ESTIMATION IN STRATIFIED RANDOM SAMPLING IN THE PRESENCE OF ERRORS

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ABSTRACT
In this manuscript, we have proposed improved estimators for estimating the finite population mean under stratified random sampling in three different situations: At first we considered the properties of the estimators under non-response, then in the next case we studied the estimators for measurement error and in the last case we examined the estimators in the presence of both measurement error and non-response simultaneously. Expressions for mean squared errors are obtained for suggested estimators. Empirical study has been carried out to verify the results for which we have considered two real datasets.

KEYWORDS: Auxiliary variable, stratified random sampling, measurement error and non-response.

MSC: 62D05

RESUMEN
En este manuscrito, proponemos estimadores mejorados para estimar la media de una población finita bajo muestreo aleatorio estratificado en tres diferentes situaciones: primero consideramos las propiedades de los estimadores al existir no-respuestas, en el segundo caso estudiamos los estimadores ante errores de medición y en el último caso examinamos los estimadores en presencia simultánea de ambos errores. Expresiones para los errores cuadráticos medios para los estimadores sugeridos. Un estudio empírico se llevó a cabo para verificar los resultados usando dos conjuntos de datos reales.

PALABRAS CLAVE: Variable auxiliar, muestreo aleatorio estratificado, errores de medición y de no-respuesta

1. INTRODUCTION

While conducting sampling survey we come usually come across non-sampling errors like measurement error and non-response. The measurements that we get on the units for estimating the characteristic under study are seldom correct. And in practical situations the observations on these units are not correctly measured and differ from the true values of the observations. This difference between the observed value and true values on the characteristics under study are called measurement errors or observational errors and is quite frequent in survey sampling. It is a kind of non-sampling error and may arise due to the following reasons see (Tabssum (2012)):

- The respondent may not provide the required information. However, the question was meant for the proper respondent. Example- many families in Africa generally do not record a birth in the family and hence no birth certificate is made as the birth was not registered. Hence, in this case it may be possible that the respondent included in the sample may give an approximate figure for the age which may not be the actual age, as the birth was not registered.
- Sometimes it may happen that the observations can be made on the closely related substitutes called proxies, although the variable is well defined. As an example: if we are interested to know the economic status of a person and suppose the person is not willing to answer this question, then we may pool out the desired information by modifying the question. For instance, instead of asking his economic status directly; we can ask him about his educational level. However, this will be only a guess as it is not necessary that a highly educated man/women is economically well established and vice-versa.
- It may also be due to respondent has misunderstood a particular question and hence supplied the information accordingly.

Several authors for instance, (Singh and Karpe 2009), (Shalabh 1997), (Manisha 2001), (Sud and Srivastava 2000) have discussed the problem of measurement errors.

One more error that arises frequently during survey sampling are the non-response errors. These errors are also part of non-sampling errors and arises due to the following reasons; it may be due to the absence of the respondent at the time of survey or she/he refuse to answer the question or due to inability to recall the answer. Authors such as (Singh, Kumar and Kozak 2010), (Khare, and Srivastava 1997), (Hansen and Hurwitz 1946), (Kumar, Singh and Gupta 2011), Khare and Srivastava (1993), Rao, P. S. R. S. (1986), (Singh and Kumar 2008), Singh, H. P., and Kumar, S. (2010), Tripathi, T. P., and Khare, B. B. (1997), Tabasum, R., and Khan, I. A. (2006) have studied the problem of non-response.

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In order to provide a good estimate for the characteristics under study, we should take proper care and should devise such techniques and estimators so that the effect due to measurement error and non-response is minimum. Measurement errors and non-response may be present on both the study and auxiliary variables. Many authors have worked for the estimation of population parameters when there is presence of measurement error and non-response simultaneously on both the study and auxiliary variables which includes the work of (Singh and Sharma 2015), (Singh, Singh and Bouza 2018), (Zahid and Shabbir 2018). We made use of auxiliary variables on which a considerable amount of work has already been done such as those of (Perri 2007), (Koyuncu and Kadilar 2009), (Chaudhary et al. 2009), (Malik and Singh 2012) and (Mishra, Singh and Singh 2017), (Mishra, Singh and Singh 2018). Through this manuscript we have tried to study effect of measurement error, non-response, and measurement error and non-response simultaneously when they are present on both the study and auxiliary variables in stratified random sampling.

Let us consider a finite population $P = \{P_1, .., P \quad n\}$ of size N divided into L homogeneous sub-groups called strata of size $N_h, h = 1, .., L$ such that there are N_h units in the h[th] stratum and $N = \sum_{h=1}^{L} N_h$

Let Y be the study variable and X be the auxiliary variables taking values y_{hi} and $x_{hi}, h = 1, ..., L; i = 1, ..., N,$ on the i[th] unit of the h[th] stratum. Let us assume that population is divided into two mutually exclusive groups called responding and non-responding groups. And suppose that in the h[th] stratum $N_{th}, t = 1{,}2\ N_{1h}$ be the size of responding (t=1) and non-responding (2) units respectively.

The problem of non-responses has been studied first Hansen and Hurwitz (1946). The technique for dealing with non-response us assume that a simple random sample of size n_h is drawn from the h[th] stratum of size N_h and a questionnaire is mailed to them. Another subsampling rule was proposed by Bouza (1985). Let us suppose that among these n_h units let n_{1h} units respond and n_{2h} units do not respond such that $n = \sum_{h=1}^{L} n_h$. Let us again draw a sub-sample of size k_h from the non-respondents sample n_{2h} such that $k_h = \frac{n_{2h}}{g_h}, g_h > 1$. Here, g is the inverse sampling rate and k_h denotes the size of the sub sample selected from the non-respondents sample of size n_{2h} from which information will be collected by personal interview method.

Now, let (y_{hi}^*, x_{hi}^*) be the observed values on the study and auxiliary variables Y and X for the i[th] (i=1,..,N_h) unit in the h[th] stratum and let (Y_{hi}^*, X_{hi}^*) be their true values. Then the measurement or observational errors can be defined as $U_{hi}^* = y_{hi}^* - Y_{hi}^*, V_{hi}^* = x_{hi}^* - X_{hi}^*$. These errors are stochastic in nature and are uncorrelated with mean zero. Let (S_{hU}^2, S_{hV}^2) be the population variances for the error terms for the responding group and $\left(S_{hU(2)}^2, S_{hV(2)}^2\right)$ for the non-responding group, respectively. Here (S_{hY}^2, S_{hX}^2) are the population variances of the responding groups. $\left(S_{hY(2)}^2, S_{hX(2)}^2\right)$ are the population variances for the non-responding units. (C_{hY}^2, C_{hX}^2) are the population coefficients of variations of the responding groups. $\left(C_{hY(2)}^2, C_{hX(2)}^2\right)$ are the population coefficients of variations of the non-responding groups. $\left(\rho_{hYX}, \rho_{hYX(2)}\right)$ are the population correlation coefficients between the variables Y and X for the responding and non-responding groups of the population.

