & \text{if } t_{ij}^d > T \\ c_{ij}, & \text{if } t_{ij}^d \le T, \quad \text{for some } d \ge 1 \end{cases} \tag{16}$$

and
$$x_{ij} \le V_{ij}^* = \begin{cases} V_{ij}, & \text{if } t_{ij}^d > T \\ \lambda_{ij}^d, & \text{if } t_{ij}^d \le T < t_{ij}^{d+1} \quad d \ge 1 \end{cases} \tag{17}$$

Here M is the usual high cost. The eggs shipping time T is feasible time for the problem ESCMTP, if the ESCMTP has a feasible solution with a finite optimal value Z^*, otherwise it is an infeasible time.

4. ALTERING A BASIC FEASIBLE SOLUTION

If a basic feasible solution is to be updated by the introduction of a non-basic variable and the removal of basic one then alterations can only be made to the basic variables. To determine the incoming variable, select the minimum difference between the true and fictitious cost:

$$\Delta_{i_o j_o} = \min\{\Delta_{ij} \mid \Delta_{ij} < 0\} \text{ or } \Delta_{M'+k_o, j_o} = \min\{\Delta_{M'+k,j} \mid \Delta_{M'+k,j} < 0\} \tag{18}$$

By applying the selection rule (18), the variable $x_{i_o j_o}$ or $x_{M'+k_o j_o}$ then becomes a basic variable of the new basic feasible solution, and an unknown quantity θ is to be added to this variable while $\theta.n_{rs}$ or $\theta.n_{M'+y,s}$ is added to all the basic variables x_{rs} or $x_{M'+y,s}$. If the new solution satisfies the original constraints, the n's must satisfy the equations set:

$$\sum_{r=1}^{M'} n_{rs} = 0 \qquad (s = 1,2,\ldots, N') \tag{19}$$

$$\sum_{s=1}^{N'} n_{rs} = 0 \qquad (r = 1,2,\ldots, M') \tag{20}$$

$$\sum_{y=1}^{P'} f_{rsy} \cdot n_{rs} + n_{M'+y,s} = 0 \qquad (s = 1,2,\ldots, N';\ r = 1,2\ldots, M') \tag{21}$$

Here, $n_{rs} = 0$, if x_{rs} is not in the basis and $n_{M'+y,s} = 0$ if $x_{M'+y,s}$ is not in the basis. There are $N'P' + M' + N' - 1$ independent equations in the set (19), (20) and (21) and $N'P' + M' + N'$ unknown n's. It is therefore possible to solve this set of equations for the $(N'P' + M' + N' - 1)$ n's associated with basic variables in terms of $n_{i_0 j_0}$. Furthermore, the values of the variables in the updated basic feasible solution are given by $x_{rs} + n_{rs}.\theta$; $x_{M'+y,s} + n_{M'+y,s}.\theta$.

By choosing a suitable value of θ from the following equation a new updated basic feasible solution is obtained:

$$\theta = \min_{\substack{n_{rs} < 0 \\ n_{M'+y,s} < 0}} \left[-\frac{x_{rs}}{n_{rs}};\ -\frac{x_{M'+y,s}}{n_{M'+y,s}} \right] \tag{22}$$

5. THE ALGORITHM

The algorithm to solve the eggshell breakage restricted bi-criteria eggs transportation problem [P1] is divided in four phases:

Phase I: Obtaining shipping cost-time trade-off solution pair (Z_1, T_1):

In the first phase, the optimal solution X_1 of the eggs shipping cost transportation problem is determined. This is used to compute optimal total shipping cost $Z(X_1)$ and feasible shipping time T_1 which helps in obtaining eggs shipping cost-time trade-off solution pair (Z_1, T_1). The stepwise description of Phase I is as follows:

Step 1: Determine the optimal solution $X_1 = \{(X_{ij})_1, (X_{M'+k,j})_1\}$ of the problem defined by objective function (1) subject to the constraints (3), (4), (5) and (6) using the following steps:

(a) Find the initial basic feasible solution by applying the inspection method.

(b) Determine dual variables u_i, v_j and w_{jk} defined such that

$$c_{ij} - \left(u_i + v_j + \sum_{k=1}^{P'} w_{jk} \cdot f_{ijk} \right) = 0 \tag{23}$$

(for those i, j for which x_{ij} is in the basis)

and

$$w_{jk} = 0 \tag{24}$$

(for those j, k for which $x_{M'+k,j}$ is in the basis)

(c) Evaluate

$$\Delta_{ij} = c_{ij} - \left(u_i + v_j + \sum_{k=1}^{P'} w_{jk} \cdot f_{ijk} \right) \tag{25}$$

and

$$\Delta_{M'+k,j} = w_{jk} \tag{26}$$

for all non basic cells.

