MULTI- STATE SYSTEM ANALYSIS WITH IMPERFECT FAULT COVERAGE, HUMAN ERROR AND STANDBY STRATEGIES

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ABSTRACT
The present work studies the reliability measures of a standby system using coverage factor. The considered system is a combination of main unit and two standby units. Provision of standbys has been taken for smooth functioning of the system. On failure of main unit standby units take the load of main unit and if both standby units failed, system goes to in completely failed state. The failures and repairs of each unit follow exponential and general distribution respectively. The whole system has been analysed under imperfect coverage and human failure. Markov model has been developed to obtain the state transient probabilities. Various reliability measures like availability, MTTF, cost and sensitivity analysis have been evaluated with the help of supplementary variable technique and Laplace transformation. Some graphical illustrations have been taken for better understanding of the model.

KEYWORDS: Imperfect coverage, Redundancy, Human failure, Cost benefit, Markov process.

MSC: 90B25

RESUMEN
El presente paper estudia la fiabilidad de medidas de un sistema “standby” usando un factor de cubrimiento. El sistema considerado es una combinación de la unidad principal y dos unidades “standby”. Previsiones de “standbys” han sido tomados para suavizar el funcionamiento del sistema. Ante fallos de la unidad principal la unidades “standby” toman la carga de esta y si ambas unidades de “standby” fallan el sistema pasa a un estado de fallo completo. Los fallos y reparaciones de cada unidad siguen una distribución exponencial y una general respectivamente. El sistema en su totalidad ha sido analizado bajo cubrimiento imperfecto y fallo humano. Un modelo Markoviano ha sido desarrollado para obtener las probabilidades de transición del estado. Varias medidas miden la fiabilidad como la disponibilidad, MTTF, costo y análisis de sensibilidad han sido evaluadas con la ayuda de la técnica de la variable suplementaria y la transformación de Laplace. Algunas ilustraciones gráficas han sido usadas para mejor entender el modelo.

PALABRAS CLAVE: Cubrimiento Imperfecto, Redundancia, Fallo Humano, Costo beneficio, Proceso de Markov.

1. INTRODUCTION

Now- a- days the evaluation of the efficiency of the complex system is a big deal. Coverage factor is one parameter that used to specify this efficiency. Coverage factor is the probability of the system recovery to the fault occurs. (Arnold, 1973) i.e.,
Coverage factor = probability (system recovery from fault / fault occurs in the system)
More dramatic effect of coverage can be seen if one adds repairs at a constant rate for a covered failure and no repair for a non-covered failure (Dugan and Trivedi, 1989). Coverage gives the knowledge about the fault detection and system recovery capability (Ram and Manglik, 2016). Fault coverage is a proportion of a system's capability to observe fault recognition, fault locale, as well as fault recovery (Ram et al., 2013). Fault coverage can be assumed constant for simplicity. However, it may be a function of time. In a covered fault, system can recover automatically. It has been shown that the reliability of a fault-tolerant system is quite sensitive to the coverage parameter (Arnold, 1973). Pham (1992) studied a high voltage system with imperfect coverage and constant fault coverage. Levitin and Amari (2007) evaluated reliability characteristic of fault tolerance systems with multi fault coverage using the universal generating function technique.

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Prabhudeva and Verma (2007) studied general coverage model (GCM) based on hardware independent fault. GCM helps to perform failure mode effect analysis of a complex system. Hsu et al. (2009) estimated the reliability measures of a repairable system under imperfect coverage using Bayesian approach. Manglik and Ram (2014) talked about the effect of perfect coverage on the multi-state manufacturing system. Perfect coverage means that the system able to detect and removing the failure of the redundant system.

Further, some system such as aircraft, spacecraft require very high-levels of reliability. Hence, in such types of system, redundancy is used to achieve high-level of reliability (Myers, 2007). Redundancy is mostly used for improvement in reliability. There are basically two types of redundancy namely active and standby redundancy. Active redundancy is one which does not require extrinsic component to perform. In active redundancy, redundant element automatically takes the load of failed component. Active redundancy is also called load-sharing redundancy. On the other hand, standby redundancy is one which requires extrinsic component to perform and switch to another component as a replacement of failed component (Li, 2016). Wang and Chiu (2006) developed the analytic steady state result for the system having warm standby unit and imperfect coverage. Here, authors used different repair time distribution such as exponential, *k*-stage Erlang, deterministic and providing a recursive method using supplementary variable technique. Chen and Wang (2018) explored the study of repairable machining system having warm standby, with single repair and N policies. Oliveira et al. (2005) discussed the behaviour of safety system whose component are under aging with imperfect repair. Jain et al. (2019) studied the fault tolerant machining system (FTMS) consisting standbys and an experienced or trained repairman. Fard et al. (2017) analysed a one-unit repairable system having standby with imperfect coverage and calculated the reliability measures with the help of fuzzy parameters. The sensitivity analysis of real time Markovian model having standby redundancy have been analysed by Zheng et al. (2018).

The earlier researchers (Gupta and Sharma, 1993; Ram and Singh, 2010; Singh and Rawal, 2011; Shekhar et al., 2019) established different types of mathematical models with redundant systems and calculated the reliability characteristics for different types of failures and repairs. In past decades, much focus on the area of human failure (Dhillon and Liu, 2006; Sutcliffe and Gregoriades, 2007; Ram and Kumar, 2014). Human error occurs in the system due to incorrectly setup or function by unexperienced or untrained operator. Human error can damage many components of the system or completely failed to the system. Human error helps to identify human performance, reduction in cost and design to the system (Dhillon and Yang, 1992).

In the present work, the assessments of reliability measures through the Markov process have been obtained. Markov process is a very useful tool for redundant systems and has a constant failures and repairs. The considered complex system comprises of main unit and two standby units. At first, the main unit begin working, after failure of main unit, the first standby takes its place and failed main unit goes for repair. If the first standby failed, second standby takes the place of first standby and system working until the second standby fails. In case the failed main unit repair before failure of first standby it takes the load of first standby and standby goes for standby mode.

The system has completely failure states in following circumstances:

- Both standby units fail before repairing of main unit.
- Human error occurs at any stage of working.

Earlier Researchers mainly show their interest for the study of some measures like an availability, reliability, MTTF, cost analysis and sensitivity analysis but they do not talk sensitivity analysis under imperfect failure and human error. So, here authors have discussed about the sensitivity analysis of standby system under imperfect coverage, human error and supplementary variable technique with the help of Markov process.

The present paper is organised as follows: Section 2 provides the mathematical details of the model containing notations and structure of the model. Also, this section gives the description of transition states and assumptions associated with this model. Section 3 depicts the formation and solution of the model. Numerical calculations such as availability, reliability MTTF, sensitivity and profit analysis have been obtained in section 4. Result and conclusion of the present paper have been discussed at length in section 5 and 6 respectively.