2. EXISTING ESTIMATORS

The Hansen and Hurvitz (1946) estimator in stratified random sampling under measurement error and non-response for estimating population mean is given by:

$$\bar{y}_{s(HH)}^* = \sum_{h=1}^{L} P_h \bar{y}_h^* \tag{2.1}$$

The expression for the variance of $\bar{y}_{s(HH)}^*$ is given by:

$$V\left(\bar{y}_{s(HH)}^*\right) = \sum_{h=1}^{L} P_h^2 A_h \tag{2.2}$$

Here, $\bar{y}_h^* = \left(\frac{n_{1h}}{n_h}\right)\bar{y}_{1h} + \left(\frac{n_{2h}}{n_h}\right)\bar{y}_{2h}' \; and \; P_h = \frac{N_h}{N}$

$$A_h = \lambda_{2h}(S_{hY}^2 + S_{hU}^2) + \theta_h\left(S_{hY(2)}^2 + S_{hU(2)}^2\right);\ \lambda_{2h} = \frac{1}{n_h} - \frac{1}{N_h};\ \theta_h = \frac{P_{2h}(g_h - 1)}{n_h}$$

Here, $\bar{y}_h^*$ and $\bar{y}_{2h}'$ are the sample means based on n_{1h}responding and $k_h$units of sub-sample from n_{2h} non-responding groups, respectively.

- The separate ratio estimator stratified random sampling under measurement error and non-response is given by:

$$\bar{y}_{s(R)}^* = \sum_{h=1}^{L} P_h \frac{\bar{y}_h^*}{\bar{x}_h^*} \bar{X}_h \tag{2.3}$$

The expression for the Bias and MSE of $\bar{y}_{s(R)}^*$ is given by

$$Bias\left(\bar{y}^*_{s(R)}\right) \cong \sum_{h=1}^{L} \frac{P_h}{\bar{X}_h}(R_h B_h - C_h) \tag{2.4}$$

$$MSE\left(\bar{y}^*_{s(R)}\right) \cong \sum_{h=1}^{L} P_h^2 (A_h + R_h^2 B_h - 2R_h C_h) \tag{2.5}$$

where $R_h = \frac{\bar{Y}_h}{X}$; $B_h = \lambda_{2h}(S_{hX}^2 + S_{hV}^2) + \theta_h\left(S_{hX(2)}^2 + S_{hV(2)}^2\right)$; $C_h = \lambda_{2h}\rho_{hXY}S_{hY}S_{hX} + \theta_h \rho_{hXY(2)} S_{hY}(2) S_{hX(2)}$

- The separate difference estimator in stratified random sampling under measurement error and non-response is given by:

$$\bar{y}^*_{s(D)} = \sum_{h=1}^{L} P_h \left[\bar{y}_h^* + d_h(\bar{X}_h - \bar{x}_h'^*)\right] \tag{2.6}$$

where $\bar{x}_h'^* = \frac{N_h \bar{X}_h - n_h \bar{x}_h^*}{N_h - n_h}$ and d_h is a constant.

The expression for the minimum variance of $\bar{y}^*_{s(D)}$ is given by:

$$V\left(\bar{y}^*_{s(D)}\right)_{min} = \sum_{h=1}^{L} P_h^2 \left[A_h - \frac{C_h^2}{B_h}\right] \tag{2.7}$$

The optimum value of d_h is $d_{h(opt)} = -\frac{C_h}{t_h B_h}$ $where$ $t_h = \frac{n_h}{N_h - n_h}$

- Azeem and Hanif (2016) estimator under stratified random sampling is given by:

$$\bar{y}^*_{s(AH)} = \sum_{h=1}^{L} P_h\, \bar{y}_h^* \left(\frac{\bar{x}_h'^*}{\bar{X}_h}\right)\left(\frac{\bar{x}_h'^*}{\bar{x}_h}\right) \tag{2.8}$$

We have the expressions for the Bias and MSE of $\bar{y}^*_{s(AH)}$ as

$$Bias\left(\bar{y}^*_{s(AH)}\right) \cong \sum_{h=1}^{L} \frac{P_h}{\bar{X}_h}(t_h^2 R_h B_h - q_h C_h) \tag{2.9}$$

$$MSE\left(\bar{y}^*_{s(AH)}\right) \cong \sum_{h=1}^{L} P_h\ (A_h + q_h^2 R_h^2 B_h - 2q_h R_h C_h) \tag{2.10}$$

where $q_h = \frac{N_h + n_h}{N_h - n_h}$

- Zahid and Shabbir (2018) gave an estimator for population mean in stratified random sampling as:

$$\bar{y}^*_{s(P)} = \sum_{h=1}^{L} P_h \left[m_{1h}\bar{y}_h^* + m_{2h}(\bar{X}_h - \bar{x}_h'^*)\left(\frac{\bar{X}_h}{\bar{x}_h'^*}\right)^{\alpha_h} exp(1-\alpha_h)\left(\frac{\bar{X}_h - \bar{x}_h'^*}{\bar{X}_h + \bar{x}_h'^*}\right)\right] \tag{2.11}$$

where m_{1h} and m_{2h} are constants whose values are to be determined and α_h is the scalar chosen arbitrarily.

The expressions for the Bias and MSE of the estimator $\bar{y}^*_{S(P)}$ are

$$Bias\left(\bar{y}^*_{s(P)}\right) \cong \sum_{h=1}^{L} P_h \left[(m_{1h}-1)\bar{Y}_h + m_{1h}\left(\frac{e_h t_h R_h C_h}{\bar{X}_h} + \frac{f_h R_h t_h^2 B_h}{\bar{X}_h}\right) + m_{2h}\left(\frac{e_h t_h^2 B_h}{\bar{X}_h}\right)\right] \tag{2.12}$$

$$MSE\left(\bar{y}^*_{s(P)}\right) \cong \sum_{h=1}^{L} P_h^2 \left[\bar{Y}_h^2 - \frac{A_{h1}E_{h1}^2 + B_{h1}D_{h1}^2 - 2C_{h1}D_{h1}E_{h1}}{A_{h1}B_{h1} - C_{h1}^2}\right] \tag{2.13}$$

Here $A_{h1} = \bar{Y}_h^2 + A_h + e_h^2 t_h^2 R_h^2 B_h + 4e_h\ t_h\ R_h\ C_h + 2f_h t_h^2 R_h^2 B_h$; $B_{h1} = t_h^2 B_h$; $C_{h1} = t_h\ C_h + 2e_h t_h^2 R_h\ B_h$; $D_{h1} = \bar{Y}_h^2 + e_h t_h\ R_h\ C_h + f_h t_h^2 R_h^2 B_h$; $E_{h1} = e_h t_h^2 R_h B_h$ and $e_h = \frac{1+\alpha_h}{2}$; $f_h = \frac{\alpha_h^2 + 4\alpha_h + 3}{8}$.