(d) If all $\Delta_{ij} \geq 0$ and $\Delta_{M'+k,j} \geq 0$, then current basic feasible solution is cost optimal, go to Step 1(e). Otherwise improve the solution using the equations set (19), (20), (21) and (22) and go to the Step 1(b).

(e) The optimal cost solution gives the optimal eggs transportation schedule

$$X_1 = \{(X_{ij})_1, (X_{M'+k,j})_1\}$$

Step 2: Compute optimal total eggs shipping cost using (27) and determine the feasible eggs shipping time T_1 corresponding to X_1 using (28) to obtain the eggs shipping cost-time trade-off solution pair (Z_1, T_1)

$$Z(X_1) = Z_1 \tag{27}$$

$$T(X_1) = T_1 = \max\{t_{ij}(x_{ij}) \mid x_{ij} > 0\} \tag{28}$$

Phase II: Formulation of Shipping Cost Minimizing Transportation Problem (SCMTP):

In this phase, SCMTP is formulated by determining eggs shipping time T_e^* and checking its feasibility. This will be done by modifying the eggs shipping cost matrix andthe eggs capacity limitation. The stepwise description of Phase II is as follows:

Step 3: Determine eggs shipping time using (29) by increasing the value of e to (e +1). Go to the Step 4 to check its feasibility

$$T_e^* = \max_{i,j,d} \{t_{ij}^d \mid t_{ij}^d < T_{e-1}\} \tag{29}$$

Step 4: Modify the eggs shipping cost matrix c_{ij} to get c_{ij}^* using (16) and the eggs capacity limitation V_{ij} to get V_{ij}^* using (17). Now using c_{ij}^* and V_{ij}^* formulate commodity dependent shipping time based

SCMTP. A feasible solution of SCMTP is a basic feasible solution if it is associated with a working basis with the property that all non basic variables x_{ij} for $(i, j) \in J$ are either equal to 0 or V_{ij}^* in the solution. Let $\bar{X}_e$ be a basic feasible solution of SCMTP associated with a working basic set β.

Phase III: Determining optimal solution for SCMTP

To improve the solution, Δ_{ij} and $\Delta_{M'+k,j}$ are evaluated and finally the optimal solution is determined. The stepwise description of Phase III is:

Step 5: For the solution of SCMTP given in Step 4, determine the dual variables u_i, v_j and w_{jk} using (23), (24) and evaluate Δ_{ij} and $\Delta_{M'+k,j}$ for all non- basic cells from (25) and (26).

Step 6: This step defines the first optimality criteria. There may be three conditions:

a) If $\Delta_{ij} < 0$ and $(\bar{x}_{ij})_e = V_{ij}^*$, (30)

the current basic feasible solution is optimal. Go to Step 7 to check the second optimality criteria.

b)If $\Delta_{ij} < 0$ and $(\bar{x}_{ij})_e \neq V_{ij}^*$, i.e. $x_{ij} = 0$, then the solution is not optimal. Go to Steps10 to change the status of the cell.

c)If $\Delta_{ij} > 0$ and $(\bar{x}_{ij})_e = V_{ij}^*$, then go to Step 8 to change the status of the cell.

Step 7: This step defines the second optimality criteria.

If $\Delta_{ij} > 0, \quad \Rightarrow \quad (\bar{x}_{ij})_e = 0$ (31)

$\Delta_{M'+k,j} > 0, \quad \Rightarrow \quad (\bar{x}_{M'+k,j})_e = 0$ (32)

then the current basic feasible solution is optimal for the second optimality criteria.
Therefore go to Step 12 to compute eggs shipping cost-time trade-off solution pair (Z_e, T_e).

Otherwise go to Step 10 to change the status of cell. If the SCMTP has an infinite optimal value, then go to Step 13.

Step 8: For change of status, choose a non-basic cell $(i_o, j_o)_e$ that violates the first optimality criteria. In this case, $(i_o, j_o)_e \in J; \Delta_{i_0 j_0} > 0$ and $x_{i_o j_o} = V_{i_o j_o}^*$. In this situation, the value of the objective function can be decreased by decreasing the value of $x_{i_o j_o}$ from its present value of $V_{i_o j_o}^*$. Therefore an unknown quantity θ is to be added $V_{i_o j_o}^*$, i.e. the new value $x_{i_o j_o}$ will be $V_{i_o j_o}^* + \theta$, and $\theta.n_{rs}$ or $\theta.n_{M'+y,s}$ is added to all the basic variables x_{rs} or $x_{M'+y,s}$. Now if the new solution satisfies the original constraints, the n's must satisfy the equations (19), (20) and (21).