2. MATHEMATICAL MODEL DETAILS

This section gives the details of the proposed model. Taking some assumption, structure of the model has been constructed. Some notation which used throughout the paper also presented in this section.

2.1. Assumptions

The system is structured with the help of following assumptions:
(i) Initially all unit and the system are in good working condition.
(ii) After failure on main unit, standby unit starts functioning.
(iii) System has been completely failed when both standbys failed before repair of the main unit and due to human failure.
(iv) All failures and repairs taken to be constant.
(v) System has been repaired in both degraded and failed state.
(vi) After repair system performs as a new system.

2.2. System Description

The present work considered a three - unit repairable system which contains three processors- one main and two standbys. The system has three states that is to say good, degraded and failed state. Initially, the system is in good state as all units are functionally well. When main unit fails, it is immediately replaced by a standby and the main unit goes for repairing. If the failed main unit repaired before failure of the standby, it takes the load of standby and standby unit goes for standby mode. On failure of first standby unit when main unit is under repair, load goes to the second standby unit. The system has been in complete failed state if the second standby unit develops a fault before repairing of the main and first standby unit. System also failed when human error occurs in the system at any stage of functioning. It is assumed that there is no time leg between switching. If any fault occurs in the system, it immediately recovered with coverage probability *c*. But if the fault is not detected then the system goes to in completely failed state with the probability 1-*c*. Both active and standby units have been considered as repairable. All possible transitions of states for the model are shown in Figure 1.

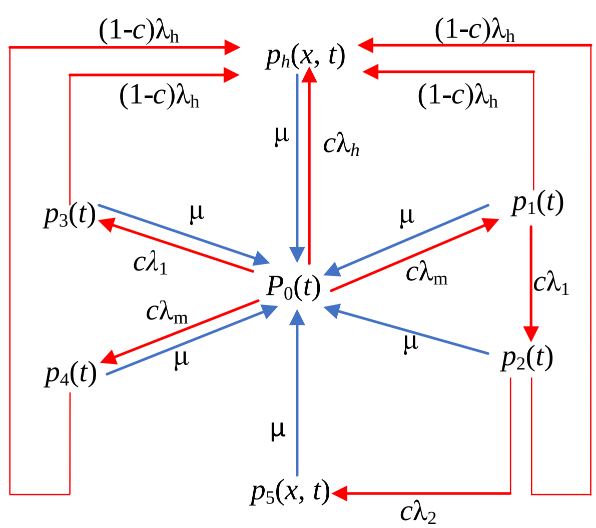


Figure1- Transition diagram of the model

2.3. Nomenclature

The following notations have been used for the considered system:

t	Time scale
s	Laplace transform variable

λ_m / λ_1 / λ_2/λ_h /c	Failure rate of main unit / Failure rate of first standby unit / Failure rate of second standby unit / human failure/ coverage factor.
μ	Repair rate of the system.
$p_0(t)$	Probability when the main and both standbys units are in good working condition.
$p_1(t)$	Probability when main unit has been failed and goes to repair, the first standby unit takes its place. Still the system is functioning.
$p_2(t)$	Probability when the system is functioning with failure of main unit and first standby unit. Second standby unit takes the load of first standby.
$p_3(t)$	Probability when first standby unit has been failed and goes to repair, main unit has repaired and takes the load of first standby unit.
$p_4(t)$	Probability when main unit has failed and goes to repair. First standby unit has repaired and takes the load of second standby unit.
$p_5(x, t)$	Probability when the system has been completely failed because of failure of main unit, first standby unit and second standby unit.
$p_h(x, t)$	Probability of the completely failed state due to human error.
k_1/ k_2	Revenue cost / Service cost per unit time.

3. MATHEMATICAL MODELLING

In this section, transition states probabilities have been calculated. Using Markov process some differential equations have been derived and solve these equations with the help of Laplace transform.

3.1. Formulation of the Model

From the state transition diagram, the following differential equations have been derived.

$$\left[\frac{\partial}{\partial t}+2c\lambda_m+c\lambda_1+c\lambda_h\right]p_0(t)=\mu\left[p_1(t)+p_2(t)+p_3(t)+p_4(t)\right]+\int_0^\infty \mu p_5(x,t)dx+\int_0^\infty \mu p_h(x,t)dx \tag{1}$$

$$\left[\frac{\partial}{\partial t}+c\lambda_1+(1-c)\lambda_h+\mu\right]p_1(t)=c\lambda_m p_0(t) \tag{2}$$

$$\left[\frac{\partial}{\partial t}+c\lambda_2+(1-c)\lambda_h+\mu\right]p_2(t)=c\lambda_1 p_1(t) \tag{3}$$

$$\left[\frac{\partial}{\partial t}+(1-c)\lambda_h+\mu\right]p_3(t)=c\lambda_1 p_0(t) \tag{4}$$

$$\left[\frac{\partial}{\partial t}+(1-c)\lambda_h+\mu\right]p_4(t)=c\lambda_m p_0(t) \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu\right]p_5(x,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu\right]p_h(x,t)=0 \tag{7}$$

Boundary condition

$$p_5(0,t)=c\lambda_2 p_2(t) \tag{8}$$

$$p_h(0,t)=c\lambda_h p_0(t)+(1-c)\lambda_h[p_1(t)+p_2(t)+p_3(t)+p_4(t)] \tag{9}$$

Initial condition

$$p_i(0)=\begin{cases}1 & ,\, i=0 \\ 0 & ,\, otherwise\end{cases} \tag{10}$$

3.2. Solution of the Model

Taking Laplace transformation from equation (1) to (9), we get the following equations

$$[s+2c\lambda_m+c\lambda_1+c\lambda_h]\,\bar{p}_0(s)=1+\mu\left[\bar{p}_1(s)+\bar{p}_2(s)+\bar{p}_3(s)+\bar{p}_4(s)\right]+\int_0^\infty \mu\bar{p}_5(x,s)dx+\int_0^\infty \mu\bar{p}_h(x,s)dx \tag{11}$$

$$\left[s+c\lambda_1+(1-c)\lambda_h+\mu\right]\bar{p}_1(s)=c\lambda_m\bar{p}_0(s) \tag{12}$$

$$\left[s+c\lambda_2+(1-c)\lambda_h+\mu\right]\bar{p}_2(s)=c\lambda_1\bar{p}_1(s) \tag{13}$$

$$\left[s+(1-c)\lambda_h+\mu\right]\bar{p}_3(s)=c\lambda_1\bar{p}_0(s) \tag{14}$$

$$\left[s+(1-c)\lambda_h+\mu\right]\bar{p}_4(s)=c\lambda_m\bar{p}_0(s) \tag{15}$$

$$\left[s+\frac{\partial}{\partial x}+\mu\right]\bar{p}_5(x,s)=0 \tag{16}$$

$$\left[s+\frac{\partial}{\partial x}+\mu\right]\bar{p}_h(x,s)=0 \tag{17}$$

$$\bar{p}_5(0,s)=c\lambda_2\bar{p}_2(s) \tag{18}$$

$$\bar{p}_h(0,s)=c\lambda_h\bar{p}_0(s)+(1-c)\lambda_h\left[\bar{p}_1(s)+\bar{p}_2(s)+\bar{p}_3(s)+\bar{p}_4(s)\right] \tag{19}$$