3. PROPOSED ESTIMATORS

3.1. The case of non-response on study and auxiliary variables.

In this case we deal with the problem of non-response for both the study and auxiliary variable case. Motivated by Mishra and Singh (2017), we propose estimators t_{p1} and t_{p2} in stratified random sampling under non-response as:

a) $$t_{p1} = \sum_{h=1}^{L} P_h \left[\bar{y}_h^* + \alpha_h \log \left(\frac{\bar{x}_h'^*}{\bar{X}_h}\right)\right] \tag{3.1}$$

b) $$t_{p2} = \sum_{h=1}^{L} P_h \left[\bar{y}_h^*(1 + w_h) + w_{2h} \log \left(\frac{\bar{x}_h'^*}{\bar{X}_h}\right)\right] \tag{3.2}$$

In order to obtain the expressions of Bias and MSE of the proposed estimators, we assume that:

$$\eta_{hY}^* = \sum_{i=1}^{n_h}(Y_{hi}^* - \bar{Y}_h), \quad \eta_{hX}^* = \sum_{i=1}^{n_h}(X_{hi}^* - \bar{X}_h), \tag{3.2a}$$

Dividing both sides of η_{hY}^{*} by n_h and then simplifying we get $\bar{y}_h^{*} = \bar{Y}_h + \frac{\eta_{hY}^{*}}{n_h}$. Similarly, we can get $\bar{x}_h^{*} = \bar{X}_h + \frac{\eta_{hX}^{*}}{n_h}$. We have:

$E\left(\frac{\eta_{hY}^{*}}{n_h}\right)^2 = \lambda_{2h}S_{hY}^2 + \theta_h S_{hY(2)}^2 = A_{h\phi}$, $\quad E\left(\frac{\eta_{hX}^{*}}{n_h}\right)^2 = \lambda_{2h}S_{hX}^2 + \theta_h S_{hX(2)}^2 = B_{h\phi}$, $\quad E\left(\frac{\eta_{hY}^{*}}{n_h}\right)E\left(\frac{\eta_{hX}^{*}}{n_h}\right) = \lambda_{2h}\rho_{hXY}S_{hY}S_{hY} + \theta_h \rho_{hXY(2)}S_{hY(2)}S_{hY(2)} = C_{h\phi}$

Now, expanding t_{p1} in terms of η_h , we get

$$t_{p1} \cong \sum_{h=1}^{L} P_h \left[\bar{Y}_h + \frac{\eta_{hY}^{*}}{n_h} + \alpha_h \log \left(\frac{\bar{X}_h + \frac{\eta_{hX}^{*}}{n_h}}{\bar{X}_h}\right)\right] \tag{3.3}$$

We know that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots, for\ |x|$ and here $\left|\frac{\eta_{hX}^{*}}{n_h\bar{X}_h}\right| < 1$. Hence,

$$\log \left(\frac{\bar{X}_h + \frac{\eta_{hX}^{*}}{n_h}}{\bar{X}_h}\right) = \frac{\eta_{hX}^{*}}{n_h\bar{X}_h} - \frac{1}{2}\left(\frac{\eta_{hX}^{*}}{n_h\bar{X}_h}\right)^2 + \left(\frac{\eta_{hX}^{*}}{n_h\bar{X}_h}\right)^3 - \cdots \tag{3.4}$$

After simplifying equation (3.3) we get

$$t_{p1} - \bar{Y} \cong \sum_{h=1}^{L} P_h \left[\frac{\eta_{hY}^{*}}{n_h} + \alpha_h \left(\frac{\eta_{hX}^{*}}{n_h\bar{X}_h} - \frac{1}{2}\left(\frac{\eta_{hX}^{*}}{n_h\bar{X}_h}\right)^2\right)\right] \tag{3.5}$$

The expressions for the Bias and MSE of the estimator t_{p1} are given as:

$$Bias(t_{p1}) \cong -\frac{1}{2}\sum_{h=1}^{L} \frac{P_h}{\bar{X}_h^2}\alpha_h B_{h\phi} \tag{3.6}$$

$$MSE(t_{p1}) \cong \sum_{h=1}^{L} P_h^2 \left(A_{h\phi} + \frac{\alpha_h^2}{\bar{X}_h^2}B_{h\phi} + 2\frac{\alpha_h}{\bar{X}_h}C_{h\phi}\right) \tag{3.7}$$

Now in order to obtain the Minimum mse of t_{p1}, we partially differentiate the equation (3.7) w.r.to $\alpha_h, (h = 1, \ldots, L)$ and equating to zero we get its optimum value as:

$$\alpha_h = \frac{C_{h\phi}\bar{X}_h}{B_{h\phi}} \tag{3.8}$$

Putting the optimum value of α_h obtained in equation (3.8) we get the minimum MSE of t_{p1} as:

$$MSE(t_{p1}) \cong \sum_{h=1}^{L} P_h^2 \left(A_{h\phi} - \frac{C_{h\phi}^2}{B_{h\phi}}\right) \tag{3.9}$$

Expanding t_{p2} in terms of η_h ,, we get

$$t_{p2} \cong \sum_{h=1}^{L} P_h \left(\bar{Y}_h + \frac{\eta_{hY}^{*}}{n_h}\right)(1 + w_{1h}) + w_{2h}\left(\frac{\frac{\eta_{hY}^{*}}{n_h}}{X_h} - \frac{1}{2}\left(\frac{\eta_{hX}^{*}}{n_h\bar{X}_h}\right)^2\right) \tag{3.10}$$

Simplifying equation (3.10) we get

$$t_{p2} - \bar{Y} \cong \sum_{h=1}^{L} P_h \left[\bar{Y}_h\ w_{1h} + \left(\frac{\eta_{hY}^{*}}{n_h}\right)(1 + w_{1h}) + w_{2h}\left(\frac{\frac{\eta_{hX}^{*}}{n_h}}{\bar{X}_h} - \frac{1}{2}\left(\frac{\eta_{hX}^{*}}{n_h\bar{X}_h}\right)^2\right)\right] \tag{3.11}$$