Choose a suitable value of θ from

$$\theta = \min_{\substack{n_{rs}<0 \\ n_{M'+y,s}<0}} \left[-\frac{x_{rs}}{n_{rs}};\ -\frac{x_{M'+y,s}}{n_{M'+y,s}};\ -V_{i_o j_o}^* \right] \tag{33}$$

So θ should be negative and as small as possible. To select the smallest value of θ, go to the Step 9.

Step 9: The smallest value that θ can have is:

Max [**(a)** $-V_{i_o j_o}^*$, then $x_{i_o j_o}$ becomes a non-basic variable whose value is 0 in the next step. Revise the values of all the basic variables in the θ-loop by substituting $-V_{i_o j_o}^*$ for θ.

(b) $> -V_{i_o j_o}^*$ and is $-\bar{x}_{i',j'}$ for some basic cell (i', j') with $a+\theta$ entry in the θ-loop. Then this cell is dropped from the working basic set and made a non-basic cell with a zero value. The variable $x_{i_o j_o}$ becomes a basic variable, with value $(V_{i_o j_o}^* - \bar{x}_{i'j'})$ in the next solution. Revise the values of all the basic variables in the θ loop by substituting the value $-\bar{x}_{i',j'}$ for θ.

(c) $> -V^*_{i_oj_o}$ and is $(\bar{x}_{i',j'} - V^*_{i_oj_o})$ for some basic cell (i', j') with $a-\theta$ entry in the θ-loop with $(i', j') \in J$, then $x_{i'j'}$ is made a non-basic variable whose value is equal to its upper bound in the next solution. The variable $x_{i_oj_o}$ becomes a basic variable in its place with its value equal to $V^*_{i_oj_o} + \theta$ in the next solution. The values of all the basic variables in the θ-loop are revised by substituting $(\bar{x}_{i',j'} - V^*_{i',j'})$ for θ.]

Go to the Step 5.

Step 10: To change the status of non basic cell in the current basic feasible solution, choose a non basic cell (i_o, j_o) or $(M'+k_o, j_o)$ that violates the second optimality criteria.

To determine entering variable, select the minimum

$$\Delta_{i_oj_o} = \min[\Delta_{ij} \mid \Delta_{ij} < 0] \quad \text{and} \quad x_{i_oj_o} = 0$$

$$\Delta_{M'+k_o,j_o} = \min[\Delta_{M'+k,j} \mid \Delta_{M'+k,j} < 0] \quad \text{and} \quad x_{M'+k_o,j_o} = 0$$

By applying the above selection rule, the variable $x_{i_oj_o}$ or $x_{M'+k_o,j_o}$ becomes an entering variable. In this situation, the value of the objective function can be decreased by increasing the value of $x_{i_oj_o}$ or $x_{M'+k_o,j_o}$ from its present value of 0.

Therefore an unknown quantity θ is to be added to non-basic variable $x_{i_oj_o}$ or $x_{M'+k_o,j_o}$ while $\theta \cdot n_{rs}$ or $\theta \cdot n_{M'+y,s}$ is added to all the basic variables x_{rs} or $x_{M'+y,s}$. Now if the new solution satisfies the original constraints, the n's must satisfy the equations (19), (20) and (21).Furthermore the values of the variables in the updated basic feasible solution are given by $x_{rs} + n_{rs}.\theta$; $x_{M'+y,s} + n_{M'+y,s}.\theta$.

Choose value of θ from:

$$\theta = \min_{\substack{n_{rs}<0 \\ n_{M'+y,s}<0}} \left[-\frac{x_{rs}}{n_{rs}};\ -\frac{x_{M'+y,s}}{n_{M'+y,s}} \right] \tag{34}$$

If $+\theta$ entry is in the cell (i_o, j_o) or $(M'+k_o, j_o)$, then go to Step 11 to select the maximum value of $x_{i_oj_o}$ or $x_{M'+k_o,j_o}$.

Step 11: The maximum value that can be given to $x_{i_oj_o}$ or $x_{M'+k_o,j_o}$ is:

Min [**(a)** $V^*_{i_oj_o}$, if $(i_o, j_o) \in J$. Make $x_{i_oj_o}$ into a non-basic variable whose value is equal to its upper bound in the next step. Revise the values of all the basic variables in the θ loop by substituting $V^*_{i_oj_o}$ for θ and then erase the θ loop. Keep the same working basis.

