A GENERAL PROCEDURE FOR ESTIMATING FINITE POPULATION MEAN USING RANKED SET SAMPLING

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ABSTRACT
In this article, we have suggested general class of estimators of the finite population mean using an auxiliary variable that is correlated with the variable of interest using ranked set sampling (RSS) method. We also obtained the biases and mean squared errors (MSEs) of various suggested estimators. We have compared newly suggested generalised ratio-cum-product type estimators to others existing estimators and shows that our suggested estimators are better than other existing estimators. This paper also provides the correct expressions of MSE of the estimators due to Mehta and Mandowara (2012, 2016) and Mandowara and Mehta (2013).

KEYWORDS: Bias, Minimum Mean Squared Error Estimators, Ranked Set Sampling, Ratio-Cum-Product Type Estimator, Simple Random Sampling.

MSC: 62D05

RESUMEN
En este artculo, hemos sugerido la clase general de estimadores de la media de la poblacin finita utilizando una variable auxiliar que se correlaciona con la variable de inters utilizando el mtodo de muestreo de conjuntos clasificados (RSS). Tambin obtuvimos los sesgos y los errores cuadrticos medios (MSE) de varios estimadores sugeridos. Hemos comparado los estimadores de tipo de producto de relacin-producto generalizados recientemente sugeridos con otros estimadores existentes y muestra que nuestros estimadores sugeridos son mejores que otros estimadores existentes. Este artculo tambin proporciona las expresiones correctas de MSE de los estimadores debido a Mehta y Mandowara (2012, 2016) y Mandowara y Mehta (2013).

PALABRAS CLAVE: Sesgo, Estimadores de error de media cuadrtica mnima, Muestreo de conjunto clasificado, Estimador de tipo de producto de proporcin ajustada, Muestreo aleatorio simple.

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1. INTRODUCTION

The literature on ranked set sampling (RSS) describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Ranked set sampling was first suggested by McIntyre (1952) to increase the efficiency of estimator of population mean. Kadilar et al. (2009) used this technique to improve ratio estimator given by Prasad (1989). Bouza (2008) has modified the product estimators.Singh et al. (2014) has further discussed the properties of the class of estimators of the population mean defined on the line of Upadhyaya et al. (1985) in ranked set sampling. For current reference in this context the reader is referred to Patil et al. (1995) and Ozturk (2016).

The problem of estimating the population mean in the presence of an auxiliary variable has been widely discussed in finite population sampling literature. Out of many ratio, product, difference and regression methods of estimation are good examples in this context.Cochran (1940, 1942) was the pioneer in using the auxiliary information at the estimation stage and envisaged the ratio estimator to estimate the population mean or total of a variable y under study. Robson (1957) and Murthy (1964) have studied the product estimator. The ratio estimator is used when the study variable y and the auxiliary variable x is positively (high) correlated.

On the other hand, if the study variable y and the auxiliary variable x are negatively (high) correlated, the product estimator can be employed more efficiently than the mean per unit estimator. Further, if the relation between y and x is a straight line passing through the neighbourhood of the origin and the variance of y about this line is proportional to x , the ratio estimator is as good as regression estimator.

However, owing to the stronger intuitive appeal statisticians are more willing towards the use of the ratio and the product estimator. Perhaps that is why substantial work has been carried out in the direction of improving the performance of ratio (product) estimator when the said regression line does not pass through the origin, for instance, see Kothwala and Gupta (1988), Naik and Gupta (1991), Singh and Ruiz-Espejo (2003) and the references cited therein. Singh et al. (2014, 2016a, 2016b) also provided generalized dual estimators based on simple random sampling (SRS) and stratified ranked set sampling (SRSS) techniques.

Singh and Agnihotri (2008) defined a family of ratio-product estimators of the population mean in simple random sampling (SRS) as:

$$\bar{y}_{RP1,SRS} = \delta\bar{y}(\frac{a\bar{X}+b}{a\bar{x}+b}) + (1-\delta)\bar{y}(\frac{a\bar{x}+b}{a\bar{X}+b}) \tag{1.1}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ are unbiased estimators of the population means $\bar{Y}$ and $\bar{X}$ respectively, $'a'$ and $'b'$ are known characterizing positive scalars and δ is a real constant to be determined such that the mean squared error (MSE) of $\bar{y}_{RP1,SRS}$ is minimum.

The class of estimators $\bar{y}_{RP1,SRS}$ due to Singh and Agnihotri (2008) is further generalised as:

$$\bar{y}_{RP2,SRS} = \delta\bar{y}(\frac{a\bar{X}+b}{a\bar{x}+b})^{p} + (1-\delta)\bar{y}(\frac{a\bar{x}+b}{a\bar{X}+b}), \tag{1.2}$$

where(δ, a, b) are the same as defined earlier; and p being a constant such that $p \in (-1,1)$.