The expressions for the Bias and MSE of the estimator t_{p2} are given as:

$$Bias(t_{p2}) \cong \sum_{h=1}^{L} P_h \left[\bar{Y}_h\ w_{1h} - \frac{w_{2h}}{2\bar{X}_h{}^2}B_{h\phi}\right] \tag{3.12}$$

$$MSE(t_{p2}) \cong \sum_{h=1}^{L} P_h^2 \left((\bar{Y}_h\ w_{1h})^2 + A_{h\phi}(1 + w_{1h})^2 + \frac{w_{2h}^2}{\bar{X}_h^2}B_{h\phi} - \frac{\bar{Y}_h\ w_{1h}w_{2h}}{\bar{X}_h^2}B_{h\phi} + 2\frac{w_{2h} + w_{1h}w_{2h}}{\bar{X}_h}C_{h\phi}\right) \tag{3.13}$$

Now in order to obtain the Minimum mse of t_{p2}, we partially differentiate the equation (3.13) w.r.to $w_{1h}\ and\ w_{2h}, (h = 1, .., L)$ and equating to zero we get the optimum values as:

$$w'_{1h} = \frac{\frac{C_{h\phi} w \bar{X}_h}{B_{h\phi}} - A_{h\phi}}{A_{h\phi} + \bar{Y}_h^2 - \frac{w^2 \bar{X}_h^2}{B_{h\phi}}} \tag{3.14}$$

$$w'_{2h} = -\frac{\bar{X}_h^2}{B_{h\phi}} \left(\frac{C_{h\phi}}{\bar{X}_h} + w w_{1h}{}' \right) \tag{3.15}$$

Here, $w = \frac{C_{h\phi}}{\bar{X}_h} - \frac{\bar{Y}_h}{2\bar{X}_h^2} B_{h\phi}$

Putting the optimum values of w_{1h} and w_{2h} in equation (3.13), we get the minimum MSE for t_{p2} as:

$$minMSE(t_{p2}) \cong \sum_{h=1}^{L} P_h^2 \left(\left(\bar{Y}_h \ w'_{1h}\right)^2 + A_{h\phi}(1 + w'_{1h})^2 + \frac{w'^2_{2h}}{\bar{X}_h^2} B_{h\phi} - \frac{\bar{Y}_h \ w'_{1h} w'_{2h}}{\bar{X}_h^2} B_{h\phi} + 2 \frac{w'_{2h} + w'_{1h} w'_{2h}}{\bar{X}_h} C_{h\phi} \right) \tag{3.16}$$

3.2. The case of measurement error on both study and auxiliary variables.

In this case we deal with the problem of measurement error for both the study and auxiliary variable. Here we consider the estimators t_{p1} and t_{p2} proposed in section 3.1 under measurement error as:

a) $t_{p1} = \sum_{h=1}^{L} P_h \left[\bar{y}_h^* + \alpha_h \log \left(\frac{\bar{x}_h^*}{\bar{X}_h} \right) \right]$ (3.17)

b) $t_{p2} = \sum_{h=1}^{L} P_h \left[\bar{y}_h^* (1 + w_{1h}) + w_{2h} log \left(\frac{\bar{x}_h^*}{\bar{X}_h} \right) \right]$ (3.18)

In order to obtain the expressions for the Bias and MSE of the proposed estimators, we assume that:

$$\eta_{hY}^* = \sum_{i=1}^{n_h} (Y_{hi}^* - \bar{Y}_h), \quad \eta_{hX}^* = \sum_{i=1}^{n_h} (X_{hi}^* - \bar{X}_h), \quad \eta_{hU}^* = \sum_{i=1}^{n_h} U_{hi}^*, \quad \eta_{hV}^* = \sum_{i=1}^{n_h} V_{hi}^*,$$

On adding η_{hY}^* and η_{hU}^*, we get $\eta_{hY}^* + \eta_{hU}^* = \sum_{i=1}^{n_h} (Y_{hi}^* - \bar{Y}_h) + \sum_{i=1}^{n_h} U_{hi}^*$. Dividing both sides by n_h and then simplifying we get $\bar{y}_h^* = \bar{Y}_h + \frac{\eta_{hY}^* + \eta_{hU}^*}{n_h}$. Similarly, we get $\bar{x}_h^* = \bar{X}_h + \frac{\eta_{hX}^* + \eta_{hV}^*}{n_h}$. We have:

$E\left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h}\right) = \lambda_{2h}(S_{hY}^2 + S_{hU}^2) = A_{h\lambda}$, $E\left(\frac{\eta_{hX}^* + \eta_{hV}^*}{n_h}\right) = \lambda_{2h}(S_{hX}^2 + S_{hV}^2) = B_{h\lambda}$, $E\left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h}\right)\left(\frac{\eta_{hX}^* + \eta_{hV}^*}{n_h}\right) = \lambda_{2h} \rho_{hXY} S_{hY} S_{hY} = C_{hY}$

Expanding t_{p1} in terms of η_h we get

$$t_{p1} \cong \sum_{h=1}^{L} P_h \left[\bar{Y}_h + \frac{\eta_{hX}^* + \eta_{hV}^*}{n_h} + \alpha_h \log \left(\frac{\bar{X}_h + \frac{\eta_{hX}^* + \eta_{hV}^*}{n_h}}{\bar{X}_h} \right) \right] \tag{3.19}$$

Here,

$$\log \left(\frac{\bar{X}_h + \frac{\eta_{hX}^* + \eta_{hV}^*}{n_h}}{\bar{X}_h} \right) = \frac{\eta_{hX}^* + \eta_{hV}^*}{n_h \bar{X}_h} - \frac{1}{2} \left(\frac{\eta_{hX}^* + \eta_{hV}^*}{n_h \bar{X}_h} \right)^2 + \frac{1}{3} \left(\frac{\eta_{hX}^* + \eta_{hV}^*}{n_h \bar{X}_h} \right)^3 - \cdots. \tag{3.20}$$

Simplifying equation (3.20) we get

$$t_{p1} - \bar{Y} \cong \sum_{h=1}^{L} P_h \left[\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h} + \alpha_h \left(\frac{\eta_{hX}^* + \eta_{hV}^*}{n_h \bar{X}_h} - \frac{1}{2} \left(\frac{\eta_{hX}^* + \eta_{hV}^*}{n_h \bar{X}_h} \right)^2 \right) \right] \tag{3.21}$$

The expressions for the Bias and MSE of the estimator t_{p1} up to the first order of approximation are given as:

$$Bias(t_{p1}) \cong -\frac{1}{2} \sum_{h=1}^{L} \frac{P_h \, \alpha_h}{\bar{X}_h{}^2} B_{h\lambda} \tag{3.22}$$

$$MSE(t_{p1}) \cong \sum_{h=1}^{L} P_h^2 \left(A_{h\lambda} + \frac{\alpha_h{}^2}{\bar{X}_h{}^2} B_{h\lambda} + 2 \frac{\alpha_h}{\bar{X}_h} C_{h\lambda} \right) \tag{3.23}$$