After solving equation (11)-(17) with the help of (18) and (19), one can get the transition states probabilities as follow:

$$\bar{p}_0(s)=\frac{1}{(s+2c\lambda_m+c\lambda_1+c\lambda_h)-\left[\left(\frac{c\lambda_m}{H_1}+\frac{c^2\lambda_1\lambda_m}{H_1H_2}+\frac{c\lambda_1}{H_3}+\frac{c\lambda_m}{H_3}\right)\left(\mu+\bar{S}(s)(1-c)\lambda_h\right)+\bar{S}(s)\left(\frac{c^3\lambda_1\lambda_2\lambda_m}{H_1H_2}+c\lambda_h\right)\right]} \tag{20}$$

$$\bar{p}_1(s)=\frac{c\lambda_m\bar{p}_0(s)}{H_1} \tag{21}$$

$$\bar{p}_2(s)=\frac{c^2\lambda_1\lambda_m\bar{p}_0(s)}{H_1.H_2} \tag{22}$$

$$\bar{p}_3(s)=\frac{c\lambda_1\bar{p}_0(s)}{H_3} \tag{23}$$

$$\bar{p}_4(s)=\frac{c\lambda_m\bar{p}_0(s)}{H_3} \tag{24}$$

$$\bar{p}_5(s)=\frac{c^3\lambda_1\lambda_2\lambda_m\bar{p}_0(s)}{H_1H_2}\left(\frac{1-\bar{S}(s)}{s}\right) \tag{25}$$

$$\bar{p}_h(s)=\bar{p}_0(s)\left(\frac{1-\bar{S}(s)}{s}\right)\left[c\lambda_h+(1-c)\lambda_h\left\{\frac{c\lambda_m}{H_1}+\frac{c^2\lambda_1\lambda_m}{H_1H_2}+\frac{c\lambda_1}{H_3}+\frac{c\lambda_m}{H_3}\right\}\right] \tag{26}$$

Where, $H_1=s+c\lambda_1+(1-c)\lambda_h+\mu$, $H_2=s+c\lambda_2+(1-c)\lambda_h+\mu$, $H_3=s+(1-c)\lambda_h+\mu$

and $\bar{S}(s)=\int_0^\infty \mu e^{-sx-\int_0^x \mu dx}dx$.

Up and down states probabilities are as follow:

$$\bar{p}_{up}(s) = \bar{p}_0(s) + \bar{p}_1(s) + \bar{p}_2(s) + \bar{p}_3(s) + \bar{p}_4(s)$$

$$= \bar{p}_0(s)\left[1 + \frac{c\lambda_m}{H_1} + \frac{c^2\lambda_1\lambda_m}{H_1H_2} + \frac{c\lambda_1}{H_3} + \frac{c\lambda_m}{H_3}\right] \tag{27}$$

$$\bar{p}_{down}(s) = \bar{p}_5(s) + \bar{p}_h(s) \tag{28}$$

4. NUMERICAL CALCULATIONS

This section gives the numerical component analysis of the system in a standby configuration.

4.1. Availability Analysis

Availability is the probability that a device performs its work in a specified time *t*. It's depends on both failure and repair rate of the system. It is a function of time. Following is the availability expression of the considered system.

$$\bar{A}(s) = \bar{p}_0(s)\left[1 + \frac{c\lambda_m}{s + c\lambda_1 + (1-c)\lambda_h + \mu} + \frac{c^2\lambda_m\lambda_1}{(s + c\lambda_1 + (1-c)\lambda_h + \mu)(s + c\lambda_2 + (1-c)\lambda_h + \mu)} + \frac{c\lambda_1}{s + (1-c)\lambda_h + \mu} + \frac{c\lambda_m}{s + (1-c)\lambda_h + \mu}\right] \tag{29}$$

4.1.1. Availability of the system with coverage factor.

To get the availability of the proposed system under coverage factor, putting the values of all failures and repairs as $\lambda_m = 0.03$, $\lambda_1 = 0.02$, $\lambda_2 = 0.015$, $\lambda_h = 0.012$ and $\mu = 1$ in Equation (29) and then taking the inverse Laplace transformation and coverage value *c* = 0.2, 0.4 , 0.6 and 0.8 respectively.

Table 1- Availability of the system with coverage factor

Time (*t*)	Availability $P_{up}(t)$			
	c = 0.2	*c* = 0.4	*c* = 0.6	*c* = 0.8
0	1.0000	1.0000	1.0000	1.0000
1	0.9985	0.9969	0.9955	0.9941
2	0.9979	0.9958	0.9939	0.9920
3	0.9976	0.9954	0.9933	0.9913
4	0.9976	0.9952	0.9931	0.9911
5	0.9975	0.9952	0.9930	0.9909
6	0.9975	0.9952	0.9929	0.9909
7	0.9975	0.9952	0.9929	0.9909
8	0.9975	0.9952	0.9929	0.9909
9	0.9975	0.9952	0.9929	0.9909
10	0.9975	0.9952	0.9929	0.9909

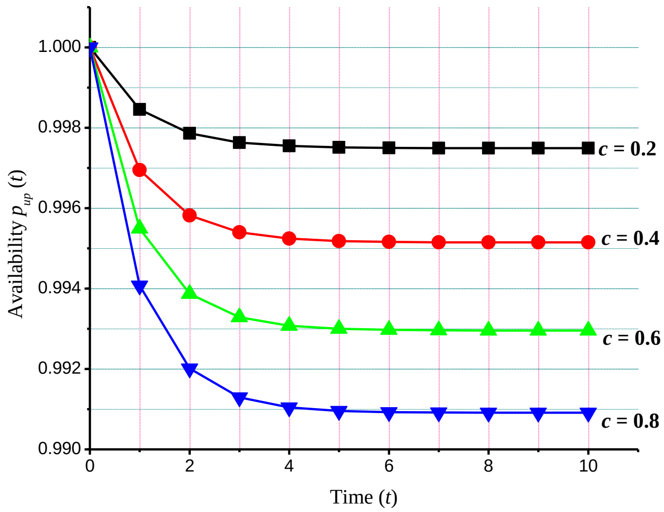


Figure 2- Availability vs Time with coverage factor c = 0.2, 0.4, 0.6, 0.8.