(b) $< V^*_{i_oj_o}$ and turns out to be $\bar{x}_{i'j'}$ or $\bar{x}_{M'+k',j'}$, where (i', j') or $(M'+k', j')$ is a basic cell with $a-\theta$ entry in the θ-loop. Drop the cell (i', j') or $(M'+k', j')$ from the working basic set and make the cell (i_o, j_o) or $(M'+k_o, j_o)$ a basic cell. Thus change the working basic set. Make the value of $x_{i_oj_o}$ or $x_{M'+k_o,j_o}$ equal to $\bar{x}_{i'j'}$ or $\bar{x}_{M'+k',j'}$, and revise the values of all the basic variables in the θ-loop by substituting $\bar{x}_{i'j'}$ or $\bar{x}_{M'+k',j'}$, for θ. Make (i', j') or $(M'+k', j')$ a zero valued non-basic cell.

(c) $< V^*_{i_oj_o}$ and turns out to be $(V^*_{i_oj_o} - \bar{x}_{i'j'})$ for a basic cell (i', j'), such that $(i', j') \in J$ and (i', j') is in the θ-loop with $a+\theta$ entry. (i', j') is dropped from the working basic set and make it a non-basic cell at its upper bound in the next solution. Make $x_{i_o j_o}$ or $x_{M'+k_o, j_o}$ equal to $(V^*_{i'j'} - \bar{x}_{i'j'})$ and revise the values of all the basic variables in the θ-loop by substituting the same value for θ. Make (i_o, j_o) or $(M' + k_o, j_o)$ a basic cell.]

Go to the Step 5.

Phase IV: Computation of (Z_e, T_e) for optimal solution of SCMTP

In this phase, eggs shipping time T_e and pair (Z_e, T_e) is determined and its feasibility is checked. The stepwise description of Phase IV is:

Step 12: If $(\bar{X}_e) = \{(\bar{x}_{ij})_e, (\bar{x}_{M'+k,j})_e\}$ is an optimal solution of SCMTP, then compute:

Finite total optimal eggs shipping cost = Z_e

and feasible eggs shipping time = T_e

Obtain eggs shipping cost-time trade-off solution pair (Z_e, T_e) and go to Step 3.

Step 13: The eggs shipping time T_e^* is thus an infeasible shipping time.

Step 14: Examine the obtained eggs shipping cost-time trade-off solution pairs for redundant solutions (if any) i.e. those eggs shipping cost-time trade-off solutions which have the same optimal eggs shipping cost but different feasible eggs shipping times. Select those eggs shipping cost-time trade-off solution pairs which have a better or smaller feasible eggs shipping time and obtain final eggs shipping cost-time trade-off solution pairs.

Most of the developed algorithm in the area of bi-criteria transportation problem have the fundamental assumption that the transportation time required for transporting a positive amount in a route is independent of the actual amount transported in that route. However the author has developed a new algorithm which discusses a more realistic and general assumption that the eggs shipping time of bi-criteria eggs transportation problem depends on the quantity of eggs transported and for each route is an increasing function. To the best knowledge of the author, this increasing piecewise constant function has not been exploited much in literature. The author also noticed that in Suguna Poultry Farm of Tamil Nadu (a state in India), more eggs are broken during shipping than in any other step; therefore, losses due to eggshell breakage are maximum during shipping stage. Hence the author introduced additional eggshell breakage restrictions in the formulation of bi-criteria eggs transportation problem. The developed algorithm generates all eggs shipping cost-time solution pairs that are pareto optimal with respect to the eggs shipping time and eggs shipping cost. The algorithm terminates when modification of total eggs shipping cost results in there being no feasible solution, that is, there exists an insufficient number of unprohibited routes to enable demands to be met from the available supplies. The algorithm indicates that a successive reduction in the eggs shipping time is there at the cost of an increase in the minimum total eggs shipping cost. Such decreasing (but not necessarily convex) behavior is a general result from application of our algorithm.

The above algorithm terminates in a finite number of steps because only a finite number of different eggs shipping time are to be checked for their feasibility. The author may also like to point out the following advantages of the proposed algorithm:

1. In our algorithm, the consideration of the eggs shipping cost–time trade-offs will give greater insight into the structure and sensitivity of the bi-criteria eggs transportation problem and subsequently to more rational decisions, especially in emerging economy like India where application of the proposed algorithm is relevant.
2. In our algorithm the decision maker conducts the search by introducing upper bounds on the values of the eggs commodity and adding one additional eggshell breakage constraints to the original /standard transportation structure. Hence usual algorithms for solving transportation problems cannot be used.
3. The construction of a sequence of solutions having different objective values as well as quality helps in cases not only where the eggs shipping time objective is an equally crucial factor besides cost but also when analyzing the practicability and sensitivity of an existing transportation situation.
4. The algorithm takes into account the special structure of the problem and helps the decision maker by eliminating all the inefficient solutions.