In order to obtain the Minimum MSE of t_{p1}, we partially differentiate the equation (3.23) w.r.to α_h $(h = 1, \ldots, L)$ and equating to zero we get its optimum value as:

$$\alpha_{h'} = \frac{C_{h\lambda}\bar{X}_h}{B_{h\lambda}} \tag{3.24}$$

Putting the optimum value of $_h\ \alpha_h$in equation (3.23) we get the minimum MSE of t_{p1}as:

$$MSE(t_{p1}) \cong \sum_{h=1}^{L} P_h^2 \left(A_{h\lambda} - \frac{C_{h\lambda}{}^2}{B_{h\lambda}}\right) \tag{3.25}$$

Expanding t_{p2} in terms of η_h , we get

$$t_{p2} \cong \sum_{h=1}^{L} P_h \left[\left(\bar{Y}_h + \frac{\eta_{hY}^* + \eta_{hU}^*}{n_h}\right)(1 + w_{1h}) + w_{2h}\left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h\bar{X}_h} - \frac{1}{2}\left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h\bar{X}_h}\right)^2\right)\right] \tag{3.26}$$

Simplifying equation (3.26) we get

$$t_{p2} - \bar{Y} \cong \sum_{h=1}^{L} P_h \left[\bar{Y}_h\ w_{1h} + \left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h}\right)(1 + w_{1h}) + w_{2h}\left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h\bar{X}_h} - \frac{1}{2}\left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h\bar{X}_h}\right)^2\right)\right] \tag{3.27}$$

The expressions for the Bias and MSE of the estimator t_{p2} up to the first order of approximation are given as

$$Bias(t_{p2}) \cong \sum_{h=1}^{L} P_h \left[\bar{Y}_h\ w_{1h} - \frac{1}{2\bar{X}_h{}^2} w_{2h} B_{h\lambda}\right] \tag{3.28}$$

$$MSE(t_{p2}) \cong \sum_{h=1}^{L} P_h{}^2 \left[\bar{Y}_h^2 w_{1h}{}^2 + A_{h\lambda}(1 + w_{1h})^2 + \frac{\left(w_{2h}{}^2 - \bar{Y}_h\ w_{1h}w_{2h}\right)B_{h\lambda}}{\bar{X}_h^2} + 2\frac{C_{h\lambda}}{\bar{X}_h}(w_{2h} + w_{1h}w_{2h})\right] \tag{3.29}$$

In order to obtain the Minimum MSE of t_{p2}, we partially differentiate the equation (3.29) w.r.to w_{1h} and w_{2h} (h=1,…,L) and equating to zero we get the optimum values as:

$$w'_{1h} = \frac{\frac{C_{h\lambda}w\bar{X}_h}{B_{h\phi}} - A_{h\lambda}}{A_{h\lambda} + \bar{Y}_h^2 - \frac{w^2\bar{X}_h^2}{B_{h\lambda}}} \tag{3.30}$$

$$w'_{2h} = -\frac{\bar{X}_h^2}{B_{h\lambda}}\left(\frac{C_{h\lambda}}{\bar{X}_h} + w w_{1h'}\right) \tag{3.31}$$

Here, $w = \frac{C_{h\lambda}}{\bar{X}_h} - \frac{\bar{Y}_h}{2\bar{X}_h^2}B_{h\lambda}$. Putting the optimum values of w_{1h} and w_{2h} (h=1,…,L) in equation (3.29), we get the minimum MSE for t_{p2}

$$minMSE(t_{p2}) \cong \sum_{h=1}^{L} P_h^2 \left(\left(\bar{Y}_h\ w'_{1h}\right)^2 + A_{h\lambda}(1 + w'_{1h})^2 + \frac{w'^2_{2h}}{\bar{X}_h^2}B_{h\lambda} - \frac{\bar{Y}_h\ w'_{1h}w'_{2h}}{\bar{X}_h^2}C_{h\lambda} + 2\frac{w'_{2h} + w'_{1h}w'_{2h}}{\bar{X}_h}C_{h\lambda}\right) \tag{3.32}$$

3.3: Case of measurement error and non-response simultaneously on both the study and auxiliary variables

In this case we deal with the problem of measurement error and non-response simultaneously for both the study and auxiliary variables. We consider the estimators t_{p1} and t_{2p} as well as $\eta_{hY}^*, \eta_{hX}^*, \eta_{hU}^*, \eta_{hV}^*, \eta_{hY}^* + \eta_{hU}^*$ proposed in section 3.1 and 3.2. We get, dividing both sides by n_h that simplifying we get $\bar{y}_h^* = \bar{Y}_h + \frac{\eta_{hY}^*}{n_h}$ and $\bar{x}_h^* = \bar{X}_h + \frac{\eta_{hX}^*}{n_h}$. We have:

$$E\left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h}\right) = \lambda_{2h}(S_{hY}^2 + S_{hU}^2) + \theta_h\left(S_{hY(2)}^2 + S_{hU(2)}^2\right) = A_{h\tau},\ E\left(\frac{\eta_{hX}^* + \eta_{hV}^*}{n_h}\right) = \lambda_{2h}(S_{hX}^2 + S_{hV}^2) + \theta_h\left(S_{hX(2)}^2 + S_{hV(2)}^2\right) = B_{h\tau},\quad E\left[\left(\frac{\eta_{hY}^* + \eta_{hU}^*}{n_h}\right)\left(\frac{\eta_{hX}^* + \eta_{hV}^*}{n_h}\right)\right] = \lambda_{2h}\rho_{hXY}S_{hY}S_{hX} + \theta_h\rho_{hXY(2)}S_{hY(2)}S_{hX(2)} = C_{h\tau} \tag{3.33}$$

Now, expanding t_{p1} in terms of η_h , we get

$$t_{p1} \cong \sum_{h=1}^{L} P_h \left[\bar{Y}_h + \frac{\eta_{hY}^* + \eta_{hU}^*}{n_h} + \alpha_h log\left(\frac{n_h\bar{X}_h + \eta_{hX}^* + \eta_{hV}^*}{n_h\bar{X}_h}\right)\right] \tag{3.34}$$

Here , again $\log\left(\left(\frac{n_h\bar{X}_h+\eta^*_{hX}+\eta^*_{hV}}{n_h\bar{X}_h}\right)\right)=\frac{\eta^*_{hX}+\eta^*_{hV}}{n_h\bar{X}_h}-\frac{1}{2}\left(\frac{\eta^*_{hX}+\eta^*_{hV}}{n_h\bar{X}_h}\right)^2+\frac{1}{3}\left(\frac{\eta^*_{hX}+\eta^*_{hV}}{n_h\bar{X}_h}\right)^3-\cdots$. After simplifying equation we get

$$t_{p1}-\bar{Y}\cong\sum_{h=1}^{L}P_h\left[\frac{\eta^*_{hY}+\eta^*_{hU}}{n_h}+\alpha_h\left(\frac{\eta^*_{hX}+\eta^*_{hV}}{n_h\bar{X}_h}-\frac{1}{2}\left(\frac{\eta^*_{hX}+\eta^*_{hV}}{n_h\bar{X}_h}\right)^2\right)\right] \tag{3.35}$$