4.1.2. Availability of the system without coverage factor

Putting the values of all failures and repairs as $\lambda_m = 0.03, \lambda_1 = 0.02, \lambda_2 = 0.015, \lambda_h = 0.012$ and $\mu = 1$ in Equation (29) (don't take coverage parameter c here for calculate availability, without coverage factor) and then taking the inverse Laplace transformation to get availability expression as follow:

$$A = 0.98814 + 0.00243e^{(-1.092t)} - 0.04651e^{(-1.032t)} + 0.04854e^{(-1.027t)} + 0.00741e^{(-1.012t)} \ldots\ldots \quad (30)$$

Now Table 2 is obtained by varying t = 0 to 10 in Equation (30).

Table 2- Availability of the system without coverage factor

Time (t)	Availability $P_{up}(t)$
0	1.0000
1	0.9925
2	0.9897
3	0.9887
4	0.9883
5	0.9882
6	0.9882
7	0.9881
8	0.9881
9	0.9881
10	0.9881

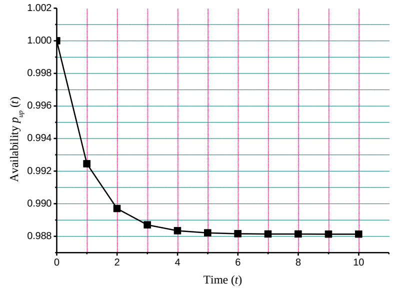


Figure 3- Availability of the system without coverage factor

4.2. Reliability Analysis

Reliability is a probability that concerned how long a system performs well once it starts works. It is a function of time. Taking repair rate $\mu = 0$ in Equation (27) to get the following reliability function.

$$\overline{R}(s) = \frac{1}{s+2c\lambda_m + c\lambda_1 + c\lambda_h}\left[1 + \frac{c\lambda_m}{s+c\lambda_1+(1-c)\lambda_h} + \frac{c^2\lambda_m\lambda_1}{(s+c\lambda_1+(1-c)\lambda_h)(s+c\lambda_2+(1-c)\lambda_h)} + \frac{c\lambda_1}{s+(1-c)\lambda_h} + \frac{c\lambda_m}{s+(1-c)\lambda_h}\right] \quad (31)$$

4.2.1. Reliability with coverage factor

Putting $\lambda_m = 0.03$, $\lambda_1 = 0.02$, $\lambda_2 = 0.015$, $\lambda_h = 0.012$ and coverage factor c=0.2, 0.4, 0.6 and 0.8 respectively in Equation (31) , we may get the reliability at c = 0.2, c = 0.4, c = 0.6 and c = 0.8.

$R_{c=0.2}$= $-0.5242e^{(-0.0184t)}-3.750e^{(-0.0136t)}+4.1379e^{(-0.0126t)}+1.1364e^{(-0.0096t)}$ (31a)
$R_{c=0.4}$= $-0.0429e^{(-0.0368t)}-1.6667e^{(-0.0152t)}+2.0338e^{(-0.0132t)}+0.6756e^{(-0.0072t)}$ (31b)
$R_{c=0.6}$= $0.0718e^{(-0.0552t)}-1.4062e^{(-0.0168t)}+1.7391e^{(-0.0138t)}+0.5952e^{(-0.0048t)}$ (31c)
$R_{c=0.8}$= $0.1209e^{(-0.0736t)}-1.3043e^{(-0.0184t)}+1.6216e^{(-0.0144t)}+0.5617e^{(-0.0024t)}$ (31d)
Now Table 3 is obtained by varying time t = 0 to 10.

Table 3- Reliability of the system with coverage factor

Time (t)	Reliability R (t)			
	c = 0.2	c = 0.4	c = 0.6	c = 0.8
0	1.0000	1.0000	1.0000	1.0000
1	0.9976	0.9952	0.9929	0.9907
2	0.9949	0.9903	0.9859	0.9819
3	0.9923	0.9854	0.9791	0.9735
4	0.9896	0.9804	0.9724	0.9656
5	0.9867	0.9754	0.9659	0.9581
6	0.9838	0.9703	0.9595	0.9509
7	0.9807	0.9653	0.9532	0.9441
8	0.9776	0.9601	0.9469	0.9376
9	0.9744	0.9549	0.9409	0.9314
10	0.9711	0.9498	0.9349	0.9254

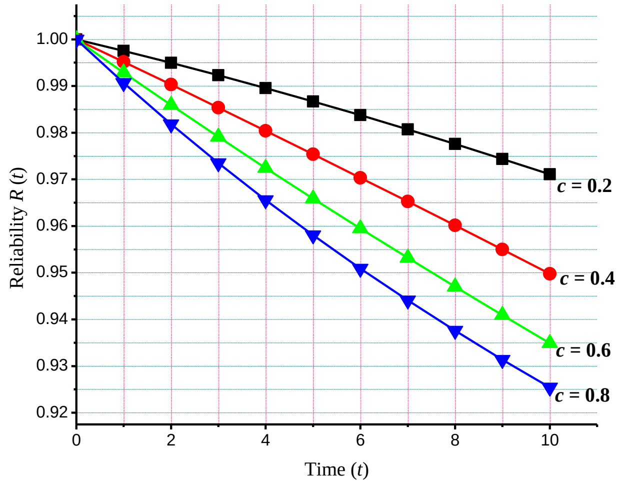


Figure 4- Reliability vs Time with coverage factor c = 0.2, 0.4, 0.6, 0.8.

4.2.2. Reliability without coverage factor

Putting $\lambda_m = 0.03$, $\lambda_1 = 0.02$, $\lambda_2 = 0.015$, $\lambda_h = 0.012$ and varying time t = 0 to t =10 in Equation (32), we may get the Table 4.

$$R = \frac{1}{s+2\lambda_m+\lambda_1+\lambda_h}\left[1+\frac{\lambda_m}{s+\lambda_1+\lambda_h}+\frac{\lambda_m\lambda_1}{(s+\lambda_1+\lambda_h)(s+\lambda_2+\lambda_h)}+\frac{\lambda_1}{s+\lambda_h}+\frac{\lambda_m}{s+\lambda_h}\right] \tag{32}$$

Table 4- Reliability of the system without coverage factor

Time (t)	Reliability R (t)
0	1.0000
1	0.9880
2	0.9763
3	0.9646
4	0.9531
5	0.9416
6	0.9303
7	0.9190
8	0.9079
9	0.8969
10	0.8859

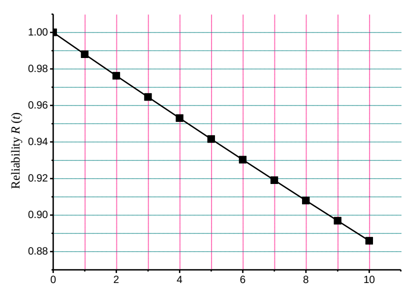


Figure 5- Reliability of the system without coverage factor

4.3. Mean time to failure (MTTF) Analysis

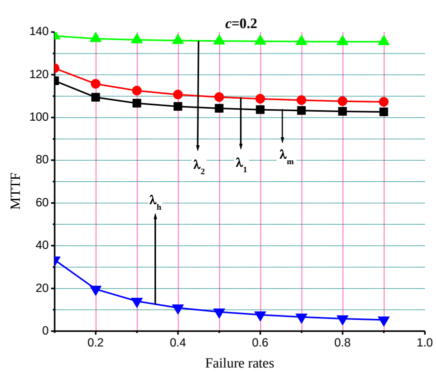
MTTF of a system represents how long a system can reasonably be expected to perform. To obtain MTTF taking $\mu = 0$ and s tends to zero in Equation (27)