5. The algorithm will prove to be useful in making the transportation problem formulated more realistic in logic and other application areas.

6. REAL LIFE EGGS SHIPPING PROBLEM

The algorithm is illustrated by the following real life eggs shipping problem of Suguna Poultry Farm, Tamil Nadu, India:

The Indian egg consumers of different locations of Tamil Nadu state receive eggs through wholesalers, wholesalers-cum-retailers and retail outlets which in turn receive a fixed quantity of eggs from main central distribution warehouses. The Central Distribution Warehouses (j) located at four different major cities: Chennai, Madurai, Tiruchirappalli and Rameshwaram of Tamil Nadu state of India have different types of egg distribution units in their Warehouses. These multi-locational egg distribution units are receiving regular supply of eggs from four major branches of Suguna Poultry Farm (i) located at Nagercoil, Coimbatore, Namakkal and Salem of Tamil Nadu state of India. The basic goal is to generate eggs shipping cost-time trade-off solution pairs by determining all feasible eggs transportation schedules which minimizes the total eggs shipping cost c_{ij} from four major branches of Suguna Poultry Farm i to Central Distribution Warehouses j and also minimizes the maximum of eggs shipping time while satisfying the extra requirement that the amount of eggshell breakage present in egg cartons or trays is less than a certain specific level. Also the eggs shipping time $t_{ij}(x_{ij})$ from four major branches of Suguna Poultry Farm i to Central Distribution Warehouses j depends on the actual amount of eggs quantity x_{ij} transported and is an increasing piecewise constant function.

In Table 1 for each route (i, j) the eggs shipping cost c_{ij} are written in the top left corner while the partition $0 = \lambda_{ij}^{o} < \lambda_{ij}^{1} < ... < \lambda_{ij}^{e} = V_{ij}$ and the corresponding eggs shipping times $t_{ij}^{1} < t_{ij}^{2} < ... < t_{ij}^{e}$ are shown on the top right corner. Availabilities a_i and the eggshell breakage contents p_i are listed in the last two columns while requirements b_j and maximum eggshell breakage contents L_j are shown in the last two rows respectively. Let x_{ij} be the amount of the eggs commodity transported from major branches of Suguna Poultry Farm i to Central Distribution Warehouses j then it is required to

$$\min \quad Z(X) = \sum_{i=1}^{4}\sum_{j=1}^{4} c_{ij} x_{ij}$$

$$\min \quad T(X) = \max_{i,j}\{t_{ij}(x_{ij})\}$$

subject to

$$\sum_{j=1}^{4} x_{ij} = a_i , \qquad a_i > 0 \qquad (i = 1,2,3,4)$$

$$\sum_{i=1}^{4} x_{ij} = b_j , \qquad b_j > 0 \qquad (j = 1,2,3,4)$$

$$\sum_{i=1}^{4} p_i \cdot x_{ij} \le L_j b_j$$

$$x_{ij} \ge 0 , \qquad \text{for all } i \text{ and } j$$

also

$$x_{ij} \le V_{ij} , \qquad \text{for } (i, j) \in J$$

Table 1: Eggs shipping problem of Suguna Poultry Farm, Tamil Nadu, India

		Central Distribution Warehouses j				a_i	p_i
		1	2	3	4		
SUGUNA FARM i	1	C_{11}=1 19(0-15) 25(15-30) 31(30-60)	C_{12}=2 12(0-25) 15(25-50)	C_{13}=7 16(0-15) 17(15-25) 18(25-30)	C_{14}=4 7(0-30) 10(30-60)	70	4
	2	C_{21}=9 8(0-20) 9(20-30) 10(30-60)	C_{22}=4 11(0-30) 12(30-65) 13(65-100)	C_{23}=5 13(0-30) 14(30-50) 16(50-60)	C_{24}=8 5(0-20) 6(20-60)	110	8
	3	C_{31}=5 20(0-32) 25(32-35) 31(35-40)	C_{32}=6 17(0-30) 18(30-47) 20(47-55)	C_{33}=3 17(0-25) 20(25-32) 25(32-50)	C_{34}=5 3(0-30) 4(30-60)	90	6
	4	C_{41}=1 1(0-25) 2(25-40)	C_{42}=5 8(0-30) 11(30-45) 16(45-60)	C_{43}=3 17(0-20) 21(20-40)	C_{44}=6 14(0-12) 16(12-30)	30	7
b_j		60	100	80	60		
L_j		7	7	7	7		

The following starting basic feasible solution is determined by applying the Inspection method:

x_{11} = 40 , x_{12} = 25, x_{14} = 5, x_{22} = 75, x_{23} = 35, x_{33} = 35,

x_{34} = 55, x_{41} = 20, x_{43} = 10, x_{51} = 120, x_{54} = 70.