The approximate expressions obtained for the Bias and MSE of this estimator, up to the first order of approximation, are given as:

$$Bias(t_{p1})\cong-\frac{1}{2}\sum_{h=1}^{L}\frac{P_h\,\alpha_h}{\bar{X}_h{}^2}B_{h\tau} \tag{3.36}$$

$$MSE(t_{p1})\cong\sum_{h=1}^{L}P_h^2\left(A_{h\tau}+\frac{\alpha_h{}^2}{\bar{X}_h{}^2}B_{h\tau}+2\frac{\alpha_h}{\bar{X}_h}C_{h\tau}\right) \tag{3.37}$$

Using the same optimization procedure its optimum value is:

$$\alpha_{h'}=\frac{C_{h\tau}\bar{X}_h}{B_{h\tau}} \tag{3.38}$$

Placing $\alpha_{h'}$ in equation (3.37) we have:

$$minMSE(t_{p1})\cong\sum_{h=1}^{L}P_h^2\left(A_{h\tau}-\frac{C_{h\tau}{}^2}{B_{h\lambda}}\right) \tag{3.39}$$

Performing a similar analysis of t_{p2} we have that:

$$Bias(t_{p2})\cong\sum_{h=1}^{L}P_h\left[\bar{Y}_h\ w_{1h}-\frac{1}{2\bar{X}_h{}^2}w_{2h}B_{h\tau}\right] \tag{3.40}$$

$$MSE(t_{p2})\cong\sum_{h=1}^{L}P_h{}^2\left[\bar{Y}_h^2w_{1h}{}^2+A_{h\tau}(1+w_{1h})^2+\frac{\left(w_{2h}{}^2-\bar{Y}_h\ w_{1h}w_{2h}\ \right)B_{h\tau}}{\bar{X}_h^2}+2\frac{C_{h\tau}}{\bar{X}_h}(w_{2h}+w_{1h}w_{2h})\right]$$

The Minimum MSE is obtained when we use:

$$w'_{1h}=\frac{\frac{C_{h\tau}w\bar{X}_h}{B_{h\tau}}-A_{h\tau}}{A_{h\tau}+\bar{Y}_h^2-\frac{w^2\bar{X}_h^2}{B_{h\tau}}} \tag{3.41}$$

$$w'_{2h}=-\frac{\bar{X}_h^2}{B_{h\tau}}\left(\frac{C_{h\tau}}{\bar{X}_h}+ww_{1h'}\right) \tag{3.42}$$

$$w=\frac{C_{h\tau}}{\bar{X}_h}-\frac{\bar{Y}_h}{2\bar{X}_h^2}B_{h\tau} \tag{3.43}$$

Hence

$$minMSE(t_{p2})\cong\ \sum_{h=1}^{L}P_h^2\left(\left(\bar{Y}_h\ w'_{1h}\right)^2+A_{h\tau}(1+w'_{1h})^2+\frac{w'^2_{2h}}{\bar{X}_h^2}B_{h\tau}-\frac{\bar{Y}_h\ w'_{1h}w'_{2h}}{\bar{X}_h^2}C_{h\tau}+2\frac{w'_{2h}+w'_{1h}w'_{2h}}{\bar{X}_h}C_{h\tau}\right) \tag{3.44}$$

4. EMPIRICAL STUDY

In this section we have carried out an empirical study for which we have considered two natural population data sets.

Population-1 (Särndal, C. E., Swensson, B., & Wretman, J. (2003)**)**

Y**:** Production of wheat (in tons), X: Area of wheat (in hectares)

No. of strata=4.

$N_1=47$, $N_2=30$, $N_3=29$, $N_4=13$, $n_1=15$, $n_2=10$, $n_3=10$, $n_4=5$, $\bar{Y}_1=443.5447$, $\bar{Y}_2=68.68276$, $\bar{Y}_3=17.06667$, $\bar{Y}_4=52.52308$, $\bar{X}_1=160.2362$, $\bar{X}_2=29.70345$, $\bar{X}_3=11.54667$,

$\overline{X}_4 = 23.62308$, $S_{1Y}^2 = 74026.75$, $S_{2Y}^2 = 2871.781$, $S_{3Y}^2 = 244.1292$, $S_{4Y}^2 = 4451.124$

$S_{1X}^2 = 8377.401$, $S_{2X}^2 = 316.4532$, $S_{3X}^2 = 91.45775$, $S_{4X}^2 = 682.9703$, $\rho_{1YX} = 0.9583838$,

$\rho_{2YX} = 0.779071$. $\rho_{3YX} = 0.8719665$, $\rho_{4YX} = 0.9922591$

Population-2 (FBS, Crops area production by districts, Islamabad; 2011)

Y: 1983 Population (in millions), X: 1982 gross national product

No of strata=5

$N_1 = 38$, $N_2 = 14$, $N_3 = 11$, $N_4 = 33$, $N_5 = 24$ $n_1 = 17$, $n_2 = 6$, $n_3 = 4$, $n_4 = 12$, $n_5 = 11$,

$\overline{Y}_1 = 13.03684$, $\overline{Y}_2 = 27.35$, $\overline{Y}_3 = 23.13636$, $\overline{Y}_4 = 79.65455$, $\overline{Y}_5 = 20.28333$ $\overline{X}_1 = 1029.158$,

$\overline{X}_2 = 25671.57$, $\overline{X}_3 = 5028.818$, $\overline{X}_4 = 7533.939$, $X_5 = 16315.25$ $S_{1Y}^2 = 270.9083$,

$S_{2Y}^2 = 3906.929$, $S_{3Y}^2 = 1339.405$, $S_{4Y}^2 = 45082.17$, $S_{5Y}^2 = 368.9423$ $S_{1X}^2 = 3667896$,

$S_{2X}^2 = 6568461403$, $S_{3X}^2 = 63348743$, $S_{4X}^2 = 440717912$, $S_{5X}^2 = 408441212$

$\rho_{1YX} = 0.7439544$, $\rho_{2YX} = 0.969956$. $\rho_{3YX} = 0.9768227$, $\rho_{4YX} = 0.2948897$,

$\rho_{5YX} = 0.9011072$

The MSE expressions for the existing estimators for the sections 1 and 2 i.e. for the cases of non-response and measurement errors can be obtained from the section of existing estimators by using the appropriate notations from section 1 and 2 respectively.