To the first degree of approximation [ignoring finite population correction (fpc) term] the bias and

MSE expressions of (1.2) are given by

$$B(\bar{y}_{RP2,SRS}) = \bar{Y}\theta\alpha[K\{1-\delta(1+p)\} + \frac{\alpha\delta p(p+1)}{2}]C_x^2 \tag{1.3}$$

and

$$MSE(\bar{y}_{RP2,SRS}) = \bar{Y}^2\theta[C_y^2 + \alpha\{1-\delta(1+p)\}C_x^2[\alpha\{1-\delta(1+p)\}+2K]], \tag{1.4}$$

where $\theta = \frac{1}{n}$,$K = \rho\frac{C_y}{C_x}$, $\rho = \frac{S_{xy}}{S_xS_y}$, $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, $\alpha = \frac{a\bar{X}}{a\bar{X}+b}$, $S_{xy} = \sum_{i=1}^{N}\frac{(x_i-\bar{X})(y_i-\bar{Y})}{N-1}$, $S_x^2 = \sum_{i=1}^{N}\frac{(x_i-\bar{X})^2}{N-1}$ and $S_y^2 = \sum_{i=1}^{N}\frac{(y_i-\bar{Y})^2}{N-1}$.
The $MSE(\bar{y}_{RP2,SRS})$ is minimum when

$$\delta^* = -K, \tag{1.5}$$

where $\delta^* = \{1-\delta(1+p)\}$.
Thus the resulting minimum MSE of $\bar{y}_{RP2,SRS}$ is given by

$$minMSE(\bar{y}_{RP2,SRS}) = \theta S_y^2(1-\rho^2). \tag{1.6}$$

For $p = 1$, $\bar{y}_{RP2,SRS}$ reduces to the class of estimators $\bar{y}_{RP1,SRS}$ and other members can be generated just by assigning different values to p in the proposed class of estimators $\bar{y}_{RP2,SRS}$ given by (1.2). Some of them are given in Table 1.

In the Table 1, C_x and $\beta_2(x)$ respectively represent the known values of the coefficient of variation and coefficient of kurtosis of an auxiliary variable x.

In this paper, we transferred (1.2) into a RSS method in the presence of an auxiliary variable x. We obtained the bias and the MSE expressions for newly suggested estimators on RSS. The proposed class of estimators includes number of known estimators based on the transformed auxiliary variable x.

2. SUGGESTED ESTIMATOR BASED ON RANKED SET SAMPLING (RSS)

In ranked set sampling (RSS) m independent random sets are chosen (each of size m) and the units in each set are selected with equal probability and with replacement from the population. The members of each random set are ranked with respectto the characteristic of the study variable or auxiliary variable. Then, the smallest unit isselected from the first ordered set and the second smallest unit is selected from the secondordered set. By this way, this procedure is continued until the unit with the largest rankis chosen from the m-th set. This cycle may be repeated r times, so $mr(= n)$ units havebeen measured during this process.
Thus, RSS and SRS have equivalent sample sizes n for comparison of their biases and efficiencies. When we rank on the auxiliary variable, let $(y_{[i]}, x_{(i)})$ denote the i-th judgment ordering of the study variable and the i-th perfect ordering for the auxiliary variable in the i-th set, where $i = 1, 2, ..., m$.

Table 1: **Members of the estimator for different choices of** $(a, b, \delta_i, \alpha_i, p_i)$**:**

S. No.	**Estimators**	**Values of Constants** $(a, b, \delta_i, \alpha_i, p_i)$				
		a	b	δ_i	α_i	p_i
1	$\bar{y}^{(1)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$ Cochran (1940) estimator	1	0	1	1	1
2	$\bar{y}^{(2)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$ Murthy (1964) estimator	1	0	0	1	1
3	$\bar{y}^{(3)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ Sisodia and Dwivedi (1981) estimator	1	C_x	1	$\frac{\bar{X}}{\bar{X}+C_x}$	1
4	$\bar{y}^{(4)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)$ Singh and Kakran (1993) estimator	1	$\beta_2(x)$	1	$\frac{\bar{X}}{\bar{X}+\beta_2(x)}$	1
5	$\bar{y}^{(5)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)}\right)$ Upadhyaya and Singh (1999) estimator	C_x	$\beta_2(x)$	1	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	1
6	$\bar{y}^{(6)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{x}C_x+\beta_2(x)}{\bar{X}C_x+\beta_2(x)}\right)$ Upadhyaya and Singh (1999) estimator	C_x	$\beta_2(x)$	0	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	1
7	$\bar{y}^{(7)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x}\right)$ Upadhyaya and Singh (1999) estimator	$\beta_2(x)$	C_x	1	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	1
8	$\bar{y}^{(8)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{x}\beta_2(x)+C_x}{\bar{X}\beta_2(x)+C_x}\right)$ Upadhyaya and Singh (1999) estimator	$\beta_2(x)$	C_x	0	$\frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+C_x}$	1
9	$\bar{y}^{(9)}_{RP2,SRS} = \bar{y}[\delta_9\left(\frac{\bar{X}}{\bar{x}}\right) + (1-\delta_9)(\frac{\bar{x}}{\bar{X}})]$ Singh and Espejo (2003) Estimator	1	0	δ_9	1	1
10	$\bar{y}^{(10)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)^{p_{10}}$ Singh et al. (2004) estimator	1	$\beta_2(x)$	1	$\frac{\bar{X}}{\bar{X}+\beta_2(x)}$	p_{10}
11	$\bar{y}^{(11)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x}\right)^{p_{11}}$ Singh et al. (2008) estimator	$\beta_2(x)$	C_x	1	$\frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+C_x}$	p_{11}
12	$\bar{y}^{(12)}_{RP2,SRS} = \bar{y}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)}\right)^{p_{12}}$ Singh et al. (2008) estimator	C_x	$\beta_2(x)$	1	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	p_{12}
13	$\bar{y}^{(13)}_{RP2,SRS} = \bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right)$ Singh and Agnihotri (2008) estimator	a	b	1	$\frac{a\bar{X}}{a\bar{X}+b}$	1
14	$\bar{y}^{(14)}_{RP2,SRS} = \bar{y}\left(\frac{a\bar{x}+b}{a\bar{X}+b}\right)$ Singh and Agnihotri (2008) estimator	a	b	0	$\frac{a\bar{X}}{a\bar{X}+b}$	1
15	$\bar{y}^{(15)}_{RP2,SRS} = \bar{y}[\delta_{15}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)}\right)$ $+(1-\delta_{15})\left(\frac{\bar{x}C_x+\beta_2(x)}{\bar{X}C_x+\beta_2(x)}\right)]$ Tailor and Sharma (2009) estimator	C_x	$\beta_2(x)$	δ_{15}	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	1
16	$\bar{y}^{(17)}_{RP2,SRS} = \bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}+b}\right)^{p_{16}}$	a	b	1	$\frac{a\bar{X}}{a\bar{X}+b}$	p_{16}
17	$\bar{y}^{(18)}_{RP2,SRS} = \bar{y}\left(\frac{a\bar{x}+b}{a\bar{X}+b}\right)^{p_{17}}$	a	b	0	$\frac{a\bar{X}}{a\bar{X}+b}$	p_{17}