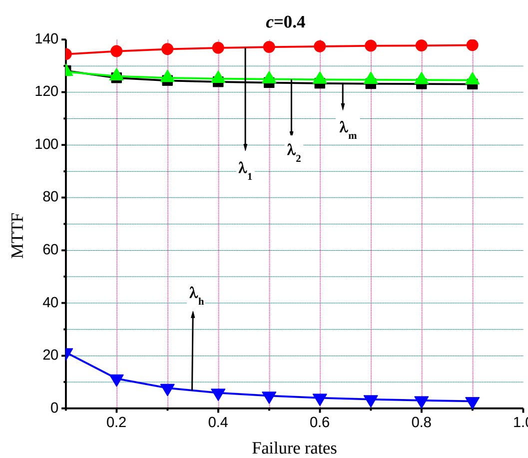
$$MTTF = \frac{1}{2c\lambda_m + c\lambda_1 + c\lambda_h}\left[1 + \frac{c\lambda_m}{c\lambda_1 + (1-c)\lambda_h} + \frac{c^2\lambda_1\lambda_m}{\left(c\lambda_1 + (1-c)\lambda_h\right)\left(c\lambda_2 + (1-c)\lambda_h\right)} + \frac{c\lambda_1}{(1-c)\lambda_h} + \frac{c\lambda_m}{(1-c)\lambda_h}\right] \qquad (33)$$

Now to get the table of MTTF as Table 5 with respect to $\lambda_m, \lambda_1, \lambda_2, \lambda_h$ at coverage factor c=0.2, 0.4, 0.6and 0.8, setting λ_1=0.02, λ_2=0.015, λ_h=0.012 and varying λ_m=0.1 to 0.9. Now setting λ_m=0.03, λ_2=0.015, λ_h=0.012 and varying λ_1=0.1 to 0.9, setting λ_m=0.03, λ_1=0.02, λ_h=0.012 and varying λ_2=0.1 to 0.9, setting λ_m=0.03, λ_1=0.02, λ_2=0.015, and varying λ_h=0.1 to 0.9 respectively.

Table 5- MTTF of the system

Variation in λ_m, λ_1, λ_2, λ_h	MTTF															
	c = 0.2				c = 0.4				c = 0.6				c = 0.8			
	λ_m	λ_1	λ_2	λ_h	λ_m	λ_1	λ_2	λ_h	λ_m	λ_1	λ_2	λ_h	λ_m	λ_1	λ_2	λ_h
0.1	117.186	123.046	138.177	33.329	128.159	134.402	127.746	21.267	162.909	181.546	154.345	20.194	270.359	336.066	261.199	27.778
0.2	109.470	115.747	136.871	19.643	125.435	135.565	126.078	11.307	161.474	190.860	152.617	9.506	268.203	365.149	259.504	11.431
0.3	106.638	112.523	136.315	14.035	124.435	136.326	125.459	7.748	160.947	195.421	152.011	6.136	267.411	378.861	258.928	6.752
0.4	105.167	110.704	136.008	10.938	123.916	136.815	125.137	5.903	160.673	198.101	151.702	4.512	266.999	386.817	258.638	4.671
0.5	104.266	109.536	135.812	8.9655	123.598	137.149	124.938	4.770	160.506	199.863	151.515	3.563	266.748	392.008	258.463	3.527
0.6	103.658	108.722	135.677	7.5980	123.383	137.393	124.804	4.003	160.392	201.107	151.389	2.941	266.578	395.662	258.346	2.815
0.7	103.219	108.122	135.578	6.593	123.229	137.577	124.708	3.449	160.311	202.034	151.298	2.503	266.455	398.374	258.263	2.333
0.8	102.889	107.662	135.503	5.824	123.112	137.720	124.635	3.030	160.249	202.749	151.231	2.178	266.363	400.465	258.199	1.987
0.9	102.630	107.298	135.443	5.215	123.021	137.836	124.577	2.702	160.201	203.319	151.178	1.927	266.291	402.128	258.151	1.728





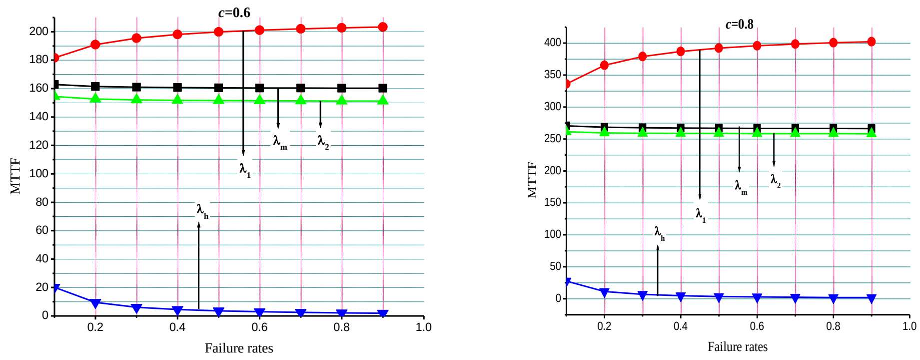


Figure 6- MTTF as a function of Failure rate at coverage factor c = 0.2, 0.4, 0.6 and 0.8.

4.4. Expected Profit

The Equation of expected profit for the interval [0, t) is given by

$$E_p(t) = k_1 \int_0^t p_{up}(t)dt - tk_2 \tag{34}$$

Substituting p_{up} in Equation (34) and integrate with respect to time t to get the expected profit for coverage factor c=0.2, 0.4, 0.6 and 0.8.

$$E_{c=0.2} = k_1\left(0.99749t + 0.00930e^{(-1.0184t)} + 0.04964e^{(-1.0136t)} - 0.05084e^{(-1.0126t)} - 0.01070e^{(-1.0096t)} + 0.00260\right) - tk_2 \tag{34a}$$

$$E_{c=0.4} = k_1\left(0.99515t + 0.00146e^{(-1.0368t)} + 0.02458e^{(-1.0152t)} - 0.02615e^{(-1.0132t)} - 0.00479e^{(-1.0072t)} + 0.00489\right) - tk_2 \tag{34b}$$

$$E_{c=0.6} = k_1\left(0.99296t - 0.00356e^{(-1.0552t)} + 0.02285e^{(-1.0168t)} - 0.02335e^{(-1.0138t)} - 0.00282e^{(-1.0048t)} + 0.00689\right) - tk_2 \tag{34c}$$

$$E_{c=0.8} = k_1\left(0.99091t - 0.00772e^{(-1.0736t)} + 0.02314e^{(-1.0184t)} - 0.02269e^{(-1.0144t)} - 0.00134e^{(-1.0024t)} + 0.00862\right) - tk_2 \tag{34d}$$

Table 6- Expected profit of the system.