To determine the Percent Relative Efficiency (PRE), of the estimators w.r.to. the usual estimator $(\bar{y}_{HH}^*, \bar{y}_{st}^*)$ we have used the given formula:

$$PRE(t, usual\ estimator) = \frac{MSE(usual\ estimator)}{MSE(t)} \times 100, t = \bar{y}_{SR,}^* \bar{y}_{S(D)}^*, \ \bar{y}_{S(AH)}^*, \bar{y}_{SP}^*, t_{p1}, t_{p2},$$

Case-1:

Table 1: MSE and PRE of estimators when there is presence of non-response on both the study and auxiliary variables for population-1

g_h=2		
Estimator	**MSE**	**PRE**
$\bar{y}_{HH}^*$	551.8020	100.0000
$\bar{y}_{SR,}^*$	61.7760	893.2299
$\bar{y}_{S(D)}^*$	61.5828	896.0318
$\bar{y}_{S(AH)}^*$	467.9744	117.9129
$\bar{y}_{SP}^*\ (\alpha_h = 0)$	61.3250	899.7985
$\bar{y}_{SP}^*\ (\alpha_h = 1)$	61.3900	898.8466
$\bar{y}_{SP}^*\ (\alpha_h = -1)$	61.3910	898.8313
t_{p1}	61.5858	896.0318
t_{p2}	58.3504	945.6685
g_h=4		
Estimator	**MSE**	**PRE**
$\bar{y}_{HH}^*$	567.8053	100.0000
$\bar{y}_{SR,}^*$	88.6968	640.1639
$\bar{y}_{S(D)}^*$	88.5978	640.8790
$\bar{y}_{S(AH)}^*$	530.3673	107.0589
$\bar{y}_{SP}^*\ (\alpha_h = 0)$	88.1623	644.0449
$\bar{y}_{SP}^*\ (\alpha_h = 1)$	88.2567	643.3562
$\bar{y}_{SP}^*\ (\alpha_h = -1)$	88.2590	643.3396
t_{p1}	88.5978	640.8790

t_{p2}	84.7477	669.9945
g_h=8		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	599.8118	100.0000
$\bar{y}^*_{SR,}$	142.5385	420.8069
$\bar{y}^*_{S(D)}$	140.8417	425.8766
$\bar{y}^*_{S(AH)}$	655.1529	91.5529
$\bar{y}^*_{SP}$ $(\alpha_h = 0)$	139.9058	428.7255
$\bar{y}^*_{SP}$ $(\alpha_h = 1)$	140.0595	428.2550
$\bar{y}^*_{SP}$ $(\alpha_h = -1)$	140.0662	428.2345
t_{p1}	140.8417	425.8766
t_{p2}	135.6444	442.1943

Table 2: MSE and PRE of estimators when there is presence of non-response on both the study and auxiliary variables for population-2

g_h=2		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	192.2504	100.0000
$\bar{y}^*_{SR,}$	288.7807	66.5731
$\bar{y}^*_{S(D)}$	169.2728	113.5743
$\bar{y}^*_{S(AH)}$	1025.251	18.7515
$\bar{y}^*_{SP}$ $(\alpha_h = 0)$	120.9315	158.9746
$\bar{y}^*_{SP}$ $(\alpha_h = 1)$	120.6644	159.3265
$\bar{y}^*_{SP}$ $(\alpha_h = -1)$	125.6208	153.0403
t_{p1}	169.2728	113.5743
t_{p2}	109.0567	176.2843
g_h=4		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	198.2250	100.0000
$\bar{y}^*_{SR,}$	301.6620	65.7109
$\bar{y}^*_{S(D)}$	175.813	112.7476
$\bar{y}^*_{S(AH)}$	1066.2370	18.5910
$\bar{y}^*_{SP}$ $(\alpha_h = 0)$	124.3443	159.4162
$\bar{y}^*_{SP}$ $(\alpha_h = 1)$	1239130	159.9711
$\bar{y}^*_{SP}$ $(\alpha_h = -1)$	129.3531	153.2433
t_{p1}	175.813	112.7476
t_{p2}	112.0998	176.8290
g_h=8		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	210.1741	100.0000
$\bar{y}^*_{SR,}$	327.4247	64.1900
$\bar{y}^*_{S(D)}$	188.7929	111.3252
$\bar{y}^*_{S(AH)}$	1148.21	18.3045
$\bar{y}^*_{SP}$ $(\alpha_h = 0)$	130.8596	160.6104
$\bar{y}^*_{SP}$ $(\alpha_h = 1)$	129.9988	161.6793
$\bar{y}^*_{SP}$ $(\alpha_h = -1)$	136.5215	153.9495
t_{p1}	188.7929	111.3252
t_{p2}	117.9026	178.2608

From Tables 1 and 2, we see that for both of the populations 1 and 2 and for each of the value of $g = 2, 4, and\, 8$, the proposed estimator t_{p2} has got the highest PRE as compared to the existing estimators $\bar{y}^*_{HH}, \bar{y}^*_{SR}, \bar{y}^*_{S(D)}, \bar{y}^*_{S(AH)}, \bar{y}^*_{SP}, t_{p1}, t_{p2}$ and the other proposed estimator t_{p1} while t_{p1} has got PRE equal to the difference type estimator $\bar{y}^*_{S(D)}$.

Case-2:

When there is presence of measurement error on both the study and auxiliary variables:

In this case the usual estimator is given by:

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h \tag{4.1}$$

And its MSE is given by:

$$MSE(\bar{y}_{st}) = \sum_{h=1}^{L} P_h^2 A_h \tag{4.2}$$

Table 3: MSE and PRE of estimators when there is presence of measurement error on both the study and auxiliary variables for population-1

Estimator	MSE with meas. error	PRE with meas. error	MSE without meas. error	PRE without meas. error
$\bar{y}^*_{HH}$	554.6764	100	573.8004	100
$\bar{y}^*_{SR,}$	73.2135	757.6149	48.3156	1187.609
$\bar{y}^*_{S(D)}$	73.1508	758.2643	47.8290	1199.691
$\bar{y}^*_{S(AH)}$	500.5492	110.8136	436.7780	131.3712
$\bar{y}^*_{SP}\ (\alpha_h = 0)$	72.8258	761.6482	47.6403	1204.443
$\bar{y}^*_{SP}\ (\alpha_h = 1)$	72.9030	760.8417	47.6906	1203.173
$\bar{y}^*_{SP}\ (\alpha_h = -1)$	72.9045	760.826	47.6913	1203.155
t_{p1}	73.1508	758.2643	47.8290	1199.691
t_{p2}	69.6804	796.0293	44.8904	1278.225