Motivated by Singh and Agnihotri (2008), we proposed a generalised ratio-cum-product type estimator for $\bar{Y}$ using RSS as:

$$\bar{y}_{RP,RSS} = \delta \bar{y}_{[n]} \left(\frac{a\bar{X}+b}{a\bar{x}_{(n)}+b} \right)^p + (1-\delta)\bar{y}_{[n]} \left(\frac{a\bar{x}_{(n)}+b}{a\bar{X}+b} \right), \tag{2.1}$$

where (δ, a, b, p) are the same as defined earlier in equation (1.2); and $\bar{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$ and $\bar{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$ are the ranked set sample means for variables x and y, respectively.
To obtain the bias and MSE of (2.1), we write $\bar{y}_{[n]} = \bar{Y}(1+e_0)$ and $\bar{x}_{(n)} = \bar{X}(1+e_1)$ such that $E(e_i) = 0, i = 0, 1$ as $E(\bar{y}_{[n]}) = \bar{Y}$ and $E(\bar{x}_{(n)}) = \bar{X}$ under RSS. Since $\bar{y}_{[n]}$ and $\bar{x}_{(n)}$ are unbiased estimators of population mean $\bar{Y}$ and $\bar{X}$ respectively, therefore $E(e_i) = 0, \forall\, i = 0, 1$. Moreover,

$$Var(e_0) = E(e_0^2) = \frac{Var(\bar{y}_{[n]})}{\bar{Y}^2} = \frac{1}{mr}\frac{1}{\bar{Y}^2}(S_y^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{y_{[i]}}^2) = (\theta C_y^2 - W_{y_{[i]}}^2).$$

Similarly

$$Var(e_1) = E(e_1^2) = \frac{Var(\bar{x}_{(n)})}{\bar{X}^2} = \frac{1}{mr}\frac{1}{\bar{X}^2}(S_x^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{x_{(i)}}^2) = (\theta C_x^2 - W_{x_{(i)}}^2)$$

and

$$Cov(e_0, e_1) = E(e_0 e_1) = \frac{Cov(\bar{y}_{[n]}, \bar{x}_{(n)})}{\bar{X}\bar{Y}} = \frac{1}{mr}\frac{1}{\bar{X}\bar{Y}}(S_{yx} - \frac{1}{m}\sum_{i=1}^{m}\tau_{yx_{(i)}}^2) = (\theta\rho C_x C_y - W_{yx_{(i)}}),$$

where $W_{x_{(i)}}^2 = \frac{1}{m^2 r \bar{X}^2}\sum_{i=1}^{m}\tau_{x_{(i)}}^2$, $W_{y_{[i]}}^2 = \frac{1}{m^2 r \bar{Y}^2}\sum_{i=1}^{m}\tau_{y_{[i]}}^2$, $W_{yx_{(i)}} = \frac{1}{m^2 r \bar{X}\bar{Y}}\sum_{i=1}^{m}\tau_{yx_{(i)}} = \frac{1}{m^2 r \bar{X}\bar{Y}}\sum_{i=1}^{m}\tau_{y_{[i]}}\tau_{x_{(i)}}$, $\tau_{x_{(i)}} = (\mu_{x_{(i)}} - \bar{X})$, $\tau_{y_{[i]}} = (\mu_{y_{[i]}} - \bar{Y})$, $\tau_{yx_{(i)}} = (\mu_{x_{(i)}} - \bar{X})(\mu_{y_{[i]}} - \bar{Y}) = \tau_{x_{(i)}}\tau_{y_{[i]}}$, $\mu_{x_{(i)}} = E(X_{(i)})$ and $\mu_{y_{[i]}} = E(Y_{[i]})$.
Expressing (2.1) in terms of e's, we have

$$\bar{y}_{RP,RSS} = \bar{Y}(1+e_0)[\delta(1+\alpha e_1)^{-p} + (1-\delta)(1+\alpha e_1)]. \tag{2.2}$$