Time (t)	Expected profit $E_p(t)$											
	c = 0.2			c = 0.4			c = 0.6			c = 0.8		
	k_2 = 0.1	k_2 = 0.3	k_2 = 0.6	k_2 = 0.1	k_2 = 0.3	k_2 = 0.6	k_2 = 0.1	k_2 = 0.3	k_2 = 0.6	k_2 = 0.1	k_2 = 0.3	k_2 = 0.6
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0.8991	0.6991	0.3991	0.8982	0.6982	0.3982	0.8974	0.6974	0.3974	0.8965	0.6965	0.3965
2	1.7972	1.3972	0.7972	1.7945	1.3945	0.7945	1.7919	1.3919	0.7919	1.7894	1.3894	0.7894
3	2.6949	2.0949	1.1949	2.6900	2.0901	1.1901	2.6855	2.0855	1.1855	2.6809	2.0809	1.1809
4	3.5925	2.7925	1.5925	3.5854	2.7854	1.5854	3.5786	2.7786	1.5786	3.5721	2.7721	1.5721
5	4.4900	3.4900	1.9900	4.4806	3.4806	1.9806	4.4717	3.4717	1.9717	4.4631	3.4631	1.9631
6	5.3875	4.1875	2.3875	5.3758	4.1758	2.3758	5.3646	4.1646	2.3646	5.3541	4.1541	2.3541
7	6.2850	4.8850	2.7850	6.2709	4.8709	2.7709	6.2576	4.8576	2.7576	6.2449	4.8449	2.7449
8	7.182	5.582	3.182	7.166	5.566	3.166	7.150	5.550	3.150	7.135	5.535	3.135

	5	5	5	1	1	1	6	6	6	8	9	9
9	8.0800	6.2800	3.5800	8.0612	6.2612	3.5613	8.0435	6.2435	3.5435	8.0268	6.2268	3.5268

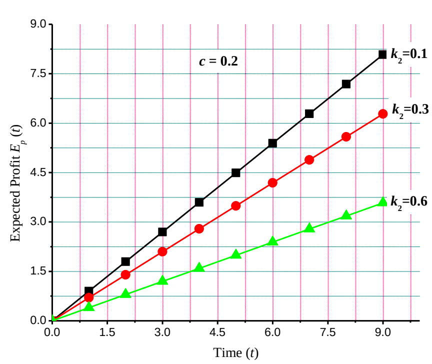
Now taking k_1=1 and k_2=0.1, 0.3, 0.6 in Equations (32a) - (32d) to get the expected profit as shown in Table 6.

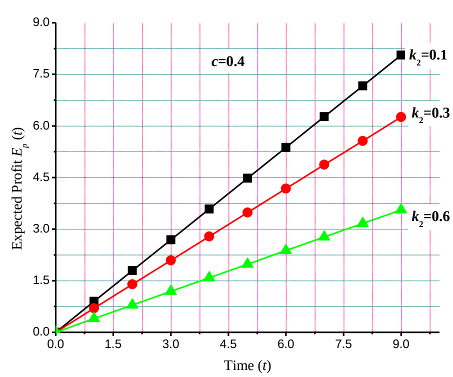
Table 7- Sensitivity of reliability of the system

Time (t)	Sensitivity of reliability															
	c = 0.2				c = 0.4				c = 0.6				c = 0.8			
	$\frac{\partial R(t)}{\partial \lambda_m}$	$\frac{\partial R(t)}{\partial \lambda_1}$	$\frac{\partial R(t)}{\partial \lambda_2}$	$\frac{\partial R(t)}{\partial \lambda_h}$	$\frac{\partial R(t)}{\partial \lambda_m}$	$\frac{\partial R(t)}{\partial \lambda_1}$	$\frac{\partial R(t)}{\partial \lambda_2}$	$\frac{\partial R(t)}{\partial \lambda_h}$	$\frac{\partial R(t)}{\partial \lambda_m}$	$\frac{\partial R(t)}{\partial \lambda_1}$	$\frac{\partial R(t)}{\partial \lambda_2}$	$\frac{\partial R(t)}{\partial \lambda_h}$	$\frac{\partial R(t)}{\partial \lambda_m}$	$\frac{\partial R(t)}{\partial \lambda_1}$	$\frac{\partial R(t)}{\partial \lambda_2}$	$\frac{\partial R(t)}{\partial \lambda_h}$
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-0.0014	-0.0007	0.0001	-0.2042	-0.0009	-0.0004	0.0620	-0.4012	0.0014	0.0007	0.0001	-0.5910	0.0055	0.0027	0.0001	-0.7738
2	-0.0056	-0.0028	0.0062	-0.4167	-0.0037	-0.0019	0.0001	-0.8046	0.0053	0.0026	0.0002	-1.1647	0.0207	0.0102	0.0004	-1.4980
3	-0.0124	-0.0062	0.00002	-0.6369	-0.0081	-0.0041	0.0002	-1.2098	0.0113	0.0054	0.0006	-1.7219	0.0441	0.0214	0.0013	-2.1765
4	-0.0217	-0.0109	0.0001	-0.8647	-0.0139	-0.0072	0.0004	-1.6165	0.0192	0.0089	0.0013	-2.2633	0.0743	0.0357	0.0029	-2.8126
5	-0.0334	-0.0167	0.0001	-1.0994	-0.0213	-0.0110	0.0007	-2.0243	0.0286	0.0131	0.0023	-2.7898	0.1099	0.0523	0.0055	-3.4098
6	-0.0473	-0.0237	0.0002	-1.3408	-0.0300	-0.0156	0.0012	-2.4329	0.0394	0.0177	0.0040	-3.3019	0.1502	0.0705	0.0092	-3.9709
7	-0.0635	-0.0319	0.0003	-1.5884	-0.0399	-0.0209	0.0019	-2.8419	0.0511	0.0225	0.0062	-3.8005	0.1938	0.0899	0.0143	-4.4988
8	-0.0816	-0.0409	0.0004	-1.8420	-0.0509	-0.0269	0.0028	-3.2513	0.0638	0.0274	0.0091	-4.2861	0.2400	0.1099	0.0207	-4.9959
9	-0.1018	-0.0511	0.0005	-2.1012	-0.0630	-0.0334	0.0039	-3.6605	0.0771	0.0324	0.0126	-4.7592	0.2881	0.1302	0.0286	-5.4648

4.5. Sensitivity Analysis

The Sensitivity measure is the rate of change of the function with respect to input parameters (Ram and Kumar, 2015). Here input parameters are failure rate. Sensitivity analysis helps in system performance and detect which parameter is good for elevate and contribute the most to the system performance.