Table 4: MSE and PRE of estimators when there is presence of measurement error on both the study and auxiliary variables for population-2

Estimator	MSE with meas error	PRE with meas. error	MSE without meas error	PRE without meas. error
$\bar{y}^*_{HH}$	193.0484	100.0000	189.2632	100.0000
$\bar{y}^*_{SR,}$	293.0089	65.8848	282.3400	67.03379
$\bar{y}^*_{S(D)}$	170.4529	113.2562	165.9888	114.0217
$\bar{y}^*_{S(AH)}$	1042.0360	18.5260	1004.7580	18.8367
$\bar{y}^*_{SP}\ (\alpha_h = 0)$	121.5285	158.8503	119.1831	158.8004
$\bar{y}^*_{SP}\ (\alpha_h = 1)$	121.2299	159.2416	118.9849	159.0649
$\bar{y}^*_{SP}\ (\alpha_h = -1)$	126.3194	152.8256	123.7142	152.9842
t_{p1}	170.4520	113.2568	165.9888	114.0217
t_{p2}	109.6072	176.1275	107.4974	176.063

From Table 3 and 4 we see that for both the populations the proposed estimator t_{p1} has got the minimum mse as compared to the existing estimators $\bar{y}^*_{HH}, \bar{y}^*_{SR}, \bar{y}^*_{S(D)}, \bar{y}^*_{S(AH)}, \bar{y}^*_{SP}, t_{p1}, t_{p2}$ and the other proposed estimator t_{p1} while t_{p1} has got MSE equal to the difference type estimator $\bar{y}^*_{S(D)}$

Case-3

Table 5: MSE and PRE of estimators when there is presence of measurement error and non-response simultaneously on both the study and auxiliary variables for population-1

g_h=2

Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	562.83810	100.00000
$\bar{y}^*_{SR,}$	87.75406	641.38126
$\bar{y}^*_{S(D)}$	87.37282	644.17977
$\bar{y}^*_{S(AH)}$	535.74030	105.05801
$\bar{y}^*_{SP}\ (\alpha_h = 0)$	86.92352	647.50956
$\bar{y}^*_{SP}\ (\alpha_h = 1)$	87.01909	646.79842
$\bar{y}^*_{SP}\ (\alpha_h = -1)$	87.02173	646.77879
t_{p1}	87.37283	644.17977
t_{p2}	83.53502	673.77499
g_h=4		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	579.1614	100.0000
$\bar{y}^*_{SR,}$	116.8352	495.7079
$\bar{y}^*_{S(D)}$	114.8885	504.1073
$\bar{y}^*_{S(AH)}$	606.1224	95.5518
$\bar{y}^*_{SP}\ (\alpha_h = 0)$	114.1589	507.3291
$\bar{y}^*_{SP}\ (\alpha_h = 1)$	114.2909	506.7432
$\bar{y}^*_{SP}\ (\alpha_h = -1)$	114.2971	506.7157
t_{p1}	114.8885	504.1073
t_{p2}	110.3114	525.0238
g_h=8		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	611.8080	100.0000
$\bar{y}^*_{SR,}$	174.9973	349.6099
$\bar{y}^*_{S(D)}$	167.6361	364.9618
$\bar{y}^*_{S(AH)}$	746.8867	81.9144
$\bar{y}^*_{SP}\ (\alpha_h = 0)$	166.2149	368.0824
$\bar{y}^*_{SP}\ (\alpha_h = 1)$	166.4198	367.6293
$\bar{y}^*_{SP}\ (\alpha_h = -1)$	166.4369	367.5915
t_{p1}	167.6361	364.9618
t_{p2}	161.5060	378.8143

Table 6: MSE and PRE of estimators when there is presence of measurement error and non-response simultaneously on both the study and auxiliary variables for population-2

g_h=2		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	196.0954	100
$\bar{y}^*_{SR,}$	299.5093	65.4722
$\bar{y}^*_{S(D)}$	173.7767	112.8433
$\bar{y}^*_{S(AH)}$	1062.5890	18.4544
$\bar{y}^*_{SP}\ (\alpha_h = 0)$	123.2653	159.0840
$\bar{y}^*_{SP}\ (\alpha_h = 1)$	122.8827	159.5793
$\bar{y}^*_{SP}\ (\alpha_h = -1)$	128.2176	152.9395
t_{p1}	173.7767	112.8433
t_{p2}	111.1571	176.4128
g_h=4		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	202.1895	100.0000
$\bar{y}^*_{SR,}$	312.5101	64.6985

$\bar{y}^*_{S(D)}$	180.4008	112.0779
$\bar{y}^*_{S(AH)}$	1103.695	18.3193
$\bar{y}^*_{SP}\ (\alpha_h = 0)$	126.6579	163.5071
$\bar{y}^*_{SP}\ (\alpha_h = 1)$	126.0812	160.3645
$\bar{y}^*_{SP}\ (\alpha_h = -1)$	131.9364	153.2477
t_{p1}	180.4008	112.0779
t_{p2}	114.1829	177.0751
g_h=8		
Estimator	**MSE**	**PRE**
$\bar{y}^*_{HH}$	214.3776	100.0000
$\bar{y}^*_{SR,}$	338.5118	63.3294
$\bar{y}^*_{S(D)}$	193.5574	110.7566
$\bar{y}^*_{S(AH)}$	1185.906	18.0771
$\bar{y}^*_{SP}\ (\alpha_h = 0)$	133.1420	161.0143
$\bar{y}^*_{SP}\ (\alpha_h = 1)$	132.0741	162.3162
$\bar{y}^*_{SP}\ (\alpha_h = -1)$	139.0870	154.1320
t_{p1}	193.5574	110.7566
t_{p2}	119.9559	178.7137

From Tables 5 and 6, we see that for both of the populations 1 and 2 and for each of the value of $g = 2, 4, \text{and } 8$, the proposed estimator t_{p2} has got the highest PRE as compared to the existing estimators $\bar{y}^*_{HH,}\bar{y}^*_{SR,}\bar{y}^*_{S(D)},\ \bar{y}^*_{S(AH)},\bar{y}^*_{SP},t_{p1},t_{p2}$proposed estimator t_{p1} while t_{p1} has got PRE equal to the difference type estimator $\bar{y}^*_{S(D)}$

5. CONCLUSION

The empirical study reveals that for all the three cases the proposed estimators t_{p2} outperforms the existing estimators considered in this paper and also is better than the proposed estimator t_{p1} because MSE of the estimator t_{p2} is minimum and has highest PRE while t_{p1} is equally efficient as the difference type estimator $\bar{y}^*_{S(D)}$. Hence it is preferable to use the proposed estimators in practice.

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