Using this solution, the associated dual variables u_i, v_j, w_{jk} $(i=1,2,3,4;\ j=1,2,3,4;\ k=1)$; Δ_{ij}, $\Delta_{M'+k,j}$ are calculated as explained in Step 1 and finally first optimal cost solution X_1 is obtained.

Now First total optimal eggs shipping cost $Z(X_1) = Z_1$ = 1015

First feasible eggs shipping time $T(X_1) = T_1$ = 31

And first eggs shipping cost-time trade-off solution pair (Z_1, T_1) = (1015, 31)

Determine eggs shipping time $T_e^* = \max_{i,j,d}\{t_{ij}^d \mid t_{ij}^d < T_{e-1}\}$ = 25

Modify the eggs cost matrix c_{ij} to get c_{ij}^* using (16) and also modify the eggs capacity limitation V_{ij} to get V_{ij}^* using (17) to formulate ESCMTP. Use last optimal solution of problem as starting solution of modified ESCMTP. The solution X_1 having actual amount $x_{ij}, x_{M'+k,j}$ of eggs are shown in the bottom left corner while maximum amount of eggs V_{ij}^* that can be transported are shown, wherever necessary, at the bottom right corner of the cell of Table 2.

The modified eggs shipping cost c_{ij}^* are written in the top left corner while eggs commodity partition and the corresponding times are shown in the top right corner of the cell. L_j and b_j are displayed in the bottom two rows while p_i and a_i are shown in the marginal right column respectively. Table 3 shows the upper bound restriction in X_1, and after some iteration the second cost optimal solution X_2 is obtained.

Table 2: Eggs Shipping Problem with X_1

	Central Distribution Warehouses j	a_i	p_i

		1		2		3		4			
S U G U N A	1	C_{11}=M	19(0-15) 25(15-30) 31(30-60)	C_{12}=2	12(0-25) 15(25-50)	C_{13}=7	16(0-15) 17(15-25) 18(25-30)	C_{14}=4	7 (0-30) 10(30-60)	70	4
		x_{11}=40		x_{12}=25				x_{14}=5			
	2	C_{21}=9	8(0-20) 9(20-30) 10(30-60)	C_{22} = 4	11(0-30) 12(30-65) 13(65-100)	C_{23}=5	13(0-30) 14(30-50) 16(50-60)	C_{24}=8	5(0-20) 6(20-60)	110	8
				x_{22}=75		x_{23}=35					
F A R M	3	C_{31}=5	20(0-32) 25(32-35) 31(35-40)	C_{32}=6	17(0-30) 18(30-47) 20(47-55)	C_{33}=3	17(0-25) 20(25-32) 25(32-50)	C_{34}=5	3(0-30) 4(30-60)	90	6
			V^*_{31}= 35			x_{33}=35	**V^*_{33}=32**	x_{34}=55			
i	4	C_{41}=1	1(0-25) 2(25-40)	C_{42}=5	8 (0-30) 11(30-45) 16(45-60)	C_{43}=3	17(0-20) 21(20-40)	C_{44} = 6	14(0-12) 16(12-30)	30	7
		x_{41}=20				x_{43}=10					
		x_{51}=120						x_{54}=70			
b_j		60		100		80		60			
L_j		7		7		7		7			

Now Second total optimal eggs shipping cost $Z(X_2) = Z_2 = 1130$

Second feasible eggs shipping time $T(X_2) = T_2 = 25$

and Second eggs shipping cost-time trade-off solution pair (Z_2, T_2) = (1130, 25