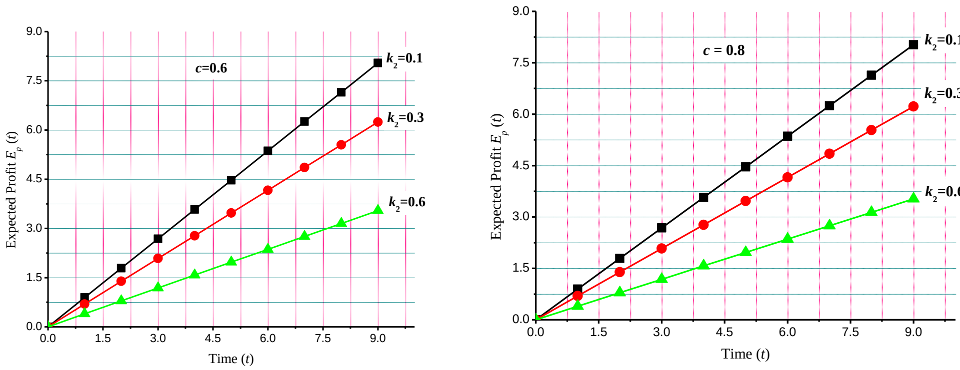


Figure 7- Expected Profit at coverage factor c = 0.2, 0.4, 0.6, 0.8.

4.5.1. Sensitivity of reliability

Sensitivity of reliability is obtained by differentiate reliability function with respect to failure rates. Then putting $\lambda_m, \lambda_1, \lambda_2, \lambda_h$ in the partial derivatives, we get Table 7 as follow:

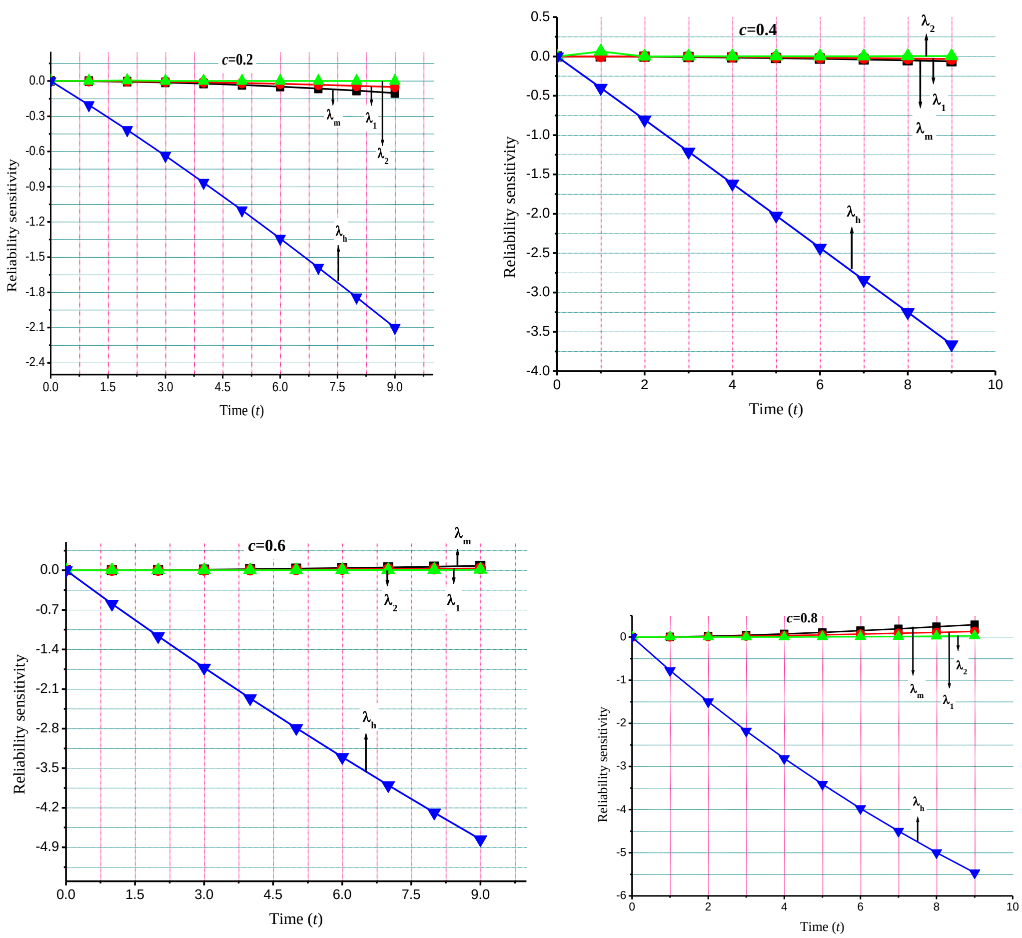


Figure 8- Sensitivity of reliability at coverage factor c=0.2, 0.4, 0.6, 0.8.

4.5.2. Sensitivity of MTTF

Sensitivity of MTTF is obtained by partial differentiation of equation (33) with respect to failure rates. Then putting the values $\lambda_m, \lambda_1, \lambda_2, \lambda_h$ in that partial derivatives and get the values of sensitivity of MTTF at coverage factor c=0.2, 0.4, 0.6 and 0.8 (Table 8).

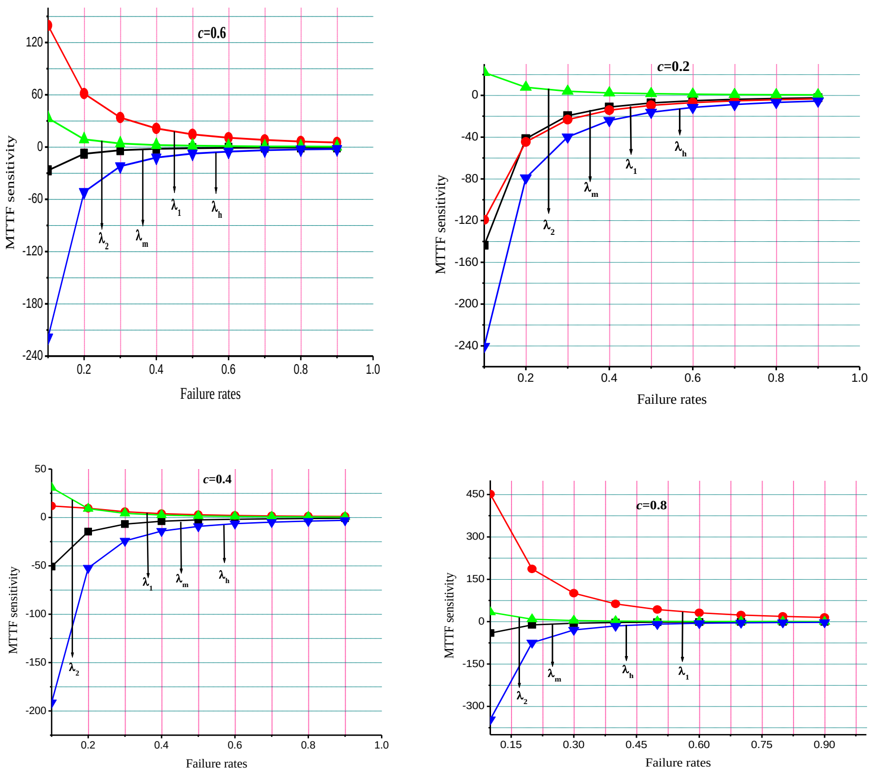


Figure 9- Sensitivity of MTTF at coverage factor c=0.2, 0.4, 0.6, 0.8.