We assume that $|\alpha e_1| < 1$ so that $(1+\alpha e_1)^{-p}$ is expandable. From (2.2) we have
$\bar{y}_{RP,RSS} = \bar{Y}(1+e_0)[\delta(1 - \alpha e_1 p + \frac{p(p+1)}{2}\alpha^2 e_1^2 - ...) + (1-\delta)(1+\alpha e_1)]$
$= \bar{Y}(1+e_0)[1 + \{1-\delta(1+p)\}e_1\alpha + \frac{p(p+1)\delta}{2}\alpha^2 e_1^2 + ...]$
$= \bar{Y}[1 + e_0 + \{1-\delta(1+p)\}e_1\alpha + \{1-\delta(1+p)\}e_1 e_0\alpha + \frac{p(p+1)\delta}{2}\alpha^2 e_1^2 + \frac{p(p+1)\delta}{2}\alpha^2 e_1^2 e_0 + ...]$
We assume that the contribution of terms involving powers in e_0 and e_1 higher than the second is negligible, being of order $1/n^\nu$, where $\nu > 1$. Thus, form the above expression we write to a first approximation,
$\bar{y}_{RP,RSS} \cong \bar{Y}[1 + e_0 + \{1-\delta(1+p)\}e_1\alpha + \{1-\delta(1+p)\}e_1 e_0\alpha + \frac{p(p+1)\delta}{2}\alpha^2 e_1^2]$
or

$$\left(\bar{y}_{RP,RSS} - \bar{Y}\right) = \bar{Y}\left[e_0 + \{1-\delta(1+p)\}e_1\alpha + \{1-\delta(1+p)\}e_1 e_0\alpha + \frac{p(p+1)\delta}{2}\alpha^2 e_1^2\right] \tag{2.3}$$

Taking expectation of both the sides of (2.3), we obtain the bias of $\bar{y}_{RP,RSS}$ to the first degree of approximation, as:

$$
\begin{aligned}
B\left(\bar{y}_{RP,RSS}\right) &= \bar{Y}\left[\{1-\delta(1+p)\}\alpha\left\{\theta\rho C_x C_y - W_{yx_{(i)}}\right\} + \frac{p(p+1)\delta\alpha^2}{2}\left\{\theta C_x^2 - W_{x_{(i)}}^2\right\}\right] \qquad (2.4)\\
&= \bar{Y}\alpha\left[\{1-\delta(1+p)\}\left\{\theta\rho C_x C_y - W_{yx_{(i)}}\right\} + \tfrac{p(p+1)\delta\alpha}{2}\left\{\theta C_x^2 - W_{x_{(i)}}^2\right\}\right]\\
&= \bar{Y}\alpha\left[\theta C_x^2\left\{\tfrac{p(p+1)\delta\alpha}{2} + \{1-\delta(1+p)\}K\right\} - \tfrac{1}{m^2 r}\left\{\tfrac{\theta\alpha p(1+p)}{2\bar{X}^2}\sum_{i=1}^m \tau_{x_{(i)}}^2 + \tfrac{\{1-\delta(1+p)\}}{\bar{X}\bar{Y}}\sum_{i=1}^m \tau_{yx_{(i)}}\right\}\right].
\end{aligned}
$$

Squaring both sides of (2.3) and neglecting terms of e's having power grater than two, then, we have

$$
\left(\bar{y}_{RP,RSS} - \bar{Y}\right)^2 = \bar{Y}\left[e_0^2 + \{1-\delta(1+p)\}^2 e_1^2\alpha^2 + 2\alpha\{1-\delta(1+p)\}e_1 e_0\right]. \qquad (2.5)
$$

Taking expectation of both sides of (2.5), we get the MSE of $\bar{y}_{RP,RSS}$ to the first degree of approximation as:

$$
\begin{aligned}
MSE\left(\bar{y}_{RP,RSS}\right) = \bar{Y}^2[\theta&\left\{C_y^2 + \{1-\delta(1+p)\}^2\alpha^2 C_x^2 + 2\alpha\{1-\delta(1+p)\}KC_x^2\right\}\\
&-W_{y_{[i]}}^2 - \{1-\delta(1+p)\}^2\alpha^2 W_{x_{(i)}}^2 - 2\alpha\{1-\delta(1+p)\}W_{yx_{(i)}}].
\end{aligned} \qquad (2.6)
$$

Expression (2.6) can be written in terms of $\tau's$ as:

$$
MSE\left(\bar{y}_{RP,RSS}\right) = MSE\left(\bar{y}_{RP2,SRS}\right) - \frac{\bar{Y}^2\theta}{m}\sum_{i=1}^m\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} + \frac{\{1-\delta(1+p)\}\alpha\tau_{x_{(i)}}}{\bar{X}}\right)^2. \qquad (2.7)
$$

Putting $\delta^* = \{1-\delta(1+p)\}$ in (2.6), we have

$$
MSE\left(\bar{y}_{RP,RSS}\right) = \bar{Y}^2\left[\left(\theta C_y^2 - W_{y_{[i]}}^2\right) + \delta^{*2}\left(\theta C_x^2 - W_{x_{(i)}}^2\right) + 2\delta^*\left(\theta KC_x^2 - W_{yx_{(i)}}\right)\right]. \qquad (2.8)
$$

The optimum value of δ^* is obtained as

$$
\frac{dMSE\left(\bar{y}_{RP,RSS}\right)}{d\delta^*} = 0 \implies \delta^* = -\frac{\left(\theta KC_x^2 - W_{yx_{(i)}}\right)}{\left(\theta C_x^2 - W_{x_{(i)}}^2\right)} \qquad (2.9)
$$

Then the minimum MSE $\bar{y}_{RP,RSS}$ is given by

$$
min\,MSE\left(\bar{y}_{RP,RSS}\right) = \bar{Y}^2\left[\left(\theta C_y^2 - W_{y_{[i]}}^2\right) - \frac{\left(\theta KC_x^2 - W_{yx_{(i)}}\right)^2}{\left(\theta C_x^2 - W_{x_{(i)}}^2\right)}\right]. \qquad (2.10)
$$

Thus we state the following theorem.