Table 3: Eggs Shipping Problem with X_1

		Central Distribution Warehouses j								a_i	p_i
		1		2		3		4			
S U G U N A	1	**C_{11}=M**	19(0-15) 25(15-30) 31(30-60)	C_{12}=2	12(0-25) 15(25-50)	C_{13}=7	16(0-15) 17(15-25) 18(25-30)	C_{14}=4	7(0-30) 10(30-60)	70	4
		x_{11}= 5		x_{12}=25				x_{14}= 40			
	2	C_{21}=9	8(0-20) 9(20-30) 10(30-60)	C_{22}=4	11(0-30) 12(30-65) 13(65-100)	C_{23}=5	13(0-30) 14(30-50) 16(50-60)	C_{24}=8	5(0-20) 6(20-60)	110	8
				x_{22}=75		X_{23}=35					
F A R M i	3	C_{31}=5	20(0-32) 25(32-35) 31(35-40)	C_{32}=6	17(0-30) 18(30-47) 20(47-55)	C_{33}=3	17(0-25) 20(25-32) 25(32-50)	C_{34}=5	3(0-30) 4(30-60)	90	6
		$\boxed{x_{31}=35}$	**V^*_{31}= 35**			X_{33}=35	**V^*_{33}=32**	x_{34}=20			
	4	C_{41}=1	1(0-25) 2(25-40)	C_{42}=5	8(0-30) 11(30-45) 16(45-60)	C_{43}=3	17(0-20) 21(20-40)	C_{44}=6	14(0-12) 16(12-30)	30	7
		x_{41}=20				X_{43}=10					

	1		2		3		4		
	x_{51}=50						x_{54}=140		
b_j	60		100		80		60		
L_j	7		7		7		7		

Second cost optimal solution X_2 is shown in Table 4.

Determine the next eggs shipping time $T_e^* = \max_{i,j,d}\{t_{ij}^d \mid t_{ij}^d < T_{e-1}\}$ = 20

Table 5 shows the modified ESCMTP and after some iteration the third cost optimal solution X_3 is obtained.

Now Third total optimal eggs shipping cost $Z(X_3) = Z_3$ = 1365

Third feasible eggs shipping time $T(X_3) = T_3$ = 20

And Third eggs shipping cost-time trade-off solution pair (Z_3, T_3) = (1365, 20)

Table 4: Eggs Shipping Problem with X_2

		Central Distribution Warehouses j								a_i	p_i
		1		2		3		4			
SUGUNA	1	**C_{11}=M**	19(0-15) 25(15-30) 31(30-60)	C_{12}=2	12(0-25) 15(25-50)	C_{13}=7	16(0-15) 17(15-25) 18(25-30)	C_{14}=4	7(0-30) 10(30-60)	70	4
				x_{12}=20				x_{14}= 50			
	2	C_{21}=9	8(0-20) 9(20-30) 10(30-60)	C_{22}=4	11(0-30) 12(30-65) 13(65-100)	C_{23}=5	13(0-30) 14(30-50) 16(50-60)	C_{24}=8	5(0-20) 6(20-60)	110	8
				x_{22}=70		x_{23}=40					
FARM	3	C_{31}=5	20(0-32) 25(32-35) 31(35-40)	C_{32}=6	17(0-30) 18(30-47) 20(47-55)	C_{33}=3	17(0-25) 20(25-32) 25(32-50)	C_{34}=5	3(0-30) 4(30-60)	90	6
i		x_{31}=30	**V^*_{31}= 35**	x_{32}=10		x_{33}=40	**V^*_{33}=32**	x_{34}=10			
	4	C_{41}=1	1(0-25) 2(25-40)	C_{42}=5	8(0-30) 11(30-45) 16(45-60)	C_{43}=3	17(0-20) 21(20-40)	C_{44} = 6	14(0-12) 16(12-30)	30	7
		x_{41}=30									
		x_{51}=30						x_{54}=160			
b_j		60		100		80		60			
L_j		7		7		7		7			

The third cost optimal solution X_3 is shown in Table 6.

Determine the next eggs shipping time $T_e^* = \max_{i,j,d}\{t_{ij}^d \mid t_{ij}^d < T_{e-1}\}$ = 18

A further modification of eggs shipping cost results in the infeasibility of eggs shipping time i.e. there exists an insufficient number of unprohibited routes to enable demands of Central Distribution Warehouses to be met from the available major branches of Suguna Poultry Farm. Therefore, the algorithm terminates.