Table 8- Sensitivity of MTTF with respect to failures rates

Variation in λ_m, λ_1, λ_2, λ_h	Sensitivity of MTTF															
	c = 0.2				c = 0.4				c = 0.6				c = 0.8			
	$\frac{\partial(MTTF)}{\partial\lambda_m}$	$\frac{\partial(MTTF)}{\partial\lambda_1}$	$\frac{\partial(MTTF)}{\partial\lambda_2}$	$\frac{\partial(MTTF)}{\partial\lambda_h}$	$\frac{\partial(MTTF)}{\partial\lambda_m}$	$\frac{\partial(MTTF)}{\partial\lambda_1}$	$\frac{\partial(MTTF)}{\partial\lambda_2}$	$\frac{\partial(MTTF)}{\partial\lambda_h}$	$\frac{\partial(MTTF)}{\partial\lambda_m}$	$\frac{\partial(MTTF)}{\partial\lambda_1}$	$\frac{\partial(MTTF)}{\partial\lambda_2}$	$\frac{\partial(MTTF)}{\partial\lambda_h}$	$\frac{\partial(MTTF)}{\partial\lambda_m}$	$\frac{\partial(MTTF)}{\partial\lambda_1}$	$\frac{\partial(MTTF)}{\partial\lambda_2}$	$\frac{\partial(MTTF)}{\partial\lambda_h}$
0.1	-143.679	-119.244	21.893	-240.616	-50.719	11.850	30.815	-191.392	-26.734	139.489	33.282	-218.732	-40.165	451.481	33.409	-346.257
0.2	-41.438	-44.711	7.796	-79.076	-14.627	9.585	9.028	-52.248	-7.710	61.468	8.973	-51.563	-11.584	186.614	8.601	-74.345
0.3	-19.361	-23.256	3.959	-39.858	-6.835	5.983	4.243	-24.282	-3.602	33.788	4.092	-21.956	-5.412	100.712	3.861	-29.061
0.4	-11.172	-14.228	2.389	-24.088	-3.943	3.981	2.456	-14.032	-2.079	21.263	2.332	-11.995	-3.123	62.834	2.182	-14.824
0.5	-7.261	-9.594	1.596	-16.147	-2.563	2.818	1.599	-9.143	-1.351	14.588	1.504	-7.518	-2.029	42.893	1.401	-8.797
0.6	-5.095	-6.905	1.142	-11.582	-1.799	2.094	1.123	-6.432	-0.948	10.622	1.050	-5.140	-1.424	31.128	0.975	-5.755
0.7	-3.771	-5.206	0.857	-8.715	-1.331	1.614	0.832	-4.771	-0.702	8.076	0.774	-3.731	-1.054	23.614	0.717	-4.029
0.8	-2.904	-4.066	0.667	-6.796	-1.025	1.282	0.641	-3.680	-0.540	6.346	0.595	-2.829	-0.812	18.524	0.549	-2.964
0.9	-2.304	-3.262	0.533	-5.448	-0.813	1.042	0.509	-2.925	-0.429	5.12	0.471	-2.218	-0.644	14.919	0.435	-2.265

4. RESULT DISCUSSION

In this research work, various reliability measures like availability, reliability, MTTF, profit and sensitivity have been discussed. Study of this paper and graphical representations give the following interpretations:

- Availability of the considered system decreases slowly and later seems to move in a uniform manner as time increases. From the Table 1 and Figure 2 it is clearly shown that how coverage factor effects the availability of the system, that is availability of the system lies between 1 and 0.9 in each cases of coverage factor. But it decreases up to 0.8 when there is no coverage factor as shown in Figure 3. On comparing Figure 2 and Figure 3, it is concluded that availability of the system is high in case of coverage factor.
- Table 3 and Figure 4 demonstrates the reliability of the proposed system with respect to time t. It is well known that the numerical value of reliability lies between 0 and 1. Hence, from Figure 4 it is clearly shown that in each cases of coverage factor, reliability is very high as if one compares it with Figure 5, which shows the reliability of the proposed system without coverage factor.
- Table 5 and Figure 6 yields the MTTF of the system with respect to variation in failure rates. From Figure 6 it is concluded that MTTF increases as coverage values increases. MTTF of the system is lowest with respect to human failure and highest with respect to standbys unit in each cases of coverage factor. Figure 6 shows that MTTF decreases with respect to all failure rate, but only at $c = 0.6$ and $c = 0.8$ it increases with respect to first standby unit failure.
- Table 6 gives the value of expected profit at different value of coverage and Figure 7 shows the cost function at different values of coverage. It is easily seen from Figure 7 that service cost is inversely proportional to the profit i.e., maximum profit requires minimum service cost and minimum profit requires maximum service cost, therefore to get maximum profit, service cost should be controlled.
- Table 7 and Table 8 gives the value of reliability and MTTF sensitivity respectively at different values of coverage. Figure 8 shows that reliability sensitivity decreases with respect to each failure rates and the system is highly sensitive with respect to human failure. Figure 9 shows the MTTF sensitivity behaviour of the system. With the close look of Figure 9 it is easily observe that MTTF sensitivity increases with respect to main unit failure and human failure, and decrease with respect to standby unit failure, but at $c = 0.2$, MTTF sensitivity increases with respect to first standby unit failure.

6. CONCLUSION

The present work investigated the performance of repairable system supported by standbys and repair facility by incorporating the concept of imperfect coverage. A correct estimate of the coverage inclined powerfully to the analysis of the dependability of a system as it improves the availability, reliability and reduce the cost, which is very important factor of reliability measures. The state transient probabilities of the system and other measures have been evaluated with the help of Markov process and Laplace transformation. Further, the system is highly sensitive with respect to human failure. Hence, human failure is the critical component of the system that is sensitive to error. Therefore, to improve the system reliability human failure should be controlled. Study of sensitivity analysis may be helpful for system engineers for the further improvements of the concerned system. This study is very much useful to industries also, where the system has standby units like aircraft, spacecraft, telecommunication, power feeding system etc. The present work can be extended by incorporating the concept of waiting time to repair and work is under process in that direction.

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