Theorem 1: To the first degree of approximation,

$MSE\left(\bar{y}_{RP,RSS}\right) \geq \bar{Y}^2\left[\left(\theta C_y^2 - W_{y_{[i]}}^2\right) - \frac{\left(\theta KC_x^2 - W_{yx_{(i)}}\right)^2}{\left(\theta C_x^2 - W_{x_{(i)}}^2\right)}\right]$, if $\delta^* = -\frac{\left(\theta KC_x^2 - W_{yx_{(i)}}\right)}{\left(\theta C_x^2 - W_{x_{(i)}}^2\right)} = -\delta_0^*$(say).

The $min\,MSE\left(\bar{y}_{RP,RSS}\right)$ can be approximately expressed as

$$
min\,MSE\left(\bar{y}_{RP,RSS}\right) \cong \theta S_y^2\left(1-\rho^2\right) - \theta\bar{Y}^2\left[K^2C_x^2\left(\frac{W_{x(i)}}{\theta C_x^2} - \frac{W_{yx(i)}}{\theta KC_x^2}\right)^2 + \frac{1}{m}\sum_{i=1}^m\left(\frac{\tau_{y[i]}}{\bar{Y}} - \frac{K\tau_{x(i)}}{\bar{X}}\right)^2\right] \qquad (2.11)
$$

or

$min\,MSE\left(\bar{y}_{RP,RSS}\right) \cong min\,MSE\left(\bar{y}_{RP2,SRS}\right) - \theta\bar{Y}^2 A,$

where
$A = \left[K^2 C_x^2 \left(\frac{W_{x(i)}}{\theta C_x^2} - \frac{W_{yx(i)}}{\theta K C_x^2}\right)^2 + \frac{1}{m}\sum_{i=1}^{m}\left(\frac{\tau_{y[i]}}{\bar{Y}} - \frac{K\tau_{x(i)}}{\bar{X}}\right)^2\right]$.

Expression (2.11) clearly shows that the proposed class of estimators $\bar{y}_{RP,RSS}$ is more efficient than $\bar{y}_{RP2,SRS}$[or usual regression estimator in SRS].■

Remark 1: The optimum value of p can be easily obtained from (1.5). For the sake of convenience we below give the optimum value of p that minimize the $MSE\left(\bar{y}_{RP,RSS}\right)$ at (2.8):

$p_{opt} = \left[\frac{1}{\delta}\left(1 + \frac{Q}{\alpha}\right) - 1\right]$,

where $Q = \frac{\left(\theta K C_x^2 - W_{yx_{(i)}}\right)}{\left(\theta C_x^2 - W_{x_{(i)}}^2\right)}$.▲

Some known members of the proposed class of estimators $\bar{y}_{RP,RSS}$ are shown in Table 2.

Thus all the results given in Mehta and Mandowara (2012, 2016) and Mandowara and Mehta (2013) are erroneous, which need to be corrected.

The affirmation has been supported by fixing the error in the original results. To illustrate this we have first give the results of MSE of the ratio estimator $\bar{y}_{RP,RSS}^{(1)}$ obtained by Mehta and Mandowara (2012, Eq. (2.4), p.555).

Result due to Mehta and Mandowara (2012, Eq. (2.4), p.555):

The MSE of the ratio estimator to the first degree of approximation obtained by Mehta and Mandowara (2012, Eq. (2.4), p.555) is:

$\bar{Y}^2\left[\theta\left\{C_y^2 + C_x^2(1-2K)\right\} - \frac{1}{m^2 r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - W_{x_{(i)}}\right)^2\right]$,

which is not correct.

Proof of the correct result:

The ratio estimator $\bar{y}_{RP,RSS}^{(1)}$ is defined by

$\bar{y}_{RP,RSS}^{(1)} = \bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right)$

Expressing $\bar{y}_{RP,RSS}^{(1)}$ in terms of e's we have

$\bar{y}_{RP,RSS}^{(1)} = \bar{Y}(1+e_0)\frac{\bar{X}}{\bar{X}(1+e_1)} = \bar{Y}(1+e_0)(1+e_1)^{-1}$.

We assume that $|e_1| < 1$ so that $(1+e_1)^{-1}$expandable. Expanding the right hand side of the above expression and neglecting terms of e's having power greater than two, we have

$\bar{y}_{RP,RSS}^{(1)} \cong \bar{Y}\left(1 + e_0 - e_1 - e_0 e_1 + e_1^2\right)$

or

$\left(\bar{y}_{RP,RSS}^{(1)} - \bar{Y}\right) \cong \bar{Y}\left(e_0 - e_1 - e_0 e_1 + e_1^2\right)$

Squaring both sides of the above expression and neglecting terms of e's having power greater than two we have

$\left(\bar{y}_{RP,RSS}^{(1)} - \bar{Y}\right)^2 \cong \bar{Y}^2\left(e_0^2 + e_1^2 - 2e_0 e_1\right)$

Taking expectations of both sides of the above expression we get the MSE of the ratio estimator $\bar{y}_{RP,RSS}^{(1)}$ to the first degree of approximation as

$MSE\left(\bar{y}_{RP,RSS}^{(1)}\right) = E\left(\bar{y}_{RP,RSS}^{(1)} - \bar{Y}\right)^2 = \bar{Y}^2\left[\theta\left\{C_y^2 + C_x^2(1-2K)\right\} - \left\{W_{y_{[i]}}^2 + W_{x_{(i)}}^2 - 2W_{yx_{(i)}}\right\}\right]$

$= \bar{Y}^2\left[\theta\left\{C_y^2 + C_x^2(1-2K)\right\} - \left\{\frac{1}{m^2 r}\frac{1}{\bar{Y}^2}\sum_{i=1}^{m}\tau_{y_{[i]}}^2 + \frac{1}{m^2 r}\frac{1}{\bar{X}^2}\sum_{i=1}^{m}\tau_{x_{(i)}}^2 - \frac{2}{m^2 r}\frac{1}{\bar{Y}\bar{X}}\sum_{i=1}^{m}\tau_{yx_{(i)}}\right\}\right]$