Table 5: Eggs Shipping Problem with X_2

		Central Distribution Warehouses j								a_i	p_i
		1		2		3		4			
SUGUNA FARM i	1	**C_{11}=M**	19(0-15) 25(15-30) 31(30-60)	C_{12}=2	12(0-25) 15(25-50)	C_{13}=7	16(0-15) 17(15-25) 18(25-30)	C_{14}=4	7(0-30) 10(30-60)	70	4
				x_{12}=20				x_{14}= 50			
	2	C_{21}=9	8(0-20) 9(20-30) 10(30-60)	C_{22} = 4	11(0-30) 12(30-65) 13(65-100)	C_{23}=5	13(0-30) 14(30-50) 16(50-60)	C_{24}=8	5(0-20) 6(20-60)	110	8
				x_{22}=70		x_{23}=40					
	3	C_{31}=5	20(0-32) 25(32-35) 31(35-40)	C_{32}=6	17(0-30) 18(30-47) 20(47-55)	**C_{33}=M**	17(0-25) 20(25-32) 25(32-50)	C_{34}=5	3(0-30) 4(30-60)	90	6
		x_{31}=30	**V^*_{31}= 32**	x_{32}=10		x_{33}=40	**V^*_{33}=32**	x_{34}=10			
	4	C_{41}=1	1(0-25) 2(25-40)	C_{42}=5	8(0-30) 11(30-45) 16(45-60)	C_{43}=3	17(0-20) 21(20-40)	C_{44} = 6	14(0-12) 16(12-30)	30	7
		x_{41}=30									
		x_{51}=30						x_{54}=160			
b_j		60		100		80		60			
L_j		7		7		7		7			

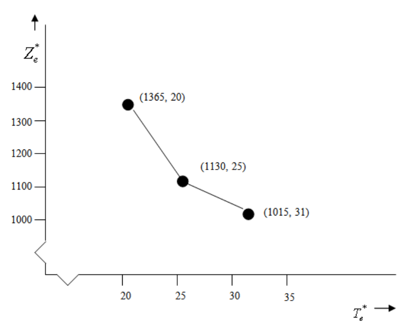

Figure 1: Eggs shipping cost-time trade-off curve

The set of efficient eggs shipping cost-time trade-off solution pairs is (1015, 31); (1130, 25); (1365, 20), and represented graphically in Figure 1 which gives a picture of the trade-offs that have been made in the sequence of solutions to this eggshell breakage restricted bi-criteria eggs transportation problem.

Table 6: Eggs Shipping Problem with X_3

	Central Distribution Warehouses j				a_i	p_i
	1	2	3	4		

S U G U N A	1	**C_{11}=M**	19(0-15) 25(15-30) 31(30-60)	C_{12} =2	12(0-25) 15(25-50)	C_{13} =7	16(0-15) 17(15-25) 18(25-30)	C_{14} =4	7(0-30) 10(30-60)	70	4
				x_{12}= 3/2		x_{13}= 39/2		x_{14}=49			
	2	C_{21} =9	8(0-20) 9(20-30) 10(30-60)	C_{22} =4	11(0-30) 12(30-65) 13(65-100)	C_{23} =5	13(0-30) 14(30-50) 16(50-60)	C_{24} =8	5(0-20) 6(20-60)	110	8
				x_{22}= 103/2		x_{23}= 117/2					
F A R M	3	C_{31} =5	20(0-32) 25(32-35) 31(35-40)	C_{32}=6	17(0-30) 18(30-47) 20(47-55)	**C_{33} =M**	17(0-25) 20(25-32) 25(32-50)	C_{34} =5	3(0-30) 4(30-60)	90	6
		$\boxed{x_{31}=32}$	**V^*_{31}= 32**	x_{32}=47			**V^*_{33}=32**	x_{34}=11			
i	4	C_{41}=1	1(0-25) 2(25-40)	C_{42}=5	8(0-30) 11(30-45) 16(45-60)	C_{43} =3	17(0-20) 21(20-40)	C_{44}=6	14(0-12) 16(12-30)	30	7
		x_{41}=28				x_{43}= 2					
		x_{51}=32						x_{54}=158			
b_j		60		100		80		60			
L_j		7		7		7		7			

This indicates that successive reduction in the eggs shipping time is there at the cost of an increase in the minimum total eggs shipping cost. With these egg shipping cost-time trade-off solution pairs set, the eggs transportation system decision maker is simply asked to select the best point of the set and then is given one of the associated solutions.

7. CONCLUDING REMARKS

In this paper an algorithm is developed for solving eggshell breakage restricted bi-criteria eggs transportation problem and all eggs shipping cost-time trade-off solution pairs that are pareto-optimal with respect to the eggs shipping cost and the eggs shipping time are also generated. The algorithm, which is very helpful for real-life multi-decision priority problems, takes into account the special structure of the eggs shipping cost-time transportation problem under consideration. The paper also discusses a more realistic and general assumption that the eggs shipping time of bi-criteria eggs transportation problem depends on the quantity of the eggs transported and is an increasing piecewise constant function.

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