$= \bar{Y}^2\left[\theta\left\{C_y^2 + C_x^2(1-2K)\right\} - \frac{1}{m^2 r}\sum_{i=1}^{m}\left\{\frac{\tau_{y_{[i]}}^2}{\bar{Y}^2} + \frac{\tau_{x_{(i)}}^2}{\bar{X}^2} - 2\frac{\tau_{yx_{(i)}}}{\bar{Y}\bar{X}}\right\}\right]$

Table 2: **Members of the $\bar{y}_{RP,RSS}$ for different choices of** $(a, b, \delta_i, \alpha_i, p_i)$**:**

S. No.	**Estimators**	**Values of Constants** $(a, b, \delta_i, \alpha_i, p_i)$				
		a	b	δ_i	α_i	p_i
1	$\bar{y}^{(1)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right)$ Swami and Muttlak (1996) estimator	1	0	1	1	1
2	$\bar{y}^{(2)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}}{\bar{X}}\right)$ Bouza (2008) estimator	1	0	0	1	1
3	$\bar{y}^{(3)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}+C_x}{\bar{x}_{(n)}+C_x}\right)$ Mehta and Mandowara (2012,2016) estimator	1	C_x	1	$\frac{\bar{X}}{\bar{X}+C_x}$	1
4	$\bar{y}^{(4)}_{RP,RSS} = \bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}_{(n)}+\beta_2(x)}\right)$ Mehta and Mandowara (2012) estimator	1	$\beta_2(x)$	1	$\frac{\bar{X}}{\bar{X}+\beta_2(x)}$	1
5	$\bar{y}^{(5)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}_{(n)}C_x+\beta_2(x)}\right)$ Mehta and Mandowara (2012,2016) estimator	C_x	$\beta_2(x)$	1	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	1
6	$\bar{y}^{(6)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}C_x+\beta_2(x)}{\bar{X}C_x+\beta_2(x)}\right)$ Mehta and Mandowara (2016) estimator	C_x	$\beta_2(x)$	0	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	1
7	$\bar{y}^{(7)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}_{(n)}\beta_2(x)+C_x}\right)$ Mandowara and Mehta (2013) estimator	$\beta_2(x)$	C_x	1	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	1
8	$\bar{y}^{(8)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}\beta_2(x)+C_x}{\bar{X}\beta_2(x)+C_x}\right)$	$\beta_2(x)$	C_x	0	$\frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+C_x}$	1
9	$\bar{y}^{(9)}_{RP,RSS} = \bar{y}_{[n]}[\delta_9\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right) +(1-\delta_9)(\frac{\bar{x}_{(n)}}{\bar{X}})]$	1	0	δ_9	1	1
10	$\bar{y}^{(10)}_{RP,RSS} = \bar{y}_{[n]}[\delta_{10}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}_{(n)}C_x+\beta_2(x)}\right) +(1-\delta_{10})(\frac{\bar{x}_{(n)}C_x+\beta_2(x)}{\bar{X}C_x+\beta_2(x)})]$ Mehta and Mandowara (2016) estimator	C_x	$\beta_2(x)$	δ_{10}	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	1
11	$\bar{y}^{(11)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}_{(n)}+\beta_2(x)}\right)^{p_{11}}$ Mehta and Mandowara (2012) estimator	1	$\beta_2(x)$	1	$\frac{\bar{X}}{\bar{X}+\beta_2(x)}$	p_{11}
12	$\bar{y}^{(12)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}_{(n)}\beta_2(x)+C_x}\right)^{p_{12}}$ Mandowara and Mehta (2013) estimatorr	$\beta_2(x)$	C_x	1	$\frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+C_x}$	p_{12}
13	$\bar{y}^{(13)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}_{(n)}C_x+\beta_2(x)}\right)^{p_{13}}$ Mandowara and Mehta (2013) estimatorr	C_x	$\beta_2(x)$	1	$\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$	p_{13}
14	$\bar{y}^{(14)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{a\bar{X}+b}{a\bar{x}_{(n)}+b}\right)$	a	b	1	$\frac{a\bar{X}}{a\bar{X}+b}$	1
15	$\bar{y}^{(15)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{a\bar{x}_{(n)}+b}{a\bar{X}+b}\right)$	a	b	0	$\frac{a\bar{X}}{a\bar{X}+b}$	1
16	$\bar{y}^{(16)}_{RP,RSS} = \bar{y}\left(\frac{a\bar{X}+b}{a\bar{x}_{(n)}+b}\right)^{p_{16}}$	a	b	1	$\frac{a\bar{X}}{a\bar{X}+b}$	p_{16}
17	$\bar{y}^{(17)}_{RP,RSS} = \bar{y}\left(\frac{a\bar{x}_{(n)}+b}{a\bar{X}+b}\right)^{p_{17}}$	a	b	0	$\frac{a\bar{X}}{a\bar{X}+b}$	p_{17}

The MSE of some known members of the proposed class of estimators $\bar{y}_{RP,RSS}$ are given in Table 3.

Table 3: **The correct and incorrect MSE expressions of different known estimators:**

S. No.	Estimators	Correct MSE expression of the estimator	Incorrect MSE expression of the estimator
1	$\bar{y}^{(1)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right)$	$\bar{Y}^2[\theta\left\{C_y^2 + C_x^2\left(1-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + C_x^2\left(1-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - W_{x_{(i)}}\right)^2]$ Obtained by Mehta and Mandowara (2012)
2	$\bar{y}^{(2)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}}{\bar{X}}\right)$	$\bar{Y}^2[\theta\left\{C_y^2 + C_x^2\left(1-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} + \frac{\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + C_x^2\left(1-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} + W_{x_{(i)}}\right)^2]$ Obtained by Mehta and Mandowara (2016)
3	$\bar{y}^{(3)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}+C_x}{\bar{x}_{(n)}+C_x}\right)$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_3 C_x^2\left(\alpha_3-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_3\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_3 C_x^2\left(\alpha_3-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_3 W_{x_{(i)}}\right)^2]$ Obtained by Mehta and Mandowara (2016)
4	$\bar{y}^{(4)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}_{(n)}+\beta_2(x)}\right)$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_4 C_x^2\left(\alpha_4-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_4\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_4 C_x^2\left(\alpha_4-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_4 W_{x_{(i)}}\right)^2]$ Obtained by Mehta and Mandowara (2012)
5	$\bar{y}^{(5)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}_{(n)}C_x+\beta_2(x)}\right)$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_5 C_x^2\left(\alpha_5-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_5\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_5 C_x^2\left(\alpha_5-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_5 W_{x_{(i)}}\right)^2]$ Obtained by Mehta and Mandowara (2016)
6	$\bar{y}^{(6)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}C_x+\beta_2(x)}{\bar{X}C_x+\beta_2(x)}\right)$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_6 C_x^2\left(\alpha_6+2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_6\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_6 C_x^2\left(\alpha_6+2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_6 W_{x_{(i)}}\right)^2]$ Obtained by Mehta and Mandowara (2016)
7	$\bar{y}^{(7)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}_{(n)}\beta_2(x)+C_x}\right)$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_7 C_x^2\left(\alpha_7+2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_7\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_7 C_x^2\left(\alpha_7+2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_7 W_{x_{(i)}}\right)^2]$ Obtained by Mandowara and Mehta (2013)
8	$\bar{y}^{(10)}_{RP,RSS} = \bar{y}_{[n]}[\delta_{10}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}_{(n)}C_x+\beta_2(x)}\right)$ $+(1-\delta_{10})\left(\frac{\bar{x}_{(n)}C_x+\beta_2(x)}{\bar{X}C_x+\beta_2(x)}\right)]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_{10} C_x^2\left(1-2\delta_{10}\right)\left(\left(1-2\delta_{10}\right)\alpha_{10}+2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_{10}(1-2\delta_{10})\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_{10} C_x^2\left(1-2\delta_{10}\right)\left(\left(1-2\delta_{10}\right)\alpha_{10}+2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_{10}\left(1-2\delta_{10}\right)W_{x_{(i)}}\right)^2]$ Obtained by Mehta and Mandowara (2016)
9	$\bar{y}^{(11)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}_{(n)}+\beta_2(x)}\right)^{p_{11}}$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_{11}p_{11} C_x^2\left(\alpha_{11}p_{11}-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_{11}p_{11}\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_{11}p_{11} C_x^2\left(\alpha_{11}p_{11}-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_{11}p_{11}W_{x_{(i)}}\right)^2]$ Obtained by Mehta and Mandowara (2012)
10	$\bar{y}^{(12)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}_{(n)}\beta_2(x)+C_x}\right)^{p_{12}}$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_{12}p_{12} C_x^2\left(\alpha_{12}p_{12}-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_{12}p_{12}\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_{12}p_{12} C_x^2\left(\alpha_{12}p_{12}-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_{12}p_{12}W_{x_{(i)}}\right)^2]$ Obtained by Mandowara and Mehta (2013)
11	$\bar{y}^{(13)}_{RP,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}_{(n)}C_x+\beta_2(x)}\right)^{p_{13}}$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_{13}p_{13} C_x^2\left(\alpha_{13}p_{13}-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\alpha_{13}p_{13}\tau_{x_{(i)}}}{\bar{X}}\right)^2]$	$\bar{Y}^2[\theta\left\{C_y^2 + \alpha_{13}p_{13} C_x^2\left(\alpha_{13}p_{13}-2K\right)\right\}$ $-\frac{1}{m^2r}\sum_{i=1}^{m}\left(W_{y_{[i]}} - \alpha_{13}p_{13}W_{x_{(i)}}\right)^2]$ Obtained by Mandowara and Mehta (2013)

$= \bar{Y}^2 \left[\theta \left\{ C_y^2 + C_x^2 (1-2K) \right\} - \frac{1}{m^2 r} \sum_{i=1}^{m} \left\{ \frac{\left(\mu_{y_{[i]}} - \bar{Y}\right)^2}{\bar{Y}^2} + \frac{\left(\mu_{x_{(i)}} - \bar{X}\right)^2}{\bar{X}^2} - \frac{2\left(\mu_{x_{(i)}} - \bar{X}\right)\left(\mu_{y_{\{i\}}} - \bar{Y}\right)}{\bar{Y}\bar{X}} \right\} \right]$
$= \bar{Y}^2 \left[\theta \left\{ C_y^2 + C_x^2 (1-2K) \right\} - \frac{1}{m^2 r} \sum_{i=1}^{m} \left\{ \frac{\left(\mu_{y_{[i]}} - \bar{Y}\right)^2}{\bar{Y}^2} + \frac{\left(\mu_{x_{(i)}} - \bar{X}\right)^2}{\bar{X}^2} \right\} \right]$
$= \bar{Y}^2 \left[\theta \left\{ C_y^2 + C_x^2 (1-2K) \right\} - \frac{1}{m^2 r} \sum_{i=1}^{m} \left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\tau_{x_{(i)}}}{\bar{X}} \right)^2 \right]$,
which is the correct MSE of the ratio estimator $\bar{y}_{RP,RSS}^{(1)}$.
Similarly, the correct results of the MSEs of the estimators $\bar{y}_{RP,RSS}^{(j)}$, $j = 2\, to\, 7\, and\, 10\, to\, 13$, shown in Table 3 can be easily obtained.

3. EFFICIENCY COMPARISON

On comparing (1.4) and (2.6), we obtain

$$MSE\left(\bar{y}_{RP2,SRS}\right) - MSE\left(\bar{y}_{RP,RSS}\right) = \frac{\bar{Y}^2\theta}{m} \sum_{i=1}^{m} \left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\{1-\delta(1+p)\}\,\alpha\tau_{x_{(i)}}}{\bar{X}} \right)^2 \geq 0 \qquad (3.1)$$

It is easily observed from (3.1) that the suggested estimator $\bar{y}_{RP,RSS}$ based on the RSS method have smaller MSE as compared to the SRS method.
It follows that the members of the class of estimators $\bar{y}_{RP,RSS}$ is better than the corresponding members of the class of estimator $\bar{y}_{RP2,SRS}$.
Further, larger the value:

$$\left(\frac{\tau_{y_{[i]}}}{\bar{Y}} - \frac{\{1-\delta(1+p)\}\,\alpha\tau_{x_{(i)}}}{\bar{X}} \right), \qquad (3.2)$$

of the difference in (3.1) will be larger the gain in efficiency by using the proposed class of estimators $\bar{y}_{RP,RSS}$ based on the RSS over the class of estimators $\bar{y}_{RP2,SRS}$ based on SRS.
We note that the expression (3.2) depends on $\tau_{y_{[\bullet]}}$ and $\tau_{x_{(\bullet)}}$. So from (3.2) it follows that for obtaining more gain in efficiency the value of $\tau_{y_{[\bullet]}}$ must be considerably larger than the value of $\tau_{x_{(\bullet)}}$.
Now we give the comparison between the two estimators based on the RSS scheme.
It is well known that the estimator $\bar{y}_{[n]}$ is unbiased for the population mean $\bar{Y}$ whose variance/ MSE's is given by

$$MSE\left(\bar{y}_{[n]}\right) = Var\left(\bar{y}_{[n]}\right) = \bar{Y}^2 \left(\theta C_y^2 - W_{y_{[i]}}^2 \right). \qquad (3.3)$$

It follows from (2.8) and (3.3) that
$MSE\left(\bar{y}_{RP,RSS}\right) < MSE\left(\bar{y}_{[n]}\right)$, $(i \neq j = 1, 2, ..., 17)$ if

$$\left\{ either\ 0 < \delta^* < 2\delta_0^* \quad or\ 2\delta_0^* < \delta^* < 0 \right. \qquad (3.4)$$

or equivalently

$$min.\left(0, 2\delta_0^*\right) < \delta^* < max.\left(0, 2\delta_0^*\right). \qquad (3.5)$$

We note from (2.8) that

$MSE\left(\bar{y}^{(i)}_{RP,RSS}\right) < MSE\left(\bar{y}^{(j)}_{RP,RSS}\right), i \neq j$ if
$\delta_i^{*2} + 2\delta_i^*\delta_0^* < \delta_j^{*2} + 2\delta_j^*\delta_0^*$
i.e. if $(\delta_i^* + \delta_0^*)^2 < \left(\delta_j^* + \delta_0^*\right)^2$
i.e. if

$$|\delta_i^* + \delta_0^*| < \left|\delta_j^* + \delta_0^*\right|, \tag{3.6}$$

where
$\delta_i^* = \alpha_i \{1 - \delta_i (1 + p_i)\}$ and $\delta_j^* = \alpha_j \{1 - \delta_j (1 + p_j)\}$.
Since $Cov\left(\bar{x}_{(n)}, \bar{y}_{[n]}\right) = \beta Var\left(\bar{x}_{(n)}\right)$,

$$\delta_0^* = \left(\frac{\theta K C_x^2 - W_{yx_{(i)}}}{\theta C_x^2 - W^2_{x_{(i)}}}\right) = K\left(= \frac{\rho C_y}{C_x}\right), \tag{3.7}$$

where $\beta = \frac{S_{yx}}{S_x^2}$ is the population regression coefficient of y on x.
Thus using (3.7) and (3.6) we obtained that
$MSE\left(\bar{y}^{(i)}_{RP,RSS}\right) < MSE\left(\bar{y}^{(j)}_{RP,RSS}\right), i \neq j$ if
$|\delta_i^* + K| < \left|\delta_j^* + K\right|$,
i. e. if

$$\left\{ either\ K < \frac{-\left(\delta_i^* + \delta_j^*\right)}{2};\ if\ \delta_i^* > \delta_j^* \quad or\ K > \frac{-\left(\delta_i^* + \delta_j^*\right)}{2};\ if\ \delta_i^* < \delta_j^*\ . \right. \tag{3.8}$$

4. CONCLUSIONS

This paper suggested a general class of estimator $\bar{y}_{RP,SSS}$ based on the transformation in an auxiliary variable x in RSS. The proposed class of estimators includes several known estimator based on transformation in auxiliary variable x. The bias and MSE expressions of the proposed class of estimators have been obtained under large sample approximation. It is interesting to note that this study unifies several estimators with their properties at one place. Also the use some known parameters associated with auxiliary variable x in defining the classes of estimators play important role in reducing the MSE of the estimators. Thus such class of estimators based on some known parameters will definitely be beneficial in terms of less mean squared errors.

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